Double Sigma Model

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Reference

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Action

$$S_{\text{bulk}} = \frac{1}{2}\int d^2\sigma\,\left(\partial_1 X^A \mathcal{H}_{AB}\partial_1 X^B - \partial_1 X^A \eta_{AB}\partial_0 X^B\right).$$

The notations are $\alpha = 0, 1$ (We use the Greek indices to indicate the worldsheet coordinates.), $A = 0, 1, \cdots, 2D-1$ (We define the double target indices from A to K.), and

$$X^A \equiv \begin{pmatrix} \tilde{X}_m \\ X^m \end{pmatrix},$$

$$\mathcal{H}^{-1} \equiv \mathcal{H}_{\bullet\bullet} = \left(\mathcal{H}^{AB}\right)^{-1} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}.$$

The index $m = 0, 1, \cdots, D-1$ (We define the non-double target indices from m to z.). The ordinary coordinates are defined to be X^m and dual coordinates are defined to be $\tilde{X}_m$.

We also define

$$\mathcal{H} \equiv \mathcal{H}^{\bullet\bullet}.$$

The name for $\mathcal{H}$ is generalized metric. For double target indices, we use $\eta \equiv \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$ to raise and lower indices for the $O(D, D)$ tensors. The index α is raised and lowered by the flat metric. The worldsheet metric is $(-, +)$ signature.

Using the strong constraints $\tilde{\partial}^m = 0$ ($\partial_m \equiv \frac{\partial}{\partial x^m}$, $\tilde{\partial}^m \equiv \frac{\partial}{\partial \tilde{x}_m}$ and $\partial_A \equiv \begin{pmatrix} \tilde{\partial}^m \\ \partial_m \end{pmatrix}$.) and a self-duality relation

$$\mathcal{H}^m{}_B \partial_1 X^B - \eta^m{}_B \partial_0 X^B = 0$$

to guarantee classical equivalence with the ordinary sigma model. The ordinary sigma model is

$$\frac{1}{2}\int d^2\sigma \left(\partial_\alpha X^m g_{mn} \partial^\alpha X^n - \epsilon^{\alpha\beta}\partial_\alpha X^m B_{mn} \partial_\beta X^n\right).$$

Boundary Conditions

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The boundary conditions on σ^1-direction (The Neumann boundary condition) are

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and the boundary condition on σ^0-direction (The Dirichlet boundary condition) is

$$\delta X^m = 0.$$

Low-Energy Effective Action

$$
\begin{aligned}
S_T &= S_1 + \alpha S_2 \\
&= \int dx\ d\tilde{x}\ \Bigg[e^{-d}\Big(-\det(\mathcal{H}_{mn})\Big)^{\frac{1}{4}} \\
&\quad +\alpha e^{-2d}\Big(\frac{1}{8}\mathcal{H}^{AB}\partial_A\mathcal{H}^{CD}\partial_B\mathcal{H}_{CD} - \frac{1}{2}\mathcal{H}^{AB}\partial_B\mathcal{H}^{CD}\partial_D\mathcal{H}_{AC} \\
&\quad -2\partial_A d\partial_B\mathcal{H}^{AB} + 4\mathcal{H}^{AB}\partial_A d\partial_B d\Big)\Bigg],
\end{aligned}
$$

where α is an arbitrary constant and

$$
e^{-d} \equiv \Big(-\det g\Big)^{\frac{1}{4}} e^{-\phi}.
$$

Quantum Equivalence with the Strong Constraints

When we perform Gaussian integration, the result of the integration on the exponent is equivalent to using

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Then we integrate out the dual coordinates:

$$\begin{aligned} & \frac{1}{2}\partial_1 X^m\left(g - Bg^{-1}B\right)_{mn}\partial_1 X^n + \partial_1 X^m\left(Bg^{-1}\right)_m{}^n\partial_1\tilde{X}_n \\ = \quad & -\frac{1}{2}\partial_0 X^m g_{mn}\partial_0 X^n + \frac{1}{2}\partial_1 X^m g_{mn}\partial_1 X^n + \partial_1 X^m B_{mn}\partial_0 X^n. \end{aligned}$$

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$$\int DX^A$$

becomes

$$\int DX^m \sqrt{\det g} \equiv \int D'X^m$$

when we integrate out the dual coordinates. We obtain the diffemorphism invariant measure $(D'X^m)$ with shift symmetry.

