RG running in the presence of spacetime curvature

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Outline

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1-loop Effective potential

- Massive self-interacting scalar field [1]

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

$$\Rightarrow$$

$$V_{\text{eff}}(\phi) = \underbrace{\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4}_{\text{classical}} + \underbrace{\frac{M(\phi)^4}{64\pi^2}\left[\log\left(\overbrace{\frac{M(\phi)^2}{\mu^2}}^{\text{effective mass}}\right) - \frac{3}{2}\right]}_{\text{quantum}}$$

$$; M(\phi)^2 = m^2 + \frac{\lambda}{2}\phi^2$$

- μ is the *renormalization scale*
- Similarly one may derive the potential for the SM Higgs

[1] Coleman & Weinberg (1972)

Effective potential for the SM Higgs

$$V_{\text{eff}}(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \sum_{i=1}^{5}\frac{n_i}{64\pi^2}M_i^4(\phi)\left[\log\frac{M_i^2(\phi)}{\mu^2} - c_i\right]$$

$$;M_i^2(\phi) = \kappa_i\phi^2 - \kappa_i'$$

Φ	i	n_i	κ_i	κ_i'	c_i
$W^\pm$	1	6	$g^2/4$	0	$5/6$
Z^0	2	3	$(g^2+g'^2)/4$	0	$5/6$
t	3	-12	$y_t^2/2$	0	$3/2$
ϕ	4	1	3λ	m^2	$3/2$
χ_i	5	3	λ	m^2	$3/2$

- Explicit μ dependence?

Callan-Symanzik equation for massless $\lambda\phi^4$ theory

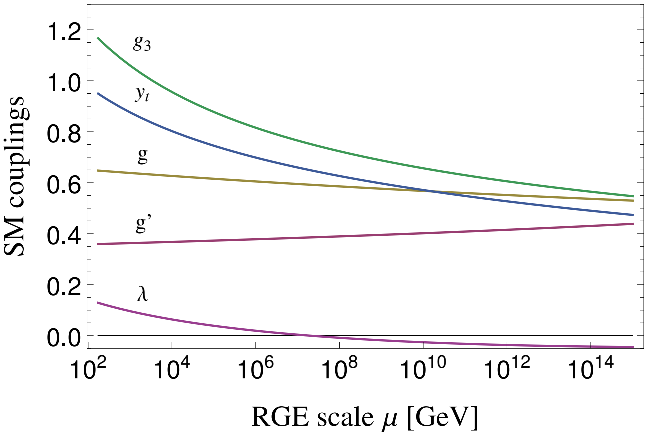
- The effective potential is renormalized at a scale μ
  $\lambda_0 \to \lambda_R, \quad \phi \to Z^{1/2}\phi$
- However, the physical result must not depend on μ:

$$\frac{d}{d\mu}V_{\text{eff}}(\phi) = 0 \quad \Leftrightarrow \quad \left\{\mu\frac{\partial}{\partial\mu} + \beta_\lambda\frac{\partial}{\partial\lambda} + \gamma_\phi\phi\frac{\partial}{\partial\phi}\right\}V_{\text{eff}}(\phi) = 0$$

$$;\beta_\lambda \equiv \mu\frac{\partial\lambda}{\partial\mu}, \quad \gamma_\phi \equiv \mu\frac{\partial\ln Z^{1/2}}{\partial\mu}$$

- This can be used to improve the perturbative result
- Leads to a potential with *running parameters*
  $\bar{V}_{\text{eff}}\big(\phi(t), \lambda(t)\big) \qquad ;t \equiv \log(\mu/M_t), \quad \phi(t) \equiv Z(t)^{1/2}\phi$

SM running (1-loop)



- For large scales, the potential is dominated by the quartic term $\lambda\phi^4$

Scale independence of V_{eff}

- A perturbative result is scale invariant only up to higher order corrections [2]

$$\frac{d}{d\mu}\bar{V}_{\text{eff}} = 0 + \mathcal{O}(\hbar^2)$$

- μ should be chosen such that the error is small [3]

The optimal choice for $\phi \gg m$

$$\mu \sim \phi$$

$\Rightarrow$ No large logarithms!

