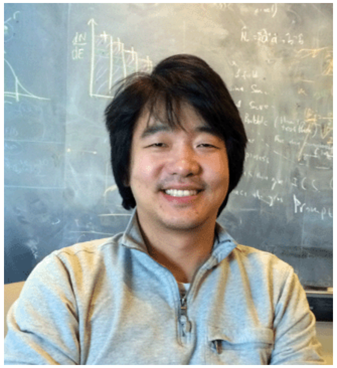
Blue Isocurvature Perturbations

Daniel J. Chung

5/22/2015



1501.05618 [w/ Hojin Yoo]
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Non-curvaton Isocurvature Sample

Multi-field inflation: Axenides, Brandenberger, Turner 83; Linde 85; Starobinsky 85; Silk, Turner 87; Polarski, Starobinsky 94; **Linde, Mukhanov 97**; Langlois 99

Axions: Turner, Wilczek, Zee 83; Steinhardt, Turner 83; Axenides, Brandenberger, Turner 83; Linde 84, 85; Seckel, Turner 85; Efstathiou, Bond 86; Hogan, Rees 88; Lyth 90; Linde, Lyth 90; Turner, Wilczek 91; Linde 91; Lyth 92; Kolb, Tkachev 94; Fox, Pierce, Thomas 04; Beltran, Garcia-Bellido, Lesgourgues 06; Hertzberg, Tegmark, Wilczek 08; **Kasuya, Kawasaki 09**; Marsh, Grin, Hlozek, Ferreira 13; Choi, Jeong, Seo 14

B-genesis/Affleck-Dine: Bond, Kolb, Silk 82; Enqvist, McDonald 99, 00; Kawasaki, Takahashi 01; Kasuya, Kawasaki, Takahashi 08; Harigaya, Kamada, Kawasaki, Mukaida, Yamada 14; Hertzberg, Karouby 14

SUSY Moduli: Yamaguchi 01; Moroi and Takahashi 01; Lemoine, Martin, Yokoyama 09; Iliesiu, Marsh, Moodley, Watson 13

WIMPZILLAs: Chung, Kolb, Riotto, Senatore 04; Chung, Yoo 11; Chung, Yoo, Zhou 13

fermionic isocurvature perturbations: Chung, Yoo, Zhou 13

dark energy isocurvature: Malquarti, Liddle 02; Gordon, Hu 04 a small sample of observational focus:

Hu, Grin 15; Gordon, Pritchard 09; Kawasaki, Sekiguchi, Takahashi 11; **Takeuchi, Chongchitnan 13**; Sekiguchi, Tashiro, Silk, Sugiyama 13; Holder, Nollett, Engelen 09; Grin, Dore, Kamionkowski 11; Hikage, Kawasaki, Sekiguchi, Takahashi 12; Planck 2013 results XXII; Chlub and Grin 13; Grin, Hanson, Holder, Dore, Kamionkowski 13; Kawasaki and Yokoyama 14; **Valviita, Muhonen 03; Ferrer, Rasanen, Valviita 04; Kurki-Suonio, Muhonen, Valviita 05; Keskitalo, Kurki-Suonio, Muhonen, Valviita 07; Beltran, Garcia-Belliodo, Lesgourgues, Riazuelo 04; Beltran, Garcia-Bellido, Lesgourgues, Viel 05; Sollom, Challinor, Hobson 09**

Fluid dynamics and general field theory parameterizations:

Bardeen 1980; review of Kodama, Sasaki 84; Gotdon, Wands, Basset, Maartens, 00; review of Malik, Wands 09; Kolb and Turner, EARLY UNIVERSE

+ Many more papers on curvaton and non-Gaussianities; apologies for omissions

What this talk is about.

Question: What do we learn about high energy theory if we measure isocurvature perturbations with a large blue spectral index ($n > 2.4$)?

relatively "novel"

answer:

1) A field with a time-dependent mass existing during inflation
2) Most likely a pseudo-Nambu-Goldstone boson dark matter

A brief review of isocurvature perturbations

classical Einstein gravity coupled to fluids

need initial conditions for PDE

At the **linear perturbation** level:

superposition of adiabatic & isocurvature initial conditions

CDM-photon isocurvature

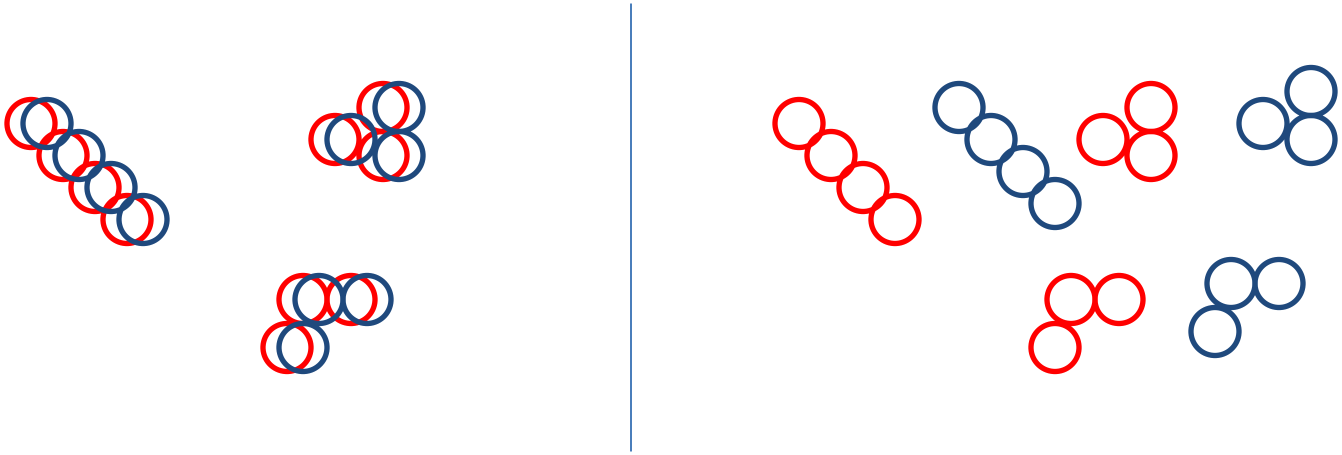
$$S_{DM,\gamma} = \frac{\delta n_{DM}}{n_{DM}} - \frac{\delta n_\gamma}{n_\gamma}$$

adiabatic density inhomogeneity

$$S_{DM,\gamma} = 0$$

isocurvature density inhomogeneity

$$S_{DM,\gamma} \neq 0$$



Special in cosmology: these initial conditions are **time evolution approximate solutions** at early times of relevant Fourier modes of linearized fluid equations, i.e. $S_{DM,\gamma} = \text{constant}$

Single field ϕ inflation **cannot** generate isocurvature perturbations.

intuition:

all inhomogeneities are tied to one field

$$\delta\phi(t,\vec{x}) \to \delta n_X(t,\vec{x})$$

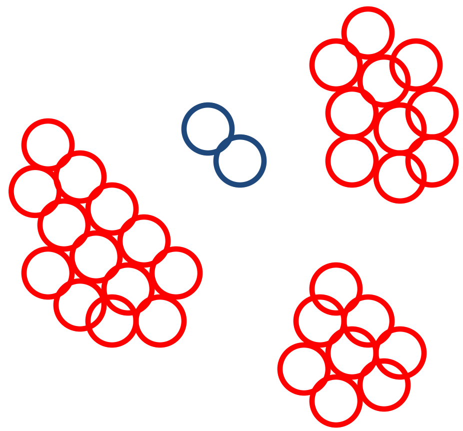
$$\delta\phi(t,\vec{x}) \to \delta n_\gamma(t,\vec{x})$$

N. B. The physical solution is continuously connected by $k \to 0$ to a pure diffeomorphism gauge mode.

[clearest reference is Weinberg 04]

Multiple dynamical degrees of freedom naturally lead to isocurvature initial conditions in fields which the inflaton field and its decay products are **very weakly** coupled (e.g. **non-thermal DM candidates**)

$$\delta\phi(t,\vec{x}) \to \delta n_\gamma(t,\vec{x})$$



$$\delta X(t,\vec{x}) \to \delta n_{DM}(t,\vec{x})$$

Multiple dynamical degrees of freedom naturally lead to isocurvature initial conditions in fields which the inflaton field and its decay products **are very weakly** coupled to (e.g. **non-thermal DM candidates**)

