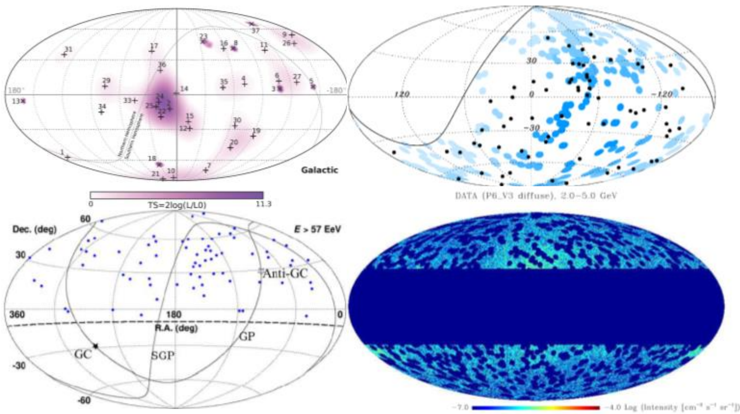


The Sensitivity of Astrophysical Anisotropy Signals to Dark Matter Annihilation

Sheldon Campbell
The Ohio State University
May 21, 2015

Arnowitt Memorial Symposium and
Mitchell Workshop on Collider and Dark Matter Physics

The Ohio State University's Center for Cosmology and AstroParticle Physics

THE OHIO STATE UNIVERSITY

Prof. Arnowitt's Suggestion

During 2007 seminar by Eiichiro Komatsu

- Look for imprint in gamma-rays of dark matter annihilation in large scale structure.

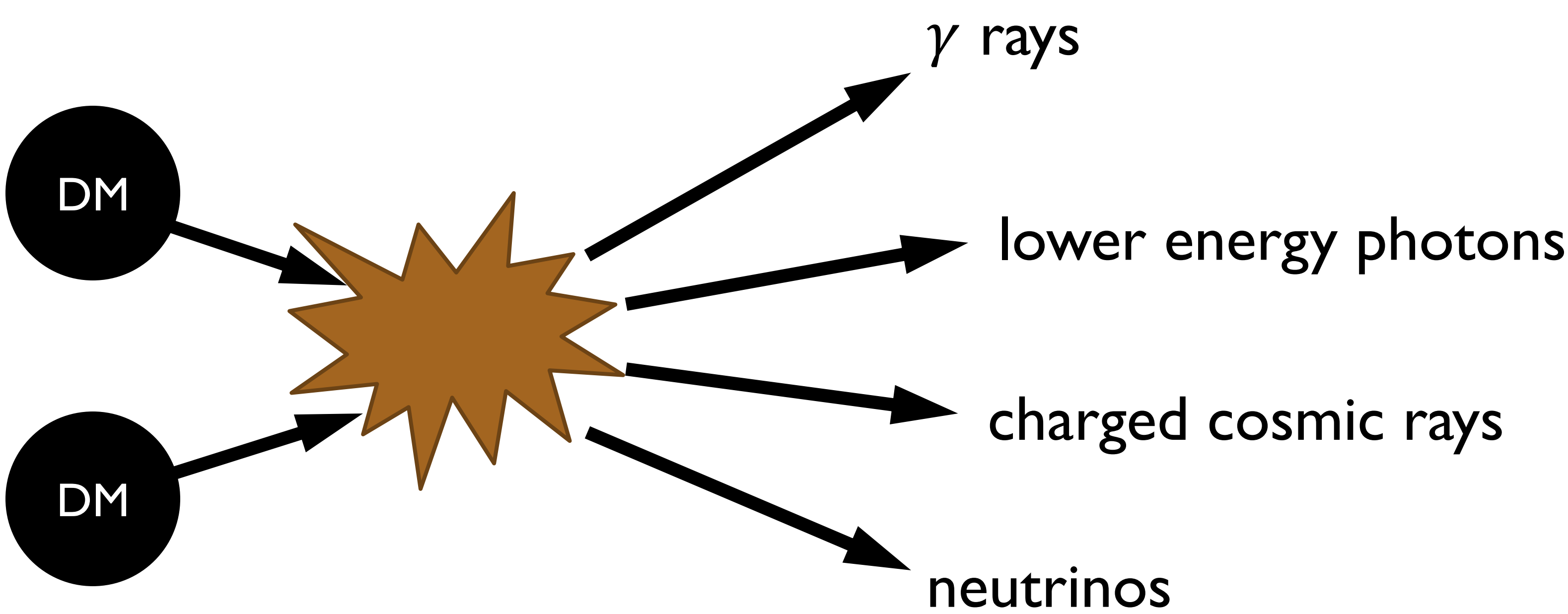
Arnowitt: What if dark matter annihilates through a p-wave?

$$\sigma_{\rm ann} v = a + b v^2$$

Answer: This would affect the gamma-ray signature in small scale anisotropies.

Indirect Detection of Dark Matter

- The astrophysical distribution of dark matter may be annihilating/decaying.



- High energy products detected as discrete events in sky.

Observing "Points" in the Sky

- High-Energy Radiation Events
  - Gamma-Rays
  - Cosmic Ray Shower Events
  - Cosmic Neutrinos

Inference radiation sources, cosmic ray acceleration, ray propagation, etc.

- Celestial Objects
  - Galaxies
  - AGN
  - X-ray Clusters
  - …

Inference cosmic expansion history, large scale structure, galaxy formation, etc.

Potential radiation sources!

Specify distribution of a class of events/objects in the sky.

- objects in a redshift range, radiation events in an energy bin, etc.

If Signal Dominated by Known Structures

- Best strategy is a stacking analysis.
  - E.g., low-luminosity dwarf galaxies

    Fermi-LAT Collaboration, Astrophys.J. 712 (2010) 147-158

    Geringer-Sameth, Koushiappas, Phys.Rev.Lett. 107 (2011) 241303

    Geringer-Sameth, Koushiappas, Walker, Phys.Rev. D91 (2015) 083535

    Fermi-LAT Collaboration, arXiv:1503.02641

  - Cross-correlation of astrophysical signals with source catalogs.

    Ando, JCAP 1410 (2014) 10, 061

    Shirasaki, Horiuchi, Yoshida, Phys.Rev. D90 (2014) 6, 063502

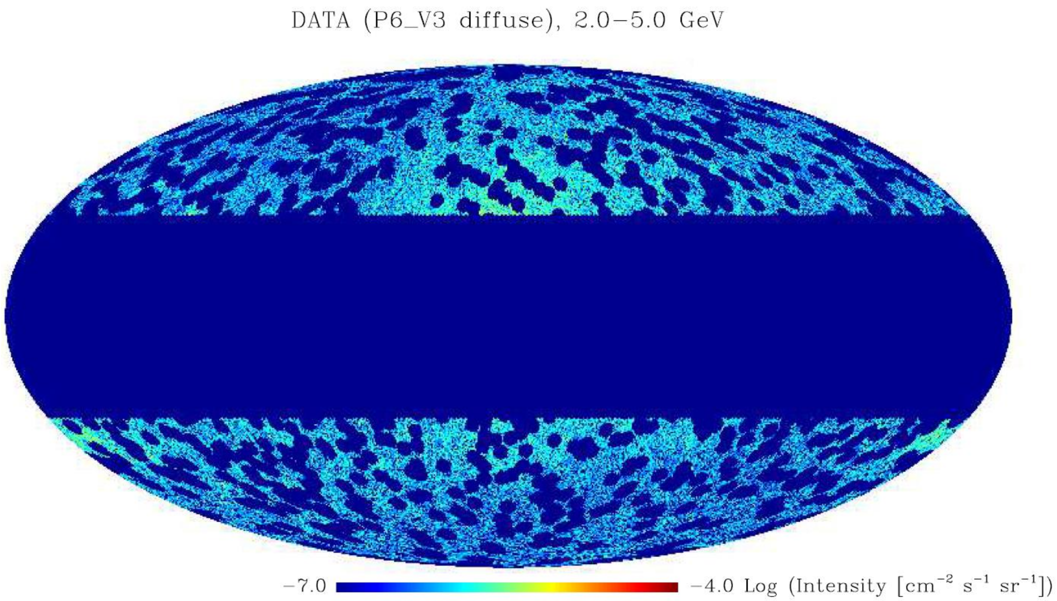
    Fornengo et al., Astrophys.J. 802 (2015) 1, L1

    Camera et al., arXiv:1411.4651

    Regis et al., arXiv:1503.05922

If Contributions from Unknown Structures

- Do a full sky "blind" search.
- Strategy: look for features in the flux spectrum and flux spatial distribution.



Regions of γ-ray sky without resolved point sources.

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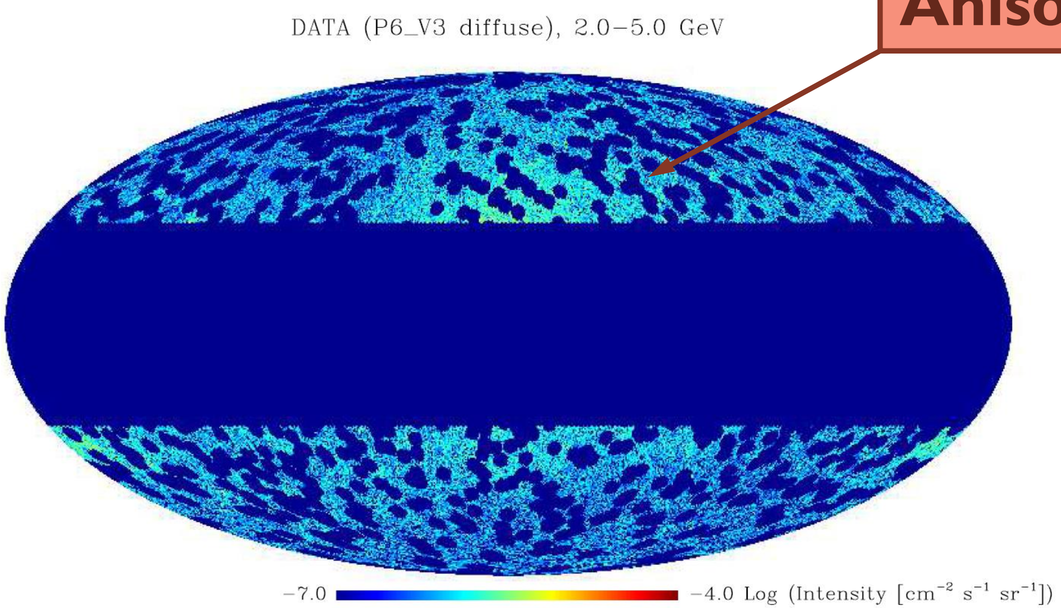
- Strategy: look for features in the flux spectrum and flux spatial distribution.