Do curvature effects change this?

[2] Casas et. al. (1994)
[3] Ford et. al. (1993)

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Curved space field theory

- QFT in curved spacetime
  - No graviton loops

$$Z[J, g^{\mu\nu}] = \int \mathcal{D}\varphi e^{\,iS[\varphi, g^{\mu\nu}]+i\int J\varphi}$$

$$S[\varphi, g^{\mu\nu}] \equiv S_g[g^{\mu\nu}] + S_m[\varphi, g^{\mu\nu}]$$

- Is renormalizable but requires generalization the Einstein-Hilbert action

$$\mathcal{L}_g[g^{\mu\nu}] = \Lambda + \alpha R$$
$$\rightarrow \Lambda_0 + \alpha_0 R + \beta_0 R^2 + \epsilon_{1,0} R_{\alpha\beta} R^{\alpha\beta} + \epsilon_{2,0} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$$

- Importantly, loops generate a term $\propto R\phi^2$

$\Rightarrow$ $\xi R\phi^2$ is a part of the SM!

UV expanding the mode

- We must solve Klein-Gordon equation in *curved space*

$$\left[-\Box + m^2 + \xi R\right]\hat{\phi} = 0; \qquad \Box \equiv \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu)$$

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a(t)d\mathbf{x}^2$$

- Using an UV ansatz for the Fourier mode,

$$\phi = \int d^3k\left[\hat{a}_\mathbf{k}u_\mathbf{k} + \hat{a}^\dagger_\mathbf{k}u^*_\mathbf{k}\right], \quad u_\mathbf{k} = \frac{1}{\sqrt{a(t)^3W}}e^{-i\int^t Wdt'}e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\Rightarrow \quad W^2 = \frac{k^2}{a(t)^2} + m^2 + \xi R - \frac{R}{6} + \mathcal{O}(k^{-2})$$

Mass shift $\propto R$ (scalar curvature)

- Works also for spinors and gauge fields
- Coincides with the resummed heat kernel method [4]

[4] Jack & Parker (1985)

Summary of curved space effects

- All effective masses acquire shifts $\propto R$
- A non-minimal ξ-term is generated by loop corrections
  - Virtually unbounded by the LHC, $\xi_{\rm EW} < 10^{15}$ [5]
- RG equation for β_ξ
  - Can easily be calculated from $V_{\rm eff}(\phi)$

The optimal scale in curved space

$$\mu^2 \sim \phi^2 + R$$

$\Rightarrow$ Spacetime curvature effects how the couplings run

[5] Atkins & Calmet (2012)

1-loop Effective potential in curved space

$$V_{\text{eff}}(\phi, R) = -\frac{1}{2}m^2(t)\phi(t)^2 + \frac{1}{2}\xi(t)R\phi(t)^2 + \frac{1}{4}\lambda(t)\phi(t)^4$$

$$+\sum_{i=1}^{9}\frac{n_i}{64\pi^2}M_i^4(t)\left[\log\frac{|M_i^2(t)|}{\mu^2(t)} - c_i\right] \qquad ; M_i^2(t) = \kappa_i\phi(t)^2 - \kappa_i' + \theta_i R$$

Φ	i	n_i	κ_i	κ_i'	θ_i	c_i
$W^\pm$	1	2	$g^2/4$	0	$1/12$	$3/2$
	2	6	$g^2/4$	0	$-1/6$	$5/6$
	3	-2	$g^2/4$	0	$-1/6$	$3/2$
Z^0	4	1	$(g^2+g'^2)/4$	0	$1/12$	$3/2$
	5	3	$(g^2+g'^2)/4$	0	$-1/6$	$5/6$
	6	-1	$(g^2+g'^2)/4$	0	$-1/6$	$3/2$
t	7	-12	$y_t^2/2$	0	$1/12$	$3/2$
ϕ	8	1	3λ	m^2	$\xi - 1/6$	$3/2$
χ_i	9	3	λ	m^2	$\xi - 1/6$	$3/2$

