Holographic Entanglement entropy and central charge in CFT

Mohsen Alishahiha

School of physics, Institute for Research in Fundamental Sciences (IPM)

Duality Symmetries in String and M-Theories

3-14 August 2015 CERN

Based on

M. A., A. F. Astaneh, P. Fonda, F. Omidi "Entanglement Entropy for Singular Surfaces in Hyperscaling violating Theories," arXiv:1507.05897.

See also

Rong-Xin Miao, "A holographic proof of the universality of corner entanglement for CFTs," 1507.06283.

P. Bueno, R. C. Myers, W. Witczak-Krempa, "Universal corner entanglement from twist operators," 1507.06997.

P. Bueno and R. C. Myers, "Universal entanglement for higher dimensional cones," 1508.00587.

In 1+1 D CFT there is a central charge appearing in different places

- The symmetry algebra $[L_m, L_n] = (m-n)L_{m+n} + \frac{m(m^2-1)}{12}\, c\, \delta_{m+n,0}$.

- Two point function of energy momentum tensor $\langle T(x)T(y)\rangle = \frac{c/2}{|x-y|^4}$.

- The Weyl anomaly $\langle T^\mu_\mu \rangle = -\frac{c}{12} R$.

- Casimir energy of a cylinder $E = -\frac{\pi}{12}\, c$.

- Entropy: putting the theory on a circle with radius $\beta = T^{-1}$ the entropy is

$$S = \frac{\pi^2}{3} c T$$

.

- Entanglement entropy $S_E = \frac{c}{3} \log \ell/\epsilon$.

Zamolodchikov's c-theorem $\Longrightarrow$ $c_{\text{uv}} \geq c_{\text{ir}}$

In $d+1$ dimensional CFT

- The symmetry group is $SO(2,d+1)$ and has no central charge.
- Two point function of energy momentum tensor

$$\langle T_{ab}(x)T_{cd}(y)\rangle = \frac{C_T}{|x-y|^{2(d+1)}} G_{abcd}(x,y).$$

- Weyl anomaly for even dimensions (odd d)

$$\langle T^\mu_\mu\rangle = a\ E_{d+1} + \sum_i c_i I_i + \nabla\cdot J$$

- Entropy at finite temperature $S_{th} = S_0 T^d$
- Entanglement entropy

$$S_E = \sum_{i=0}^{[\frac{d}{2}]-1} \frac{A_{2i}}{d-2i-1}\frac{1}{\varepsilon^{d-2i+1}} + \delta_{2[\frac{d}{2}]+1,d}\ A_{2[\frac{d}{2}]} \log\frac{H}{\varepsilon} + \text{finite terms.}$$

Which one may have a "c"-Theorom?

a-theorem

In four dimensional space time ($d = 3$) the coefficient that multiplies the Euler density, a, always decreases along RG flow (Card 1988, Komargodski, Schwimmer 2011)

In any higher (even) dimensions, the coefficient of E_{d+1} in the anomaly may be considered as generalization of a-theorem: It is has natural monotonic flow.

If one considers the entanglement entropy for a sphere in even dimensions

$$A_{2[\frac{d}{2}]} = a$$

In odd dimensions there is no anomaly, though one could still define $A_{2[\frac{d}{2}]}$ as the universal term in the entanglement entropy of a sphere. Of course there is no log term.

The universal term in the entanglement entropy for a sphere could provide a monotonic function. (Myers, Sinha 2010)

Entanglement entropy

Consider a generic quantum system with a Hilbert space $\mathcal{H}$. For a pure state $|\psi\rangle$ in the system which evolves in time by its Hamiltonian H the density matrix is given by

$$\rho_{\text{total}} = |\psi\rangle\langle\psi|.$$

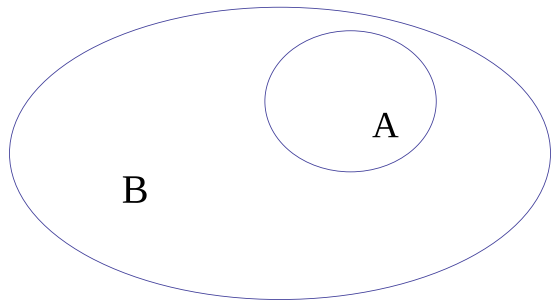
Physical quantities are computed as expectation values of operators as follows

$$\langle O\rangle = \langle\psi|O|\psi\rangle = \text{Tr}(\rho_{\text{total}} O)$$

In mixed states, the system is described by a density matrix ρ. An example of a mixed state is the canonical distribution

$$\rho = \frac{e^{-\beta H}}{\text{Tr}(e^{-\beta H})}$$

Assume that the quantum system has multiple degrees of freedom and so one can decompose the total system into two subsystems A and B



$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

The reduced density matrix of the subsystem A

$$\rho_A = \mathrm{Tr}_B(\rho_{\mathrm{total}})$$

Then the entanglement entropy is defined as the von Neumann entropy for ρ_A

$$S_A = -\mathrm{Tr}(\rho_A \ln \rho_A)$$

A measure how much a given quantum state is quantum mechanically entangled.

