Radiative effects in decay of metastable vacua: a Green's function approach

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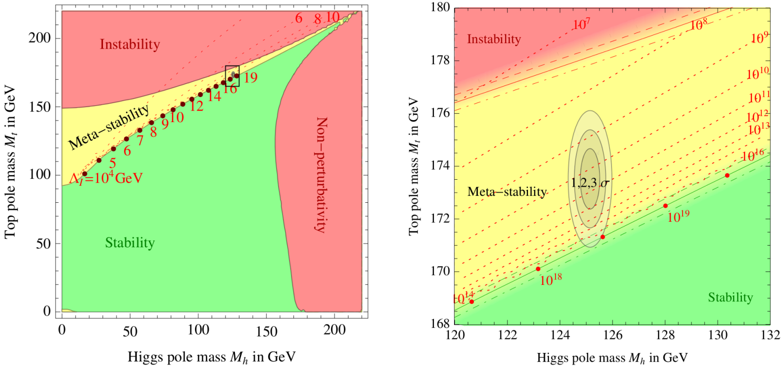
Outline

- Introduction and motivation
- The **Coleman bounce** and the semi-classical **tunnelling rate**
- **Green's function** approach:
  - **thin-** and **planar-wall** approximations
  - **heat kernel** method
- Example:
  - one-loop correction, the **functional determinant**
  - two-loop correction, the **quantum-corrected bounce**
- Concluding remarks

Introduction and motivation

The **perturbatively**-calculated SM effective potential develops an instability at a scale $\sim 10^{11}$ GeV, given a ~ 125 GeV Higgs boson.

[Cabibbo, Maiani, Parisi, Petronzio, NPB**158** (1979) 295; Sher, PR**179** (1989) 273 & PLB**317** (1993) 159 [hep-ph/9307342]; Isidori, Ridolfi, Strumia, NPB**609** (2001) 387 [hep-ph/0104016]; Elias-Moro, Espinosa, Giudice, Isodori, Riotto, Strumia, PLB**709** (2012) 222 [1112.3022]; Degrassi, Di Vita, Elias-Moro, Espinosa, Giudice, Isidori, Strumia, JHEP**1208** (2012) 098 [1205.6497]; Alekhin, Djouadi, Moch, PLB**716** (2012) 214 [1207.0980]]



[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia, JHEP**1312** (2013) 089 [1307.3536]]

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  - incorporation of the inhomogeneity of the solitonic background

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It is provident to consider methods of finding tunnelling rates that account for radiative corrections, particular if these are dominant.

Semi-classical tunnelling rate

Archetype: Euclidean ϕ^4 theory with tachyonic mass $\mu^2 > 0$

$$\mathcal{L} \;=\; \frac{1}{2!}\left(\partial_\mu\phi\right)^2 \;-\; \frac{1}{2!}\,\mu^2\phi^2 \;+\; \frac{1}{3!}\,g\phi^3 \;+\; \frac{1}{4!}\,\lambda\varphi^4 \;+\; U_0$$

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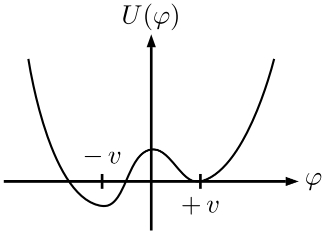
Non-degenerate minima:

$$\varphi \;=\; v_\pm \;\approx\; \pm\, v \;-\; \frac{3g}{2\lambda}\,, \qquad v^2 \;=\; \frac{6\mu^2}{\lambda}$$



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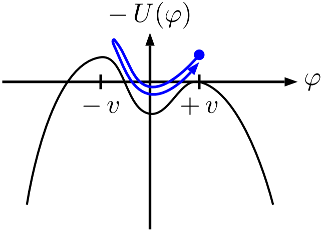
There exists a solution — the **Coleman bounce** — satisfying

[Coleman, PRD**15** (1977) 2929; Callan, Coleman, PRD**16** (1977) 1762; Coleman Subnucl. Ser. **15** (1979) 805; Affleck, De Luccia, PRD**20** (1979) 3168; Coleman, De Luccia, PRD**21** (1980) 3305; Konoplich, Theor. Math. Phys. **73** (1987) 1286]

$$\varphi\big|_{x_4\,\to\,\pm\,\infty} \;=\; +\,v\;, \qquad \dot{\varphi}\big|_{x_4\,=\,0} \;=\; 0\;, \qquad \varphi\big|_{|\mathbf{x}|\,\to\,\infty} \;=\; +\,v\;.$$