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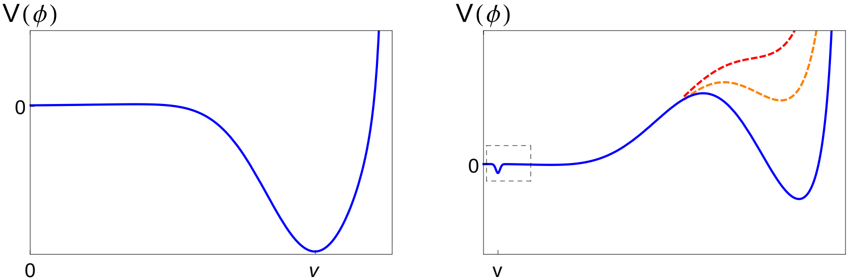
Standard Model Higgs potential



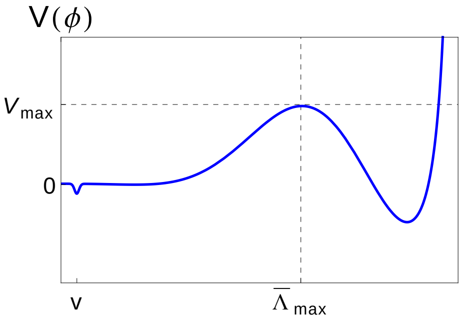
- Minimum at $\phi = v$
  - A vacuum at $\phi \neq v$ incompatible with observations
  - *Meta* stable at 99% CL [6]
    - Lifetime much longer than $13.8 \cdot 10^9$ years
- Sensitive to M_h and M_t

- Is this also true for the early Universe (inflation)?
- New physics needed to stabilize the vacuum?

[6] Buttazzo et al. (2013); Spencer-Smith (2014)

Inflation and the Standard Model

- In principle we can assume the SM to be valid
  - Energy-density is dominated by decoupled physics
- Inflation induces fluctuations to the Higgs field $\Delta\phi \sim H$
  - Important if $\overline{\Lambda}_{\rm max} \lesssim H$
  - State of the art calculations [7]: $\overline{\Lambda}_{\rm max} \sim 10^{11}\text{GeV}$



$r = \mathcal{P}_T/\mathcal{P}_\mathcal{R}$

$\overline{\Lambda}_{\rm max} \sim H$

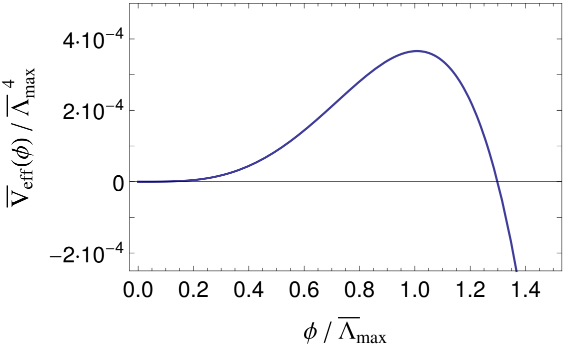
$\Rightarrow r \sim 2 \cdot 10^{-7}$

BICEP2:

$\overline{\Lambda}_{\rm max} \ll H$

[7] Degrazzi et. al.(2013); Buttazzo et. al. (2013)

Stability results (Minkowski)



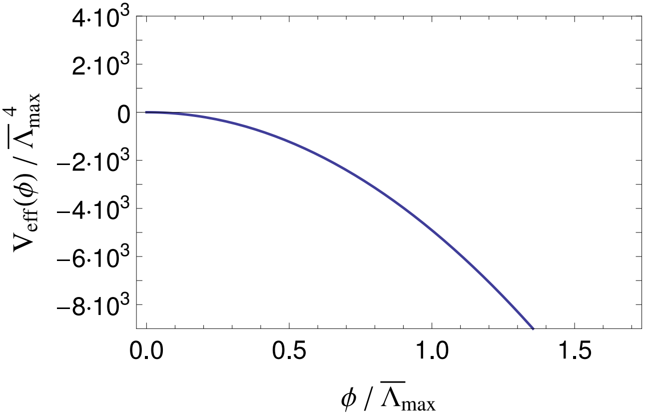
- For large H $(\sim 10^3\overline{\Lambda}_{\rm max})$, the SM is not stable [8]

What about curvature corrections?

