Construction of Ghost Free & Singularity Free Theory of Gravity

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Phys. Rev. Lett. (2012), JCAP (2012, 2011), JCAP (2006)
CQG (2013), Phys. Rev. D (2014), 1412.3467, 1503.05568

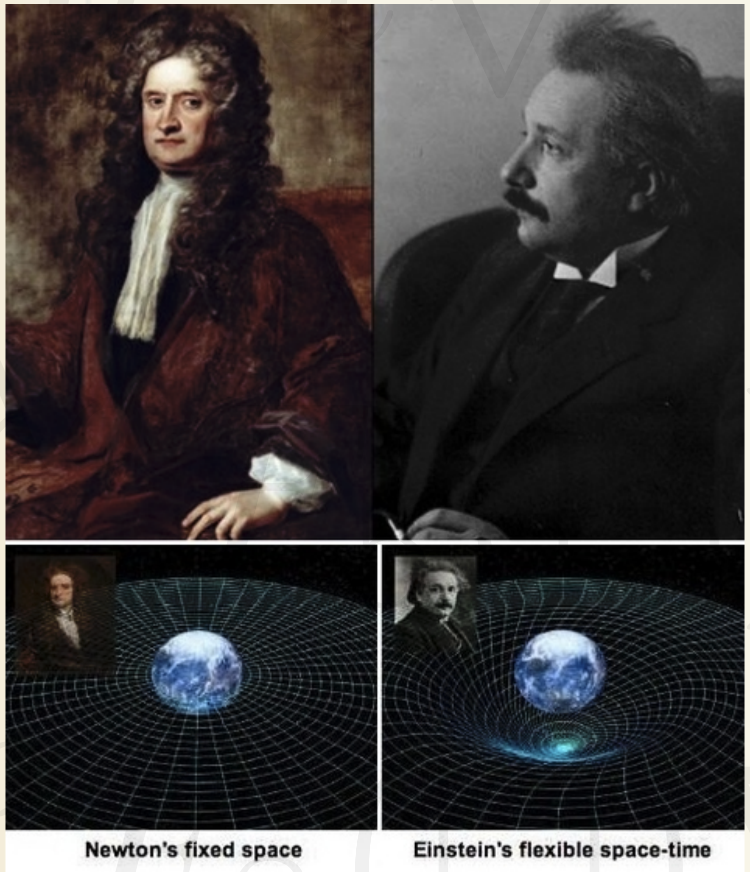
Einstein's GR is well behaved in IR, but UV is Pathetic; Aim is to address the UV aspects of Gravity

Many Contributors

Born, Enfeld, Utiyama, Efimov, Tseytlin, Siegel, Grisaru, Biswas, Krasnov, Anselmi, DeWitt, Desser, Stelle, Witten, Sen, Zwiebach, Kostelecky, Samuel, Frampton, Okada, Olson, Freund, Tomboulis, Talaganis, Khoury, Modesto, Bravisnky, Koivisto, Cline, Barnaby, Kamran, Woodard, Vernov, Kapusta, Daffayet, Arefeva, Dvali, Arkani-Hamed, Koshelev, Conroy, Craps, Sagnotti, Rubakov, …

Many contributors are present in this room …

Classical Singularities



UV is Pathological,

IR Part is Safe

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} + \cdots \right)$$

What terms shall we add such that gravity behaves better at small distances and at early times ?

While keeping the General Covariance

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} \right)$$

Motivations

Resolution to Blackhole Singularity

Resolution to Cosmological Big Bang Singularity

While Keeping IR Property of GR Intact

Bottom-up approach

- **Higher derivative gravity & ghosts**
- **Covariant extension of higher derivative ghost-free gravity**
- **Singularity free theory of gravity - "Classical Sense"**
- **Divergence structures in 1 and 2-loops in a scalar Toy model**

4th Derivative Gravity & Power Counting renormalizability

$$I = \int d^4x \sqrt{g} \left[\lambda_0 + k\,R + a\,R_{\mu\nu}R^{\mu\nu} - \tfrac{1}{3}(b+a)R^2 \right]$$

$$D \propto \frac{1}{k^4 + Ak^2} = \frac{1}{A}\left(\frac{1}{k^2} - \frac{1}{k^2 + A}\right)$$

Massive Spin-0 & Massive Spin-2 (Ghost) Stelle (1977)

Utiyama, De Witt (1961), Stelle (1977)

Modification of Einstein's GR

Modification of Graviton Propagator

Extra propagating degree of freedom

Challenge: to get rid of the extra dof

Ghosts

Higher Order Derivative Theory Generically Carry Ghosts (-ve Risidue) with real "m" (No-Tachyon)

$$S = \int d^4x \, \phi\Box(\Box + m^2)\phi \Rightarrow \Box(\Box + m^2)\phi = 0$$

$$\Delta(p^2) = \frac{1}{p^2(p^2+m^2)} \sim \frac{1}{p^2} - \frac{1}{(p^2+m^2)}$$

Propagator with first order poles

Ghosts cannot be cured order by order, finite terms in perturbative expansion will always lead to Ghosts !!

$$\Box e^{-\Box}\phi = 0$$

No extra states other than the original dof.

Higher Derivative Action

$$S = S_E + S_q$$

$$S_q = \int d^4x\sqrt{-g}\,[R_{\dots\cdot}\mathcal{O}^{\dots\cdot}_{\dots\cdot}R^{\dots\cdot} + R_{\dots\cdot}\mathcal{O}^{\dots\cdot}_{\dots\cdot}R^{\dots\cdot}\mathcal{O}^{\dots\cdot}_{\dots\cdot}R^{\dots\cdot} + R_{\dots\cdot}\mathcal{O}^{\dots\cdot}_{\dots\cdot}R^{\dots\cdot}\mathcal{O}^{\dots\cdot}_{\dots\cdot}R^{\dots\cdot}\mathcal{O}^{\dots\cdot}_{\dots\cdot}R^{\dots\cdot} + \cdots]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad R \sim \mathcal{O}(h)$$

$$S_q = \int d^4x\sqrt{-g}R_{\mu_1\nu_1\lambda_1\sigma_1}\mathcal{O}^{\mu_1\nu_1\lambda_1\sigma_1}_{\mu_2\nu_2\lambda_2\sigma_2}R^{\mu_2\nu_2\lambda_2\sigma_2}$$

