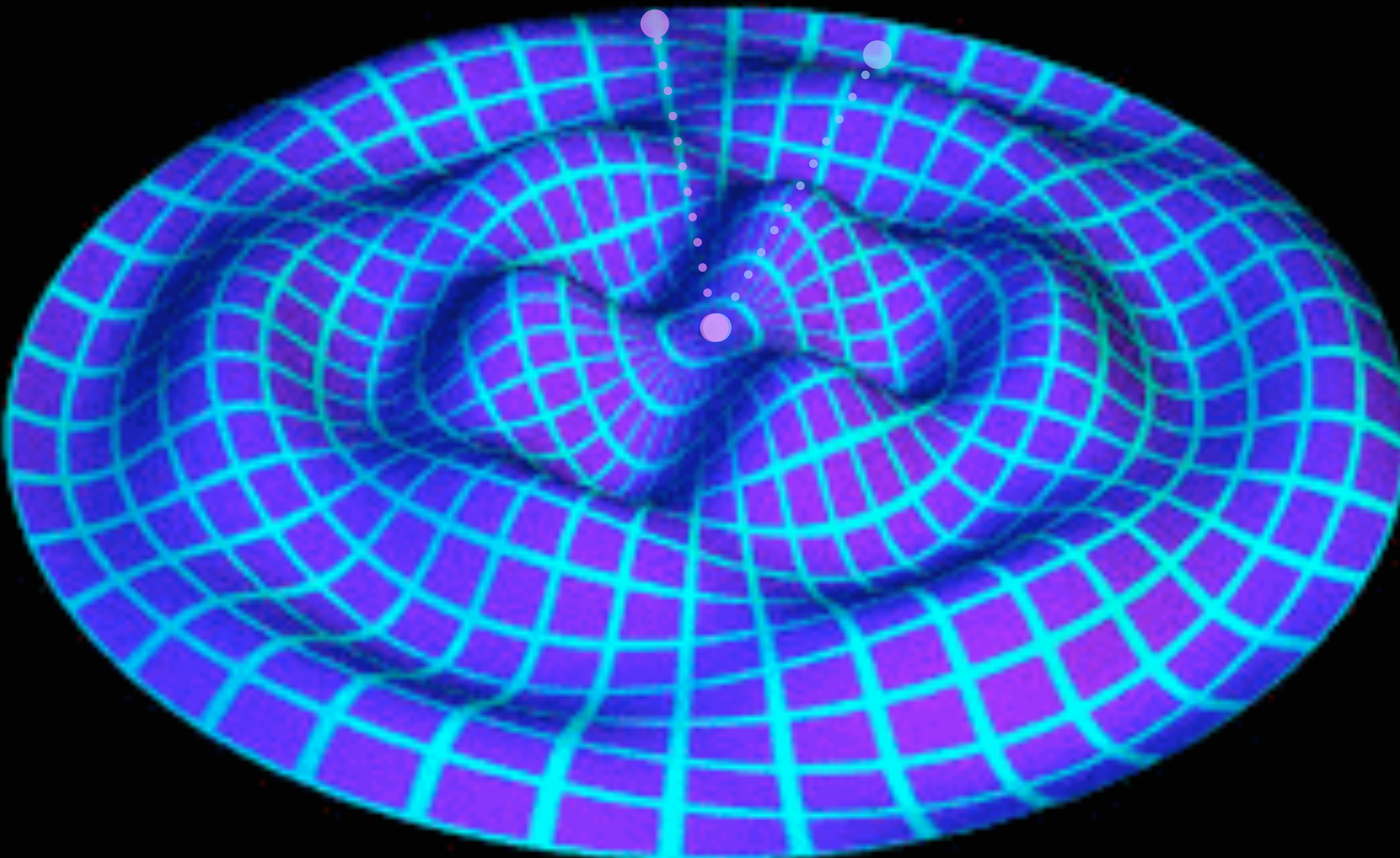
Background independent ERG for conformally reduced gravity

UK QFT Imperial 30/3/15

Tim Morris,
Physics & Astronomy,
University of Southampton, UK.