[8] Kobakhidze & Spencer-Smith (2014); Hook et. al. (2014); Fairbairn & Hogan (2014); Enqvist, Meriniemi & Nurmi (2014)

Stability results (curved space) I

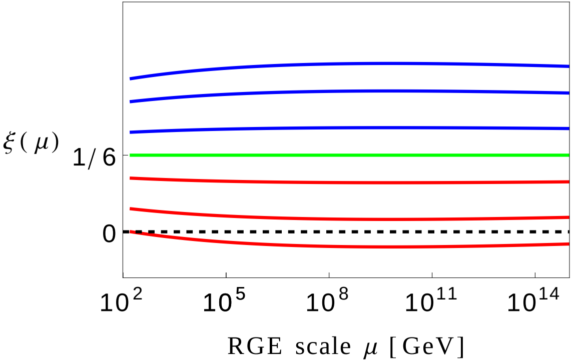
- First attempt, set $\xi_{\rm EW} = 0$ and $H \sim 10^{10}{\rm GeV}$ $(\sim 10^3 \overline{\Lambda}_{\rm max}$)



- Potential negative *everywhere*
  - Clearly unstable (worse than with Minkowski QFT)
- What causes this?

Stability results (curved space) II

- In curved space $\lambda(\mu) < 0$ earlier since $\mu^2 \sim \phi^2 + R$
- ξ Can become positive or negative depending on $\xi_{\rm EW}$

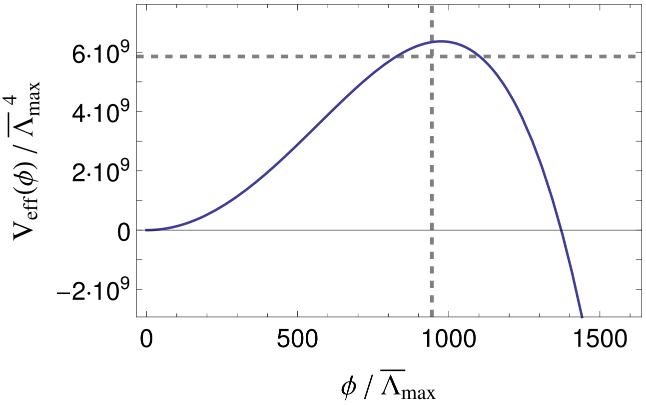


ξ_{EW}

$0,\ 0.05,\ 0.12,\ 1/6,\ 0.22,\ 0.28,\ 0.33$

Stability results (curved space) III

- Now choosing $\xi_{\text{EW}} = 0.1$ [9]



$$\Lambda_{\max} \simeq H\left(\frac{12\xi}{|\lambda|}\right)^{1/2}$$

$$V_{\max}^{1/4} \simeq H\frac{(6\xi)^{1/2}}{|\lambda|^{1/4}}$$

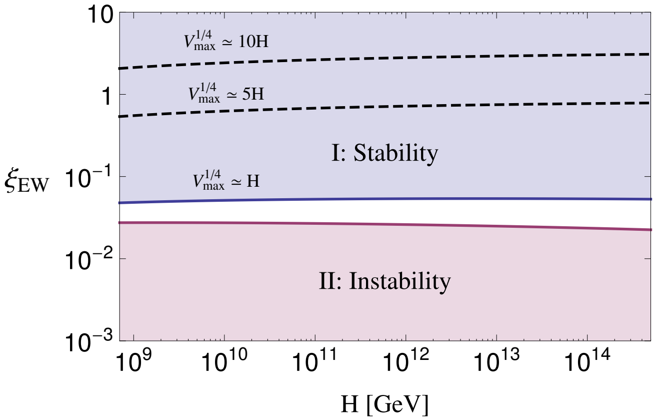
- $V_{\max}(\text{curved}) \gg V_{\max}(\text{flat})$ (and at a higher scale)