Covariant derivatives

Unknown Infinite Functions of Derivatives

Redundancies

$$\begin{aligned} S_q = & \int d^4x\sqrt{-g}[RF_1(\Box)R + RF_2(\Box)\nabla_\mu\nabla_\nu R^{\mu\nu} + R_{\mu\nu}F_3(\Box)R^{\mu\nu} + R^\nu_\mu F_4(\Box)\nabla_\nu\nabla_\lambda R^{\mu\lambda} \\ & + R^{\lambda\sigma}F_5(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\lambda R^{\mu\nu} + RF_6(\Box)\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\lambda}F_7(\Box)\nabla_\nu\nabla_\sigma R^{\mu\nu\lambda\sigma} \\ & + R^\rho_\lambda F_8(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\rho R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1}F_9(\Box)\nabla_{\mu_1}\nabla_{\nu_1}\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} \\ & + R_{\mu\nu\lambda\sigma}F_{10}(\Box)R^{\mu\nu\lambda\sigma} + R^\rho_{\mu\nu\lambda}F_{11}(\Box)\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\rho_1\nu\sigma_1}F_{12}(\Box)\nabla^{\rho_1}\nabla^{\sigma_1}\nabla_\rho\nabla_\sigma R^{\mu\rho\nu\sigma} \\ & + R^{\nu_1\rho_1\sigma_1}_\mu F_{13}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1\rho_1\sigma_1}F_{14}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_{\mu_1}\nabla_\mu\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} \end{aligned}$$

$$= \int d^4x\sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

$$\Delta\mathcal{L} = \sqrt{-g}\left(\alpha R^2 + \beta R^2_{\mu\nu} + \gamma R^2_{\alpha\beta\mu\nu}\right)$$

$$\int \mathrm{d}^4x\sqrt{-g}(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\alpha\beta})$$

Gauss-Bonet Gravity

$$= \int d^4x\sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$S_q = -\int d^4x\Big[\frac{1}{2}h_{\mu\nu}a(\Box)\Box h^{\mu\nu} + h^\sigma_\mu b(\Box)\partial_\sigma\partial_\nu h^{\mu\nu} \qquad (3)$$
$$+ hc(\Box)\partial_\mu\partial_\nu h^{\mu\nu} + \frac{1}{2}hd(\Box)\Box h + h^{\lambda\sigma}\frac{f(\Box)}{\Box}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\mu\nu}\Big]$$

$$\begin{aligned}
a(\Box) &= 1 - \frac{1}{2}\mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box\\
b(\Box) &= -1 + \frac{1}{2}\mathcal{F}_2(\Box)\Box + 2\mathcal{F}_3(\Box)\Box\\
c(\Box) &= 1 + 2\mathcal{F}_1(\Box)\Box + \frac{1}{2}\mathcal{F}_2(\Box)\Box\\
d(\Box) &= -1 - 2\mathcal{F}_1(\Box)\Box - \frac{1}{2}\mathcal{F}_2(\Box)\Box\\
f(\Box) &= -2\mathcal{F}_1(\Box)\Box - \mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box.
\end{aligned}$$

$$\begin{aligned}
R_{\mu\nu\lambda\sigma} &= \frac{1}{2}(\partial_{[\lambda}\partial_\nu h_{\mu\sigma]} - \partial_{[\lambda}\partial_\mu h_{\nu\sigma]})\\
R_{\mu\nu} &= \frac{1}{2}(\partial_\sigma\partial_{(\nu}h^\sigma_{\mu)} - \partial_\nu\partial_\mu h - \Box h_{\mu\nu})\\
R &= \partial_\nu\partial_\mu h^{\mu\nu} - \Box h
\end{aligned}$$

$$a + b = 0$$
$$c + d = 0$$
$$b + c + f = 0$$

$\mathcal{F}_3(\Box)$ is redundant

Graviton Propagator

$$a(\Box)\Box h_{\mu\nu} + b(\Box)\partial_\sigma\partial_{(\nu}h^\sigma_{\mu)} + c(\Box)(\eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \partial_\mu\partial_\nu h)$$
$$+\eta_{\mu\nu}d(\Box)\Box h + \frac{1}{4}f(\Box)\Box^{-1}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\lambda\sigma} = -\kappa\tau_{\mu\nu}$$

$$-\kappa\tau\nabla_\mu\tau^\mu_\nu = 0 = (c \overset{=0}{+} d)\Box\partial_\nu h + (a \overset{=0}{+} b)\Box h^\mu_{\nu,\mu} + (b + c \overset{=0}{+} f)h^{\alpha\beta}_{,\alpha\beta\nu}$$

Bianchi Identity

$$a + b = 0$$
$$c + d = 0$$
$$b + c + f = 0$$

Biswas, Koivisto, AM
1302.0532

$$\Pi^{-1\lambda\sigma}_{\mu\nu} h_{\lambda\sigma} = \kappa\tau_{\mu\nu} \qquad h = h^{TT} + h^L + h^T$$

$$\Pi = \frac{P^2}{ak^2} + \frac{P^0_s}{(a-3c)k^2}$$

Spin projectors

Let us introduce

$$\begin{aligned}
\mathcal{P}^2 &= \frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma}+\theta_{\mu\sigma}\theta_{\nu\rho})-\frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}\,,\\
\mathcal{P}^1 &= \frac{1}{2}(\theta_{\mu\rho}\omega_{\nu\sigma}+\theta_{\mu\sigma}\omega_{\nu\rho}+\theta_{\nu\rho}\omega_{\mu\sigma}+\theta_{\nu\sigma}\omega_{\mu\rho})\,,\\
\mathcal{P}^0_s &= \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}\,,\quad \mathcal{P}^0_w=\omega_{\mu\nu}\omega_{\rho\sigma}\,,\\
\mathcal{P}^0_{sw} &= \frac{1}{\sqrt{3}}\theta_{\mu\nu}\omega_{\rho\sigma}\,,\quad \mathcal{P}^0_{ws}=\frac{1}{\sqrt{3}}\omega_{\mu\nu}\theta_{\rho\sigma}\,,
\end{aligned}\tag{16}$$

where the transversal and longitudinal projectors in the momentum space are respectively

$$\theta_{\mu\nu}=\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2},\qquad \omega_{\mu\nu}=\frac{k_\mu k_\nu}{k^2}.$$

Note that the operators $\mathcal{P}^i$ are in fact 4-rank tensors, $\mathcal{P}^i_{\mu\nu\rho\sigma}$, but we have suppressed the index notation here.