Rényi entropies

It is also useful to compute Rényi entropies

$$S_n = \frac{1}{1-n} \log \mathrm{Tr}\rho^n$$

Then the entanglement entropy is given by

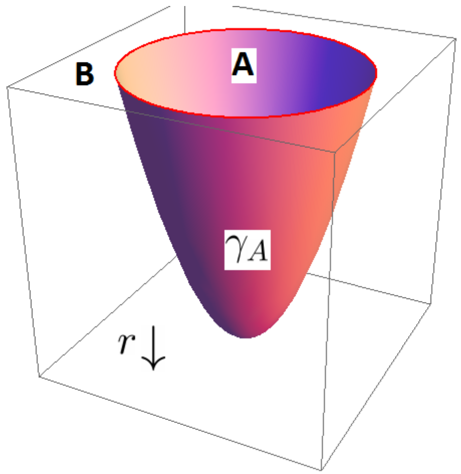
$$S_E = \lim_{n \to 1} S_n$$

Practically one may first compute $\mathrm{Tr}(\rho^n)$ by making use the replica trick and then

$$S_E = -\partial_n \mathrm{Tr}\rho^n|_{n=1}$$

Holographic Formula for Entanglement Entropy

For static background and fixed time divide the boundary into A and B. Extend this division $A \cup B$ to of the bulk spacetime. Extend ∂A to a surface γ_A in the entire spacetime such that $\partial \gamma_A = \partial A$.



$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N^{(d+2)}}$$

S. Ryu and T. Takayanagi, "Holographic derivation of entanglement entropy from AdS/CFT," Phys. Rev. Lett. **96**, 181602 (2006) [hep-th/0603001].

Entanglement entropy

$$S_E = \sum_{i=0}^{[\frac{d}{2}]-1} \frac{A_{2i}}{d-2i-1} \frac{1}{\varepsilon^{d-2i+1}} + \delta_{2[\frac{d}{2}]+1,d}\, A_{2[\frac{d}{2}]} \log \frac{H}{\varepsilon} + \text{finite terms.}$$

$A_{2[\frac{d}{2}]} = a$ for sphere.

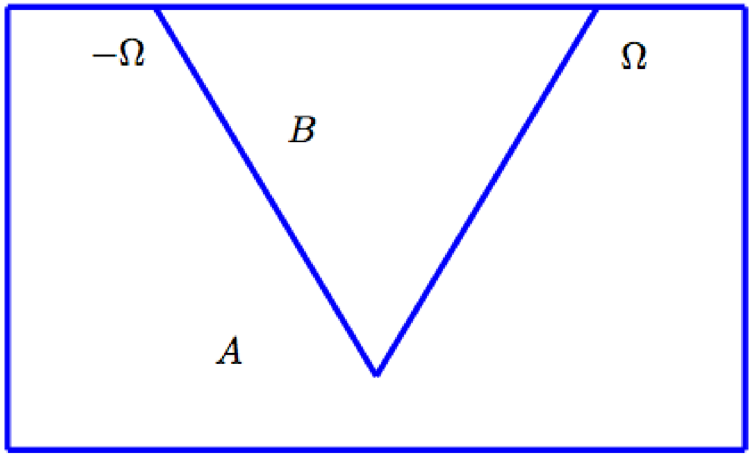
Natural Questions

• Is there any other universal term in the expressions of entanglement entropy?

• Is there any other relations between these parameters? Specially in odd dimensions where anomaly is zero?

• If other surfaces can be also useful ?

Consider a vacuum state of a three dimensional CFT whose gravity dual is provided by an AdS_4 geometry. And an entangling region with cusp.



In general one finds

$$S_E = \frac{Area}{\varepsilon} + a(\Omega)\log\frac{H}{\varepsilon} + A_0$$

where H is a length scale, ε a uv cut off.

Drukker, Gross, Ooguri 1999 – Hirata, Takayanagi 2006 – Myers, Singh 2012

$$S_E = \frac{Area}{\varepsilon} + a(\Omega) \log \frac{H}{\varepsilon} + A_0$$

• When the entangling region is a smooth sphere, $a(\Omega) = 0$ and A_0 is just the one could provide a monotonic function (central charge).

• When there is a cusp, one has a universal term. Of course there are certain constraints on $a(\Omega)$.