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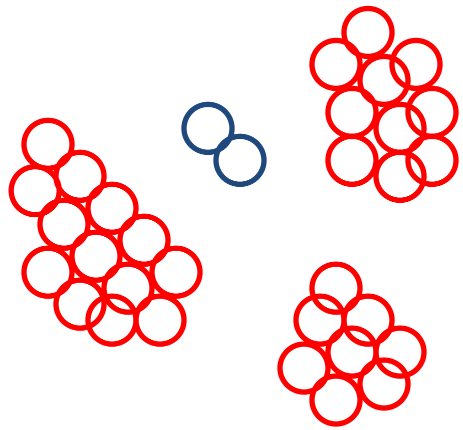
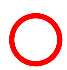
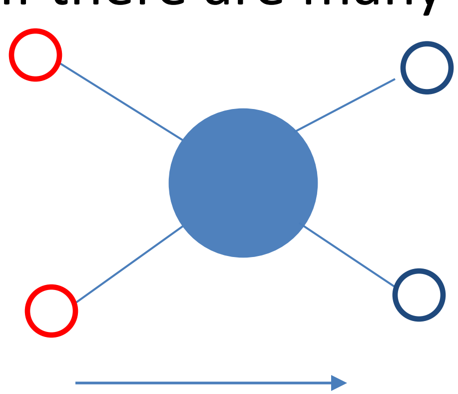


$$\delta X(t,\vec{x}) \rightarrow \delta n_{DM}(t,\vec{x})$$

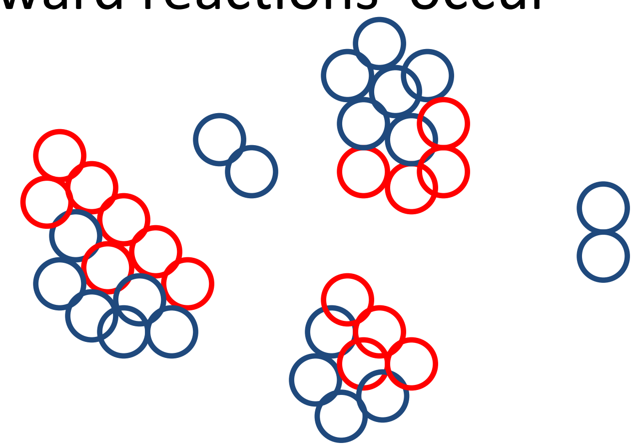
Importance of **hidden** character

If there are many more  than , and forward reactions occur

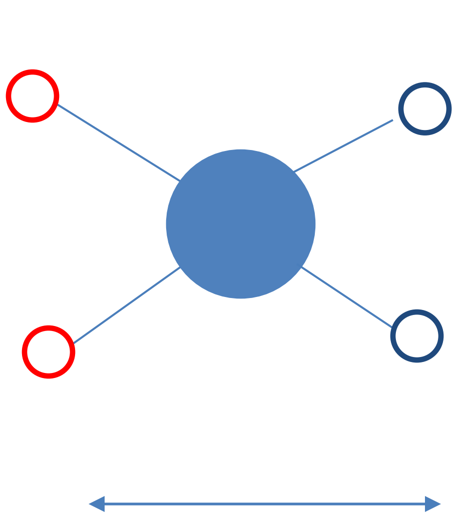


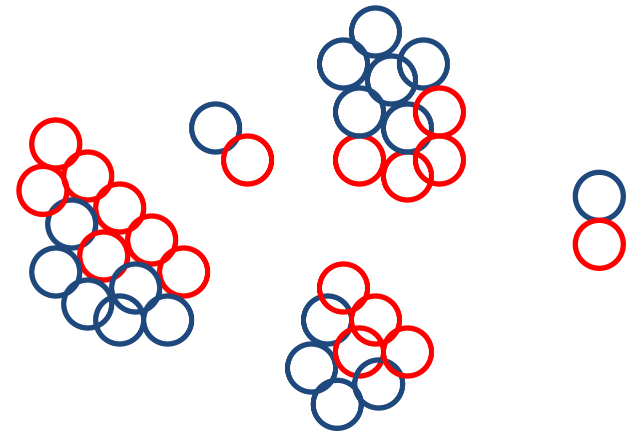
makes enough blue s.t.

adiabatic again



If thermal equilibrium, then even more accurately adiabatic





If $\langle \sigma v \rangle \lesssim \frac{1}{\Lambda^2}$, then $\left(\frac{200\ \mathrm{TeV}}{\Lambda}\right)^2 \left(\frac{T_{RH}}{m_\chi}\right) < 1$

example dark matter types:

- hidden sector fields
- also easily satisfied by QCD axions

→

observables are
$\langle SS \rangle, \langle S\zeta \rangle, \ldots$
observed through SM probes w/ gravitational coupling

Define spectral index: n

$$\Delta^2_{s_\chi}(k) = \frac{k^3}{2\pi^2}\int \frac{d^3k'}{(2\pi)^3}\langle S_\chi(t,\vec{k})S_\chi(t,\vec{k}')\rangle \quad \propto k^{n-1}$$

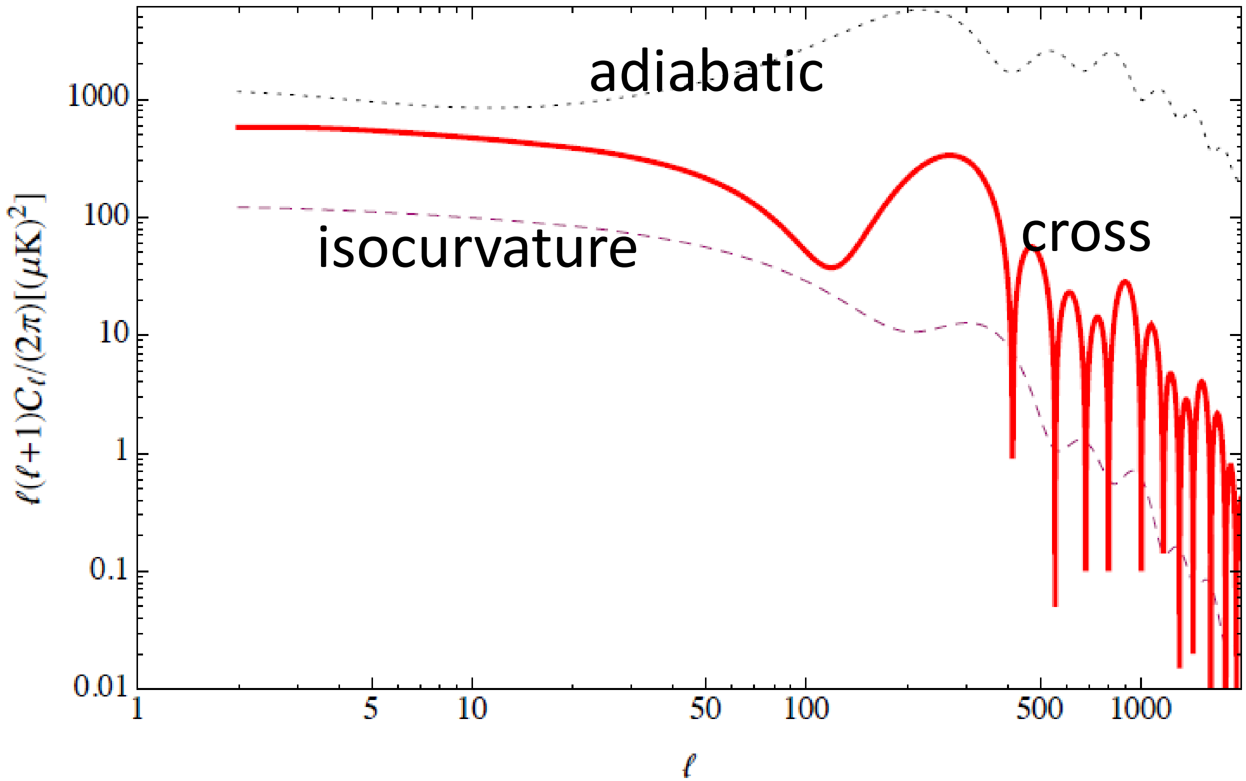
Why **blue** $n-1 \sim O(1)$ instead of scale invariant $n-1 \ll 1$?