Out of the six operators four of them, $\{\mathcal{P}^2,\mathcal{P}^1,\mathcal{P}^0_s,\mathcal{P}^0_w\}$, form a complete set of projection operators:

$$\mathcal{P}^i_a\mathcal{P}^j_b=\delta^{ij}\delta_{ab}\mathcal{P}^i_a \quad \text{and} \quad \mathcal{P}^2+\mathcal{P}^1+\mathcal{P}^0_s+\mathcal{P}^0_w=1\,,\tag{17}$$

$$\mathcal{P}^0_{ij}\mathcal{P}^0_k=\delta_{jk}\mathcal{P}^0_{ij}\,,\quad \mathcal{P}^0_{ij}\mathcal{P}^0_{kl}=\delta_{il}\delta_{jk}\mathcal{P}^0_k\,,\quad \mathcal{P}^0_k\mathcal{P}^0_{ij}=\delta_{ik}\mathcal{P}^0_{ij}\,,$$

For this action, see:

Biswas, Koivisto, AM 1302.0532

Tree level Graviton Propagator

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2}$$

No new propagating degree of freedom other than the massless Graviton

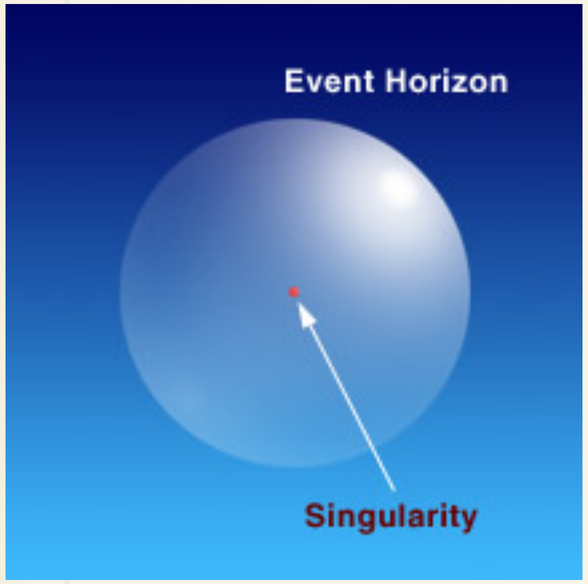
$$a(\Box) = c(\Box) \Rightarrow 2\mathcal{F}_1(\Box) + \mathcal{F}_2(\Box) + 2\mathcal{F}_3(\Box) = 0$$

Biswas, Gerwick, Koivisto, AM

PRL (2012)

(gr-qc/1110.5249)

(1) Gravitational Entropy



$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$

$$S_W = -8\pi \oint_{r=r_H,\ t=\mathrm{const}} \left(\frac{\partial \mathcal{L}}{\partial R_{rtrt}}\right) q(r) d\Omega^2$$

$$S_W = \frac{Area}{4G}\left[1 + \alpha\left(2\mathcal{F}_1 + \mathcal{F}_2 + 2\mathcal{F}_3\right) R\right]$$

$= 0$

Holography is an IR effect

Higher order corrections yield zero entropy, i.e. the ground state of gravity

Conroy, AM, Teimouri
1503.05568, hep-th

(2) Newtonian Limit

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2} \qquad a(\Box) = c(\Box) = e^{-\Box/M^2}$$

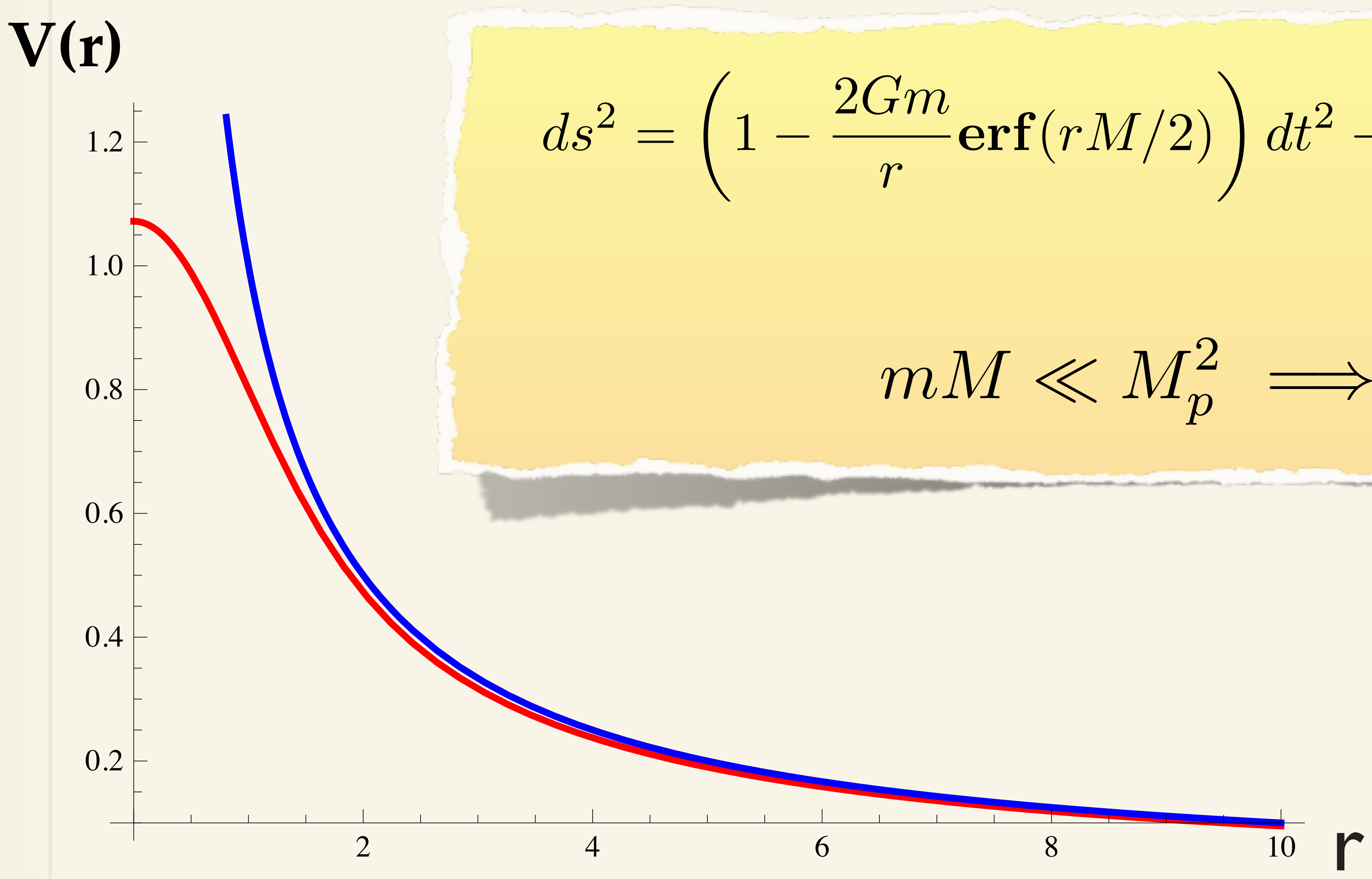
$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box} \right] R - 2R_{\mu\nu} \left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} \right] R^{\mu\nu} \right]$$

$$ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Psi)dr^2$$

$$\Phi = \Psi = \frac{Gm}{r} \mathbf{erf} \left(\frac{rM}{2} \right)$$

Biswas, Gerwick, Koivisto, AM, PRL (2012)
(gr-qc/1110.5249)