Juergen A. Dietz & TRM, arXiv:1502.0739.

What does scale 1/k mean?



Conformal field theory? How to separate scales?

What does scale invariant mean? What is large/small scale?

What is short wavelength? What is long wavelength?

How to form a continuum limit?

Use background field method:

$$\tilde{g}_{\mu\nu} = \bar{g}_{\mu\nu} + \tilde{h}_{\mu\nu}$$

cutoff:

$$R_k \sim r\left(-\frac{\bar{\nabla}^2}{k^2}\right)$$

- Now the scale k is defined through $\bar{g}_{\mu\nu}$ so the notion depends on the choice of background
- The RG itself depends on both $g_{\mu\nu}$ **and** $\bar{g}_{\mu\nu}$ so also is inherently background dependent, and necessarily bi-metric
- Is there a background independent notion of scale k?

Conformally reduced gravity

$$\phi = \varphi + \chi$$

$$g_{\mu\nu} = f(\phi)\,\delta_{\mu\nu} \qquad \text{and} \qquad \bar{g}_{\mu\nu} = f(\chi)\,\delta_{\mu\nu}$$

- Remnant diffeomorphism invariance ...

$$x^\mu \mapsto x^\mu/\lambda\,, \qquad f(\chi) \mapsto \lambda^2 f(\chi)$$

E.g. $f(\phi) = \exp(2\phi)$

$$\Gamma_k[\varphi,\chi] = \int d^dx\, f(\chi)^{\frac{d}{2}} \left(\frac{1}{2} \frac{K(\varphi,\chi)}{f(\chi)} (\partial_\mu \varphi)^2 + V(\varphi,\chi) \right)$$

$$\Gamma_k[\varphi,\chi] = \int d^dx \sqrt{\bar{g}} \left(-\frac{1}{2} K(\varphi,\chi)\, \bar{g}^{\mu\nu} \partial_\mu\varphi\, \partial_\nu\varphi + V(\varphi,\chi) \right)$$

Maintain background covariance:

- φ at $O(\partial^2)$ but slowly varying χ

$$\chi \mapsto \chi + \ln\lambda\,, \qquad \phi \mapsto \phi + \ln\lambda\,, \qquad \varphi \mapsto \varphi$$

- Conformal factor problem ⇒ wrong sign kinetic term

- K has absorbed the factor of $1/8\pi G$ and V absorbed $\Lambda/16\pi G$

$$S_k[\varphi, \bar{g}] = \frac{1}{2}\int d^dx \sqrt{\bar{g}(x)}\, \varphi(x)\, R_k[\bar{g}]\, \varphi(x)$$

Break quantum covariance:

- ... fixes factors of f(χ)

$$\chi \mapsto \chi\,, \qquad \phi \mapsto \phi + \ln\lambda\,, \qquad \varphi \mapsto \varphi + \ln\lambda$$

Flow equation

$$\int_x \equiv \int d^d x \, f(x)^{\frac{d}{2}}$$

$$\partial_t \Gamma_k[\varphi, \chi] = \frac{1}{2} \int_x \int_y \Delta(x,y) \, \partial_t R_k(y,x)$$

$$f(x) \equiv f\left(\chi(x)\right)$$

$$\Delta(x,y) = \left[f(x)^{-\frac{d}{2}} f(y)^{-\frac{d}{2}} \frac{\delta^2 \Gamma_k}{\delta\varphi(x)\delta\varphi(y)} + R_k(x,y) \right]^{-1}$$

$$t = \ln(k/\mu)$$

Broken split Ward Identity (msWI)

$$\phi = \varphi + \chi$$

$$\varphi(x) \mapsto \varphi(x) + \varepsilon(x) \qquad \chi(x) \mapsto \chi(x) - \varepsilon(x)$$

$$\int_w f^{-\frac{d}{2}} \left(\frac{\delta \Gamma_k}{\delta \chi} - \frac{\delta \Gamma_k}{\delta \varphi} \right) \varepsilon = \frac{1}{2} \int_x \int_y \Delta(x,y) \left[\frac{d}{2} \partial_\chi \ln f(y) \, R_k(y,x) + \partial_\chi R_k(y,x) \right] \varepsilon(y)$$

Sketch of derivative expansion...

