Computer Assisted BSM Model Building with LieART

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Based on:

"LieART—A Mathematica Application for Lie Algebras and Representation Theory" Robert Feger and TWK, Comput.Phys.Commun. 192 (2015) 166.

TheLieART package is a free download (as a tar.gz archive) from:

http://www.hepforge.org/downloads/lieart/

Inspired by R. Slansky, Phys. Rept., **79**, 1 (1981).

LieART: Lie Algebras and Representation Theory

The primary purpose of the LieART application is the tensor product and subalgebra decomposition of irreducible representations (irreps) of Lie algebras. These tasks are frequently needed in particle physics, where multiparticle fields are assigned to irreps. Tensor products of irreps are thus commonly used to find singlets suitable for the Lagrangian. Spontaneous symmetry breaking corresponds to the subalgebra decomposition.

Algebras

Algebra · **ProductAlgebra** · **CartanMatrix** · **MetricTensor**

Roots

RootSystem — Complete root system of an algebra

PositiveRoots — Only the positive roots of an algebra

OrthogonalSimpleRoots — Simple roots of an algebra in orthogonal coordinates

Weights

WeightSystem — Complete weight system of a representation

Representations

Irrep — Irreducible representation

Dim — Compute the dimension of a representation

YoungTableau — Displays the Young tableau corresponding to an SU(N) representation

ProductIrrep · **DimName** · **Index**

Decompositions

DecomposeProduct — Decompose tensor products of irreps

DecomposeIrrep — Decompose irrep to subalgebras

Displaying Output

IrrepPlus — Textbook style sum of a list irreps

LaTeXForm — Formatting suitable for copy&paste into a LaTeX document

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TUTORIALS

- Quick Start Tutorial

TABLES

- Representation Properties
- Tensor Products
- Branching Rules

Table A.12: SU(12) Irreps

Dynkin label	Dimension (name)	l (index)	Decality	SU(11) singlets	SU(10)×SU(2) singlets	SU(9)×SU(3) singlets	SU(7)×SU(5) singlets	SU(6)×SU(6) singlets
(10000000000)	**12**	1	1	1	0	0	0	0
(01000000000)	**66**	10	2	0	1	0	0	0
(20000000000)	**78**	14	2	1	0	0	0	0
(10000000001)	**143**	24	0	1*	1*	1*	1*	1*
(00100000000)	**220**	45	3	0	0	1	0	0
(30000000000)	**364**	105	3	1	0	0	0	0
(00010000000)	**495**	120	4	0	0	0	0	0
(11000000000)	**572**	141	3	0	0	0	0	0
(01000000001)	**780**	185	1	0	0	0	0	0
(00001000000)	**792**	210	5	0	0	0	1	0

*SU(11)×U(1) and SU(10)×SU(2)×U(1) and SU(9)×SU(3)×U(1) and SU(7)×SU(5)×U(1) and SU(6)×SU(6)×U(1) singlets resp.

Table A.45: SU(12) Tensor Products

$\overline{\mathbf{12}} \times \mathbf{12}$	$=$	$\mathbf{1} + \mathbf{143}$
$\mathbf{12} \times \mathbf{12}$	$=$	$\mathbf{66} + \mathbf{78}$
$\overline{\mathbf{66}} \times \mathbf{12}$	$=$	$\overline{\mathbf{12}} + \overline{\mathbf{780}}$
$\mathbf{66} \times \mathbf{12}$	$=$	$\mathbf{220} + \mathbf{572}$
$\overline{\mathbf{78}} \times \mathbf{12}$	$=$	$\overline{\mathbf{12}} + \overline{\mathbf{924}}$
$\mathbf{78} \times \mathbf{12}$	$=$	$\mathbf{364} + \mathbf{572}$
$\mathbf{143} \times \mathbf{12}$	$=$	$\mathbf{12} + \mathbf{780} + \mathbf{924}$