Non-singular static solution



$$ds^2 = \left(1 - \frac{2Gm}{r}\mathbf{erf}(rM/2)\right) dt^2 - \frac{dr^2}{\left(1 - \frac{2Gm}{r}\mathbf{erf}(rM/2)\right)}$$

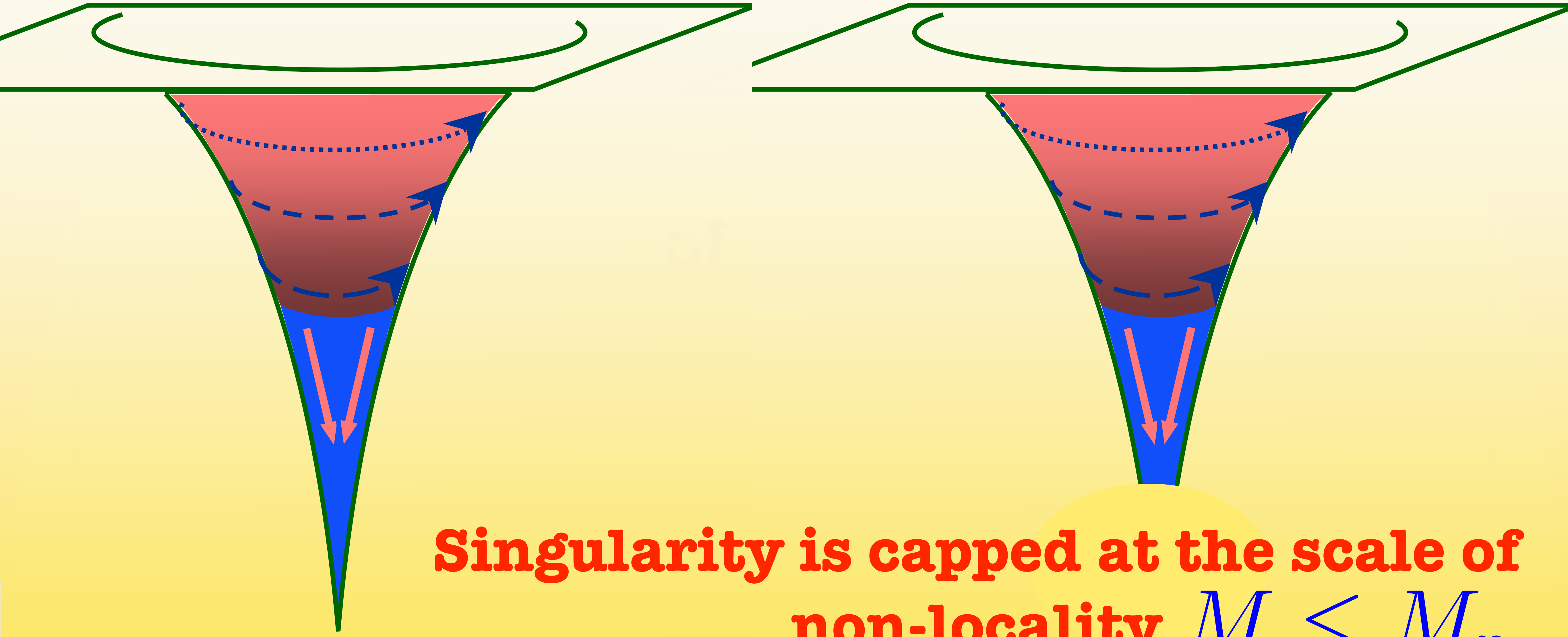
$$mM \ll M_p^2 \implies m \ll M_p$$

$$r \to 0, \qquad \mathrm{erf}(r) \to r \qquad \Phi(r) \to \mathrm{const.}$$

$$r \to \infty, \qquad \mathrm{erf}(r) \to 1 \qquad \Phi(r) \to \frac{1}{r}$$

Biswas, Gerwick, Koivisto, AM, PRL (2012)
(gr-qc/1110.5249)

Where would you expect the modifications?



Singularity is capped at the scale of non-locality $M \leq M_p$

(3) Non-singular time dependent solutions

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box} \right] R - 2R_{\mu\nu} \left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} \right] R^{\mu\nu} \right]$$