CMB

Scale invariant isocurvature

$$\frac{k^3}{2\pi^2}\int\frac{d^3p}{(2\pi)^3}\langle\frac{\Delta T(\vec{p})}{T}\frac{\Delta T^*(\vec{k})}{T}\rangle = \Delta_\zeta^2(k)\left[|c_1|^2+|c_2|^2\frac{\alpha}{1-\alpha}-2\Re\left(c_1^*c_2\beta\sqrt{\frac{\alpha}{1-\alpha}}\right)\right]$$

$\beta = -1$



$\alpha|_{\beta=0} < 0.016$ (95% CL) and $\alpha|_{\beta=-1} < 0.0011$ (95% CL)

Planck+WP 2013

A case for blue spectra:

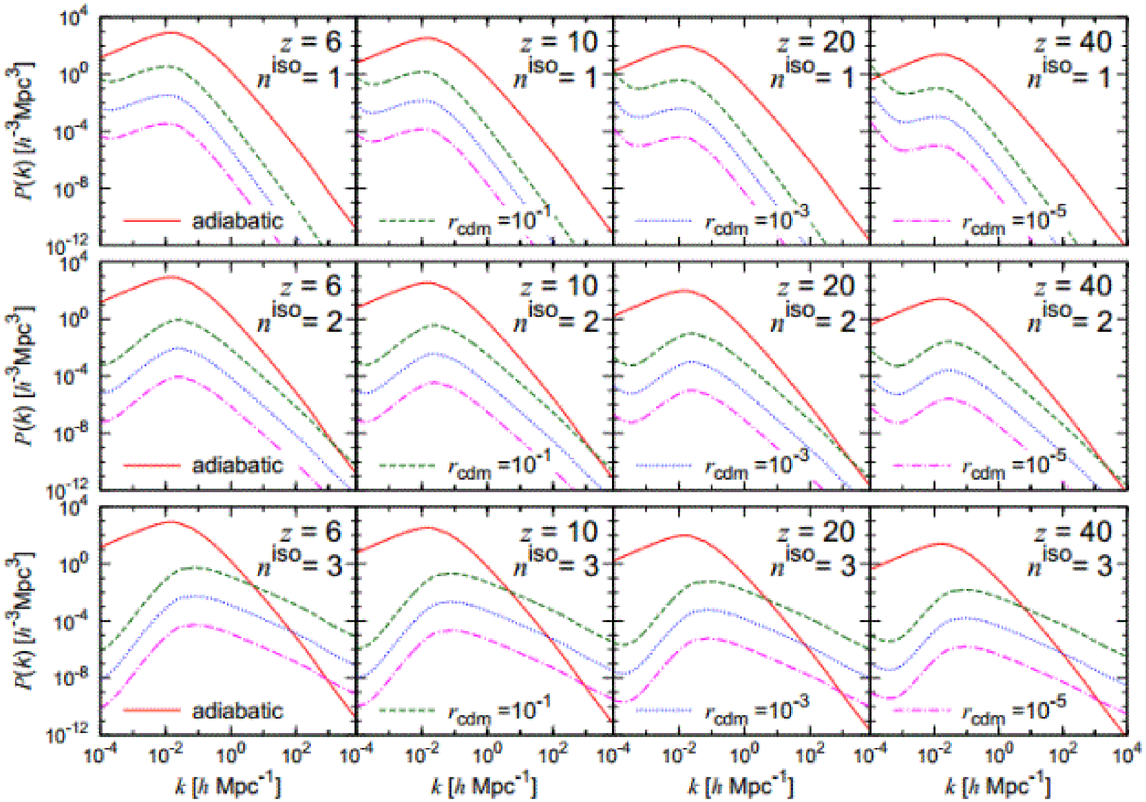


Figure 1. The matter power spectra generated by adiabatic or pure CDM isocurvature fluctuations, at redshifts $z = 6$, 10, 20 and 40 (from left column to right). The spectral indices of the isocurvature mode are as $n_s^{\rm iso} = 1$, 2 and 3 (from top row to bottom). In each panel, the different curves represent the matter power spectrum of the adiabatic fluctuations (solid/red) and the CDM isocurvature fluctuations with $r_{\rm cdm} = 10^{-1}$ (dashed/green), 10^{-3} (dotted/blue) and 10^{-5} (dot-dashed/magenta). The isocurvature spectra shown have no contribution from adiabatic fluctuations.

[Takeuchi, Chongchitnan 13]

minihalos → 21 cm experiments such as SKA

n=3 can be measured

summary of intro:

What are DM-photon isocurvature perturbations “S”?

DM inhomogeneities different from photons.

Scale invariant ($k^3 \langle SS(k) \rangle \sim k^0$) isocurvature is well constrained by large scale (CMB scale) data.

Future data will most strongly improve short scale info. Blue ($k^3 \langle SS(k) \rangle \sim k^{(n-1)>0}$) is easy to hide on CMB scales and may be seen by future data.

next:

Question: What will we learn about high energy theory if we observe blue isocurvature perturbations?

Step 1: establish quantization consistently solving gravitational consraints in the **blue** limit.

High energy theory dependence:

(correlation functions **during inflation**) X (transfer function)

classical physics (transfer function)

quantum: correlators in the presence of **gravitational constraints**. (correlation functions during inflation)

Most of isocurvature literature is regarding a scale invariant spectrum.

Hence, we have formulated some useful theorems to deal with **blue** isocurvature perturbations. $k^3\langle SS(k)\rangle \sim k^{(n-1)>0}$

[DC, H. Yoo 1501.05618]

Definition of linear spectator isocurvature field:

$$ds^2 = (1+2\Psi^{(N)})dt^2 - a^2(t)(1+2\Phi^{(N)})|d\vec{x}|^2$$

$$\chi = \boxed{\chi_0(t)} + \delta\chi^{(N)}(t,\vec{x})$$ linear

$$\chi_0^{(N)}(t_{\text{during inflation}}) \gg \frac{H}{2\pi}\Big|_{\text{during inflation}}$$

$$\frac{\delta\rho_\chi^{(N)}}{\delta\rho_{\text{dominant}}^{(N)}} = \frac{\delta T_{\chi\ 0}^{(N)0}}{\delta T_{\text{dominant}\ 0}^{(N)0}} \ll 1$$ spectator

Assume that χ_0 coherent oscillations form CDM

We formulate some useful theorems regarding this

• Theorem 1: A classically conserved quantity for spectator isocurvature with the dominant interactions given by gravity and $V_\chi = m^2\chi^2/2$ is

$$S = 2\frac{\delta\chi^{(G)}(t,\vec{k}) - \delta\chi_{ad}^{(G)}(t,\vec{k})}{\chi_0(t)}$$

$$\delta\chi_{ad}^{(G)}(t,\vec{k}) \equiv -\zeta_{\vec{k}}\frac{\dot{\chi}_0(t)}{a(t)}\int dt\, a(t) + \xi^0\partial_0\chi_0(t)$$

measures the deviation from the Newtonian gauge

Reason: an approximate symmetry in the limit that mass term dominates.

$$V_\chi'(\delta\chi) \approx V_\chi''(\delta\chi)\delta\chi$$

Conservation is valid through end of inflation and through reheating.

- Theorem 3: Quantum correlator. $\Delta^2_{s_\chi}(k) = \frac{k^3}{2\pi^2}\int \frac{d^3k'}{(2\pi)^3}\langle S_\chi(t,\vec{k})S_\chi(t,\vec{k}')\rangle \quad \propto k^{n-1}$

Bunch-Davies + $m^2 < 9H^2/4$ + $\nu\tilde{N}_k \gg |\ln f|$

$$\tilde{N}_k \in \{f/\varepsilon_k, H(t_e - t_k)\}$$

$$\Delta^2_{s_\chi}(k) \approx 4\left(\frac{2^{2\nu-1}|\Gamma(\nu)|^2}{\pi}\right)\left(\frac{H(t_{k_0})}{2\pi\chi_0(t_{k_0})}\right)^2\left(\frac{k}{k_0}\right)^{3-2\nu-2\varepsilon_{k_0}+O(\varepsilon_{k_0})g_0(m/H)}$$

$$\mathscr{E} = \exp\left[-2\nu(m_1)\tilde{N}_k\right] + \frac{\delta\rho_\chi^{(N)}}{\delta\rho^{(N)}_{\text{dominant}}} + \frac{m^2}{3\sqrt{2\varepsilon}H^2}\frac{|\chi_0(t_k)|}{M_p} + \frac{|\chi_0(t_k)|}{M_p} + \varepsilon_{k_0} + f$$

Technical challenge of the theorem proof: quantization in the presence of gravitational constraint.