One has

$$A_0 = \frac{\kappa}{\Omega} + \cdots, \qquad \text{at } \Omega \to 0,$$
$$A_0 = \sigma \left(\frac{\pi}{2} - \Omega\right)^2 + \cdots, \qquad \text{at } \Omega \to \frac{\pi}{2}.$$

More importantly

$$\frac{\sigma}{C_T} = \frac{\pi^2}{24}$$

seems universal for 3D CFT. (Bueno, Myers 2015 – Bueno, Myers, Witczak-Krempa 2015)

How universal this behavior is?

What about higher dimensions? Does this work for non conformal cases? Could this define a new charge?

We would like to partially address these questions within a specific model

Theories with hyperscaling violation

Charmousis, Gouteraux, Kim, Kiritsis, Meyer 2010– Goldstein, Iizuka, Kachru, Prakash, Trivedi, Westphal 2010 – Gouteraux, Kiritsis 2011 – Huijse, Sachdev, Swingle 2011 – Dong, Harrison, Kachru, Torroba, Wang 2012 – .....

General solution with hyperscaling factor

$$S = -\frac{1}{16\pi G_N}\int d^{d+2}x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + V_0 e^{\gamma\phi} - \frac{1}{4}\sum_{i=1}^{N_g} e^{\lambda_i\phi}F^{(i)2}\right],$$

One of the gauge field is required to produce an anisotropy while the above particular form of the potential is needed to get hyperscaling violating factor. The other gauge fields make the background charged. Let's consider $N_g = 2$.

It has exact charged black hole solutions as follows

$$ds^2 = \frac{L^2}{r^2}(\frac{r}{r_F})^{2\frac{\theta}{d}}\Big(-\frac{f(r)}{r^{2(z-1)}}dt^2 + \frac{dr^2}{f(r)} + d\vec{x}^2\Big), \quad \phi = \beta \ln r,$$

$$A_t^{(1)} = \sqrt{\frac{2(z-1)}{d-\theta+z}}\, r^{-d+\theta-z}, \qquad A_t^{(2)} = \sqrt{\frac{2(d-\theta)}{d-\theta+z-2}}\, Q r^{d-\theta+z-2},$$

with $\beta = \sqrt{2(d-\theta)(z-1-\theta/d)}$ and

$$f(r) = 1 - mr^{d-\theta+z} + Q^2 r^{2(d-\theta+z-1)}.$$

where z is the dynamical exponent and θ is the hyperscaling violation exponent.

M. A, O Colgain, Yavartanoo 2012 – Bueno, Chemissany, Meessen, Ortin, Shahba 2012.

For $Q = 0$ this geometry is a black brane background whose Hawking temperature is

$$T = \frac{d_\theta + z}{4\pi\, r_H^z},$$

where r_H is the radius of horizon.

$$S_{\text{th}} = \left(\frac{4\pi}{d_\theta + z}\right)^{\frac{d_\theta}{z}} \frac{L^d V_d}{4G\, r_F^{d-d_\theta}}\, T^{\frac{d_\theta}{z}} \equiv S_0 T^{\frac{d_\theta}{z}}$$

where $d_\theta = d - \theta$.

We shall study holographic entanglement entropy on a singular region containing an n dimensional cone c_n. It is convenient to use the following parametrization for the metric

$$ds^2 = \frac{L^2}{r_F^{2\frac{\theta}{d}}} \frac{-r^{2(1-z)}dt^2 + dr^2 + d\rho^2 + \rho^2(d\varphi^2 + \sin^2\varphi \, d\Omega_n^2) + d\vec{x}_{d-n-2}^2}{r^{2(1-\frac{\theta}{d})}}.$$

The entangling region which, in general, have the form of $c_n \times R^{d-n-2}$ may be given by

$$t = \text{fixed} \qquad 0 \leq \varphi \leq \Omega$$

When $n = 0$ the entangling region will be given by $-\Omega \leq \varphi \leq \Omega$.

Using holographic entanglement entropy prescription one can compute the corresponding entanglement entropy (Ryu, Takayanagi 2006)

Given the symmetry of both the background metric and of the shape of the entangling region, the corresponding co-dimension two hypersurface may be described by the function $r = r(\rho, \varphi)$, and therefore the induced metric on the hypersurface is

$$ds^2_{\text{ind}} = \frac{L^2}{r_F^{2\frac{\theta}{d}}} \frac{(1 + r'^2)d\rho^2 + (\rho^2 + \dot{r}^2)d\varphi^2 + 2r'\dot{r}d\rho d\varphi + \rho^2 \sin^2\varphi \, d\Omega_n^2 + d\vec{x}^2_{d-n-2}}{r^{2(1-\frac{\theta}{d})}}.$$

where $r' = \partial_\rho r$ and $\dot{r} = \partial_\varphi r$. Form this induced metric the area functional whose minimum gives the holographic entanglement entropy reads

$$\mathcal{A} = \epsilon_n \frac{\Omega_n V_{d-2-n} L^d}{r_F^\theta} \int d\rho \, d\varphi \, \frac{\rho^n \sin^n\varphi}{r^{d-\theta}} \sqrt{\rho^2(1 + r'^2) + \dot{r}^2},$$

where V_{d-n-2} is the regularized volume of R^{d-n-2} space and Ω_n is the volume of S^n sphere. Here $\epsilon_n = 1 + \delta_{n0}$ to make sure that for $n = 0$ there is a factor of 2 as the interval of integration is from 0 to Ω.