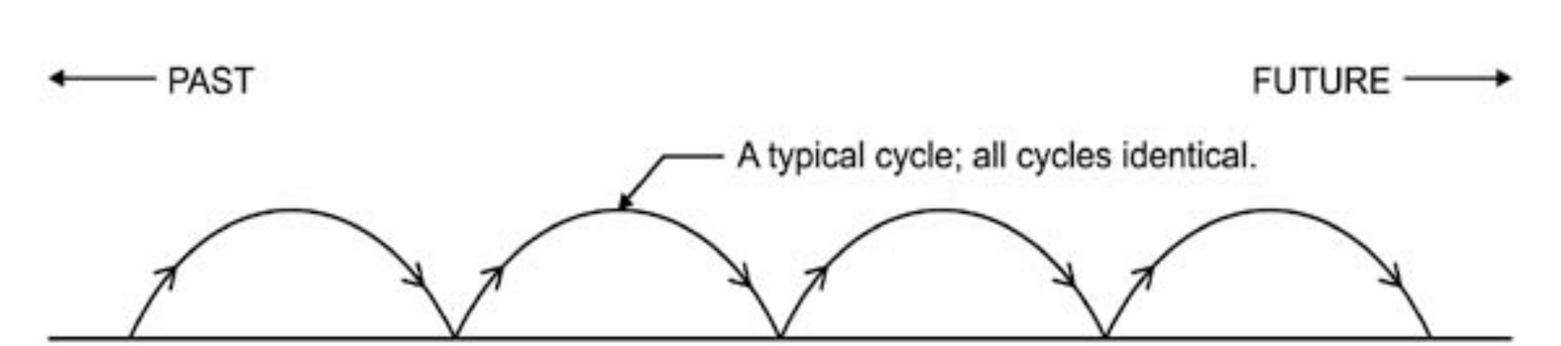


Fig 02

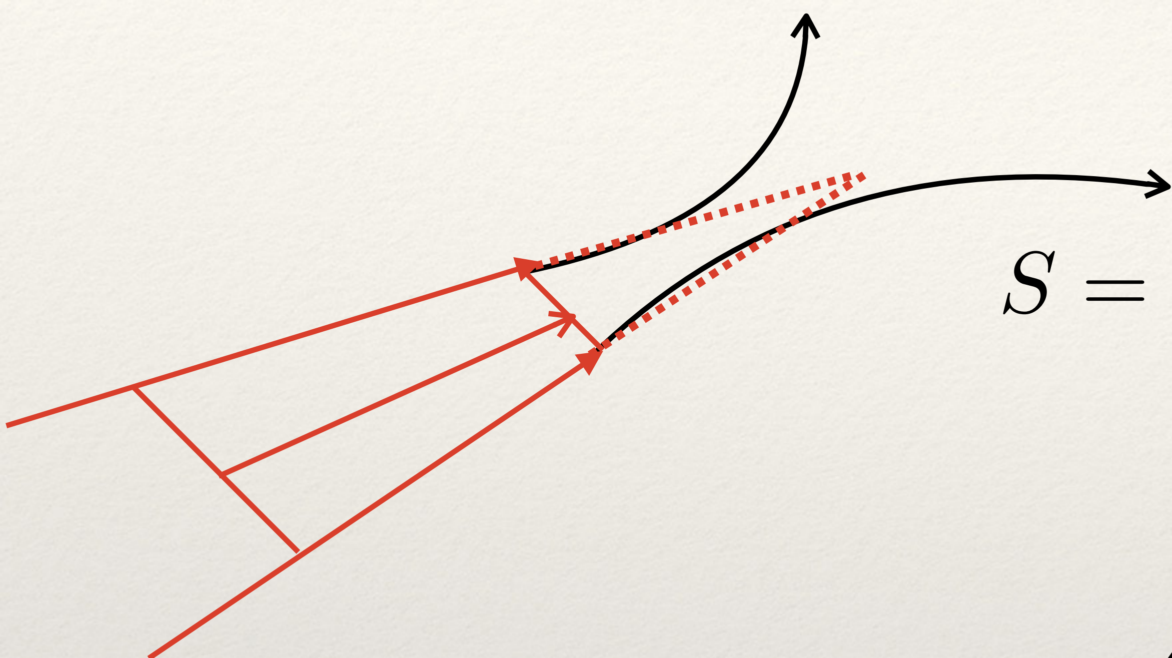
$$h \sim \mathtt{diag}(0, A\sin\lambda t, A\sin\lambda t, A\sin\lambda t) \text{ with } A \ll 1$$

Non- Singular Bouncing, Homogeneous & Isotropic Universe

Such a solution is not possible in GR

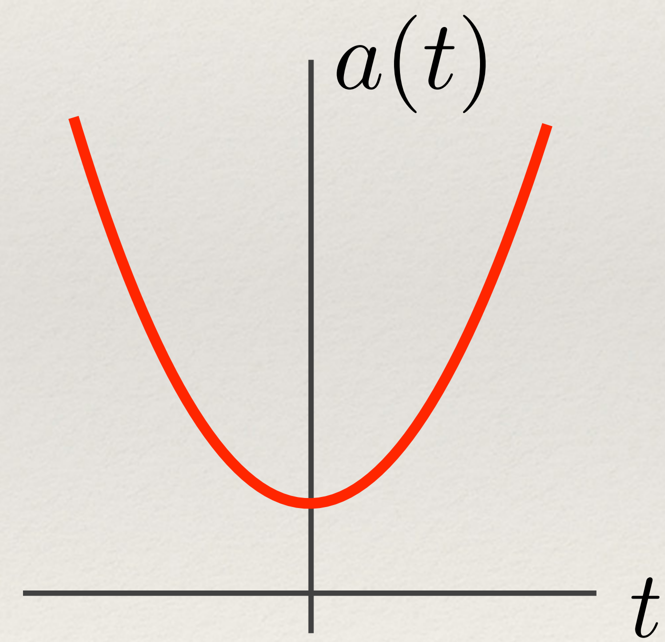
Biswas, Gerwick, Koivisto, AM, Phys. Rev. Lett. (gr-qc/1110.5249)

Cosmological non-singular solution



$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}}-1}{\Box}\right]R + \Lambda\right]$$

$$a(t) = \cosh\left(\sqrt{\frac{r_1}{2}}t\right)$$



Stay tuned: details of the
Singularity theorem by "Hawking-Penrose" will
arrive sometime this summer …
(a very nasty computation)

Biswas, Gerwick, Koivisto, AM, PRL (2012)

Biswas, AM, Siegel, JCAP (2006)

Toy model based on Symmetries

GR e.o.m : $g_{\mu\nu} \rightarrow \Omega\, g_{\mu\nu}$

Around Minkowski space the e.o.m are invariant under:

$$h_{\mu\nu} \rightarrow (1+\epsilon)h_{\mu\nu} + \epsilon\eta_{\mu\nu}$$

Construct a scalar field theory with infinite derivatives whose e.o.m are invariant under

$$\phi \rightarrow (1+\epsilon)\phi + \epsilon$$

$$S_{free} = \frac{1}{2}\int d^4x(\phi\Box a(\Box)\phi)$$

$$a(\Box) = e^{-\Box/M^2}$$

$$S_{int} = \frac{1}{M_p}\int d^4x\left(\frac{1}{4}\phi\partial_\mu\phi\partial^\mu\phi + \frac{1}{4}\phi\Box\phi a(\Box)\phi - \frac{1}{4}\phi\partial_\mu\phi a(\Box)\partial^\mu\phi\right)$$

$$\Pi(k^2) = -\frac{i}{k^2 e^{\bar{k}^2}}$$

Quantum aspects

- **Superficial degree of divergence goes as**

$$E = V - I. \ \text{Use Topological relation}: \ L = 1 + I - V$$

$$E = 1 - L \qquad\qquad E < 0, \ \text{for} \ L > 1$$

- **At 1-loop, the theory requires counter term, the 1-loop, 2 point function yields Λ^4 divergence**
- **At 2-loops, the theory does not give rise to additional divergences, the UV behaviour becomes finite, at large external momentum, where dressed propagators gives rise to more suppression than the vertex factors**

Towards understanding the ultraviolet behavior of quantum loops in infinite-derivative theories of gravity

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Abstract

In this paper we will consider quantum aspects of a non-local, infinite derivative scalar field theory - a *toy model* depiction of a covariant infinite derivative, non-local extension of Einstein's general relativity which has previously been shown to be free from ghosts around the Minkowski background. The graviton propagator in this theory gets an exponential suppression making it *asymptotically free*, thus providing strong prospects of resolving various classical and quantum divergences. In particular, we will find that at 1-loop, the 2-point function is still divergent, but once this amplitude is renormalized by adding appropriate counter terms, the ultraviolet (UV) behavior of all other 1-loop diagrams as well as the 2-loop, 2-point function remains well under control. We will go on to discuss how one may be able to generalize our computations and arguments to arbitrary loops.