SU(12) → SU(7)×SU(5)×U(1)		
$\mathbf{12}$	=	$(\mathbf{1},\mathbf{5})(-7)+(\mathbf{7},\mathbf{1})(5)$
$\mathbf{66}$	=	$(\mathbf{1},\mathbf{10})(-14)+(\mathbf{7},\mathbf{5})(-2)+(\mathbf{21},\mathbf{1})(10)$
$\mathbf{78}$	=	$(\mathbf{7},\mathbf{5})(-2)+(\mathbf{1},\mathbf{15})(-14)+(\mathbf{28},\mathbf{1})(10)$
$\mathbf{143}$	=	$(\mathbf{1},\mathbf{1})(0)+(\mathbf{7},\overline{\mathbf{5}})(12)+(\overline{\mathbf{7}},\mathbf{5})(-12)+(\mathbf{1},\mathbf{24})(0)+(\mathbf{48},\mathbf{1})(0)$
$\mathbf{220}$	=	$(\mathbf{1},\overline{\mathbf{10}})(-21)+(\mathbf{7},\mathbf{10})(-9)+(\mathbf{21},\mathbf{5})(3)+(\mathbf{35},\mathbf{1})(15)$
$\mathbf{364}$	=	$(\mathbf{7},\mathbf{15})(-9)+(\mathbf{28},\mathbf{5})(3)+(\mathbf{1},\overline{\mathbf{35}})(-21)+(\mathbf{84},\mathbf{1})(15)$
$\mathbf{495}$	=	$(\mathbf{1},\overline{\mathbf{5}})(-28)+(\mathbf{7},\overline{\mathbf{10}})(-16)+(\mathbf{21},\mathbf{10})(-4)+(\overline{\mathbf{35}},\mathbf{1})(20)+(\mathbf{35},\mathbf{5})(8)$
$\mathbf{572}$	=	$(\mathbf{7},\mathbf{10})(-9)+(\mathbf{7},\mathbf{15})(-9)+(\mathbf{21},\mathbf{5})(3)+(\mathbf{28},\mathbf{5})(3)+(\mathbf{1},\overline{\mathbf{40}})(-21)+(\mathbf{112},\mathbf{1})(15)$
$\mathbf{780}$	=	$(\mathbf{1},\mathbf{5})(-7)+(\mathbf{7},\mathbf{1})(5)+(\overline{\mathbf{7}},\mathbf{10})(-19)+(\mathbf{21},\overline{\mathbf{5}})(17)+(\mathbf{7},\mathbf{24})(5)+(\mathbf{1},\mathbf{45})(-7)+(\mathbf{48},\mathbf{5})(-7)+(\mathbf{140},\mathbf{1})(5)$
$\mathbf{792}$	=	$(\mathbf{1},\mathbf{1})(-35)+(\mathbf{7},\overline{\mathbf{5}})(-23)+(\overline{\mathbf{21}},\mathbf{1})(25)+(\mathbf{21},\overline{\mathbf{10}})(-11)+(\overline{\mathbf{35}},\mathbf{5})(13)+(\mathbf{35},\mathbf{10})(1)$