The divergent terms of the holographic entanglement entropy for $d_\theta - n \neq 2$ are given by

$$S = \epsilon_n \frac{\Omega_n V_{d-n-2} L^d}{4G r_F^\theta} \left[\sum_{i=0}^{[\frac{d_\theta}{2}]-1}{}' \frac{a_{2i}}{(n-2i+1)(d_\theta-2i-1)} \left(\frac{H^{n-2i+1}}{\varepsilon^{d_\theta-2i-1}} - \frac{h_0^{2i-n-1}}{\varepsilon^{d_\theta-n-2}} \right) \right.$$
$$+ \frac{\delta_{2[\frac{n}{2}]+1,n} a_{2[\frac{n}{2}]+2}}{(d_\theta - 2[\frac{n}{2}] - 3)} \frac{\log \frac{H h_0}{\varepsilon}}{\varepsilon^{d_\theta - 2[\frac{n}{2}]-3}} + \frac{A_0}{d_\theta - n - 2} \frac{h_0^{d_\theta-n-2}}{\varepsilon^{d_\theta-n-2}}$$
$$\left. - \frac{a_{2[\frac{d_\theta}{2}]} \delta_{2[\frac{d_\theta}{2}]+1, d_\theta}}{d_\theta - n - 2} \left(\frac{\log \left(\frac{H}{\varepsilon}\right)}{H^{d_\theta-n-2}} - \frac{1-(d_\theta-n-2)\log h_0}{(d_\theta-n-2)(\varepsilon/h_0)^{d_\theta-n-2}} \right) \right] + \text{finite terms.}$$

where the prime in the summation indicates that when n is an odd number as one computes the summation, $i = [\frac{n}{2}] + 1$ should be excluded from the summation.

From this general expression one observes that the holographic entanglement entropy for a singular surface in the shape of $c_n \times R^{d-n-2}$ contains various divergent terms including a log term which results to a universal term when d_θ is an odd number.

On the other hand when $d_\theta = n+2$ the holographic entanglement entropy gets new logarithmic divergences

$$S = \epsilon_n \frac{\Omega_n V_{d-n-2} L^d}{4 G r_F^\theta} \Bigg[\sum_{i=0}^{[\frac{d_\theta}{2}]-1}{}' \frac{a_{2i}}{(n-2i+1)(d_\theta-2i-1)} \left(\frac{H^{n-2i+1}}{\varepsilon^{d_\theta-2i-1}} - \frac{h_0^{2i-n-1}}{\varepsilon^{d_\theta-n-2}} \right)$$
$$+ \frac{\delta_{2[\frac{n}{2}]+1,n} a_{2[\frac{n}{2}]+2}}{(d_\theta - 2[\frac{n}{2}] - 3)} \frac{\log \frac{H h_0}{\varepsilon}}{\varepsilon^{d_\theta - 2[\frac{n}{2}]-3}} + A_0 \log \frac{H h_0}{\varepsilon} + \frac{a_{2[\frac{d_\theta}{2}]}}{2} \delta_{2[\frac{d_\theta}{2}]+1, d_\theta} \log^2 \left(\frac{H}{\varepsilon} \right) \Bigg]$$
$$+ \text{finite terms.}$$

$$A_0 = \sum_{i=0}^{[\frac{d_\theta}{2}]-1} \frac{-a_{2i}}{(d_\theta - 2i - 1) h_0^{d_\theta - 2i - 1}} + a_{2[\frac{d_\theta}{2}]} \delta_{2[\frac{d_\theta}{2}]+1, d_\theta} \log h_0 + \int_0^{h_0} dh\, A_{\text{reg}}$$

$$A_{\text{reg}} = \frac{\sin^n \varphi}{h^{d_\theta}} \sqrt{1 + (1+h^2)\varphi'^2} - \left(\sum_{i=0}^{[\frac{d_\theta}{2}]-1} \frac{a_{2i}}{h^{d_\theta - 2i}} + \frac{a_{2[\frac{d_\theta}{2}]}}{h} \delta_{2[\frac{d_\theta}{2}]+1, d_\theta} \right).$$