A corollary is given in the paper for mass shift. $\tilde{N}(t_c,t_k) \in \{f/\varepsilon_k, (t_c - t_k)H\}$

$$n = 4 - 3\sqrt{1 - \frac{4}{9}\frac{m^2}{H^2}} - 2\varepsilon_{k_0}$$

Example application: A clarification/correction of [Kasuya, Kawasaki 0904.3800]

QCD axion sector hidden from inflation: $W = h(\Phi_+\Phi_- - F_a^2)\Phi_0$

Khaeler induced mass terms during inflation:

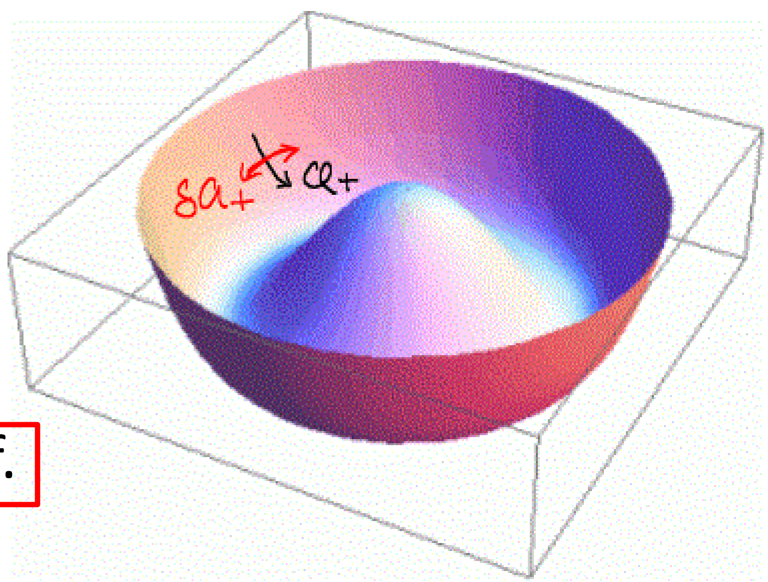
$$V_K = c_+H^2|\Phi_+|^2 + c_-H^2|\Phi_-|^2 + c_0H^2|\Phi_0|^2$$

e.g. eta problem is generic → good for blue spectrum

Large φ_+ limit:

$$\left(\Box - \frac{\Box\varphi_+}{\varphi_+}\right)a = 0$$

$$n \approx 4 - 3\sqrt{1 - \frac{4}{9}c_+} - 2\varepsilon_{k_0}$$

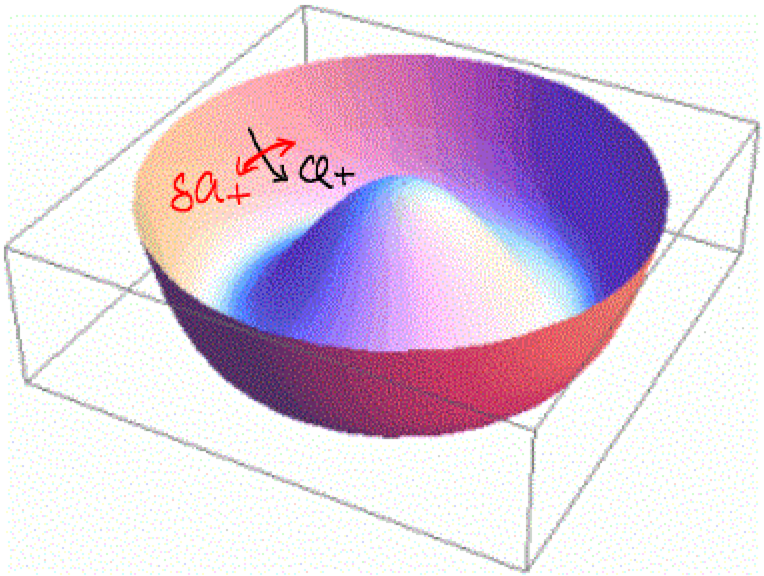


At the bottom, mass turns off.

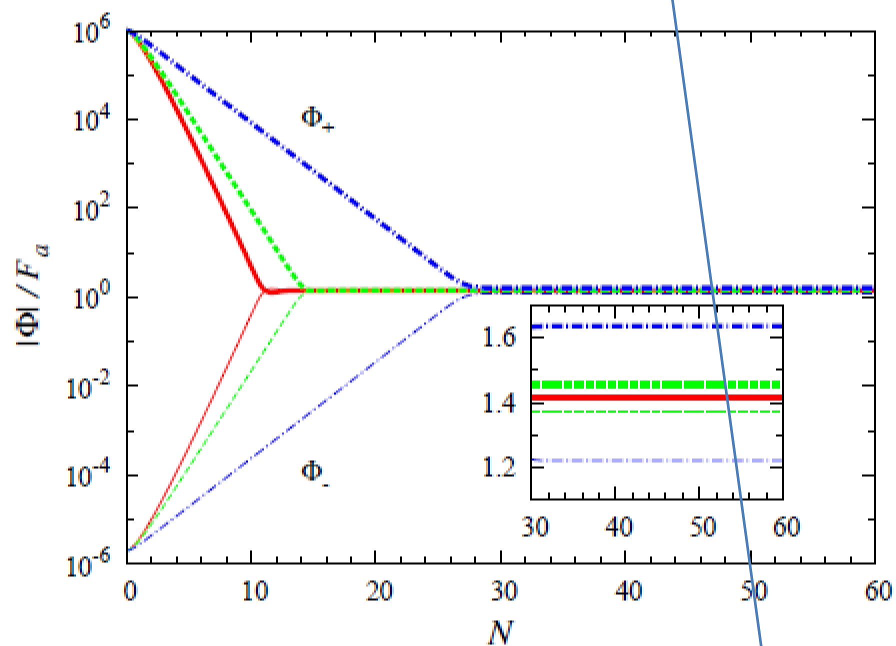
Nambu-Goldstone theorem.

Example application: A clarification/correction of [Kasuya, Kawasaki 0904.3800]

$$W = h(\Phi_+\Phi_- - F_a^2)\Phi_0$$



wrong



Why?

FIG. 1: Evolution of the fields Φ_+ (upper thick lines) and Φ_- (lower thin lines) for $c_- = 9/4$ and $c_+ = 9/4$ ($n_{\rm iso} = 4$, red solid), 2 ($n_{\rm iso} = 3$, green dashed), and 5/4 ($n_{\rm iso} = 2$, blue dotted-dashed). The inset shows the minima where the fields settle down.

Theorem 3:

$$\nu \tilde{N}_k \gg |\ln f| \longrightarrow \frac{4-n}{2} \gg \frac{|\ln 10^{-1}|}{5}$$