Write trace in terms of differential operators...

$$E.g. \quad R_{k,x} = R_k\left(-\bar{\nabla}_x^2\right) = R_k\left(-f(\chi)^{-1}\partial_x^2\right)$$

$$\int d^dx\,\Delta_x\,\mathcal{K}_x\,\delta(x-x')\big|_{x'=x} = \int d^dx \frac{d^dp}{(2\pi)^d}\underline{\left\{e^{-ip\cdot x}\Delta_x e^{ip\cdot x}\right\}}\,\mathcal{K}\left(f^{-1}p^2\right)$$

$$Q(p,x) \equiv e^{-ip\cdot x}\left[\Gamma^{(2)} + R\right]^{-1} e^{ip\cdot x}$$

satisfies differential equation:

$$\underline{\mathrm{e}^{-ip\cdot x} R\left\{\mathrm{e}^{ip\cdot x}Q\right\}} = 1 - \mathrm{e}^{-ip\cdot x}\Gamma^{(2)}\left\{\mathrm{e}^{ip\cdot x}Q\right\}$$

$$R(p^2/f)Q - 2i\partial_{p^2}R(p^2/f)p^\mu\partial_\mu Q - \partial_{p^2}R(p^2/f)\partial^2 Q$$
$$-2\partial_{p^2}^2 R(p^2/f)p^\mu p^\nu\partial_\mu\partial_\nu Q + \mathcal{O}(\partial^3)$$

iterate

Flow equations, msWIs

$$\partial_t V(\varphi,\chi) = f(\chi)^{-\frac{d}{2}} \int_0^\infty dp\, p^{d-1} \frac{\dot{R}(p^2/f)}{\partial_\varphi^2 V - K p^2/f + R(p^2/f)}$$

$$\partial_\chi V - \partial_\varphi V + \frac{d}{2}\partial_\chi \ln f\, V = f(\chi)^{-\frac{d}{2}} \int_0^\infty dp\, p^{d-1} \frac{\partial_\chi R + \frac{d}{2}\partial_\chi \ln f\, R}{\partial_\varphi^2 V - K p^2/f + R(p^2/f)}$$

$$f^{-1}\partial_t K(\varphi,\chi) = 2 f^{-\frac{d}{2}} \int_0^\infty dp\, p^{d-1}\, P \dot{R}$$

$$f^{-1}\left\{\partial_\chi K - \partial_\varphi K + \frac{d-2}{2}\partial_\chi \ln f\, K\right\} = 2 f^{-\frac{d}{2}} \int_0^\infty dp\, p^{d-1}\, P \left[\partial_\chi R + \frac{d}{2}\partial_\chi \ln f\, R\right]$$

$$\begin{aligned} P = &-\frac{1}{2}\frac{\partial_\varphi^2 K}{f} Q_0^2 + \frac{\partial_\varphi K}{f}\left(2\partial_\varphi^3 V - \frac{2d+1}{d}\frac{\partial_\varphi K}{f} p^2\right) Q_0^3 \\ &- \left[\left\{\frac{4+d}{d}\frac{\partial_\varphi K}{f} p^2 - \partial_\varphi^3 V\right\}\left(\partial_{p^2} R - \frac{K}{f}\right) + \frac{2}{d} p^2 \partial_{p^2}^2 R \left(\frac{\partial_\varphi K}{f} p^2 - \partial_\varphi^3 V\right)\right]\left(\partial_\varphi^3 V - \frac{\partial_\varphi K}{f} p^2\right) Q_0^4 \\ &- \frac{4}{d} p^2 \left(\partial_{p^2} R - \frac{K}{f}\right)^2 \left(\partial_\varphi^3 V - \frac{\partial_\varphi K}{f} p^2\right)^2 Q_0^5 \end{aligned}$$

$$Q_0 = \frac{1}{\partial_\varphi^2 V - K p^2/f + R}$$

Want momentum dependence so that:

$$\dot{R}(p^2/f) \propto \partial_\chi R + \frac{d}{2}\,\partial_\chi \ln f\, R$$