The coefficients a_{2i} appearing in these equations are

$$a_0 = \sin^n\Omega, \qquad a_2 = \varphi_2(2\varphi_2 + n\cot\Omega)\sin^n\Omega$$

$$a_4 = \frac{1}{2}[n\left(2\varphi_2^3 + \varphi_4\right)\sin 2\Omega - \varphi_2\sin^2\Omega\left(\varphi_2\left(4\varphi_2^2 + n - 4\right) - 16\varphi_4\right)$$

$$+ \; \varphi_2^2(n-1)n\cos^2\Omega]\sin^{n-2}\Omega.$$

Here

$$\varphi_2 = -\frac{n\cot\Omega}{2(d_\theta - 1)},$$

$$\varphi_4 = -\frac{n\cot\Omega[(-2n + (d_\theta - 1)^2)n\cot^2\Omega + (d_\theta - 1)^2(6 - 2d_\theta + n)]}{8(d_\theta - 3)(d_\theta - 1)^3},$$

$$nh\left(\varphi'^2 + \frac{1}{1+h^2}\right)\cot\varphi + \varphi'\left[\left(\left(h^2+1\right)d_\theta - h^2\right)\varphi'^2 + d_\theta - \frac{2h^2}{(h^2+1)}\right] - h\varphi'' = 0,$$

Therefore we get certain universal terms

For $d_\theta \neq n+2$ the the universal term are

$$S_{\text{univ}} = \delta_{2[\frac{d_\theta}{2}]+1,d_\theta}\, \epsilon_n \frac{\Omega_n V_{d-n-2} a_{2[\frac{d_\theta}{2}]} L^d H^{n+2-d_\theta}}{4(d_\theta - n - 2)\, r_F^\theta\, G} \log\left(\frac{H}{\varepsilon}\right).$$

For $d_\theta = n+2$ the universal terms are

$$S_{\text{univ}} = \epsilon_n \frac{\Omega_n V_{d-n-2} L^d}{4G\, r_F^\theta}\left[A_0 \log\frac{H h_0}{\varepsilon} + \frac{a_{2[\frac{d_\theta}{2}]}}{2}\, \delta_{2[\frac{d_\theta}{2}]+1,d_\theta} \log^2\left(\frac{H}{\varepsilon}\right)\right].$$

Using these results one may define the coefficient of the logarithmic term, normalized to the volume of the entangling region, as follows

$$C^{\text{EE}}_{\text{singular}} = -\epsilon_n \frac{3L^d}{4(d_\theta - n - 2)G}\, a_{2[\frac{d_\theta}{2}]}, \qquad \text{for } d_\theta \text{ odd, and } d_\theta \neq n+2,$$

$$C^{\text{EE}}_{\text{singular}} = -\epsilon_n \frac{3L^d}{4G}\, \frac{a_{2[\frac{d_\theta}{2}]}}{2}, \qquad \text{for } d_\theta \text{ odd, and } d_\theta = n+2,$$

$$C^{\text{EE}}_{\text{singular}} = -\epsilon_n \frac{3L^d}{4G}\, A_0, \qquad \text{for } d_\theta \text{ even, and } d_\theta = n+2,$$

It is illustrative to present explicit results

$$d_\theta = 2$$

For $d_\theta = 2$ being an even number, the holographic entanglement entropy has universal log term only for $n = 0$ in which one has

$$C^{\mathrm{EE}}_{\mathrm{singular}} = \frac{3L^d}{2G}\, A_0,$$

$$A_0 \sim \frac{\kappa}{\Omega}, \qquad\qquad \text{at } \Omega \to 0,$$

$$A_0 \sim \frac{1}{4\pi}\left(\frac{\pi}{2} - \Omega\right)^2, \qquad \text{at } \Omega \to \frac{\pi}{2}.$$

Bueno, Myers 2015 – Bueno, Myers, Witczak-Krempa 2015

$$d_\theta = 3$$

In this case when $n \neq 1$ the holographic entanglement entropy has a log term whose coefficient may be treated as a universal factor given by

$$C^{\rm EE}_{\rm singular} = \frac{3n^2L^d}{32G}\,\frac{\cos^2\Omega}{(1-n)\sin^{2-n}\Omega}.$$

On the other hand for $n=1$ the universal term should be read from $\log^2$ term with the coefficient

$$C^{\rm EE}_{\rm singular} = \frac{3L^d}{32G}\,\frac{\cos^2\Omega}{2\sin\Omega}.$$

$$C^{\rm EE}_{\rm singular} \sim \frac{\kappa}{\Omega^{2-n}}, \qquad \text{at } \Omega \to 0,$$