More detail on why n=4 is impossible

wrong

$$H_\nu^{(1)}(k/(aH))$$

$$H_\nu^{(1)}(x) = J_\nu^{(1)}(x) + iY_\nu^{(1)}(x)$$

$$Y_\nu^{(1)}(x \to 0) \to \infty$$

modes become imag

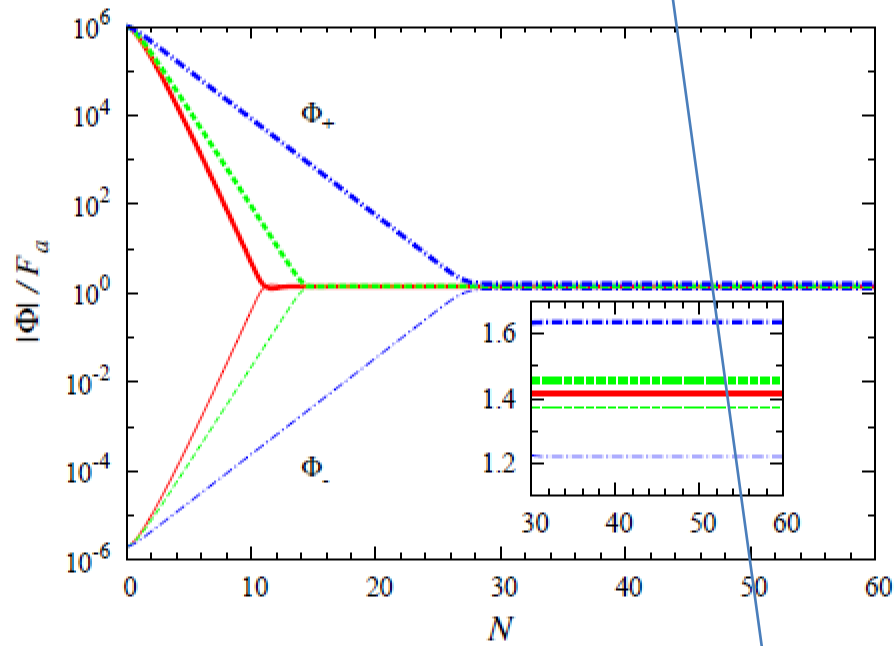
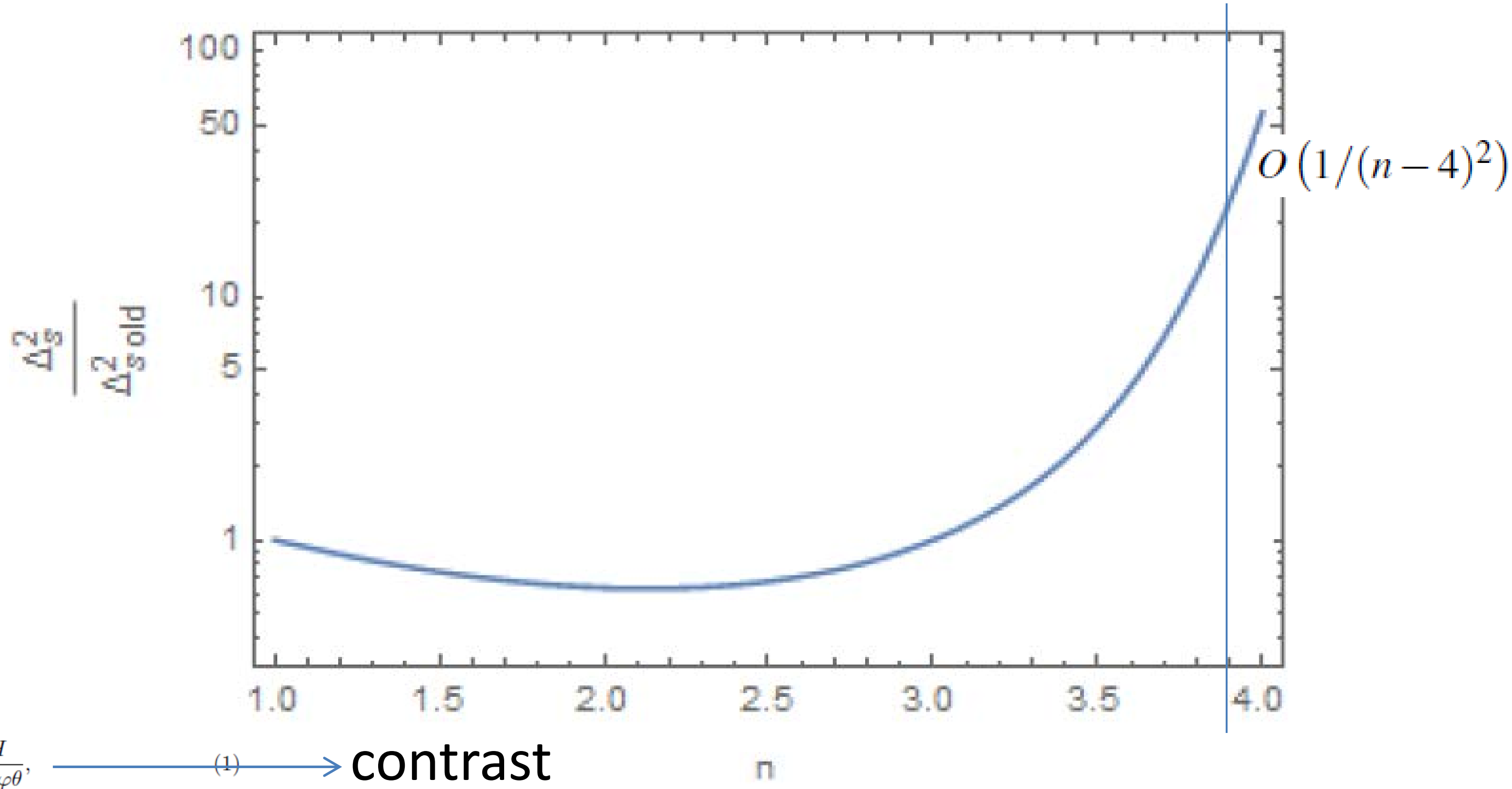


FIG. 1: Evolution of the fields Φ_+ (upper thick lines) and Φ_- (lower thin lines) for $c_- = 9/4$ and $c_+ = 9/4$ ($n_{\rm iso} = 4$, red solid), 2 ($n_{\rm iso} = 3$, green dashed), and 5/4 ($n_{\rm iso} = 2$, blue dotted-dashed). The inset shows the minima where the fields settle down.

→ classical stochastic amplitude that grows only if n<4

Comparison of our result with Kasuya, Kawasaki 0904.3800

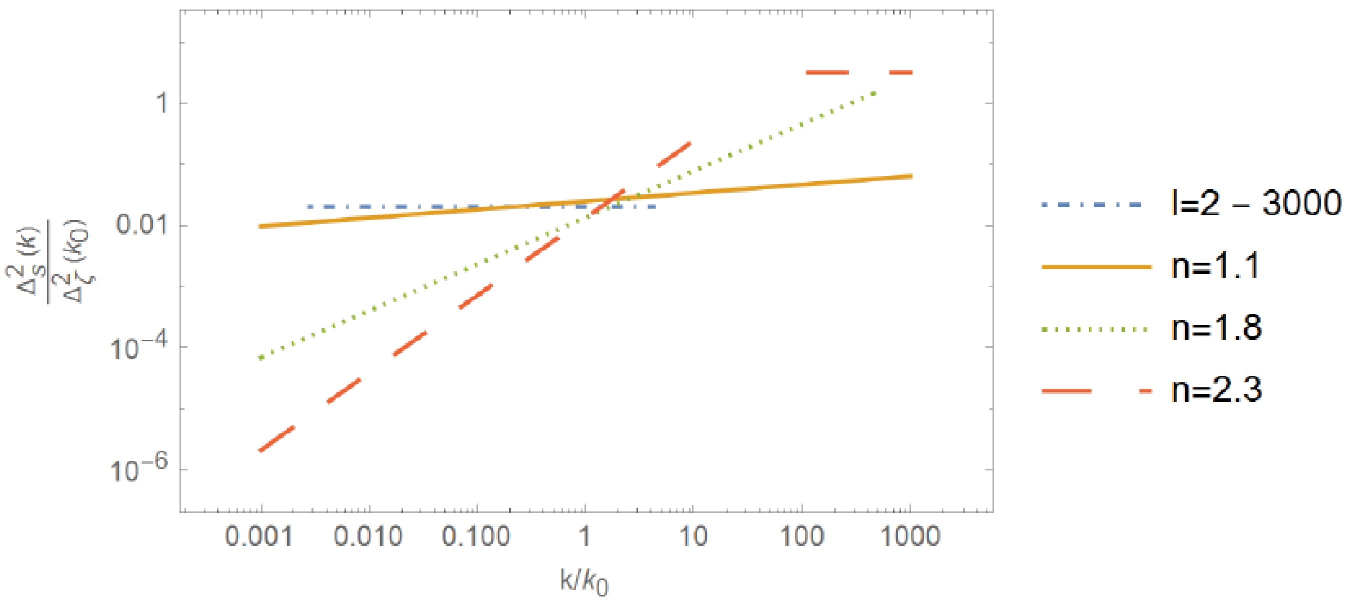


$\frac{\delta\theta}{\theta} \simeq \frac{H}{2\pi\varphi\theta}$, (1) → contrast

$$\Delta_s^2(k) = \omega_a^2 \left(\frac{2^{2\nu+} |\Gamma(\nu(\sqrt{c_+}H))|^2}{\pi} \right) \left(\frac{H(t_{k_0})}{2\pi a(t_{k_0})} \right)^2 \left(\frac{k}{k_0} \right)^{3-2\nu(\sqrt{c_+}H)-2\varepsilon_{k_0}+O(\varepsilon_{k_0})g_0(m/H)}$$