Cutoff

dimensionless cutoff profile

$$R\left(p^2/f\right) = -k^{d-\eta-\frac{d}{2}d_f}\, r\left(\frac{p^2}{k^{2-d_f} f}\right)$$

$$\implies \qquad \partial_\chi \ln f\, \dot{R} = (2-d_f)\left[\partial_\chi R + \frac{d}{2}\,\partial_\chi \ln f\, R\right] - \eta\, \partial_\chi \ln f\, R$$

- $\eta = 0$ ✓

$$\eta\, \partial_\chi \ln f\, R \propto \partial_\chi R + \frac{d}{2}\,\partial_\chi \ln f\, R \qquad \implies \qquad f\frac{\partial R}{\partial f} \propto R$$

- $\boxed{r(z) = \frac{1}{z^n}}$ ✓

Linear PDEs

$$\alpha = 2\left(1 - \frac{\eta}{d+2n}\right) - d_f$$

$$\partial_\chi \ln f\, \partial_t V = \alpha \left(\frac{d}{2}\, \partial_\chi \ln f\, V + \partial_\chi V - \partial_\varphi V \right)$$

$$\partial_\chi \ln f\, \partial_t K = \alpha \left(\partial_\chi K - \partial_\varphi K + \frac{d-2}{2}\, \partial_\chi \ln f\, K \right)$$

Solutions

$$V(k,\varphi,\chi) = f(\chi)^{-\frac{d}{2}} \tilde{V}(\tilde{k},\phi)\,,\;\; K(k,\varphi,\chi) = f(\chi)^{-\frac{d}{2}+1} \tilde{K}(\tilde{k},\phi)\,,\;\; \tilde{k} = k f(\chi)^{\frac{1}{\alpha}}$$

Dimensionless variables

$$\hat{k} = \tilde{k}^{\frac{1}{1+d_f/\alpha}} = k^{\frac{1}{1+d_f/\alpha}} f(\chi)^{\frac{1}{\alpha+d_f}}$$

$$[\hat{k}] = 1$$

$$\tilde{V} = \hat{k}^d\, \hat{V}_{\hat{k}}(\hat{\phi})\,, \quad \tilde{K} = \hat{k}^{d-2-\eta}\, \hat{K}_{\hat{k}}(\hat{\phi})\,, \quad \phi = \hat{k}^{\frac{\eta}{2}} \hat{\phi}\,, \quad p = \hat{k}\, \hat{p}$$

msWIs, flow equations

$$\partial_t V(\varphi,\chi) = f(\chi)^{-\frac{d}{2}} \int_0^\infty dp\, p^{d-1} \frac{\dot{R}(p^2/f)}{\partial_\varphi^2 V - Kp^2/f + R(p^2/f)}$$

$$\partial_\chi V - \partial_\varphi V + \frac{d}{2}\,\partial_\chi \ln f\, V = f(\chi)^{-\frac{d}{2}} \int_0^\infty dp\, p^{d-1} \frac{\partial_\chi R + \frac{d}{2}\,\partial_\chi \ln f\, R}{\partial_\varphi^2 V - Kp^2/f + R(p^2/f)}$$

$$f^{-1}\partial_t K(\varphi,\chi) = 2f^{-\frac{d}{2}} \int_0^\infty dp\, p^{d-1}\, P\dot{R}$$

$$f^{-1}\left\{\partial_\chi K - \partial_\varphi K + \frac{d-2}{2}\,\partial_\chi \ln f\, K\right\} = 2f^{-\frac{d}{2}} \int_0^\infty dp\, p^{d-1}\, P\left[\partial_\chi R + \frac{d}{2}\,\partial_\chi \ln f\, R\right]$$