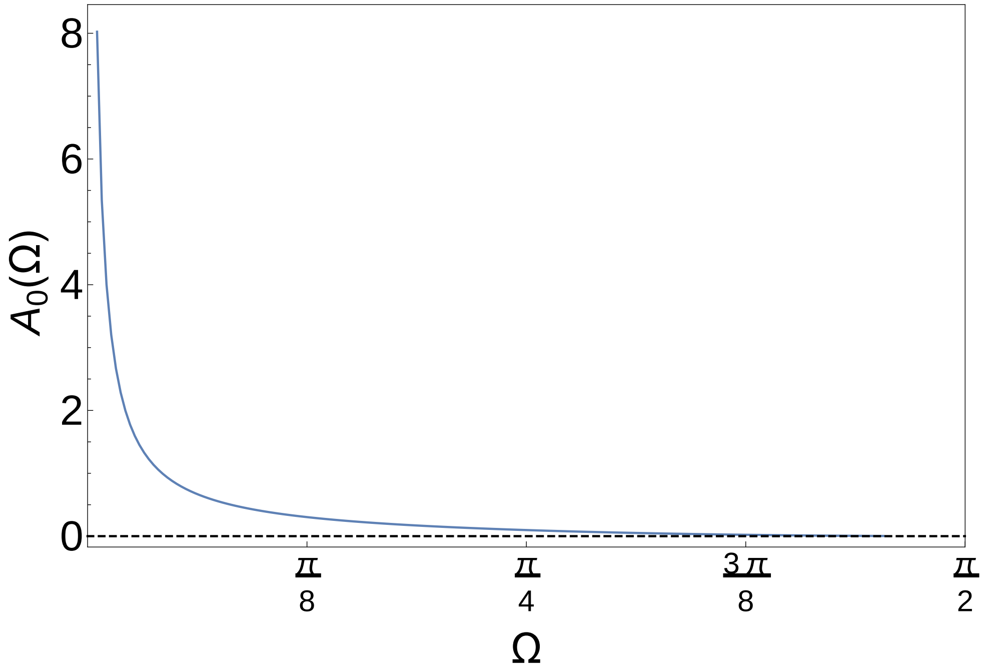
$$C^{\rm EE}_{\rm singular} \sim \sigma\left(\frac{\pi}{2}-\Omega\right)^2, \qquad \text{at } \Omega \to \frac{\pi}{2}.$$