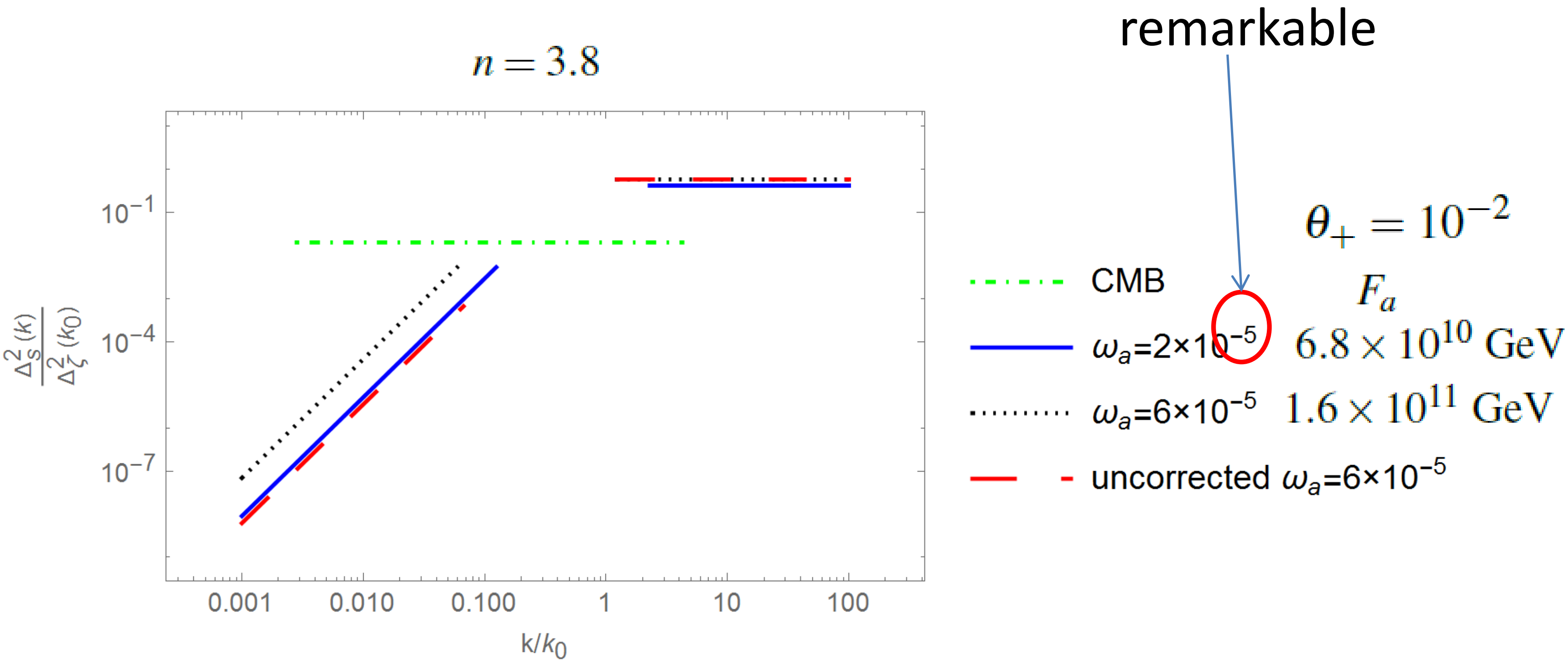
difference: quantization induced mode versus homogeneous solution

Example



1
0.01
10^{-4}
10^{-6}
$\frac{\Delta_s^2(k)}{\Delta_\zeta^2(k_0)}$
0.001
0.010
0.100
1
10
100
1000
k/k_0
l=2 – 3000
n=1.1
n=1.8
n=2.3

More Example



Measuring Large Spectral Index → Time dependent mass

Why can't we trivially generate a measurable large blue spectrum with a constant mass field?

$$n = 4 - 3\sqrt{1 - \frac{4}{9}\frac{m^2}{H^2}} - 2\varepsilon_{k_0}$$

$$\text{isocurvature observable} \propto \rho_\chi \frac{\delta\rho_\chi}{\rho_\chi} \propto \rho_\chi(t_0) S_\chi$$

$$\rho_\chi(t_0) \propto \frac{1}{2} m^2 (\chi_0)^2_{H=m}$$

$$(\chi_0^2)_{H=m} \propto \chi_0^2(t_e) \propto \boxed{e^{-(n_s-1)N_{min}}}$$

Blue spectral energy density dilutes away!

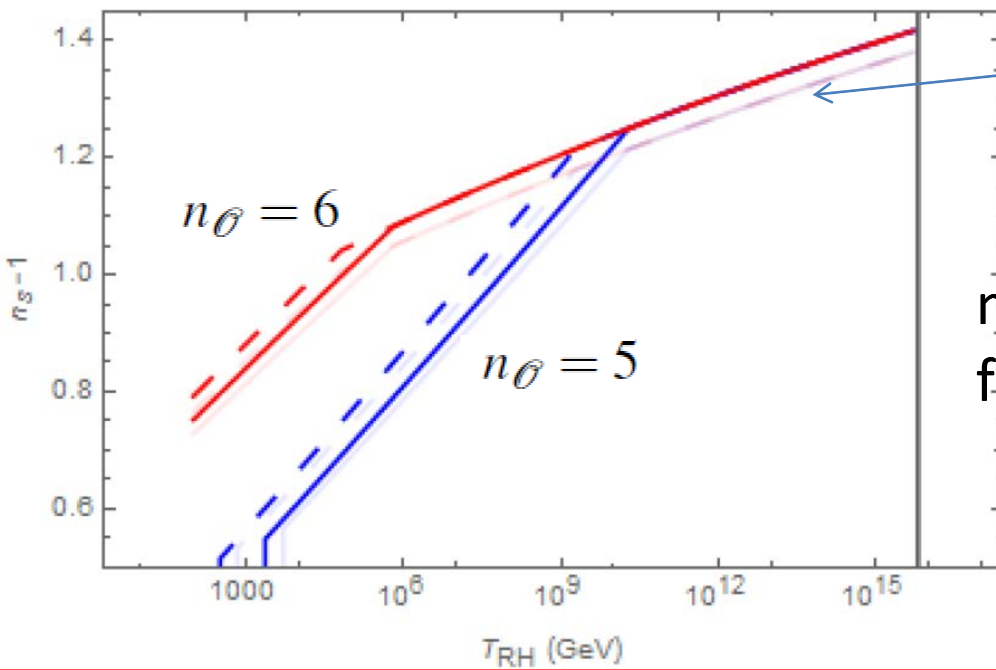
How blue is too blue for constant mass?

150X.XXXXX

related to T_{RH}/T_{eq} and experimental resolution

$$n_s - 1 \lesssim \frac{55}{N_{min}}\left(1+\frac{1}{55}\ln\left[\frac{10^{-5}}{\omega_{\chi\,\min}(n_s-1)}\left(\frac{T_{RH}}{10^{10}\,\mathrm{GeV}}\right)\right]+\frac{1}{55}\ln\left[\frac{(\chi_0)^2_{t_{k_{min}}}\, f^2(n_s-1)R}{M_p^2}\right]\right)$$

$r < 10^{-1}$ $\quad k_{\max} \sim O(100\ \mathrm{kpc})$ $\quad E_{k_{\max}} = 10^{-3}$



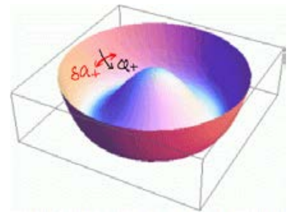
$k_{\max} \sim O(1\ \mathrm{Mpc})$

max reheating for r < 0.1

Measuring $n_s > 2.4$ is a signature of a time dependent mass of a scalar field. (SKA seems to be able to clearly measure 3)

Measuring $n_s > 2.4$ is a signature of a time dependent mass of a scalar field.

Contrast with an axion:



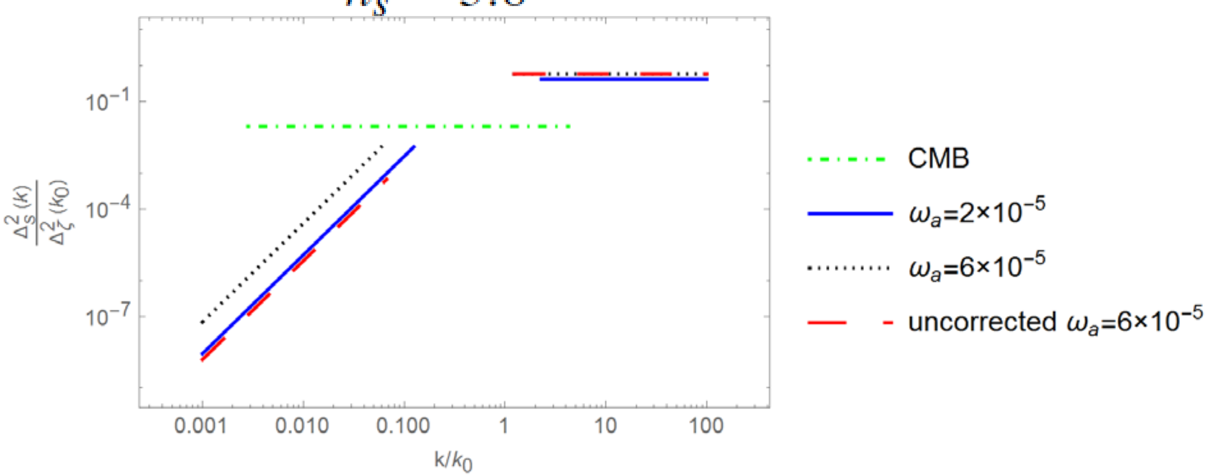
reach bottom of well

$$(a^2)_{H=m} \propto a^2(t_e) \propto e^{-(n_s-1)(N_{min}-N_c)}$$

instead of

$$(\chi_0^2)_{H=m} \propto \chi_0^2(t_e) \propto e^{-(n_s-1)N_{min}}$$

$n_s = 3.8$



massive → massless

Pseudo-NG models may be a "unique" class of natural models.