$$\begin{aligned} P = &-\frac{1}{2}\frac{\partial_\varphi^2 K}{f} Q_0^2 + \frac{\partial_\varphi K}{f}\left(2\partial_\varphi^3 V - \frac{2d+1}{d}\frac{\partial_\varphi K}{f}p^2\right) Q_0^3 \\ &- \left[\left\{\frac{4+d}{d}\frac{\partial_\varphi K}{f}p^2 - \partial_\varphi^3 V\right\}\left(\partial_{p^2} R - \frac{K}{f}\right) + \frac{2}{d}p^2 \partial_{p^2}^2 R \left(\frac{\partial_\varphi K}{f}p^2 - \partial_\varphi^3 V\right)\right]\left(\partial_\varphi^3 V - \frac{\partial_\varphi K}{f}p^2\right) Q_0^4 \\ &- \frac{4}{d}p^2\left(\partial_{p^2} R - \frac{K}{f}\right)^2 \left(\partial_\varphi^3 V - \frac{\partial_\varphi K}{f}p^2\right)^2 Q_0^5 \end{aligned}$$

$$Q_0 = \frac{1}{\partial_\varphi^2 V - Kp^2/f + R}$$

Background Independent flow equations

$$\partial_{\hat{t}}\hat{V} + d\hat{V} - \frac{\eta}{2}\hat{\phi}\hat{V}' = -\left(d - \eta + 2n\right)\int_0^\infty d\hat{p}\;\hat{p}^{d-1}\,\hat{Q}_0\,r\left(\hat{p}^2\right)$$

- No χ : $\hat{V} \equiv \hat{V}_{\hat{k}}(\hat{\phi})\,, \quad \hat{K} \equiv \hat{K}_{\hat{k}}(\hat{\phi})\,!$

$$\partial_{\hat{t}}\hat{K} + (d - 2 - \eta)\hat{K} - \frac{\eta}{2}\hat{\phi}\hat{K}' = -2(d - \eta + 2n)\int_0^\infty d\hat{p}\;\hat{p}^{d-1}\,\hat{P}\,r\left(\hat{p}^2\right)$$

- No f !

$$\hat{t} = \ln(\hat{k}/\mu)$$

$$\begin{aligned}\hat{P} = &-\frac{1}{2}\hat{K}''\hat{Q}_0^2 + \hat{K}'\left(2\hat{V}''' - \frac{2d+1}{d}\hat{K}'\hat{p}^2\right)\hat{Q}_0^3 \\ &+ \left[\left\{\frac{4+d}{d}\hat{K}'\hat{p}^2 - \hat{V}'''\right\}\left(r'(\hat{p}^2) + \hat{K}\right) + \frac{2}{d}\hat{p}^2 r''(\hat{p}^2)\left(\hat{K}'\hat{p}^2 - \hat{V}'''\right)\right]\left(\hat{V}''' - \hat{K}'\hat{p}^2\right)\hat{Q}_0^4 \\ &- \frac{4}{d}\hat{p}^2\left(r'(\hat{p}^2) + \hat{K}\right)^2\left(\hat{V}''' - \hat{K}'\hat{p}^2\right)^2\hat{Q}_0^5\end{aligned}$$

$$\hat{Q}_0 = \frac{1}{\hat{V}'' - \hat{K}\hat{p}^2 - r(\hat{p}^2)}$$

Two notions of fixed point

$$V(\varphi,\chi) = k^{d-\frac{d}{2}d_f}\,\bar{V}_k(\bar{\varphi},\bar{\chi})\,, \qquad \varphi = k^{\eta/2}\bar{\varphi}\,, \qquad \chi = k^{\eta/2}\bar{\chi}$$

$$= f(\chi)^{-\frac{d}{2}}\,\hat{k}^d\,\hat{V}_{\hat{k}}(\hat{\phi})$$

$$\partial_t \bar{V} = \left(k^{-d_f} f\right)^{\frac{d\eta}{2(d-\eta+2n)}} \left\{ \frac{1}{\alpha+d_f}\left(\alpha + \frac{\eta}{2}\chi\,\partial_\chi \ln f\right)\partial_{\hat{t}}\hat{V}(\hat{\phi}) + \left(\frac{\eta}{2}\chi\,\partial_\chi \ln f \; - d_f\right)\left(\frac{d\eta}{2(d-\eta+2n)}\hat{V} - \frac{\eta}{2(\alpha+d_f)}\hat{\phi}\hat{V}'\right)\right\}$$