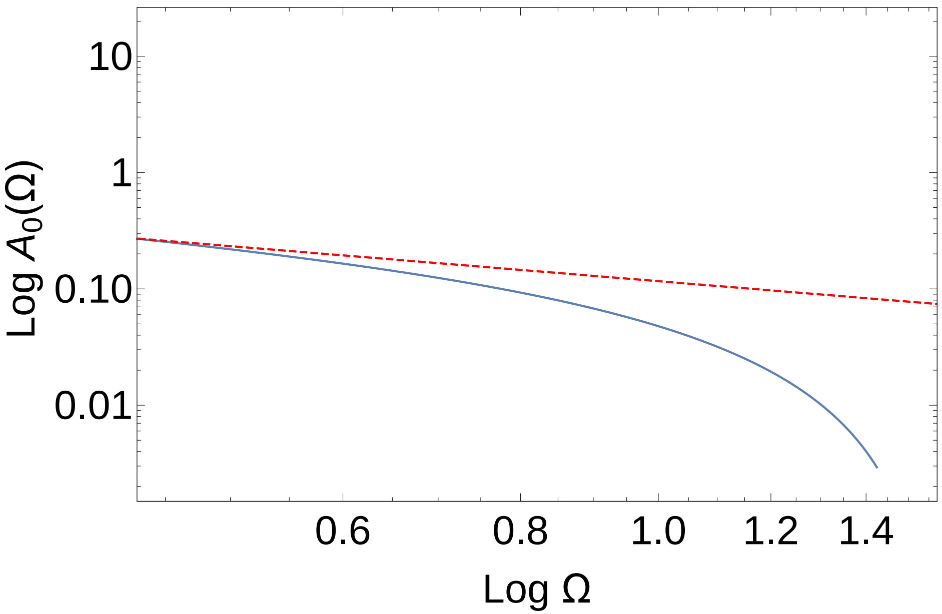
$d_\theta = 4$

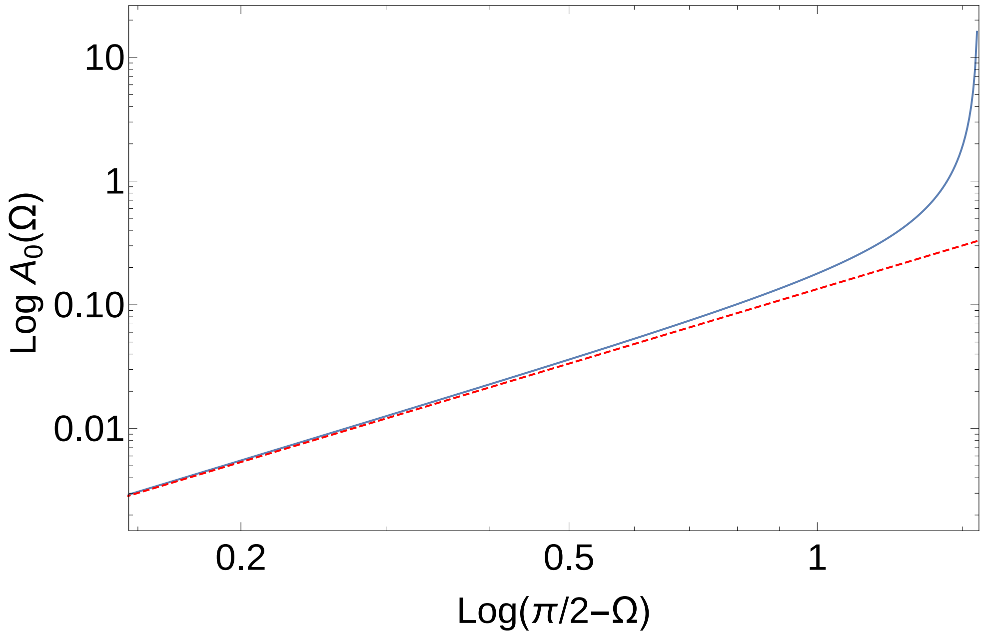
In this case in general there is no universal term except for the case where $n = 2$

$$C^{\text{EE}}_{\text{singular}} = \frac{3L^d}{4G} A_0,$$

Using the explicit form of A_0 one can find it numerically







$$A_0 \sim \frac{0.116}{\Omega}, \qquad \text{at } \Omega \to 0,$$

$$A_0 \sim \frac{1.683}{4\pi}\left(\frac{\pi}{2} - \Omega\right)^2, \qquad \text{at } \Omega \to \frac{\pi}{2}.$$

$$d_\theta = 5$$

For this case and when $n \neq 3$ the universal term must be read from log term

$$C^{\mathrm{EE}}_{\mathrm{singular}} = \frac{n^2 L^5}{16384(3-n)G} \left[\left(7n^2-64\right)\cos(2\Omega) + n(7n-32) + 64\right] \frac{\cos^2\Omega}{\sin^{4-n}\Omega}$$

while for $n = 3$ it comes from $\log^2$ term

$$C^{\mathrm{EE}}_{\mathrm{singular}} = \frac{L^5}{4G} \frac{9(31-\cos 2\Omega)}{4096} \frac{\cos^2\Omega}{\sin\Omega},$$

$$C^{\mathrm{EE}}_{\mathrm{singular}} \sim \frac{\kappa}{\Omega^{4-n}}, \qquad \text{at } \Omega \to 0,$$

$$C^{\mathrm{EE}}_{\mathrm{singular}} \sim \sigma \left(\frac{\pi}{2} - \Omega\right)^2, \qquad \text{at } \Omega \to \frac{\pi}{2}.$$

The lesson we learn from these explicit examples is that for a singular surface of the form $c_n \times R^{d-n-2}$ and for $d_\theta \geq 2$ the coefficient of the universal term has the following generic asymptotic behaviour

$$C^{\text{EE}}_{\text{singular}} \sim \begin{cases} \frac{3L^d}{4G}\frac{1}{\Omega^{d_\theta-n-1}}, & \Omega \to 0, \\ \frac{3L^d}{4G}\left(\frac{\pi}{2}-\Omega\right)^2, & \Omega \to \frac{\pi}{2}. \end{cases}$$

For a generic opening angle Ω, we can infer the following expression for the coefficient of the universal term

$$C^{\text{EE}}_{\text{singular}} = f_{d_\theta,n}(\Omega)\,\frac{3L^d}{4G}\,\frac{\cos^2\Omega}{\sin^{d_\theta-n-1}\Omega},$$

where $f_{d_\theta,n}(\Omega)$ is a function of Ω which is fixed for given d_θ and n.

Based on these results for $d_\theta \geq 2$ one may define a new charge as follows

$$C_d^n = \lim_{\Omega \to \frac{\pi}{2}} \frac{C^{\text{EE}}_{\text{singular}}}{\cos^2 \Omega}.$$

This is a well defined limit, leading to a finite quantity which is proportional to $\frac{L^d}{G}$ up to a numerical factor of order of one.

As soon as we fixed d_θ the resulting charge is independent of θ, and may be defined in any dimension by setting $n = d_\theta - 2$.