Anisotropies



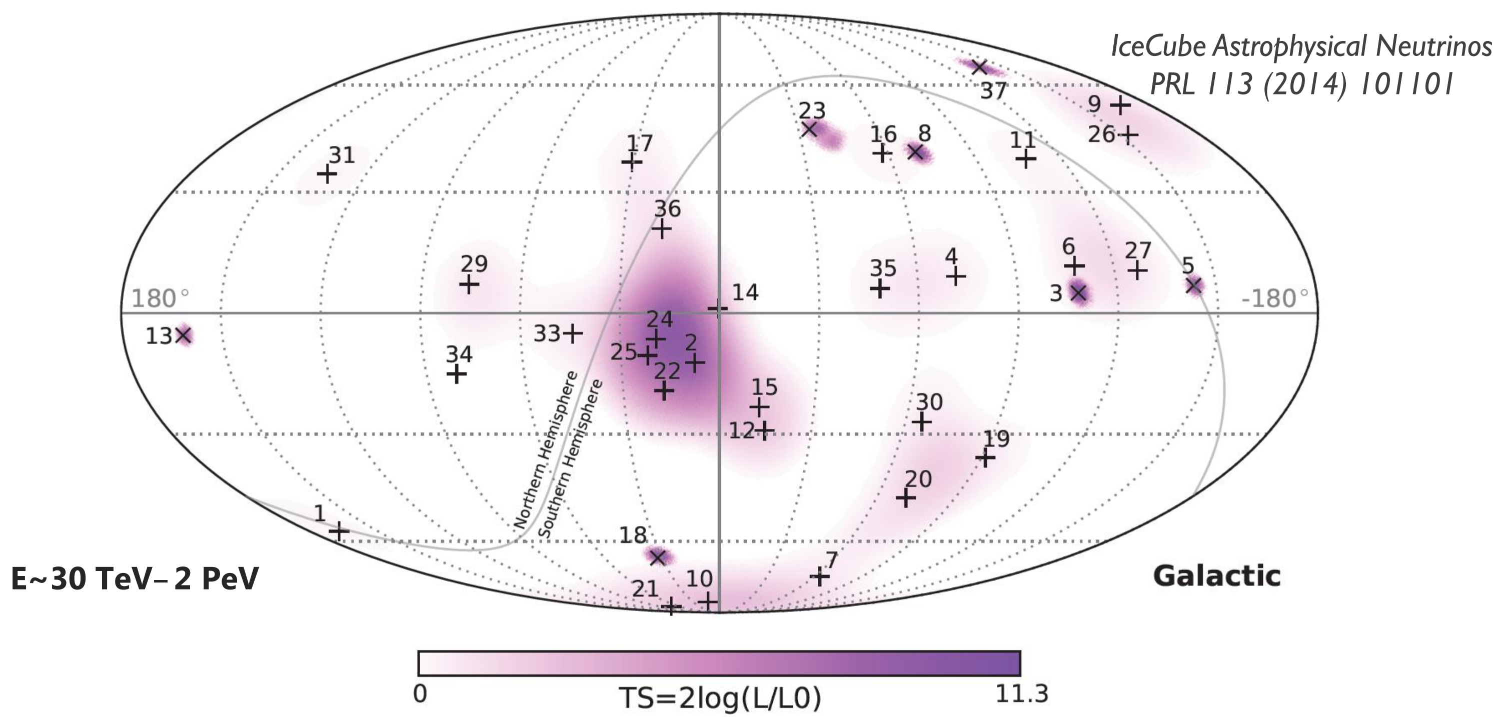
Regions of γ-ray sky without resolved point sources.

Questions to Address. Can anisotropy:

1. see dark matter? (sensitive?)
2. discover dark matter? (very sensitive?)
3. constrain particle dark matter models? (competitive?)
4. teach us anything about the nature of dark matter? (informative?)

Angular Distribution Methods

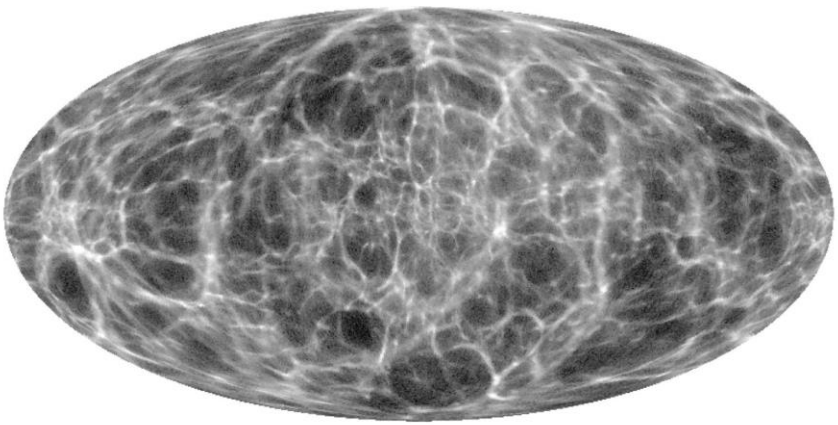
- Observation 1:
  - the angular distribution of **observed events** approaches the angular distribution of **sources** (messenger-propagated and projected) on our sky (full **skymap**).



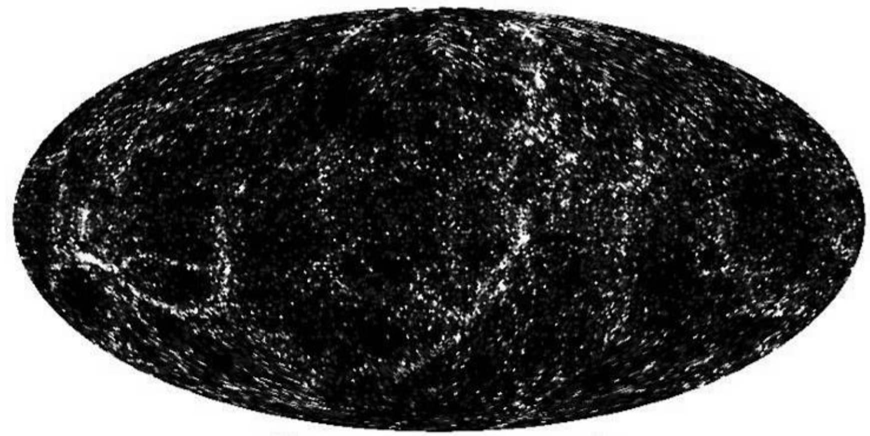
Angular Distribution Methods

- Observation 2:
  - Radiation events are a sampling of sources.

Francisco-Shu Kitaura et al., MNRAS 427, L35 (2012)



A skymap (catalog) of sources.



Sample gamma-ray events observed from those sources.

Given N events, what can we infer about the full **skymap**?

Given physical source models, how many events would distinguish them?

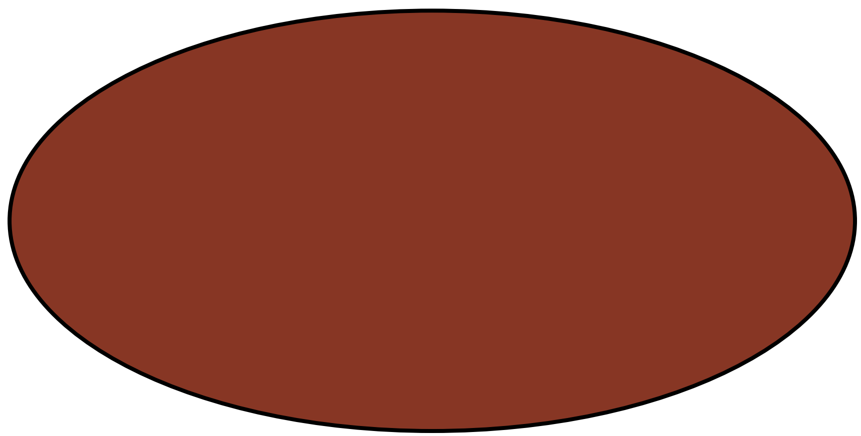
Angular Distribution Methods

- Observation 3:
  - The diffuse gamma-ray background is very isotropic.
    - remove galactic cosmic-ray foreground and resolved point sources.
    - emerging picture: dominant flux is from star formation (z~2) but anisotropy dominated by unresolved point sources (blazars).

    [Xia, Cuoco, Branchini, Viel, Astrophys.J.Suppl. 217 (2015) 1, 15]
  - The astrophysical neutrinos are also very isotropic.
    - no correlation with galactic structure.
    - no resolved anisotropy.
    - no point source detected.
      - Cosmological source. Must be very numerous and very dense. [Waxman, Murase 2015]

Anisotropy Strategy for Dark Matter

- Observe Structure in Different Energy Bins.
- With smooth background, dim foreground structures emerge easily.



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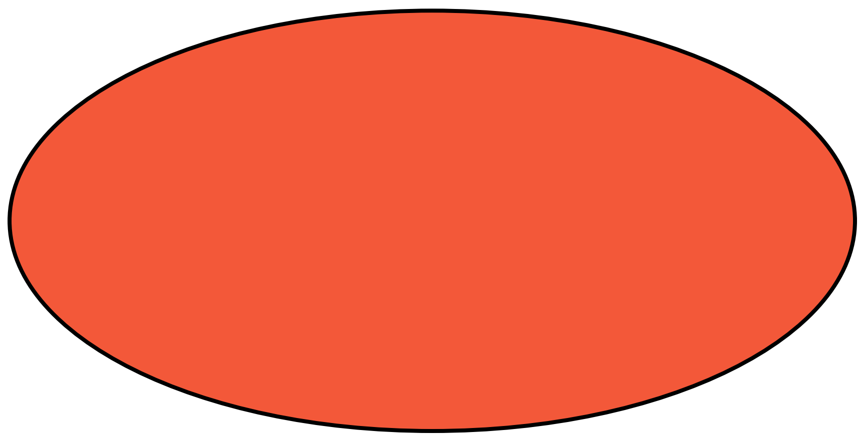

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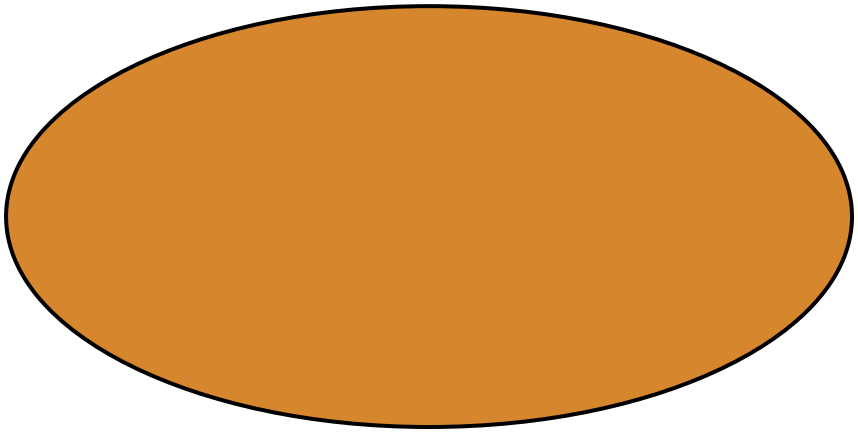