Summary

- Blue CDM-photon isocurvature pertrubations are attractive in terms of observability. Even a very tiny fraction of DM made up of 2nd dynamical degree of freedom existing in inflation can be measured.

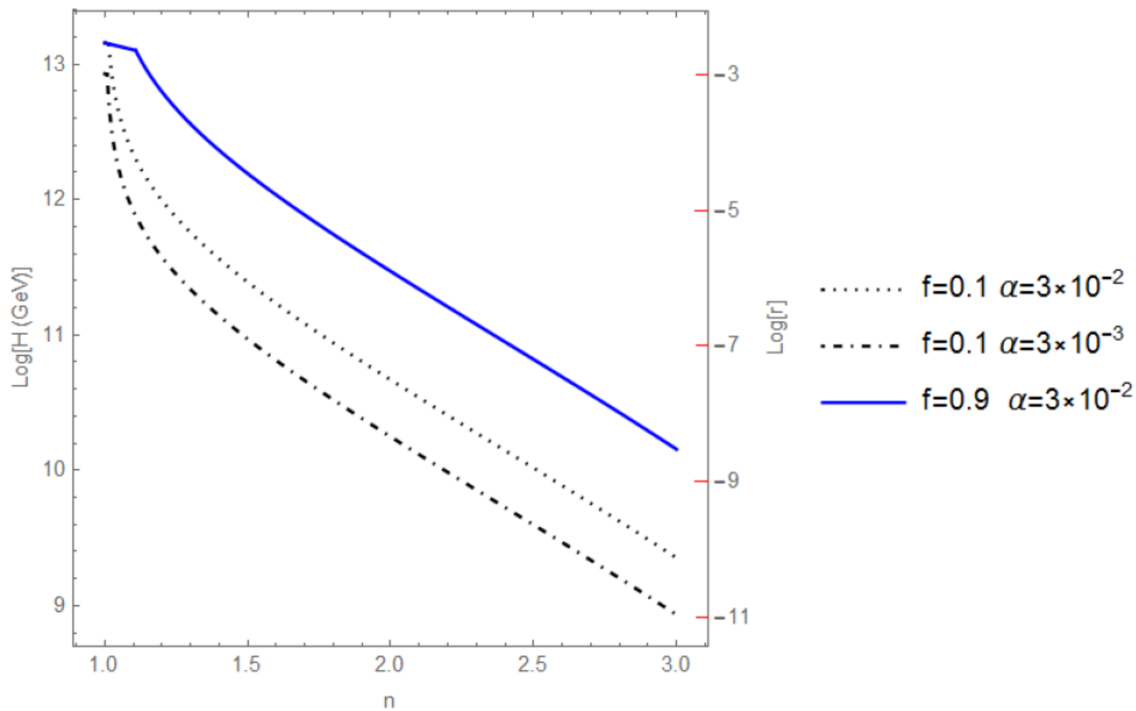
- Three theorems useful for **blue** isocurvature perturbations from spectator scalar fields are given in [1501.05618]
  - theorems 1: conserved quantity $V'_\chi(\delta\chi) \approx V''_\chi(\delta\chi)\delta\chi$
  - theorem 3: constrained quantization and secular effect
  - error bounds given

- Axion isocurvature computation of 0904.3800 (time-dep mass) was corrected.

- A measurement of a blue spectral index larger than 2.4 gives evidence for a field with time dependent mass during inflation [15xx.xxxx]. → most likely a pseudo-Nambu-Goldstone boson.

Backup

Largest H Allowed by Decoupled Linear Spectator?



Quantization decoupling constraint:

$$\frac{2}{3}\sqrt{2}\pi\left(\frac{k_{H_0}}{k_0}\right)^{-\frac{3}{2}+\nu+O(\varepsilon)g_0(m/H)} c_+\sqrt{\Delta_\zeta^2(k_{H_0})}\frac{M_p}{H}\frac{\theta_+(t_{k_0})}{f} < \frac{M_p}{\varphi_+(t_{k_0})}$$

Application 3: Is a blue spectrum a signature for Hubble scale massive scalar fields?

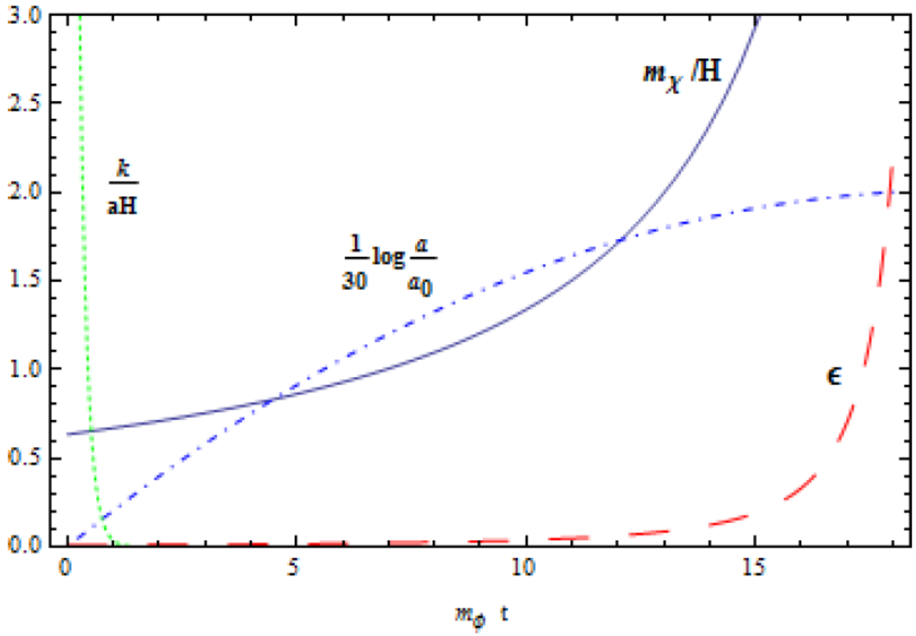
0904.3800 used a time dependent mass during inflation
→ k range of blueness limited

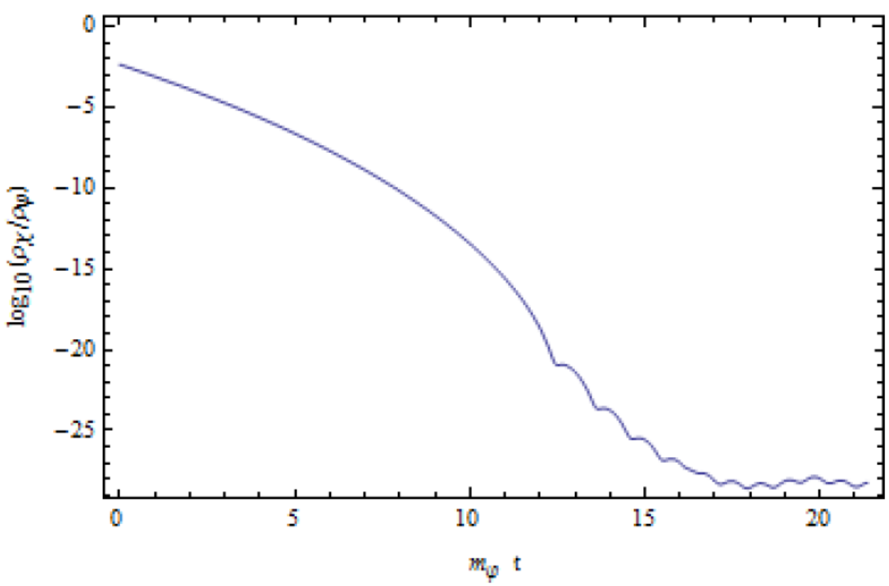
What about isocurvature with constant mass during inflation to extend the k range?

challenge: The energy density typically dilutes away.

e.g.