C_d^n may be though of a new central charge of the model

Other charges

We have already computed thermal entropy

$$S_{\rm th} = \left(\frac{4\pi}{d_\theta + z}\right)^{\frac{d_\theta}{z}} \frac{L^d V_d}{4G\, r_F^{d-d_\theta}} T^{\frac{d_\theta}{z}}. \equiv S_0 T^{\frac{d_\theta}{z}}$$

There is another one appearing in the two point function of energy momentum ternsor

$$\langle T_{ab}(x) T_{cd}(y)\rangle = \frac{C_T}{|x-y|^{2(d+1)}} G_{abcd}(x,y).$$

For $\theta = 0$ one has (for example see Liu, Tseytlin 1998)

$$C_T = \frac{L^d}{8\pi G}\, \frac{d+2}{d}\, \frac{\Gamma(d+2)}{\pi^{\frac{d+1}{2}}\Gamma\left(\frac{1+d}{2}\right)}.$$

Is there any relation between these charges?

All charges considered above are proportional to $\frac{L^d}{G}$, it is evident that their ratio is a purely numerical constant.

It was conjectured that for three dimensional CFT's the ration $\frac{C_2^0}{C_T}$ is universal.

It is thus interesting to understand whether this ratio, which could characterize whatsoever CFT of fixed dimensionality, is still universal even in the higher dimensional cases we are considering.

The easiest step we can make in this direction is to look at gravity theories with higher curvature terms in the action, and see whether the corrections alter the ratio.

To proceed let us consider an action containing the most general curvature squared corrections as follows

$$I = -\frac{1}{16\pi G}\int d^{d+2}\sqrt{-g}\Big(R + V(\phi) + \lambda_1 R^2 + \lambda_2 R_{\mu\nu}R^{\mu\nu} + \lambda_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\Big).$$

It is then straightforward, although lengthy, to compute holographic entanglement entropy for this mode. which may be obtained by minimizing the following entropy functional (Fursaev, Patrushev, Solodukhin, 2013)

$$S_A = \frac{1}{4G}\int d^d\zeta\,\sqrt{\gamma}\,\Big[1 + 2\lambda_1 R + \lambda_2\left(R_{\mu\nu}n_i^\mu n_i^\nu - \frac{1}{2}\mathcal{K}^i\mathcal{K}_i\right) + 2\lambda_3\Big(R_{\mu\nu\rho\sigma}n_i^\mu n_j^\nu n_i^\rho n_j^\sigma - \mathcal{K}^i_{\mu\nu}\mathcal{K}_i^{\mu\nu}\Big)\Big],$$

where $i = 1, 2$ denotes the two transverse directions to a co-dimension two hypersurface in the bulk, n_i^μ are two mutually orthogonal unit vectors to the hypersurface and $\mathcal{K}^{(i)}$ are the traces of two extrinsic curvature tensors

To compute higher curvature corrections to the entanglement entropy we note that in our case the normal vectors are given by

$$\begin{aligned} n_1 &= \frac{L}{r}\Big(1, 0, 0, 0 \cdots\Big), \\ n_2 &= \frac{L}{r}\frac{1}{\sqrt{1+h(\varphi)^2+h'(\varphi)^2}}\Big(0, 1, -h(\varphi), -\rho h'(\varphi), 0, \cdots\Big). \end{aligned}$$

Computing the HEE one arrives at

$$\tilde{C}_d^n = \Upsilon \, C_d^n \,,$$

where $\tilde{C}$ is the corrected central charge and

$$\Upsilon = 1 + \frac{4(d-2)}{L^2}\lambda_3 - \frac{2(d+1)}{L^2}(\lambda_2 + (d+2)\lambda_1)$$

It is also possible to compute the corresponding corrections to C_T using for example holographic renormalization method (Henningson, Skenderis 1998). Doing so one arrives at

$$\tilde{C}_T = \Upsilon C_T.$$

with the same Υ as that appearing in C_d^n.

Therefore one arrives at

$$\frac{\tilde{C}_d^{\,n}}{\tilde{C}_T} = \frac{C_d^{\,n}}{C_T}.$$

The others do not have such a simple universal relation.

Conclusions

• Entanglement entropy for surfaces with cusp might provide a good central charge to count the number of degrees of freedom.

• There is a general behavior for the universal terms (for $d_\theta \geq 2$) from which one can define

$$C_d^n = \lim_{\Omega \to \frac{\pi}{2}} \frac{C^{\text{EE}}_{\text{singular}}}{(\frac{\pi}{2} - \Omega)^2}$$

which could be defined in any dimension.

• Among all "charges" in the model it seems that the one appearing in the two point function of energy momentum tensor is related to C_d^n

$$C_d^n = \eta_d \, C_T$$

$$\eta_2 = \frac{\pi^2}{8}, \qquad \eta_3 = \frac{3\pi^3}{320}.$$

Thank you