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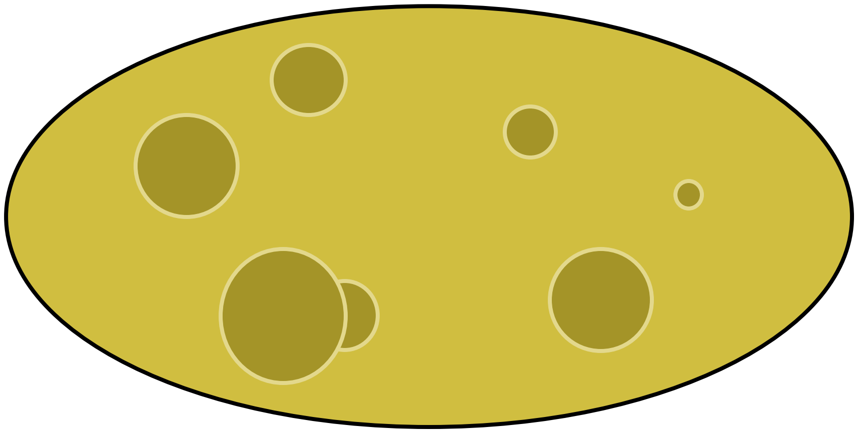

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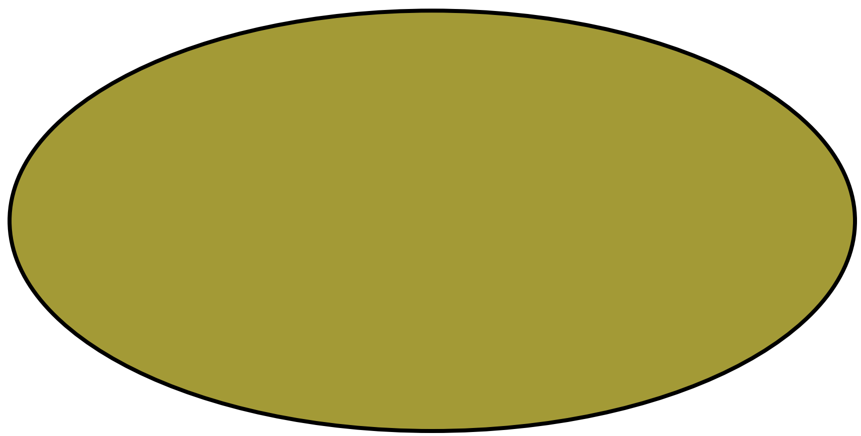


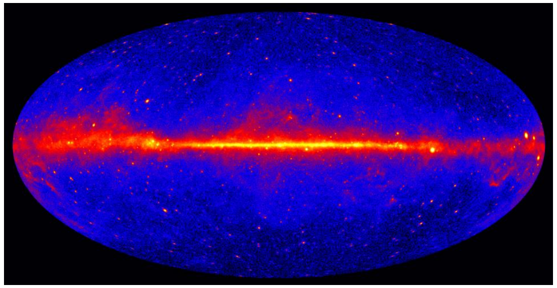
Angular Clustering of the Source Skymap

- Positive, real function on the sphere $\Phi(\boldsymbol{n})$.
- Normalize: Let $S(\boldsymbol{n}) = \frac{\Phi(\boldsymbol{n})}{\langle\Phi\rangle} - 1$.

For cosmic radiation, Φ is the apparent flux map of all sources.

- Normalized spherical transform:

$$\tilde{a}_{\ell m} = \int d\boldsymbol{n}\, Y^*_{\ell m}(\boldsymbol{n}) S(\boldsymbol{n})$$



- Angular power spectrum (dimensionless):

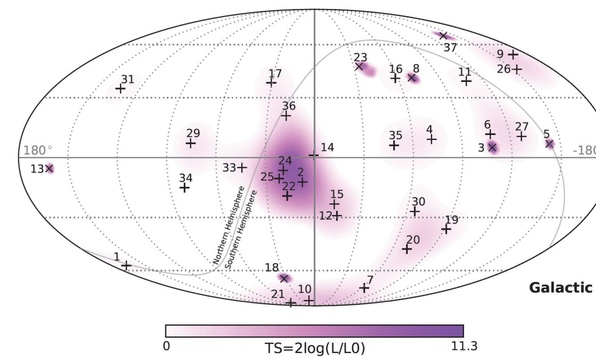
$$\tilde{C}_\ell = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |\tilde{a}_{\ell m}|^2 = 4\pi \int \frac{d\boldsymbol{n}_1}{4\pi} \frac{d\boldsymbol{n}_2}{4\pi}\, S(\boldsymbol{n}_1) P_\ell(\boldsymbol{n}_1 \cdot \boldsymbol{n}_2)\, S(\boldsymbol{n}_2)$$

- Angular bispectrum:

$$\tilde{B}_{\ell_1 \ell_2 \ell_3} = \sum_{m_1, m_2, m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \tilde{a}_{\ell_1 m_1} \tilde{a}_{\ell_2 m_2} \tilde{a}_{\ell_3 m_3}$$

Angular Clustering of N Observed Events

- Differential flux of events $\Phi_N(\boldsymbol{n}) = \frac{4\pi}{\varepsilon}\sum_{i=1}^{N}\delta(\boldsymbol{n}-\boldsymbol{n}_i)$.
  - Each term needs weights if exposure ε is not uniform.



- Normalize: $S_N(\boldsymbol{n}) = \frac{4\pi}{N}\sum_{i=1}^{N}\delta(\boldsymbol{n}-\boldsymbol{n}_i) - 1$.

- Normalized spherical transform:

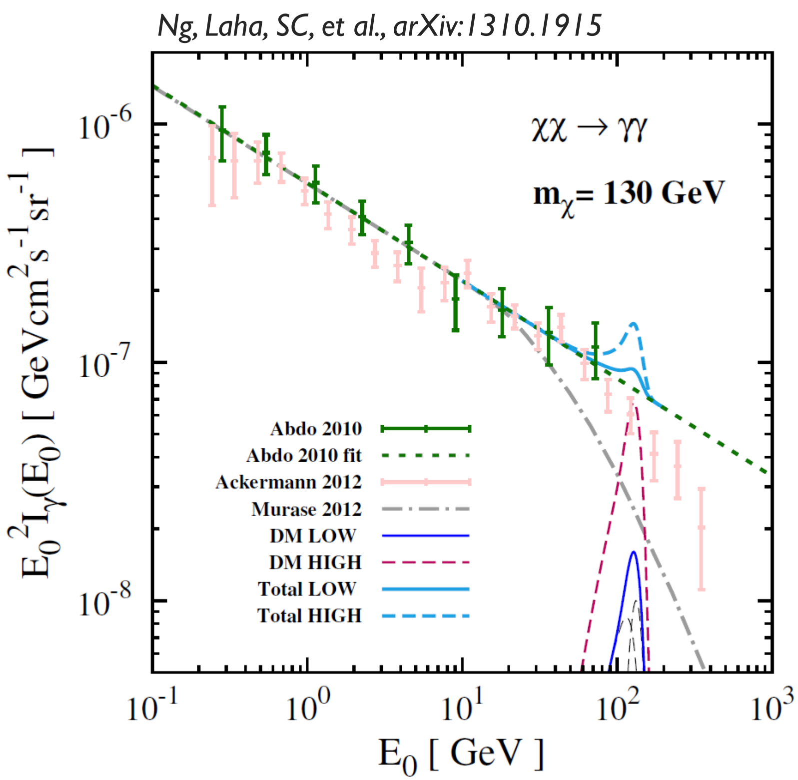
$$\tilde{a}_{\ell m,N} = \frac{4\pi}{N}\sum_{i=1}^{N} Y^{*}_{\ell m}(\boldsymbol{n}_i)$$

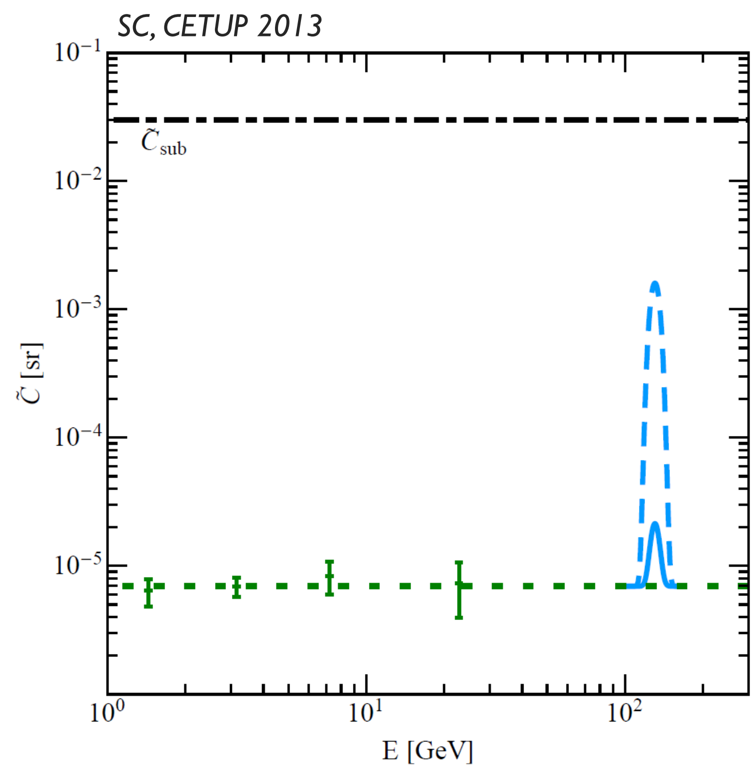
- Angular power spectrum of N events:

$$\tilde{C}_{\ell,N} = \frac{4\pi}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} P_\ell(\boldsymbol{n}_i\cdot\boldsymbol{n}_j)$$

Statistical properties of this observable tell us about the sources.