SU(12) → SU(6)×SU(6)×U(1)		
$\mathbf{12}$	=	$(\mathbf{6},\mathbf{1})(1)+(\mathbf{1},\mathbf{6})(-1)$
$\mathbf{66}$	=	$(\mathbf{6},\mathbf{6})(0)+(\mathbf{15},\mathbf{1})(2)+(\mathbf{1},\mathbf{15})(-2)$
$\mathbf{78}$	=	$(\mathbf{6},\mathbf{6})(0)+(\mathbf{21},\mathbf{1})(2)+(\mathbf{1},\mathbf{21})(-2)$
$\mathbf{143}$	=	$(\mathbf{1},\mathbf{1})(0)+(\mathbf{6},\overline{\mathbf{6}})(2)+(\overline{\mathbf{6}},\mathbf{6})(-2)+(\mathbf{35},\mathbf{1})(0)+(\mathbf{1},\mathbf{35})(0)$
$\mathbf{220}$	=	$(\mathbf{6},\mathbf{15})(-1)+(\mathbf{15},\mathbf{6})(1)+(\mathbf{20},\mathbf{1})(3)+(\mathbf{1},\mathbf{20})(-3)$
$\mathbf{364}$	=	$(\mathbf{21},\mathbf{6})(1)+(\mathbf{6},\mathbf{21})(-1)+(\mathbf{56},\mathbf{1})(3)+(\mathbf{1},\mathbf{56})(-3)$
$\mathbf{495}$	=	$(\overline{\mathbf{15}},\mathbf{1})(4)+(\mathbf{1},\overline{\mathbf{15}})(-4)+(\mathbf{6},\mathbf{20})(-2)+(\mathbf{20},\mathbf{6})(2)+(\mathbf{15},\mathbf{15})(0)$
$\mathbf{572}$	=	$(\mathbf{6},\mathbf{15})(-1)+(\mathbf{15},\mathbf{6})(1)+(\mathbf{21},\mathbf{6})(1)+(\mathbf{6},\mathbf{21})(-1)+(\mathbf{70},\mathbf{1})(3)+(\mathbf{1},\mathbf{70})(-3)$
$\mathbf{780}$	=	$(\mathbf{6},\mathbf{1})(1)+(\mathbf{1},\mathbf{6})(-1)+(\mathbf{15},\overline{\mathbf{6}})(3)+(\overline{\mathbf{6}},\mathbf{15})(-3)+(\mathbf{35},\mathbf{6})(-1)+(\mathbf{6},\mathbf{35})(1)+(\mathbf{84},\mathbf{1})(1)+(\mathbf{1},\mathbf{84})(-1)$
$\mathbf{792}$	=	$(\overline{\mathbf{6}},\mathbf{1})(5)+(\mathbf{1},\overline{\mathbf{6}})(-5)+(\mathbf{6},\overline{\mathbf{15}})(-3)+(\overline{\mathbf{15}},\mathbf{6})(3)+(\mathbf{15},\mathbf{20})(-1)+(\mathbf{20},\mathbf{15})(1)$

Dynamic, so tables can be extended, etc.

LieART - Quick Start Tutorial

The application LieART provides functions to perform the most common tasks associated with irreducible representations (irreps) of the classical and exceptional Lie algebas.

This loads the package:

```
In[5]:= << LieART`
```

LieART 1.1.1

last revised 3 February 2014

Entering Irreducible Representations

Irreducible representations (irreps) are internally decribed by their Dynkin label with a combined head of `Irrep` and the Lie algebra.

`Irrep`[*algebraClass*][*label*]	Irrep descibed by its *algebraClass* and Dynkin *label*.

Entering irreps by Dynkin label.

The *algebraClass* follows the Dynkin classification of simple Lie algebras and can only be `A`, `B`, `C`, `D` for the classical algebras and `E6`, `E7`, `E8`, `F4` and `G2` for the exceptional algebras. The precise classical algebra is determined by the length of the Dynkin label.

Entering the $\overline{\mathbf{10}}$ of SU(5) by its Dynkin label and algebra class:

```
In[1]:= Irrep[A][0, 0, 1, 0] // FullForm
```

Out[1]//FullForm=

Irrep[A][0, 0, 1, 0]

In `StandardForm` the irrep is displayed in the textbook notation of Dynkin labels :

```
In[2]:= Irrep[A][0, 0, 1, 0] // StandardForm
```

Out[2]//StandardForm=

(0010)

In `TraditionalForm` (default) the irrep is displayed by its dimensional name:

```
In[3]:= Irrep[A][0, 0, 1, 0]
```

Out[3]= $\overline{\mathbf{10}}$

The default output format type of LieART is `TraditionalForm`. The associated user setting is overwritten for the notebook LieART is loaded in. For `StandardForm` as output format type please set the global variable `$DefaultOutputForm=StandardForm`.