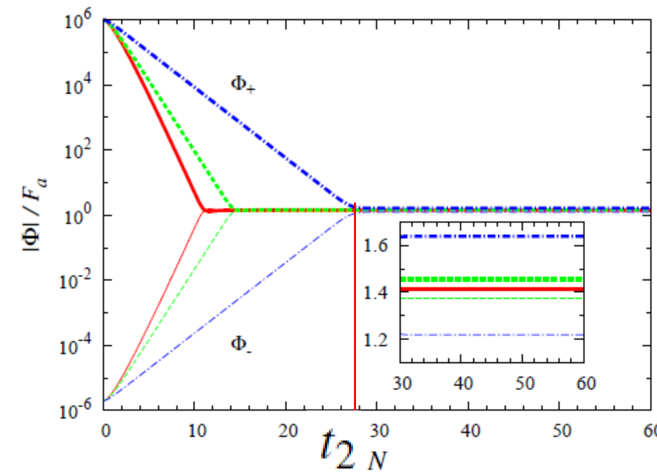
$$V_\varphi = \frac{m_\varphi^2}{2}\varphi^2$$





Although φ_+ rolls down to the minimum, a decays to a final point that depends on the initial conditions.

$$V \approx -h^2 F_a^2 \varphi_+ \varphi_- \cos\left(\frac{\sqrt{\varphi_+^2+\varphi_-^2}}{\varphi_+\varphi_-} b\right) + h^2 F_a^4 + \frac{1}{4} h^2 \varphi_-^2 \varphi_+^2$$
$$+\frac{1}{2} c_+ H^2 \varphi_+^2 + \frac{1}{2} c_- H^2 \varphi_-^2$$



$$\left(\Box - \frac{\Box\varphi_+}{\varphi_+}\right) a = 0 \longrightarrow a(t_2) \sim \theta_i F_a$$

Fortunately, for most of its evolution, the misalignment angle relaxes approximately at the same rate because of the dominance of quadratic φ_+ potential.

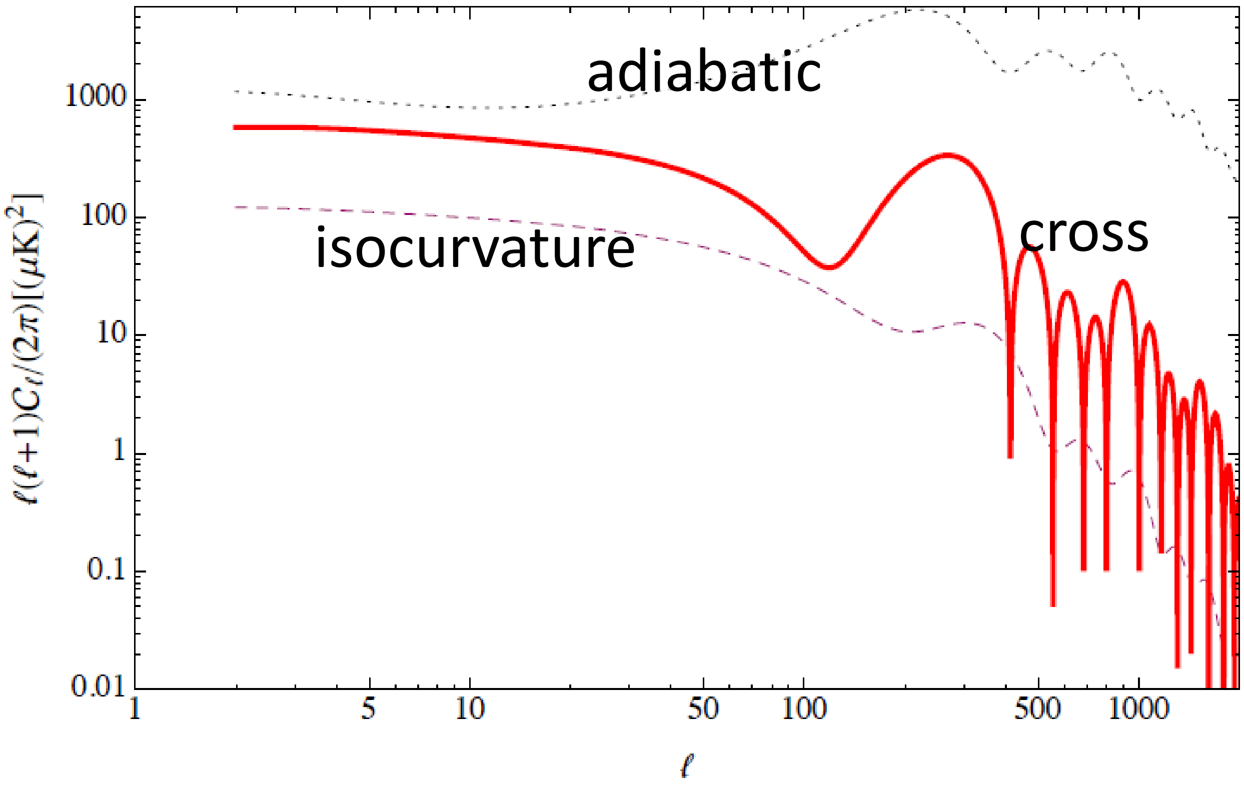
$$\Box\varphi_+ + \frac{1}{\sqrt{g}}\partial_\mu\left[\frac{a^2}{2\varphi_+^2}\sqrt{g}\partial^\mu\varphi_+\right] - \frac{1}{2\varphi_+}\frac{1}{\sqrt{g}}\partial_\mu\left[\sqrt{g}a\partial^\mu a\right] + \partial_{\varphi_+}V = 0 \longrightarrow$$

$$\left(\Box + c_+ H^2\right) a = 0$$
$$\left(\Box + c_+ H^2\right) \varphi_+ = 0$$

CMB

Scale invariant isocurvature

$\beta = -1$



$$\alpha|_{\beta=0} < 0.016 \text{ (95\% CL) and } \alpha|_{\beta=-1} < 0.0011 \text{ (95\% CL)}$$

$$\frac{k^3}{2\pi^2}\int \frac{d^3p}{(2\pi)^3}\langle \frac{\Delta T(\vec{p})}{T}\frac{\Delta T^*(\vec{k})}{T}\rangle = \Delta_\zeta^2(k)\left[|c_1|^2 + |c_2|^2\frac{\alpha}{1-\alpha} - 2\Re\left(c_1^* c_2 \beta\sqrt{\frac{\alpha}{1-\alpha}}\right)\right]$$

Planck+WP 2013

Newtonian intuition:

$$(\text{equation governing potential}) = \frac{1}{M_p^2}(\delta\rho_m + \delta\rho_\gamma)$$

Potential force and pressure push fluid components such as $\delta\rho_\gamma$ around.

e.g.

$$\frac{\delta\rho_\gamma}{\rho_\gamma}(k_s,\eta) \sim \Phi(k_s,\eta_i)\sqrt{c_s}\cos\left(k_s\int^{\eta} c_s(\eta')d\eta'\right) - S^i_{DM,\gamma}(k_s,\eta_i)\frac{\sin\left(k\eta/\sqrt{3}\right)}{k_s\eta_{eq}} \qquad k_s \gg k_{eq}$$

For scale invariant (non-blue) isocurvature spectrum,

this makes CMB sensitive primarily on large scales.

• corollary 1: Suppose a sudden transition of mass is made during inflation.

$$m^2(t) = \begin{cases} m_1 & t < t_c \\ m_2 & t > t_c \end{cases}$$

conserved quantity:

$$S = 2\frac{\delta\chi^{(G)}(t,\vec{k}) - \delta\chi_{ad}^{(G)}(t,\vec{k})}{\chi_0(t)}$$

error:

$$\mathscr{E} \equiv \max\left\{\exp\left[-2\nu(m_1)(t_c - t_k)H\right],\ \frac{\delta\rho_\chi^{(N)}}{\delta\rho_{\text{dominant}}^{(N)}}\right\}$$

$$\nu(m_1) = \frac{3}{2}\sqrt{1 - \frac{4}{9}\frac{m_1^2}{H^2}}$$

Conservation is valid through end of inflation and through reheating.

- Theorem 2: Isocurvature correlator is equal to the S correlator.

$$\delta_{s_\chi} \equiv 3(\zeta_\chi - \zeta_\gamma) \qquad \Delta^2_{s_\chi}(k) \equiv \frac{k^3}{2\pi^2} \int \frac{d^3k'}{(2\pi)^3} \langle \delta_{s_\chi}(t,\vec{k}) \delta_{s_\chi}(t,\vec{k}') \rangle$$

$$\delta_{S_\chi} \equiv \frac{\delta\rho_\chi^{(N)}|_{\text{background smoothed}}}{\langle \rho_\chi + P_\chi \rangle_{\text{time}}} - \frac{\delta\rho_\gamma^{(N)}}{\rho_\gamma + P_\gamma}$$

$$\Delta^2_{s_\chi}(k) = \frac{k^3}{2\pi^2} \int \frac{d^3k'}{(2\pi)^3} \langle S_\chi(t,\vec{k}) S_\chi(t,\vec{k}') \rangle$$

merit of the theorem: defines the sense in which $\dfrac{\delta\chi_{nad}}{\chi}$ arises from averaging of fluid quantities

This is merely a restatement of the results in [Gordon, Wands, Bassett, Maartens astro-ph/0009131; Polarski , Starobinsky astro-ph/9404061]