Spectral Line in $\Phi(\text{E})$ vs. $\tilde{C}_\ell(E)$

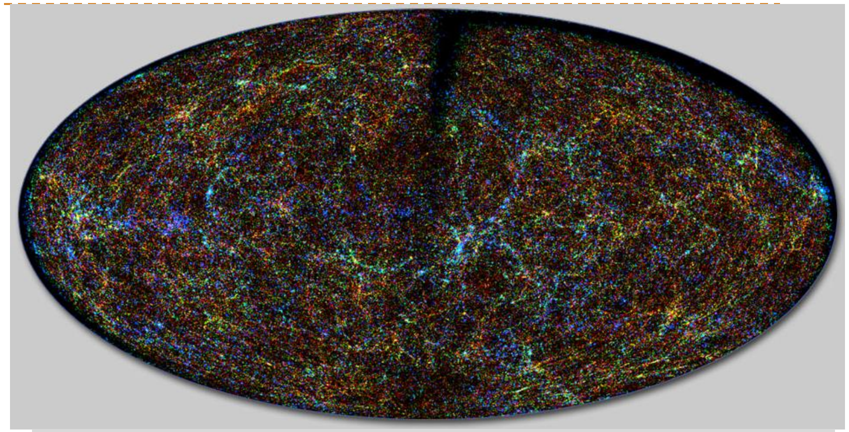




- If relative brightness of line sources is low, but structure is significant, they power spectrum can be more sensitive.
- Uncertainty of $\tilde{C}_\ell$ measurements is crucial to understand sensitivity.

The Problem of Measuring $\tilde{C}_\ell$

- Let $\tilde{C}_\ell$ be the fluctuation (normalized) APS of a **skymap**– what we are trying to measure.
- Receive N events at random, weighted by the sky map.
- Assume full sky observations with uniform exposure.



A hypothetical projected **skymap** of sources.
The 2 micron sky courtesy of the 2MASS collaboration, http://www.ipac.caltech.edu/2mass/.

- What is the angular power spectrum of the N events, $\tilde{C}_{\ell,N}$, from a full sky map with distribution $\tilde{C}_\ell$?
  - mean of $\tilde{C}_{\ell,N}$?
  - variance of $\tilde{C}_{\ell,N}$?

Simplest Model: Poisson Point Process

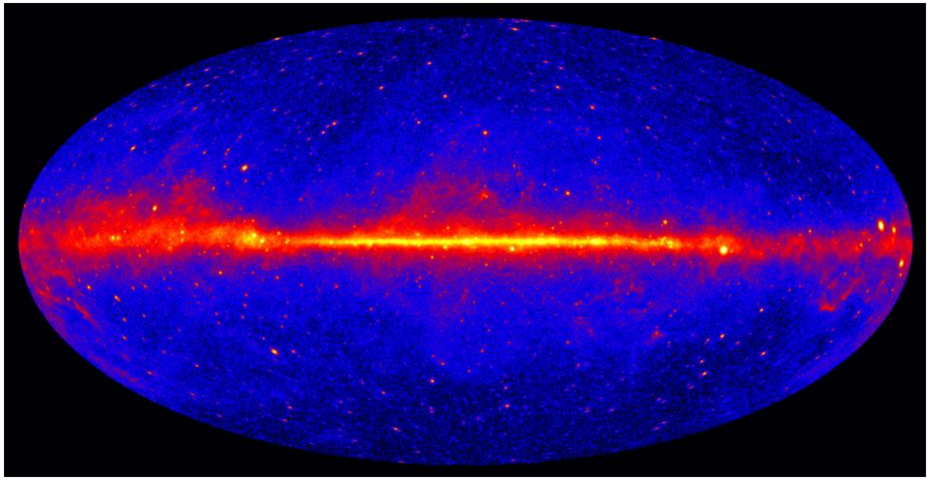
- Only 2 assumptions (need experimental justification):
  1. The skymap of sources is **stationary** over the exp. lifetime.

     Neglect transients. Their effect will depend on the timescales involved.

  2. The observed events are **independent**.

     The probability of observing an event at a given position depends on the source skymap, but not on previous events already observed.

The statistics of the observable $\tilde{C}_{\ell,N}$ are exactly solvable in this case.



Statistical Mean: Events Relate to Sources!

- The average measurement of $\tilde{C}_{\ell,N}$ from a random sample:

$$\langle \tilde{C}_{\ell,N} \rangle = \frac{4\pi}{N} + \left(1 - \frac{1}{N}\right) \tilde{C}_\ell$$

$\tilde{C}_\ell$ is now APS of source skymap, convolved with instrument PSF.

  - Angular power spectrum of events is a *biased* estimator of the source distribution.

- Therefore, an unbiased estimator $\hat{\tilde{C}}_{\ell,N}$ with $\left\langle \hat{\tilde{C}}_{\ell,N} \right\rangle = \tilde{C}_\ell$:

$$\hat{\tilde{C}}_{\ell,N} = \frac{1}{1 - \frac{1}{N}} \left[\tilde{C}_{\ell,N} - \frac{4\pi}{N} \right] = \frac{4\pi}{N(N-1)} \sum_i \sum_{j \neq i} P_\ell(\boldsymbol{n}_i \cdot \boldsymbol{n}_j)$$

  - In agreement with other existing methods!

Exact Statistical Covariance of $\hat{\hat{C}}_{\ell,N}$

$$\mathrm{Cov}\left[\hat{\hat{C}}_{\ell_1,N},\hat{\hat{C}}_{\ell_2,N}\right]=\frac{(4\pi)^2}{N(N-1)}\left\{2\left[\frac{\delta_{\ell_1,\ell_2}}{2\ell_1+1}+\tilde{C}^{(2)}_{\ell_1\ell_2}-\frac{\tilde{C}_{\ell_1}\tilde{C}_{\ell_2}}{(4\pi)^2}\right]+4(N-2)\left[\frac{\delta_{\ell_1,\ell_2}}{2\ell_1+1}\frac{\tilde{C}_{\ell_1}}{4\pi}+\frac{\tilde{C}^{(3)}_{\ell_1\ell_2}}{4\pi}-\frac{\tilde{C}_{\ell_1}\tilde{C}_{\ell_2}}{(4\pi)^2}\right]\right\}$$

$$\tilde{C}^{(2)}_{\ell_1\ell_2}=\sum_{\ell'=|\ell_1-\ell_2|}^{\ell_1+\ell_2}\frac{2\ell'+1}{4\pi}\begin{pmatrix}\ell_1 & \ell_2 & \ell'\\ 0 & 0 & 0\end{pmatrix}^2\tilde{C}_{\ell'}$$

$$\tilde{C}^{(3)}_{\ell_1\ell_2}=\frac{1}{\sqrt{(2\ell_1+1)(2\ell_2+1)}}\sum_{\ell'=|\ell_1-\ell_2|}^{\ell_1+\ell_2}\sqrt{\frac{2\ell'+1}{4\pi}}\begin{pmatrix}\ell_1 & \ell_2 & \ell'\\ 0 & 0 & 0\end{pmatrix}\tilde{B}_{\ell_1\ell_2\ell'}$$

Analytic Work Generated Higher Order Angular Spectra

$$\tilde{C}^{(2)}_{\ell_1\ell_2} = \sum_{\ell'=|\ell_1-\ell_2|}^{\ell_1+\ell_2} \frac{2\ell'+1}{4\pi} \begin{pmatrix} \ell_1 & \ell_2 & \ell' \\ 0 & 0 & 0 \end{pmatrix}^2 \tilde{C}_{\ell'}$$

$$\tilde{C}^{(3)}_{\ell_1\ell_2} = \frac{1}{\sqrt{(2\ell_1+1)(2\ell_2+1)}} \sum_{\ell'=|\ell_1-\ell_2|}^{\ell_1+\ell_2} \sqrt{\frac{2\ell'+1}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell' \\ 0 & 0 & 0 \end{pmatrix} \tilde{B}_{\ell_1\ell_2\ell'}$$

$$\tilde{C}^{(4)}_{\ell_1\ell_2} = \tilde{C}_{\ell_1}\tilde{C}_{\ell_2}$$

I know two ways to see that $\tilde{C}_\ell$ is the first order angular spectrum, and that these comprise the **complete** set of 2nd order spectra.

Higher Order Spectra: Tensor Picture

- First and Second Rank Spherical Harmonic Transforms of *S*:

$$\tilde{a}_{\ell m} = \int d\boldsymbol{n}\; Y^*_{\ell m}(\boldsymbol{n})\, S(\boldsymbol{n}), \qquad \tilde{a}_{\ell_1 m_1 \ell_2 m_2} = \int d\boldsymbol{n}\, Y^*_{\ell_1 m_1}(\boldsymbol{n}) Y^*_{\ell_2 m_2}(\boldsymbol{n})\, S(\boldsymbol{n})$$

- Raised Azimuthal Indices generated by ${Y_\ell}^m = (-1)^m\, Y^*_{\ell,-m}$:

$$\tilde{a}_{\ell_1 m_1 \ell_1}{}^{m_2} = \int d\boldsymbol{n}\, Y^*_{\ell m_1}(\boldsymbol{n}) {Y_\ell}^{m_2}(\boldsymbol{n})\, S(\boldsymbol{n}) = (-1)^{m_2}\, \tilde{a}_{\ell_1, m_1, \ell_2, -m_2}$$

- Create rank 0 (rotation invariant) tensors by contracting azimuthal indices:

$$\tilde{C}_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} {\tilde{a}_\ell}^m \tilde{a}_{\ell m}$$

Higher Order Spectra: Tensor Picture

- All possible rank 0 tensors from rank 1 and 2 transforms.

$$\tilde{C}^{(2)}_{\ell_1\ell_2} = \frac{1}{(2\ell_1+1)(2\ell_2+1)} \sum_{m_1,m_2} \tilde{a}_{\ell_1}{}^{m_1}{}_{\ell_2}{}^{m_2}\, \tilde{a}_{\ell_1 m_1 \ell_2 m_2}$$

$$\tilde{C}^{(3)}_{\ell_1\ell_2} = \frac{1}{(2\ell_1+1)(2\ell_2+1)} \sum_{m_1,m_2} \tilde{a}_{\ell_1}{}^{m_1}{}_{\ell_2}{}^{m_2}\, \tilde{a}_{\ell_1 m_1} \tilde{a}_{\ell_2 m_2}$$

$$\tilde{C}^{(4)}_{\ell_1\ell_2} = \tilde{C}_{\ell_1}\tilde{C}_{\ell_2} = \frac{1}{(2\ell_1+1)(2\ell_2+1)} \sum_{m_1,m_2} \tilde{a}_{\ell_1}{}^{m_1}\tilde{a}_{\ell_1 m_1}\tilde{a}_{\ell_2}{}^{m_2}\tilde{a}_{\ell_2 m_2}$$

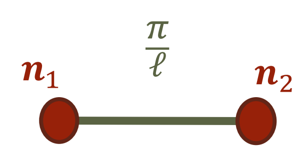
Higher Order Spectra: Field Theory Pic.