Semi-classical tunnelling rate

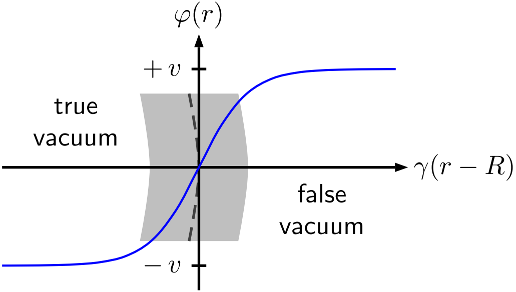
In hyperspherical coordinates, the boundary conditions are

$$\varphi\big|_{r\to\infty} = +v\,, \qquad \mathrm{d}\varphi/\mathrm{d}r\big|_{r=0} = 0\,,$$

with the bounce corresponding to the **kink**

[Dashen, Hasslacher & Neveu, PRD 10 (1974) 4114; *ibid.* 4130; *ibid.* 4138]

$$\varphi(r) = v\tanh\gamma(r-R)\,, \qquad \gamma = \mu/\sqrt{2}\,.$$



The bounce looks like a **bubble** of radius $R = 12\lambda/g/v$.

Semi-classical tunnelling rate

The tunnelling rate Γ is calculated from the path integral

$$Z[0] \;=\; \int [\mathrm{d}\phi] e^{-S[\phi]/\hbar}\,, \qquad \Gamma/V \;=\; 2|\mathrm{Im}\, Z[0]|/V/T$$

Semi-classical tunnelling rate

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Expanding around the kink, the spectrum of the operator

$$G^{-1}(\varphi) \;\equiv\; \frac{\delta^2 S[\phi]}{\delta\phi^2}\bigg|_{\phi=\varphi} \;=\; -\,\Delta^{(4)} \;+\; U''(\varphi)$$

contains four zero eigenvalues (from translational invariance) and one negative eigenvalue (from dilatation invariance).

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Thus, writing $B \equiv S[\varphi]$,

$$Z[0] \;=\; -\,\frac{i}{2}\, e^{-B/\hbar} \left| \frac{\lambda_0 \det^{(5)}\, G^{-1}(\varphi)}{(VT)^2 \big(\frac{B}{2\pi\hbar}\big)^4 (4\gamma^2)^5 \det^{(5)}\, G^{-1}(v)} \right|^{-1/2}.$$

Quantum-corrected bounce

Functionally differentiating the effective action, the **quantum-corrected bounce** $\varphi^{(1)}(x)$ must satisfy the e.o.m.

$$-\partial^2\varphi^{(1)}(x) + U'(\varphi^{(1)};x) + \Pi(\varphi;x)\varphi(x) \;=\; 0\;,$$

which includes the **tadpole correction**

$$\Pi(\varphi;x) \;=\; \frac{\lambda}{2}\,G(\varphi;x,x)\;.$$

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The **tunnelling rate** per unit volume, at order $\hbar$, is

$$\mathit{\Gamma}/V \ = \ \left(\frac{B}{2\pi}\right)^2 (2\gamma)^5 |\lambda_0|^{-\frac{1}{2}} \exp\!\left[-\frac{1}{\hbar}\left(B + \hbar B^{(1)} + \hbar B^{(2)}\right)\right],$$

where, writing $\delta\varphi(x) \equiv \varphi^{(1)}(x) - \varphi(x)$,

$$B^{(1)} \ = \ \frac{1}{2}\,\mathrm{tr}^{(5)}\Big(\ln G^{-1}(\varphi) \ - \ \ln G^{-1}(v)\Big) \ ,$$

$$B^{(2)} \ = \ -\frac{3}{2}\int \mathrm{d}^4x \ \varphi(x)\Pi(\varphi;x)\delta\varphi(x) \ .$$

Approximations

1. **Thin-wall** approximation $\mu R \gg 1$: drop the damping term

$$\frac{1}{\mu r}\frac{\mathrm{d}}{\mathrm{d}r} \sim \frac{1}{\mu R}\frac{\mathrm{d}}{\mathrm{d}r} .$$

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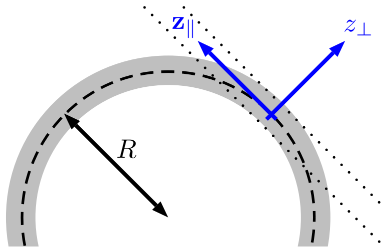
2. **Planar-wall** approximation: sum over discrete angular momenta replaced by an integral over linear momenta, i.e.