If $\eta\chi\,\partial_\chi \ln f$ varies with χ then

$$\partial_t \bar{V} = 0 \quad \Longrightarrow \quad \partial_{\hat{t}}\hat{V} = 0$$

and

$$\frac{d\eta}{2(d-\eta+2n)}\hat{V} - \frac{\eta}{2(\alpha+d_f)}\hat{\phi}\hat{V}' = 0 \qquad \Longrightarrow \qquad \hat{V} \propto \hat{\phi}^{\frac{2d}{d+2n}}$$

Two notions of fixed point

$$V(\varphi,\chi) = k^{d-\frac{d}{2}d_f}\,\bar V_k(\bar\varphi,\bar\chi)\,, \qquad \varphi = k^{\eta/2}\bar\varphi\,, \qquad \chi = k^{\eta/2}\bar\chi$$

$$= f(\chi)^{-\frac{d}{2}}\,\hat k^{d}\,\hat V_{\hat k}(\hat\phi)$$

$$\partial_t \bar V = \left(k^{-d_f} f\right)^{\frac{d\eta}{2(d-\eta+2n)}} \left\{ \frac{1}{\alpha+d_f}\left(\alpha + \frac{\eta}{2}\chi\,\partial_\chi \ln f\right)\partial_{\hat t}\hat V(\hat\phi) + \left(\frac{\eta}{2}\chi\,\partial_\chi \ln f \;-\; d_f\right)\left(\frac{d\eta}{2(d-\eta+2n)}\hat V - \frac{\eta}{2(\alpha+d_f)}\hat\phi\hat V'\right)\right\}$$

If $\eta\chi\,\partial_\chi \ln f$ varies with χ then $\partial_t\bar V \neq 0 \quad (!)$

$\partial_t \bar V = 0 \quad \Longrightarrow \quad \partial_{\hat t}\hat V = 0$ and

k-fixed points exist if:

$\hat{\text{k}}$-fixed points coincide if:

$$\frac{d\eta}{2(d-\eta+2n)}\hat V - \frac{\eta}{2(\alpha+d_f)}\hat\phi\hat V' = 0 \quad \Longrightarrow \quad \hat V \propto \hat\phi^{\frac{2d}{d+2n}}$$

- $\eta = 0$
- $f(\chi) \propto \chi^\gamma$

- $d_f = 0$
- $d_f = \gamma\,\frac{\eta}{2}$

Background independence is incompatible with k-fixed points!

$$f(\phi) = k_0^{d_f}\, F\left(\frac{\phi}{k_0^{\eta/2}}\right)$$

$$\Longrightarrow$$

$$\bar{f}(\bar{\chi}) = F\left(\bar{\chi}\,\mathrm{e}^{\frac{\eta}{2}(t-t_0)}\right)\mathrm{e}^{d_f(t_0-t)}$$

k-fixed points exist if:

- $\eta = 0$
- $f(\chi) \propto \chi^{\gamma}$

$\hat{k}$-fixed points coincide if:

- $d_f = 0$
- $d_f = \gamma\,\frac{\eta}{2}$

In conformally truncated gravity...

- Background independent notion of scale $\hat{k}$ exists
- which is also independent of the form of $f(\phi)$
- Generally k-fixed points are **incompatible** with background independence (i.e. the msWI)

...exceptions depending on form (running) of $f(\chi)$

$\hat{k}$-fixed points exist independently of all this...

....at a simpler deeper level