Remnants of stringy Gravity

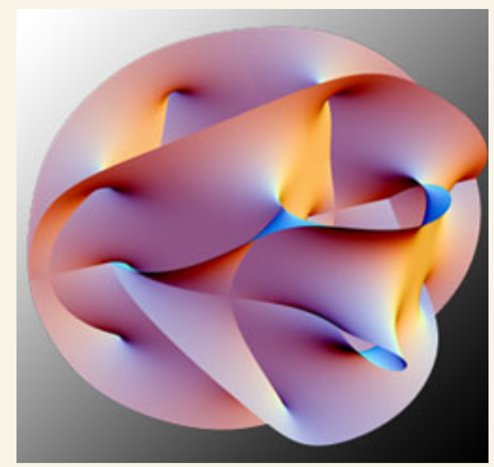
$\mathrm{M_p}$

$$\mathcal{L}^{10\mathrm{d}} \sim R + R^4 + \cdots \qquad \kappa^2 = g_s^2(\alpha')^4$$

Perturbative string theory has α' & g_s corrections

For all orders : String field theory

$\mathrm{m_W}$



$\mathrm{m_s}$

$\mathrm{m_{KK}}$

$$\mathcal{L}^{4\mathrm{d}} \sim R + \sum_i c_i R \left(\frac{\Box}{m_{kk}}\right)^i R + \cdots$$

$1-$ loop in g_s all orders in α'

Witten (1998) , Tseytlin (1995), Zwiebach (2000), Sigel (1998, 2003), ...

Conclusions

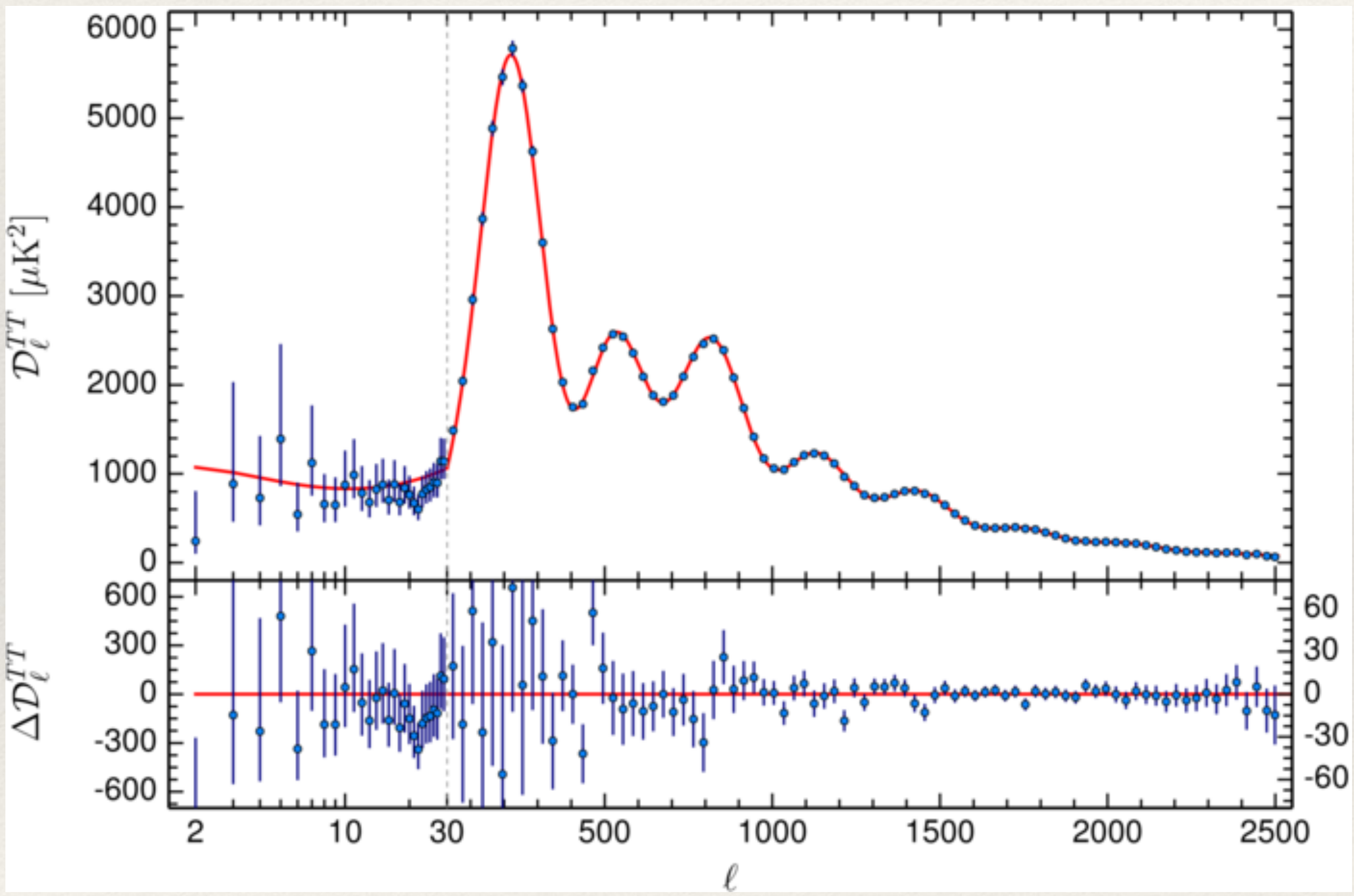
- **We have constructed a Ghost Free & Singularity Free Theory of Gravity**
- **If we can show higher loops are finite then it is a great news --** this is what we have shown up to 2 loops
- **Studying singularity theorems, positive energy theorems, Hawking radiation, Non-Singular Bouncing Cosmology , ..., many interesting problems can be studied in this framework**
- **Holography is no longer a property of UV, becomes part of an IR effect.**

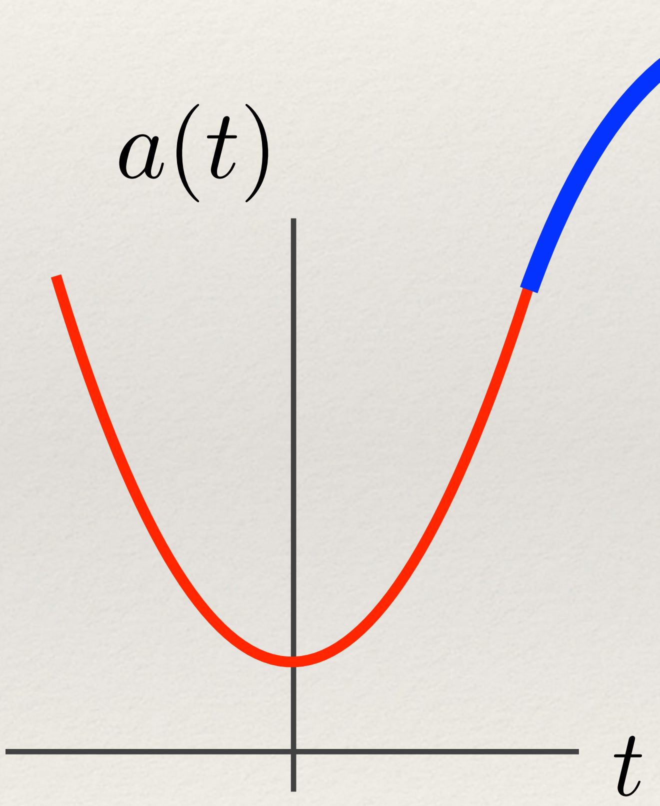
Extra Slides

Non-Singular Inflationary Trajectory

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}}-1}{\Box}\right]R + \Lambda\right]$$

Resolves Cosmological singularity





Interesting smoking gun in the Planck data !