$$P \sim \exp\left[-8\pi^2\,(V_{\max}/3H^4)\right] \quad \Rightarrow \quad \textbf{Stable!}$$

[9] Espinosa, Giudice & Riotto (2008)

Stability results (curved space) IV

- The (in)stability of the potential is determined by ξ_{EW}



Outline

Cosmological constant problem(s)

$$\Lambda \sim \int^{k_{\max}} d^3k \sqrt{k^2 + m^2} \sim M_{\rm pl}^4$$

- Zero-point energies imply a value too large by 10^{120}
  - Argument comes from non-covariant cut-off regularization
  - Not a problem in dim. reg.

$$\Lambda \sim V(\langle\phi\rangle) \sim m_{\rm h}^4$$

- EW symmetry breaking implies a value too large by 10^{55}
  - "The real magnitude of the fine-tuning" [10]

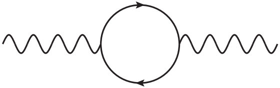
$$\mu\frac{\partial\Lambda}{\partial\mu} \equiv \beta_\Lambda \sim \sum_i m_i^4$$

- Running potentially problematic

[10] J. Martin (2012); J. Solà (2013)

Decoupling theorem

- Consider the electric charge $\beta_e = -\frac{e}{2}\mu\frac{\partial}{\partial\mu}\Pi$

$\Pi \sim$ 

- Using a physical subtraction scheme at $p^2 = -M^2$

$$\beta_e^{\text{ph}} = \beta_e^{\overline{\text{MS}}} \int_0^1 dx \, \frac{M^2 x(x-1)}{m^2 + M^2 x(x-1)}$$

Appelquist-Carazzone theorem

$$\beta_e^{\text{ph}} \to \beta_e^{\overline{\text{MS}}}, \qquad M \gg m,$$

BUT $\beta_e^{\text{ph}} \to 0, \qquad M \ll m$

- Heavy degrees of freedom decouple [11]

[11] Appelquist & Carazzone (1974)

Curvature induced running in the EW vacuum

- Consider then the SM on a dynamical curved background with $T = 0$
- If the only dynamics comes from changing R,

$$\Rightarrow \quad \mu = \mu(R)$$

- Curvature induces running of Λ [12]
- In the electroweak vacuum $R \ll m^2$, $\Rightarrow$ $\overline{\text{MS}}$ is no good
- In a scheme respecting the decoupling theorem we get

$$\Lambda_{\text{in}} \lesssim \xi^2 \frac{R_{\text{EW}}^2}{128\pi^2} \frac{48y^4 - 9g_1^4 - 6g_1^2 {g_2}^2 - 3{g_2}^4 - 64\lambda^2}{16\lambda^2} \lesssim 10^{-47}\text{GeV}^4$$

- Is the complete running of Λ compatible with observations?

[12] J.Solà & I. Shapiro (2008)

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Conclusions

Curvature effects in RG running

- Important for EW vacuum stability
  - SM with $\xi_{EW} = 0$ is unstable during inflation for large H
  - Having $\xi_{EW} \gtrsim 6 \times 10^{-2}$ stabilizes the vacuum
- Induce running of the Cosmological Constant

Thank You!

Sensitivity to the choice of μ

- A loop calculation is never fully scale invariant
- How dependent is the result on the choice $\mu^2 = \phi^2 + R$?

$$\mu^2 = \alpha\phi^2 + \beta R \qquad \alpha, \beta \in \{0.1 \cdots 10\}$$