- Use the Spherical Harmonic Addition Theorem:

$$\frac{1}{2\ell+1}\sum_{m} Y_{\ell}^{\ m}(\boldsymbol{n}_1)\, Y^{*}_{\ell m}(\boldsymbol{n}_2) = \frac{1}{4\pi} P_{\ell}(\boldsymbol{n}_1 \cdot \boldsymbol{n}_2)$$

- Angular Power Spectrum is like 2 field configurations connected by a "correlator".

$$\tilde{C}_{\ell} = 4\pi \int \frac{d\boldsymbol{n}_1}{4\pi}\frac{d\boldsymbol{n}_2}{4\pi}\, S(\boldsymbol{n}_1) P_{\ell}(\boldsymbol{n}_1 \cdot \boldsymbol{n}_2)\, S(\boldsymbol{n}_2)$$



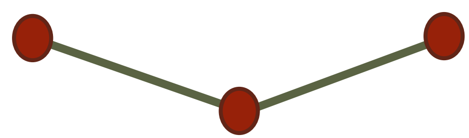
Higher Order Spectra: Field Theory Pic.

- All possible diagrams with 2 correlators.

$$\tilde{C}^{(2)}_{\ell_1 \ell_2} = \int \frac{d\boldsymbol{n}_1}{4\pi}\frac{d\boldsymbol{n}_2}{4\pi}\, S(\boldsymbol{n}_1) P_{\ell_1}(\boldsymbol{n}_1 \cdot \boldsymbol{n}_2) P_{\ell_2}(\boldsymbol{n}_1 \cdot \boldsymbol{n}_2) S(\boldsymbol{n}_2)$$

"Composite Angular Power Spectrum"

$$\tilde{C}^{(3)}_{\ell_1 \ell_2} = 4\pi \int \frac{d\boldsymbol{n}_1}{4\pi}\frac{d\boldsymbol{n}_2}{4\pi}\frac{d\boldsymbol{n}_3}{4\pi}\, S(\boldsymbol{n}_1) P_{\ell_1}(\boldsymbol{n}_1 \cdot \boldsymbol{n}_2)\, S(\boldsymbol{n}_2) P_{\ell_2}(\boldsymbol{n}_2 \cdot \boldsymbol{n}_3)\, S(\boldsymbol{n}_3)$$



"Open Angular Bispectrum"

$$\tilde{C}^{(4)}_{\ell_1 \ell_2} = \tilde{C}_{\ell_1} \tilde{C}_{\ell_2}$$

"Disjoint Angular Trispectrum"

Unbiased Estimators from N Events

$$\hat{\hat{C}}^{(2)}_{\ell_1\ell_2,N} = \frac{1}{N(N-1)}\sum_{i_1}\sum_{i_2\neq i_1} P_{\ell_1}(\boldsymbol{n}_{i_1}\cdot\boldsymbol{n}_{i_2})P_{\ell_2}(\boldsymbol{n}_{i_1}\cdot\boldsymbol{n}_{i_2}) - \frac{\delta_{\ell_1\ell_2}}{2\ell_1+1}$$

$$\hat{\hat{C}}^{(3)}_{\ell_1\ell_2,N} = \frac{4\pi}{N(N-1)(N-2)}\sum_{i_1}\sum_{i_2\neq i_1}\sum_{\substack{i_3\neq i_2\\ i_3\neq i_1}} P_{\ell_1}(\boldsymbol{n}_{i_1}\cdot\boldsymbol{n}_{i_2})P_{\ell_2}(\boldsymbol{n}_{i_2}\cdot\boldsymbol{n}_{i_3}) - \frac{\delta_{\ell_1\ell_2}}{2\ell_1+1}\hat{\hat{C}}_{\ell_1,N}$$

$$\hat{\hat{C}}^{(4)}_{\ell_1\ell_2,N} = \frac{(4\pi)^2}{N(N-1)(N-2)(N-3)}\sum_{i_1}\sum_{i_2\neq i_1}\sum_{\substack{i_3\neq i_2\\ i_3\neq i_1}}\sum_{\substack{i_4\neq i_3\\ i_4\neq i_2\\ i_4\neq i_1}} P_{\ell_1}(\boldsymbol{n}_{i_1}\cdot\boldsymbol{n}_{i_2})P_{\ell_2}(\boldsymbol{n}_{i_3}\cdot\boldsymbol{n}_{i_4})$$

Statistical Covariance of $\hat{\hat{C}}_{\ell,N}$ $(N \gg 1)$

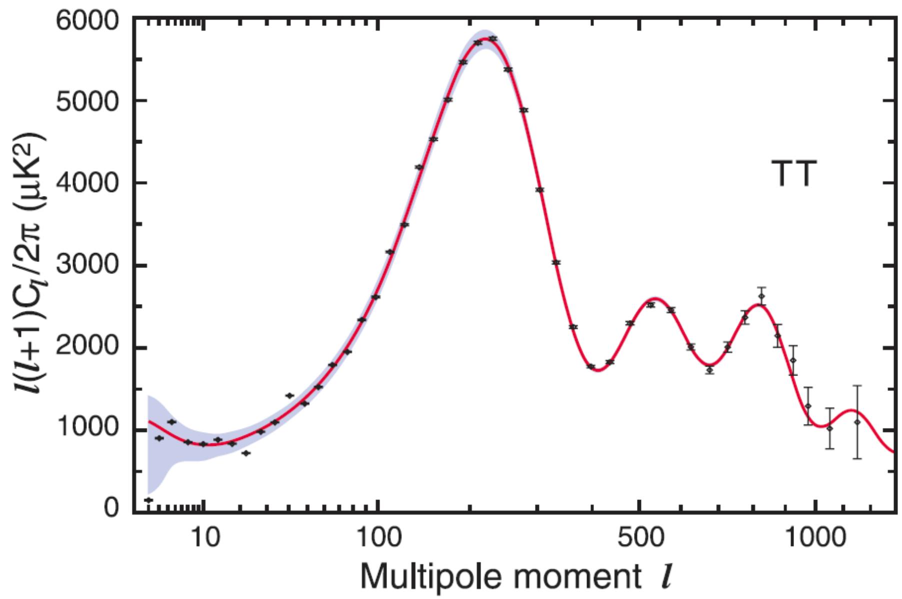
$$\mathrm{Cov}\left[\hat{\hat{C}}_{\ell_1,N}, \hat{\hat{C}}_{\ell_2,N}\right] = (4\pi)^2 \left\{ \frac{2}{N^2}\left[\frac{\delta_{\ell_1,\ell_2}}{2\ell_1+1} + \tilde{C}^{(2)}_{\ell_1\ell_2} - \frac{\tilde{C}_{\ell_1}\tilde{C}_{\ell_2}}{(4\pi)^2}\right] + \frac{4}{N}\left[\frac{\delta_{\ell_1,\ell_2}}{2\ell_1+1}\frac{\tilde{C}_{\ell_1}}{4\pi} + \frac{\tilde{C}^{(3)}_{\ell_1\ell_2}}{4\pi} - \frac{\tilde{C}_{\ell_1}\tilde{C}_{\ell_2}}{(4\pi)^2}\right] \right\}$$

- If higher-order spectra are neglected:
  - the covariance is diagonal—each multipole measurement is independent.
  - call this C_ℓ-only statistical uncertainty.

$$\mathcal{V}\mathrm{ar}\left[\hat{\hat{C}}_{\ell,N}\right] = \frac{2}{2\ell+1}\left[\left(\frac{4\pi}{N}\right)^2 + 2\left(\frac{4\pi}{N}\right)\tilde{C}_\ell\right] \qquad (C_\ell\text{-only})$$

Good Agreement with WMAP Data

$$\mathcal{V}\text{ar}\left[\hat{\tilde{C}}_{\ell,N}\right] = \frac{2}{2\ell+1}\left[\left(\frac{4\pi}{N}\right)^2 + 2\left(\frac{4\pi}{N}\right)\tilde{C}_\ell\right] \qquad (C_\ell\text{-only})$$

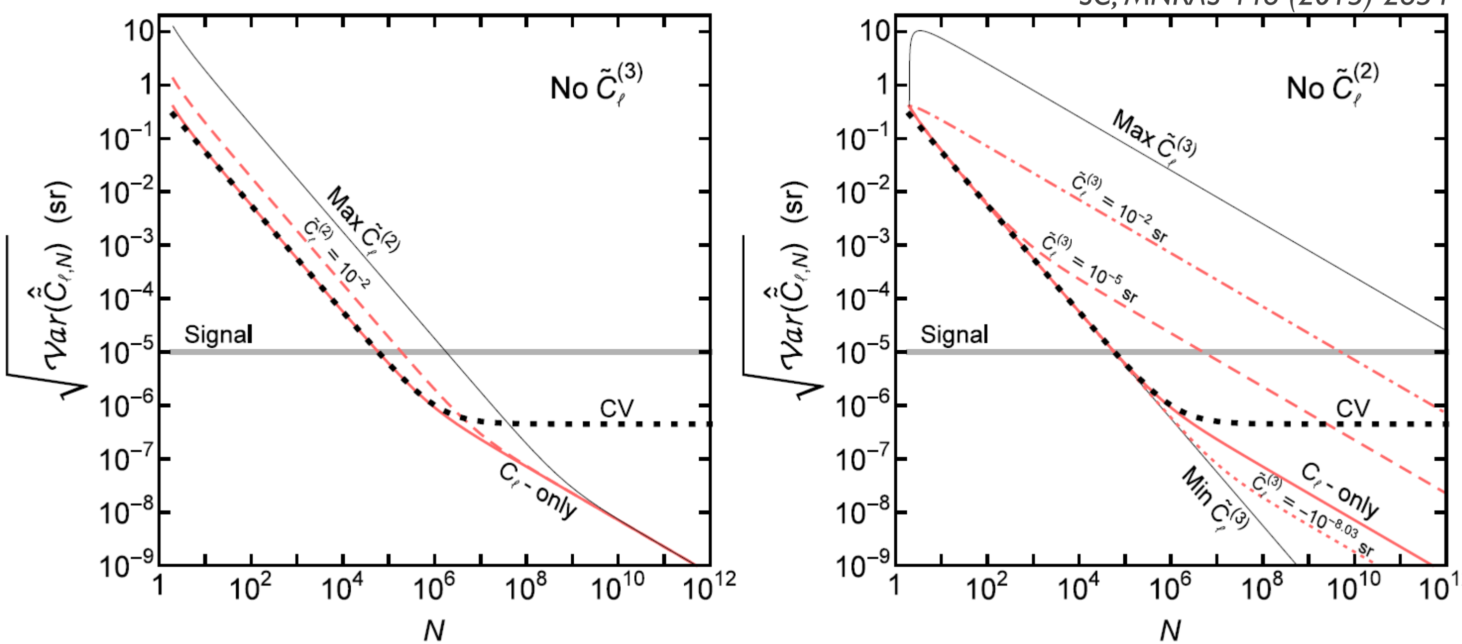


WMAP Collaboration, Astrophys.J.Suppl. 208 (2013) 20 & PTEP 2014 (2014) 6, 06B102

Fig. 5 Nine-year angular power spectrum of the CMB temperature (adapted from [37]). While we measure C_ℓ at each ℓ in $2 \le \ell \le 1200$, the points with error bars show the binned values of C_ℓ for clarity. The error bars show the standard deviation of C_ℓ from instrumental noise, $[2(2C_\ell N_\ell + N_\ell^2)/(2\ell+1)f_{\mathrm{sky},\ell}^2]^{1/2}$. The shaded area shows the standard deviation from the cosmic variance term, $[2C_\ell^2/(2\ell+1)f_{\mathrm{sky},\ell}^2]^{1/2}$ (except at very low ℓ where the 68% CL from the full non-Gaussian posterior probability is shown). The solid line shows the theoretical curve of the best-fit ΛCDM cosmological model.