Seiberg-Witten Map

$$\hat{A}(A) + \hat{\delta}_{\hat{\lambda}}(A) = \hat{A}(A + \delta_\lambda A),$$

where $\hat{A}$ is the Seiberg-Witten map, δ_λ is gauge transformation on the commutative space and $\hat{\delta}_{\hat{\lambda}}$ is gauge transformation on the non-commutative space. On the non-commutative space, field strength is given by

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + [\hat{A}_\mu, \hat{A}_\nu]_*.$$

The gauge transformations are

$$\delta_\lambda A_\mu \equiv \partial_\mu \lambda, \qquad \hat{\delta}_{\hat{\lambda}} \hat{A}_\mu \equiv \partial_\mu \hat{\lambda} - [\hat{\lambda}, \hat{A}_\mu]_*.$$

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We find a solution at leading order,

$$\hat{A}_\mu = A_\mu - \theta^{\rho\sigma}\left(A_\rho \partial_\sigma A_\mu - \frac{1}{2} A_\rho \partial_\mu A_\sigma\right), \qquad \hat{\lambda} = \lambda + \frac{1}{2}\theta^{\rho\sigma} A_\sigma \partial_\rho \lambda.$$

$$\hat{F}_{\mu\nu} \approx F_{\mu\nu} + \theta^{\rho\sigma}\left(F_{\mu\rho} F_{\nu\sigma} - A_\rho \partial_\sigma F_{\mu\nu}\right).$$

From the Poisson limit to infinite orders,

$$\begin{aligned}
\delta\hat{A}_\mu &= -\frac{1}{4}\delta\theta^{\rho\sigma}\left[\hat{A}_\rho * \left(2\partial_\sigma\hat{A}_\mu - \partial_\mu\hat{A}_\sigma\right) + \left(2\partial_\sigma\hat{A}_\mu - \partial_\mu\hat{A}_\sigma\right) * \hat{A}_\rho\right], \\
\delta\hat{\lambda} &= \frac{1}{4}\delta\theta^{\rho\sigma}\left(\partial_\rho\hat{\lambda} * \hat{A}_\sigma + \hat{A}_\rho * \partial_\sigma\hat{\lambda}\right), \\
\delta\hat{F}_{\mu\nu} &= \frac{1}{4}\delta\theta^{\rho\sigma}\left[2\hat{F}_{\mu\rho} * \hat{F}_{\nu\sigma} + 2\hat{F}_{\nu\sigma} * \hat{F}_{\mu\rho} - \hat{A}_\rho * \left(\partial_\sigma\hat{F}_{\mu\nu} + \hat{D}_\sigma\hat{F}_{\mu\nu}\right)\right. \\
&\quad \left. - \left(\partial_\sigma\hat{F}_{\mu\nu} + \hat{D}_\sigma\hat{F}_{\mu\nu}\right) * \hat{A}_\rho\right],
\end{aligned}$$

where

$$\hat{D}_\lambda\hat{F}_{\mu\nu} \equiv \partial_\lambda\hat{F}_{\mu\nu} + [\hat{A}_\lambda, \hat{F}_{\mu\nu}]_*.$$

- If we do not use the strong constraints, we should have a combination of $B - F$. Non-commutative geometry can be constructed from gauge symmetries without using action. Double sigma model should have potentials to build the non-commutative geometry on the bulk

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- We show equivalence between standard and double sigma models.
- Without using the strong constraints, we have global symmetry structures to avoid the non-gauge invariant entanglement entropy on closed string.
- Non-Commutative geometry of closed string should shed the light on all α' effects from the Moyal product.