$$\frac{j(j+2)\hbar}{\mu^2 R^2} \longrightarrow \frac{k^2}{\mu^2}\,.$$



Green's function

Green's function: $u^{(\prime)} \equiv \varphi(r^{(\prime)})/v$ and $m \equiv \left(1 + k^2/4/\gamma^2\right)^{1/2}$

$$
\begin{aligned}
G(u, u', m) \;=\; & \frac{1}{2\gamma m}\Bigg[\vartheta(u-u')\left(\frac{1-u}{1+u}\right)^{\frac{m}{2}}\left(\frac{1+u'}{1-u'}\right)^{\frac{m}{2}} \\
& \times\left(1 - 3\,\frac{(1-u)(1+m+u)}{(1+m)(2+m)}\right) \\
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$$

The renormalization is performed using the effective potential of a homogeneous false-vacuum configuration.

Renormalized tadpole correction:

$$
\Pi^R(u) \;=\; \frac{3\lambda\gamma^2}{16\pi^2}\left[6 + (1-u^2)\left(5 - \frac{\pi}{\sqrt{3}}\,u^2\right)\right]
$$

Heat-kernel method

[see e.g. Vassilevich, Phys. Rept. **388** (2003) 279 [hep-th/0306138], cf. Baacke, PRD**49** (1994); PRD**78** (2008); Baacke, Daiber, PRD**51** (1995) & using the Gel'fand-Yaglom thoerem JMP**1** 1960) 48, see Baacke, Kiselev PRD**48** (1993); Dunne, Kirsten, JPA**39** (2006) 11915 [hep-th/0607066]; Dunne, JPA**41** (2008) 304006 [0711.1178]]

Functional determinant over the positive-definite modes:

$$\mathrm{tr}^{(5)} \ln G^{-1}(\varphi; x) \;=\; -\int \mathrm{d}^4x \int_0^\infty \frac{\mathrm{d}\tau}{\tau}\, K(\varphi; x, x|\tau)\;.$$

The **heat kernel** is the solution to the **heat-flow equation**

$$\partial_\tau K(\varphi; x, x'|\tau) \;=\; G^{-1}(\varphi; x) K(\varphi; x, x'|\tau)\;,$$

with $K(\varphi; x, x'|0) = \delta^{(4)}(x - x')$.

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But it's **Laplace transform**

$$\mathcal{K}(\varphi; x, x'|s) \;=\; \int_0^\infty \mathrm{d}\tau\; e^{s\tau}\, K(\varphi; x, x'|\tau)$$

is just the **Green's function** with $k^2 \to k^2 + s$.

Example

To enhance the radiative effects, while remaining in a perturbative regime, we consider an $\boldsymbol{N}$**-field model**

$$\mathcal{L} \supset \sum_{i=1}^{N} \left[\frac{1}{2!} \left(\partial_\mu \chi_i \right)^2 + \frac{1}{2!} m_\chi^2 \chi_i^2 + \frac{\lambda}{4} \phi^2 \chi_i^2 \right].$$

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For $m_\chi^2 \gg \gamma^2$, the χ **renormalized tadpole correction** is

$$\Sigma^R(u) = \frac{\lambda \gamma^2}{8\pi^2} \frac{\gamma^2}{m_\chi^2} \left[72 + (1 - u^2)(40 - 3u^2)\right] .$$

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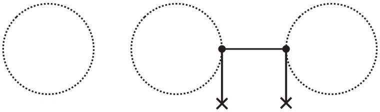
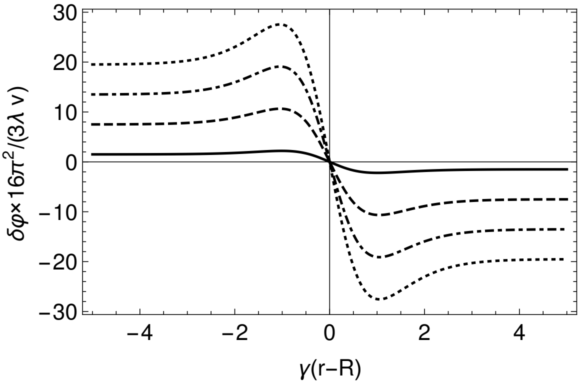
Dominant $\hbar$ corrections:



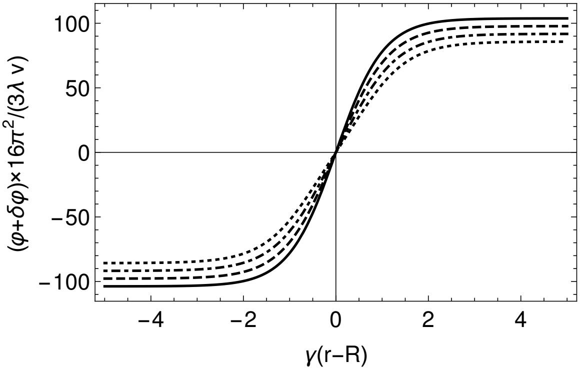
Quantum-corrected bounce

$$\delta\varphi(u) \;=\; -\,\frac{v}{\gamma}\int_{-1}^{1}\mathrm{d}u'\,\frac{u'\,G(u,u',m)\big|_{k=0}}{1-u'^2}\,\Big(\Pi^R(u') + N\Sigma^R(u')\Big)$$



$N\gamma^2/m_\chi^2$: 0 (solid), 0.5 (dashed), 1 (dash-dotted) and 1.5 (dotted)

Quantum-corrected bounce



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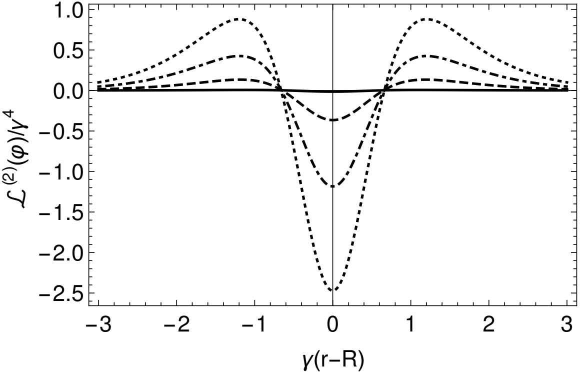
Two-loop correction to the bounce action $B^{(2)}$



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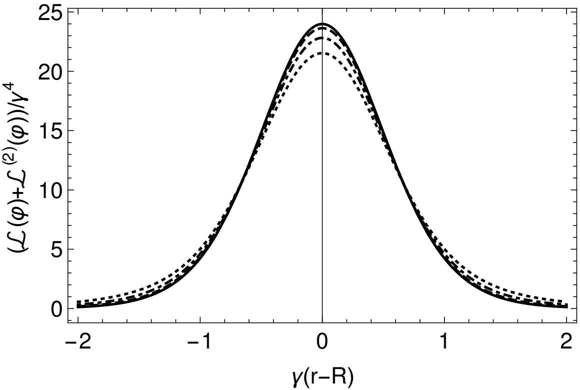

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Overall increase in the tunnelling action $\Rightarrow$ overall decrease in the tunnelling rate (opposite to impact of the functional determinant).

Concluding remarks

- Green's function approach to the calculation of radiative corrections to tunnelling rates from false vacua, including the
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  - functional determinant and
  - corrections to the bounce itself.
- In the scalar toy model considered, this calculation could be performed analytically, by employing the
  - thin-wall approximation and
  - planar-wall approximation.
- The method is well-suited to numerical analysis and may be of **particular use for radiatively-generated minima**, e.g. in the
  - instability of the SM,
  - massless Coleman-Weinberg model of SSB or

    [Coleman & Weinberg, PRD 7 (1973) 1888]
  - symmetry restoration at finite temperature.

    [Kirzhnits, Linde, PLB**42** (1972) 471; Dolan, Jackiw, PRD**9** (1974) 3320; Weinberg, PRD**9** (1974) 3357]

Back-up slides

Tunnelling rate

$$B \;=\; \frac{8\pi^2 R^3 \gamma^3}{\lambda}$$

$$B^{(1)} \;=\; -B\left(\frac{3\lambda}{16\pi^2}\right)\left[\frac{\pi}{3\sqrt{3}} + 21 + \frac{2542}{15}\frac{\gamma^2}{m_\chi^2}N\right]$$

$$\begin{aligned} B^{(2)} \;&=\; -\frac{3}{2}\int \mathrm{d}^4x\, \varphi(u)\left(\Pi^R(u) + N\Sigma^R(u)\right)\delta\varphi(u) \\ &=\; B\left(\frac{3\lambda}{16\pi^2}\right)^2\left[\frac{291}{8} - \frac{37}{4}\frac{\pi}{\sqrt{3}} + \frac{5}{56}\frac{\pi^2}{3}\right. \\ &\qquad\left. + \left(\frac{667}{2} - \frac{2897}{42}\frac{\pi}{\sqrt{3}}\right)\frac{\gamma^2}{m_\chi^2}N + \frac{5829}{14}\frac{\gamma^4}{m_\chi^4}N^2\right] \end{aligned}$$