The New Error Terms Can Be Important

SC, MNRAS 448 (2015) 2854

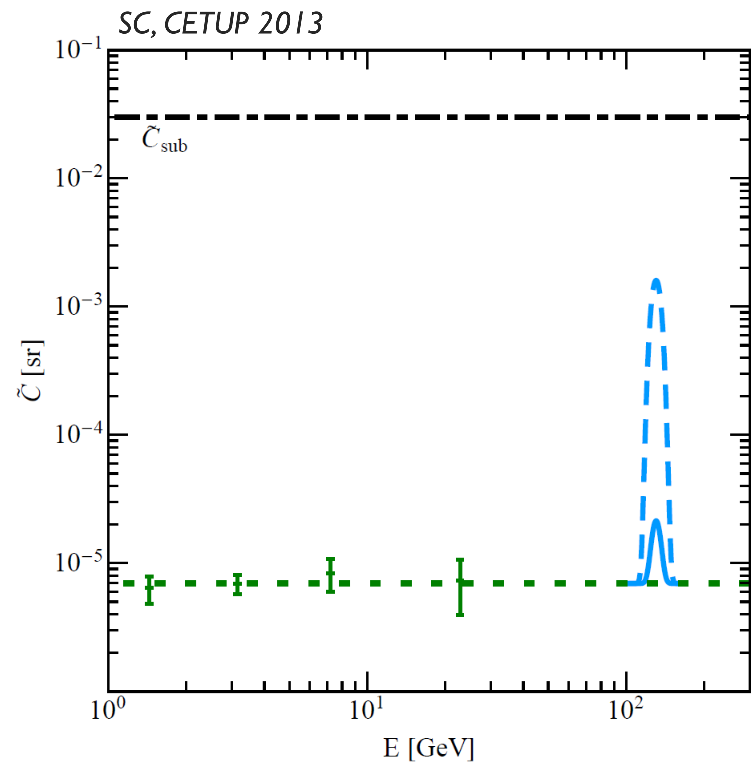


- Example uncertainty evolution at $\ell = 500$ with $\tilde{C}_\ell = 10^{-5}$ sr.

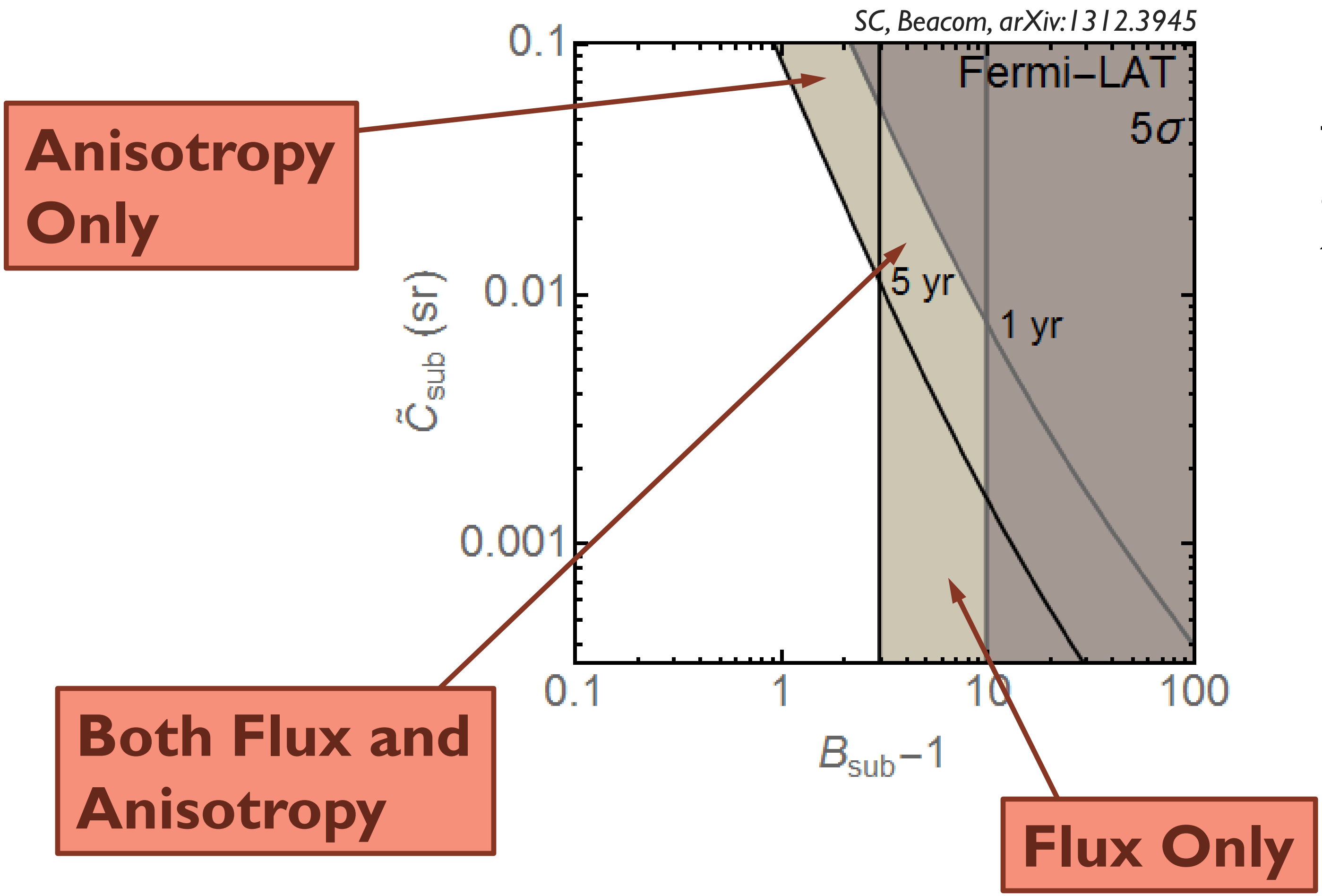
- But is this a real effect? Does a distribution with bispectrum have a different power spectrum uncertainty than one without bispectrum?

Application to the Line Search

- Model:
  - Assume background distribution is the same in each energy bin.
  - Take signal bin at width of energy resolution.
  - Measure error-weighted angular power over $155 \leq \ell \leq 504$.
  - Assume dark matter angular power is dominated by Galactic subhalos.
    - angular power $\tilde{C}_{\rm sub}$
    - intensity boost $B_{\rm sub}$



Sensitivity Depends on Structure



Theoretical constraints on this parameter plane?

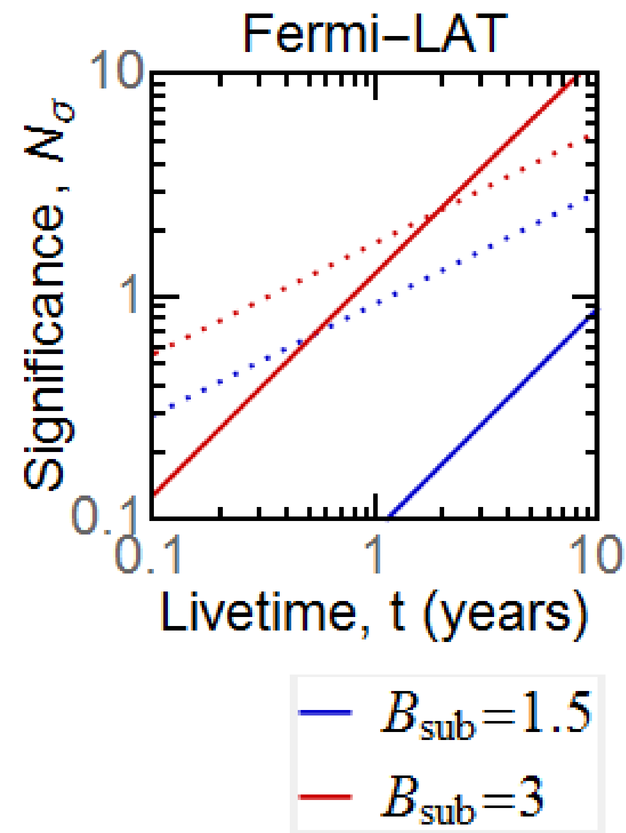
Time-Dependence of Sensitivity

- Anisotropy statistical error shrinks faster when shot-dominated.

$$\sigma_{\tilde{C}} \propto N^{-1} \propto t^{-1}$$

- A 3σ hint of a line in Φ can still be discovered at 5σ in $\tilde{C}_\ell$ first.

- A combined search can be powerful for discovery and signal interpretation!

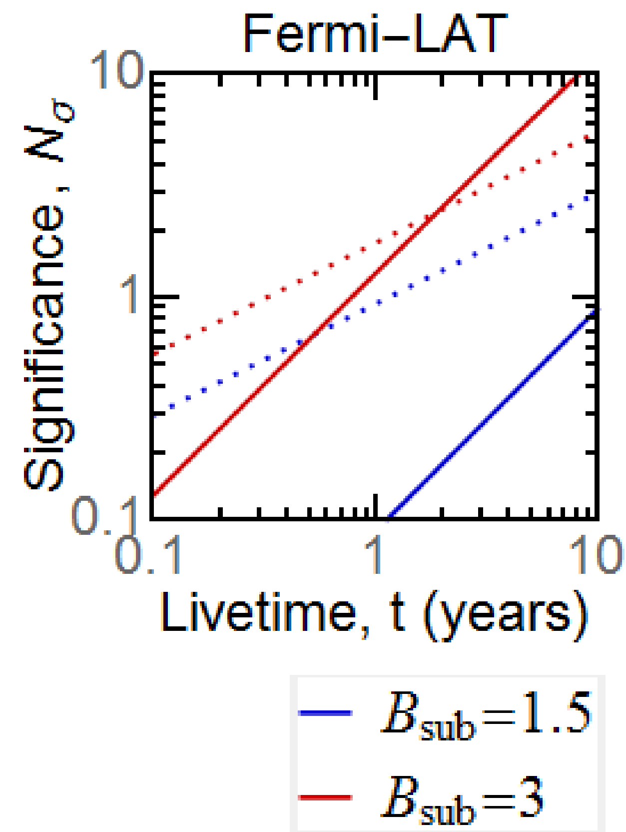


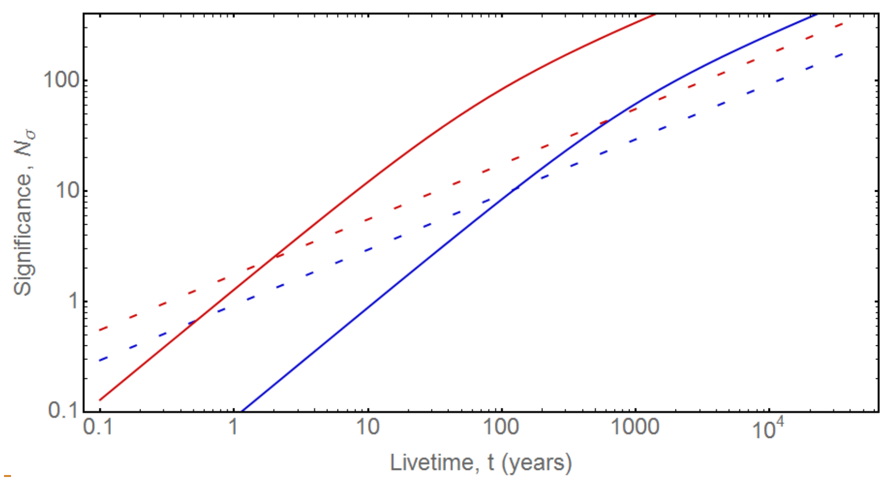
$$\tilde{C}_{\text{sub}} = 0.03 \text{ sr}$$