Blue tilt for low multipoles: on large scales power increases

Biswas, AM, PRD (2014)

Loop quantum gravity
or
CDT approach

↓

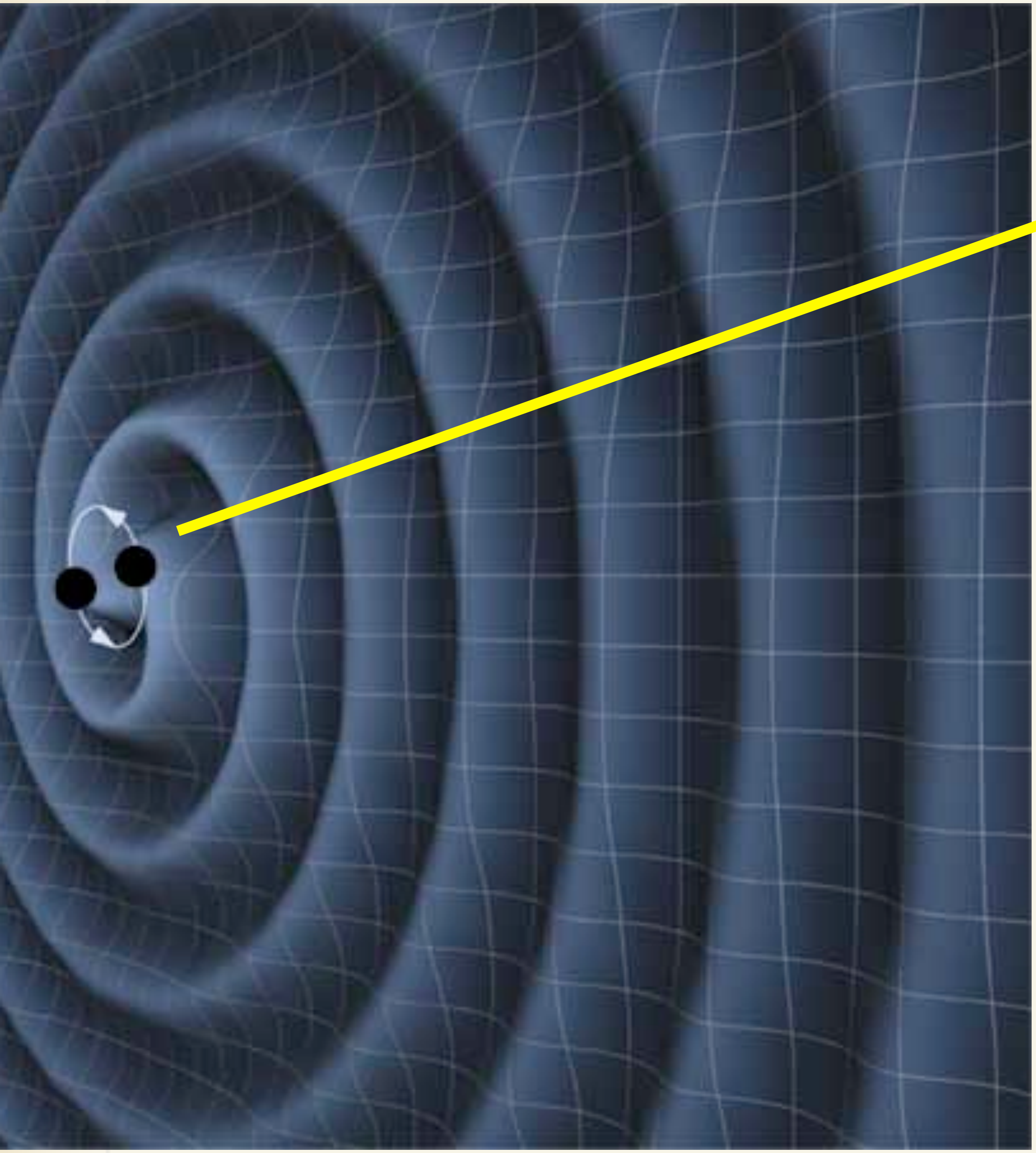
Wilson loops

↓

Non-local objects

It would be interesting to establish the connection

Gravitational Waves



$$\bar{h}_{jk} \approx G\frac{\omega^2(ML^2)}{r}$$

Large r
limit

$$\bar{h}_{jk} \approx G\frac{\omega^2(ML^2)}{r}\mathrm{erf}\left(\frac{rM_P}{2}\right)$$

$r \Longrightarrow 0,$ **No Singularity**

**Biswas, Gerwick, Koivisto, AM,
Phys. Rev. Lett. (gr-qc/1110.5249)**

Revisiting Hawking-Penrose Singularity Theorems

$$\theta = \nabla_a N^a \qquad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq -R_{ab}N^a N^b$$

General Relativity

$$R_{ab}N^a N^b = 8\pi T_{ab}N^a N^b \geq 0$$

$$\frac{d\theta}{d\tau} \leq 0 \qquad \rho + p \geq 0$$

Non-local extension of GR

$$R_{ab}N^a N^b \leq 0, \qquad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \geq 0$$

