Dynamical mechanism for ultra-light scalar Dark Matter

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- Motivations
- Convexity of the effective potential
- Explicit construction
- Relevance to BEC cold DM?

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1 Motivations

Cold Dark Matter halos = Bose-Einstein condensated *(Sin 1994)*
Formation: radiation-dominated era
(recent review: Suarez, Robles, Matos arXiv:1302.0903)

Coherent state of ultra-light non-relativistic scalar particles
Broglie wave length $\simeq$ halo radius $\longrightarrow$ typical mass $\simeq 10^{-23}$ eV

BEC stability analysed recently
(*Guth, Hertzberg, Prescod-Weinstein arXiv:1412.5930)*
$\longrightarrow$ need for a repulsive interaction of ultra-light scalars

Present talk: dynamical mechanism to provide such a small mass?

2 Convexity of the effective potential

Self-interacting scalar field
Convexity of the effective potential $U''_{eff}(\phi) \geq 0$ from

$$\frac{\delta^2 \Gamma}{\delta\phi\delta\phi} = -\left(\frac{\delta^2 W}{\delta j \delta j}\right) \geq 0$$

(Symanzik 1970; Iliopoulos, Itzykson, Martin 1975; Haymaker, Perez-Mercader 1983)

If double-well bare potential:
Convexity obtained if both minima contribute to the path integral
(Fujimoto, O'Raifeartaigh, Parravicini 1983)

3 Explicit construction

(JA, Tsapalis arXiv:1211.0921)
Bare potential

$$U_{bare}(\phi) = \frac{\lambda}{24}(\phi^2 - v^2)^2$$

Semi-classical approximation for the partition function

$$Z[j] \simeq \exp\left(-S[\phi_1] - \int j\phi_1\right) + \exp\left(-S[\phi_2] - \int j\phi_2\right)$$

where $\phi_{1,2}$ minima of $S[\phi] + \int j\phi$ for constant source j

Using Taylor expansions:

- Calculate classical field $\phi_c(j)$
- Invert $\longrightarrow j(\phi_c)$
- Define the Legendre transform (V = volume)

$$VU_{eff}(\phi_c) = W[j] - Vj\phi_c$$

Dressed potential for $|\phi| \leq v$, large volume regime $Vv^4 >> 1$

$$U_{dressed}(\phi) = \frac{1}{2V}\left(\frac{\phi}{v}\right)^2 + \frac{1}{12V}\left(\frac{\phi}{v}\right)^4 + \mathcal{O}(\phi^6)$$

Generalisation to $O(N)$ symmetric field ($\rho^2 \equiv \vec{\phi}\cdot\vec{\phi}$)

$$U^{(N)}_{dressed}(\vec{\phi}) = \frac{N}{2V}\left(\frac{\rho}{v}\right)^2 + \frac{N^2}{4(N+2)V}\left(\frac{\rho}{v}\right)^4 + \mathcal{O}(\rho^6)$$

Comments

- Maxwell construction when $V \to \infty$
- Abelian Higgs mechanism ($N = 2$):
  Gauge fixing $\longrightarrow$ no summation over different vacuua
  $\longrightarrow$ no Maxwell construction

4 Relevance to BEC cold DM?

Keep finite volume defined by the particle horizon l at cosmological time t

$$V \equiv l^4$$

Identification

$$\frac{1}{2}m^2\phi^2 + \frac{g}{24}\phi^4 \equiv \frac{1}{2V}\left(\frac{\phi}{v}\right)^2 + \frac{1}{12V}\left(\frac{\phi}{v}\right)^4$$

mass and coupling

$$m = \frac{1}{vl^2} \quad , \qquad g = \frac{2}{(vl)^4}$$

Numerical values:
Input: Natural mass scale v = Higgs vev = 246 GeV and $m = 10^{-23}$eV
Output: $l \simeq 12$cm,

Assume radiation dominated Universe $\longrightarrow t \simeq 10^{-10}$s
$\longrightarrow$ Electroweak symmetry breaking, consistent with choice of v

Large volume assumption satisfied $vl \simeq 10^{17}$
Dressed coupling $g \simeq 10^{-68} \longrightarrow$ non-interacting

Future directions

Improvement the semiclassical approximation
Convexity of effective potential in curved space time
Relation to the Higgs mechanism
Relevance to axion physics