Time-Dependence of Sensitivity

- A 3σ hint of a line in Φ can still be discovered at 5σ in $\tilde{C}_\ell$.

- A combined search can be powerful for discovery and signal interpretation!





Hypothetical extrapolation shows transition to signal-domination.

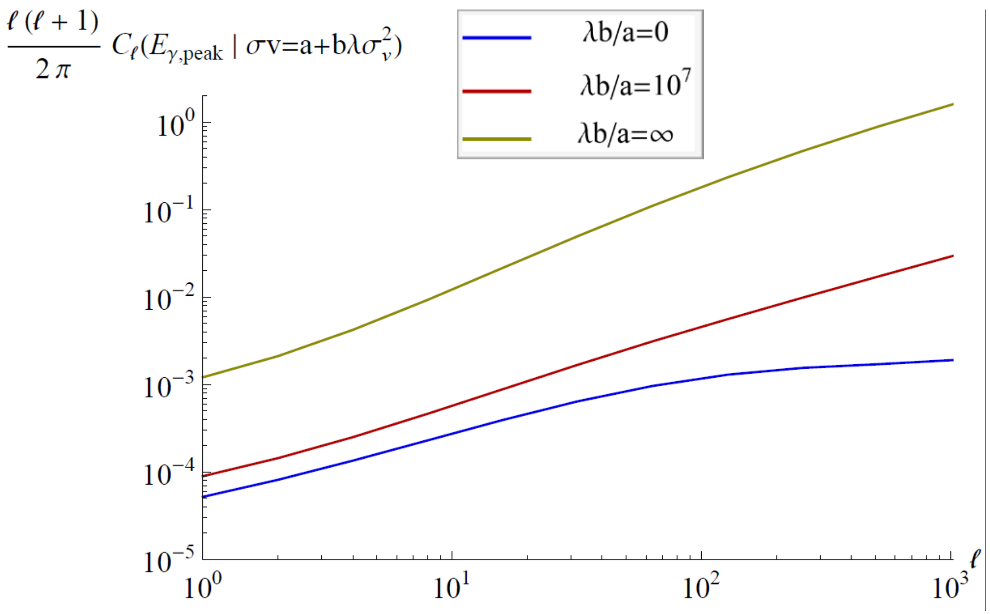
Conclusions

1. Can anisotropy see dark matter?
   Yes, with large enough counts and clustering.
2. Can anisotropy alone discover dark matter?
   Would require large clustering with low intensity boost to beat diffuse flux. "Puffy" subhalos.
   Would need many "dark" subhalos to compete with dwarf galaxies.
3. Can anisotropy constrain particle dark matter models?
   Joint particle/astrophysical distribution constraint. Robust particle constraints only possible if substructure known. Competitiveness depends on the distribution.
4. Can anisotropy teach us anything new about the nature of dark matter? Measurements of small scale distribution!

Denouement: Arnowitt's Question

Does p-wave annihilation in large scale structure affect the anisotropy of radiation?

SC, Dutta, Phys.Rev. D84 (2011) 075004



Ensemble average of smooth NFW halos.

Important energy scale is at $v \sim 10^{-3}$.

Probing spin structure of dark matter interactions.

Extra Slides

A Popular Measure of Angular Distribution: The Angular Power Spectrum

Intensity Angular Power Spectrum C_ℓ

$$I(E,\boldsymbol{n}) - \langle I(E) \rangle = \sum_{\ell,m} a_{\ell m}(E) Y_\ell^m(\boldsymbol{n}) \qquad C_\ell(E) = \frac{1}{2\ell+1} \sum_m |a_{\ell m}(E)|^2$$

- Absolute intensity fluctuations.
- Monotonically increases as sources are added.

Fluctuation Angular Power Spectrum $\widetilde{C_\ell}$

$$\frac{I(E,\boldsymbol{n}) - \langle I(E) \rangle}{\langle I(E) \rangle} = \sum_{\ell,m} \tilde{a}_{\ell m}(E) Y_\ell^m(\boldsymbol{n}) \qquad \widetilde{C_\ell}(E) = \frac{1}{2\ell+1} \sum_m |\tilde{a}_{\ell m}(E)|^2$$

- Relative intensity fluctuations.
- Constant for universal spectrum sources at fixed redshift.

Special Case: Pure Isotropic Source

- Receive N events at uniformly random positions.

$$\tilde{a}_{\ell m,N} = \frac{4\pi}{N}\sum_{i=1}^{N} Y^{*}_{\ell m}(\hat{n}_i) \qquad \tilde{C}_{\ell,N} = \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell} \left|\tilde{a}_{\ell m,N}\right|^2$$

$$\left\langle \tilde{C}_{\ell,N} \right\rangle = \tilde{C}_{P,N} = \frac{4\pi}{N}$$ Shot noise/Poisson noise.

$$\sigma_{\tilde{C}_{\ell,N}} = \sqrt{\frac{2}{2\ell+1}\frac{4\pi}{N}}$$

Error Estimate with Anisotropic Source

- Lesson from CMB: Cosmic Variance
- The dominant statistical uncertainty in CMB anisotropy.
  - Cosmic Variance ⟺ Unknown Initial Conditions
- Assuming the signal is randomly Gaussian distributed, then our estimator for $\tilde{C}_\ell$ is the maximum likelihood estimator with uncertainty:

$$\sigma_{\tilde{C}_\ell} = \sqrt{\frac{2}{2\ell + 1}}\tilde{C}_\ell$$

"Rule of Thumb" Stat. Uncertainty Est.

- Angular power spectrum from "events".
- Assume sources are approximately Gaussian distributed.
- Shot noise is a bias to be subtracted from estimator.

$$\hat{\hat{C}}_{\ell,N} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \left| \frac{4\pi}{N} \sum_{i=1}^{N} Y^*_{\ell m}(\boldsymbol{n}_i) \right|^2 - \frac{4\pi}{N}$$

$$\sigma_{\hat{\hat{C}}_{\ell,N}} = \sqrt{\frac{2}{2\ell+1}} \left(\frac{4\pi}{N} + \tilde{C}_\ell \right)$$

Knox, PRD52, 4307 (1995)

- The goal is to check these standard estimates.

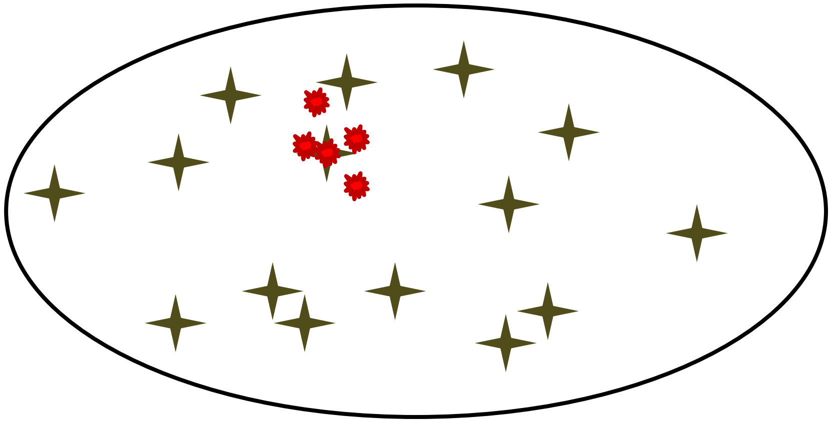
Improving Our Understanding of the Statistical Variance

- Some conceptual difficulties with using the cosmic variance as we did.
  - Cosmic variance is a theoretical error, which applies when making physical inferences about our models based on data.
  - The angular power spectrum measurement should be able to be made independently of any model.
  - We should not need to assume the signal is Gaussian-distributed.

- Investigations have led to a new formula for the model-independent statistical variance of the angular power spectrum of events from a background distribution.

Strategy for Calculation

Consider each event observed at position $\hat{n}'$ but originated from position $\hat{n}$.



1) For fixed source positions $\hat{n}_i$, average over event position $\hat{n}_i{}'$, via the instrument point spread function.

Result of this step: what is being measured is the sky map convolved with the instrument PSF.

2) Average the N events source positions, weighted by the skymap.

Compare to Gaussian Cosmic Variance

- Old method with shot noise + Gaussian cosmic variance:

$$\sigma^2_{\hat{\tilde{C}}_{\ell,N}} = \frac{2}{2\ell+1}\left(\frac{4\pi}{N}+\tilde{C}_\ell\right)^2$$
$$\simeq \left(\frac{4\pi}{N}\right)^2\left[\frac{2}{2\ell+1}+\frac{4N}{2\ell+1}\frac{\tilde{C}_\ell}{4\pi}+\frac{2N^2}{2\ell+1}\left(\frac{\tilde{C}_\ell}{4\pi}\right)^2\right]$$

- New variance formula:

$$\sigma^2_{\hat{\tilde{C}}_{\ell,N}} \simeq \left(\frac{4\pi}{N}\right)^2\left[\frac{2}{2\ell+1}+2\tilde{C}_\ell^{(2)}+\frac{4N}{2\ell+1}\frac{\tilde{C}_\ell}{4\pi}+4N\frac{\tilde{C}_\ell^{(3)}}{4\pi}-4N\left(\frac{\tilde{C}_\ell}{4\pi}\right)^2\right]$$

- The new formula agrees surprisingly well with the traditional estimate, with dominant contributions for a weak signal in precise agreement.
- New terms important at large N. Note no N-independent terms!