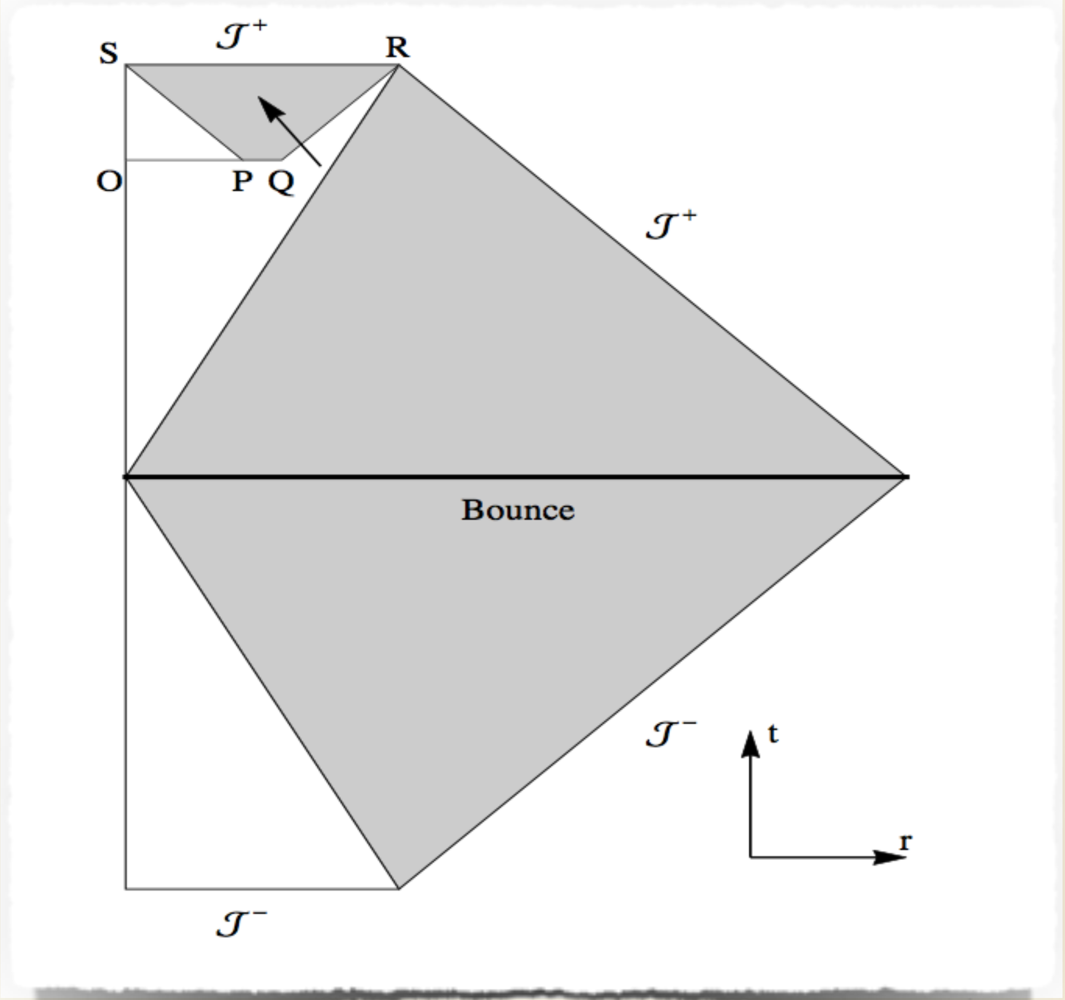
$$R_{ab}N^a N^b \neq 8\pi T_{ab}N^a N^b$$

Conroy, Koshlev, AM,
PRD (2014) (gr-qc/1408.6205)

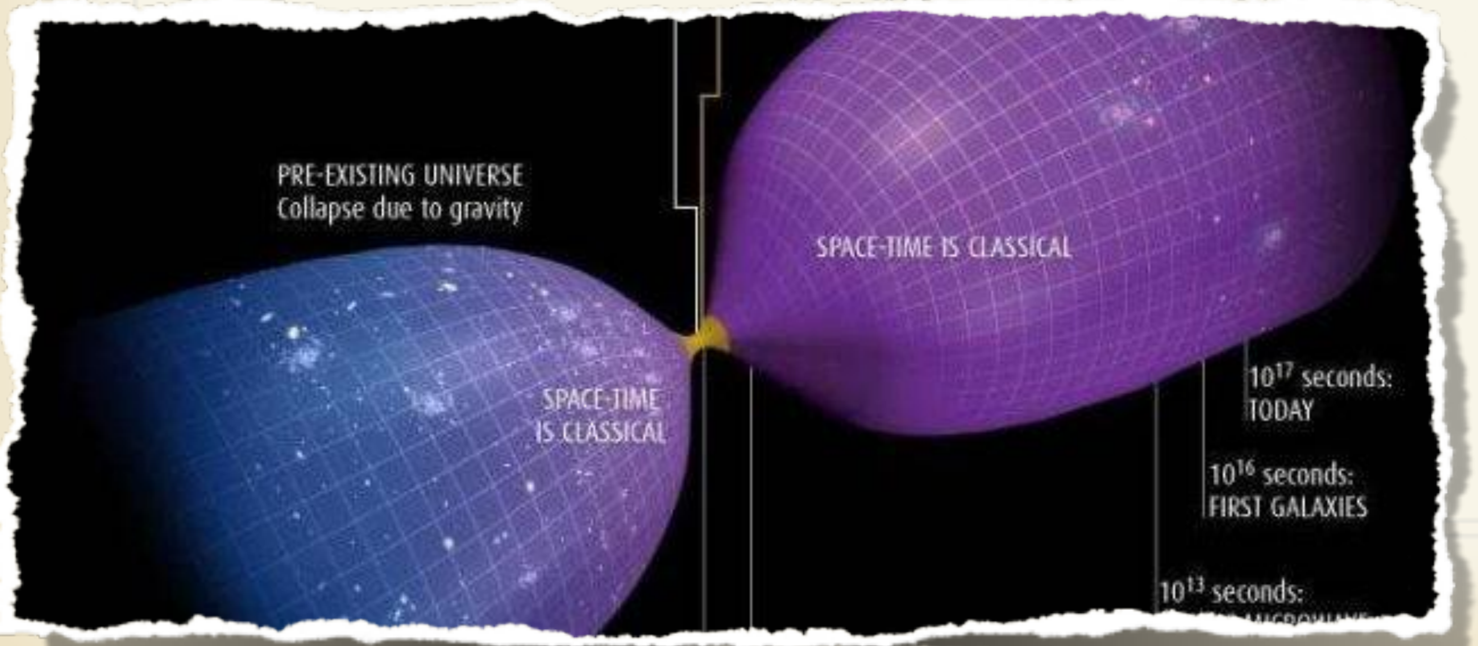
Revisiting Singularity Theorems

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \frac{R\mathcal{F}(\Box)R}{2}\right)$$

$$R_{\mu\nu}k^\mu k^\nu = (k^0)^2\frac{(\rho+p)+2\partial_t^2(\mathcal{F}(\Box)R)}{M_p^2+2\mathcal{F}(\Box)R}$$



$$R_{\mu\nu}k^\mu k^\nu \leq 0, \qquad T_{\mu\nu}k^\mu k^\nu \geq 0 \rightarrow (\rho + p \geq 0)$$



$$\frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \geq 0$$

Conjecture

$$S = \int d^4x \,\sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box}\right]R - 2R_{\mu\nu}\left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box}\right]R^{\mu\nu}\right]$$

Absence of Cosmological and Blackhole Singularities

Conjecture : The Form of Most General Action

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + \alpha_0(R, R_{\mu\nu}) + \alpha_1(R, R_{\mu\nu})R\mathcal{F}_1(\Box)R\right.$$
$$\left.+\alpha_2(R, R_{\mu\nu})R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + \alpha_3(R, R_{\mu\nu})C_{\mu\nu\lambda\sigma}\mathcal{F}_3 C^{\mu\nu\lambda\sigma}\right]$$