Gaussian-Distributed Sky Map

- Our results do not assume Gaussianity.
- If the sky map is Gaussian, then higher order spectra are determined from $\tilde{C}_\ell$ as follows:

$$\left\langle \tilde{C}_\ell^{(2)} \right\rangle = \sum_{\ell'=0}^{2\ell} \frac{2\ell' + 1}{4\pi} \begin{pmatrix} \ell & \ell & \ell' \\ 0 & 0 & 0 \end{pmatrix}^2 \langle \tilde{C}_{\ell'} \rangle$$

$$\left\langle \tilde{C}_\ell^{(3)} \right\rangle = 0$$

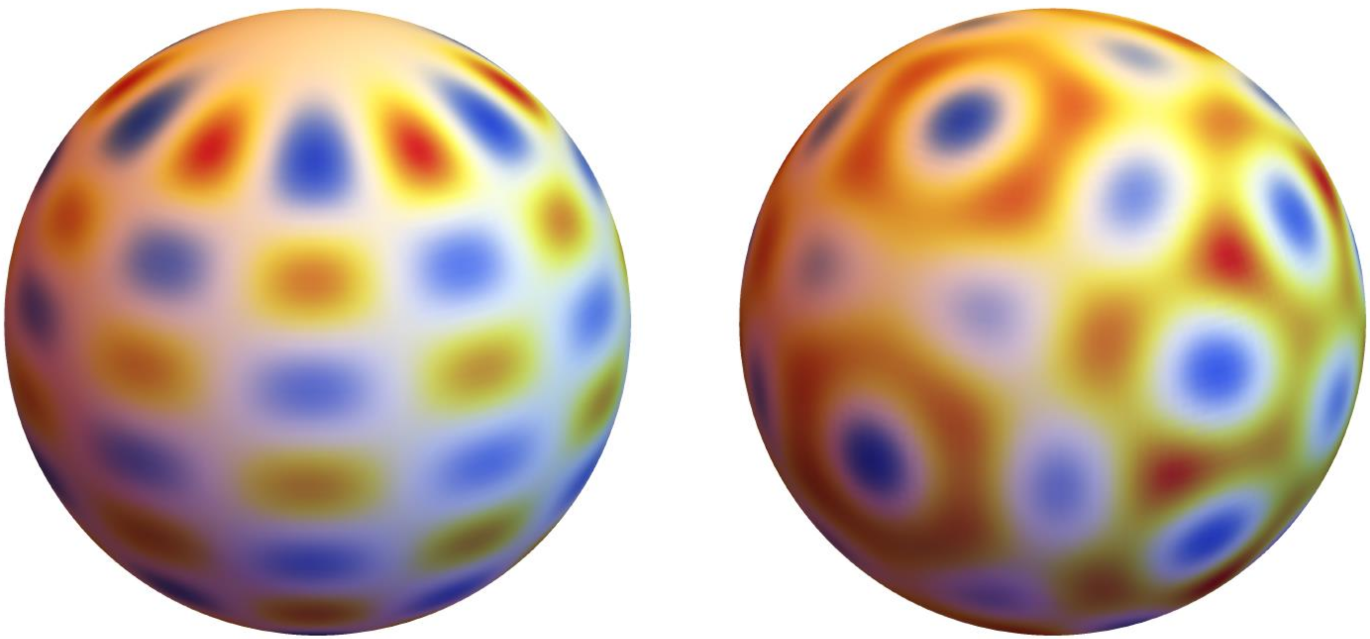
$$\left\langle \tilde{C}_\ell^{(4)} \right\rangle = \frac{2\ell + 3}{2\ell + 1} \langle \tilde{C}_\ell \rangle^2$$

Consequences of Findings

- Experiments using Monte Carlo to estimate error already take into account these new effects automatically.
- Experiments using Gaussian Cosmic Variance may be missing higher orders in the uncertainty of angular power.
  - Fermi-LAT anisotropy measurement should check estimators of these terms for possible corrections to their uncertainties.
  - Small χ^2 suggests either their errors should be smaller (possibly due to some more subtle effects) or energy bins are somehow correlated.
- This error analysis must also take into account effects of:
  - non-uniform exposure,
  - sky masking,
  - other observational bias or instrumental effects.

Test with Monte-Carlo Sampling

SC, MNRAS 448 (2015) 2854

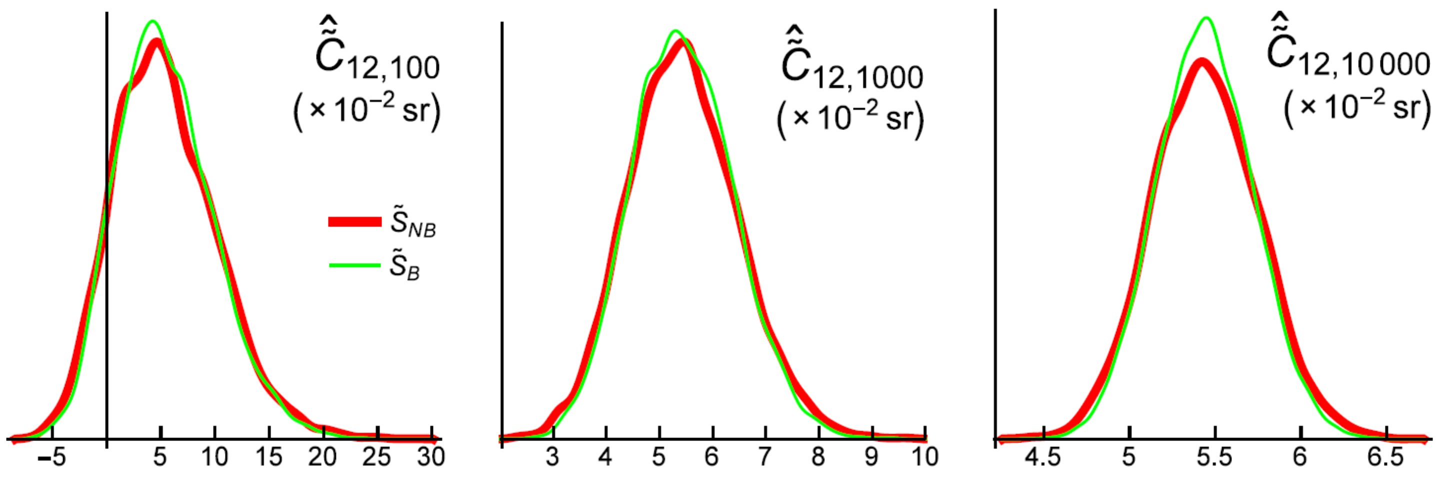


(a) $\tilde{S}_{NB}(\mathbf{n})$ (b) $\tilde{S}_{B}(\mathbf{n})$

$\tilde{C}_\ell = (0.0544 \text{ sr})\, \delta_{\ell,12}$	$\tilde{C}_\ell = (0.0544 \text{ sr})\, \delta_{\ell,12}$
$\tilde{C}^{(3)}_{\ell\ell} = 0$	$\tilde{C}^{(3)}_{\ell\ell} = (-0.000413 \text{ sr})\, \delta_{\ell,12}$

$\hat{\tilde{C}}_{\ell,N}$ Distribution of 10 000 Samplings

SC, MNRAS 448 (2015) 2854

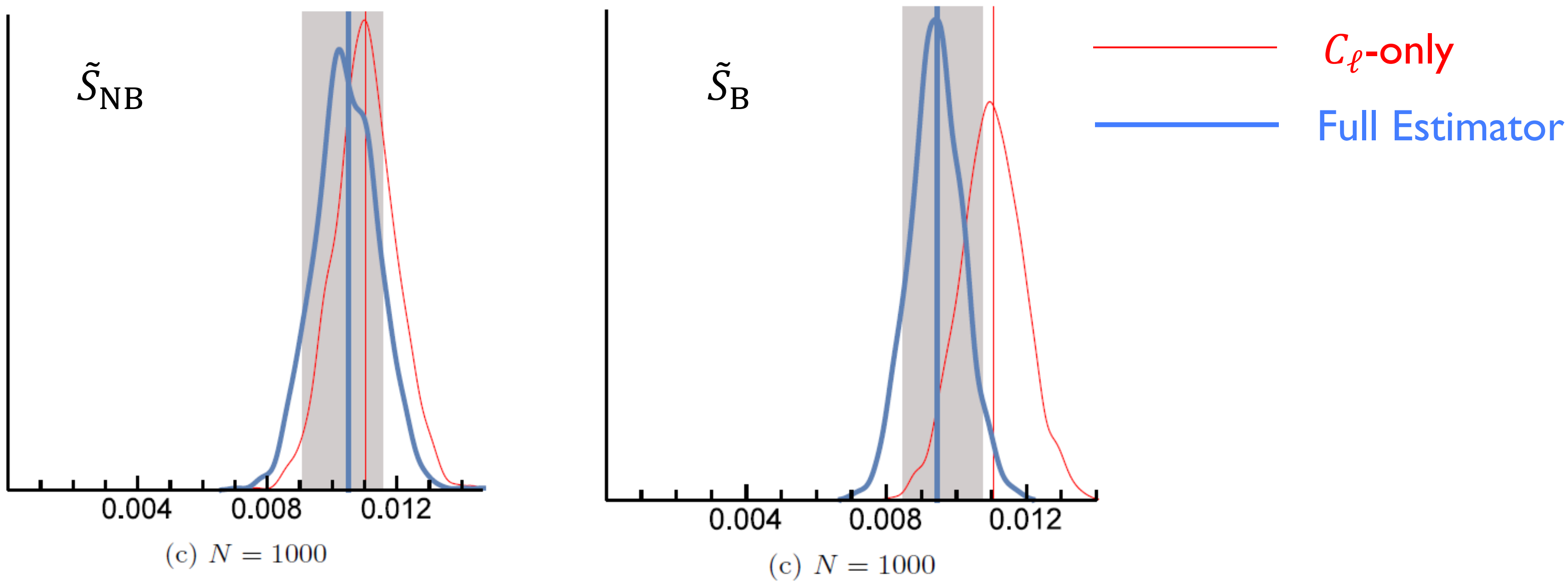


(a) $N = 100$ (b) $N = 1000$ (c) $N = 10000$

- Low counts gives very wide distribution. Shot noise subtraction can give negative power spectrum estimates.
- At high counts, the distribution becomes narrow, and the distribution with negative bispectrum is visibly narrower.

$\sigma_{\hat{\hat{C}}_{\ell,N}}$ Distribution of 10 000 Samplings

SC, MNRAS 448 (2015) 2854



- The negative bispectrum does indeed appear to lower the variance of the power spectrum measurement.
- Even the distribution without bispectrum is affected by the other higher-order spectra, but those effects are small and unresolved in this example.