Decomposing Tensor Products

`DecomposeProduct[`*irreps*`]`	Decomposes the tensor product of several *irreps*.

Tensor product decomposition.

Decompose the tensor product $\mathbf{3}\otimes\overline{\mathbf{3}}$ of SU(3):

```
In[12]:= DecomposeProduct[Irrep[SU3][3], Irrep[SU3][Bar[3]]]
```

Out[12]= $\mathbf{1}+\mathbf{8}$

```
In[4]:= DecomposeProduct[Irrep[SU3][3],
         Irrep[SU3][Bar[3]], Irrep[SU3][3], Irrep[SU3][Bar[3]]]
```

Out[4]= $2(\mathbf{1})+4(\mathbf{8})+\mathbf{10}+\overline{\mathbf{10}}+\mathbf{27}$

Decompose the tensor product $\mathbf{27}\otimes\overline{\mathbf{27}}$ of E_6:

```
In[13]:= DecomposeProduct[Irrep[E6][27], Irrep[E6][Bar[27]]]
```

Out[13]= $\mathbf{1}+\mathbf{78}+\mathbf{650}$

Decompose the tensor product $\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}$ of SU(3):

```
In[14]:= DecomposeProduct[Irrep[SU3][3], Irrep[SU3][3], Irrep[SU3][3]]
```

Out[14]= $\mathbf{1}+2(\mathbf{8})+\mathbf{10}$

Decompose the tensor product $\mathbf{8}\otimes\mathbf{8}$ of SU(3):

```
In[15]:= DecomposeProduct[Irrep[SU3][8], Irrep[SU3][8]]
```

Out[15]= $\mathbf{1}+2(\mathbf{8})+\mathbf{10}+\overline{\mathbf{10}}+\mathbf{27}$

Decomposition to Subalgebras

`DecomposeIrrep[irrep, subalgebra]`	Decomposes *irrep* to the specified *subalgebra*.
`DecomposeIrrep[pIrrep, subalgebra, pos]`	Decompose *pirrep* at position *pos* of the product irrep *pIrrep*.

Decompose irreps and product irreps.

Decompose the $\overline{\mathbf{10}}$ of SU(5) to SU(3)⊗SU(2)⊗U(1):

```
In[22]:= DecomposeIrrep[Irrep[SU5][Bar[10]], ProductAlgebra[SU3, SU2, U1]]
```

Out[22]= $(\mathbf{1}, \mathbf{1})(6) + (\mathbf{3}, \mathbf{1})(-4) + (\overline{\mathbf{3}}, \mathbf{2})(1)$

Decompose the **10** and the $\overline{\mathbf{5}}$ of SU(5) to SU(3)⊗SU(2)⊗U(1) (`DecomposeIrrep` is `Listable`):

```
In[23]:= DecomposeIrrep[{Irrep[SU5][10], Irrep[SU5][Bar[5]]}, ProductAlgebra[SU3, SU2, U1]]
```

Out[23]= $\{(\overline{\mathbf{3}}, \mathbf{1})(4) + (\mathbf{3}, \mathbf{2})(-1) + (\mathbf{1}, \mathbf{1})(-6), (\overline{\mathbf{3}}, \mathbf{1})(-2) + (\mathbf{1}, \mathbf{2})(3)\}$

Decompose the **16** of SO(10) to SU(5)⊗U(1):

```
In[24]:= DecomposeIrrep[Irrep[SO10][16], ProductAlgebra[SU5, U1]]
```

Out[24]= $(\mathbf{1})(-5) + (\overline{\mathbf{5}})(3) + (\mathbf{10})(-1)$

Decompose the **27** of E_6 to SU(3)⊗SU(3)⊗SU(3):

```
In[25]:= DecomposeIrrep[Irrep[E6][27], ProductAlgebra[SU3, SU3, SU3]]
```

Out[25]= $(\mathbf{3}, \mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{3}, \overline{\mathbf{3}}) + (\overline{\mathbf{3}}, \overline{\mathbf{3}}, \mathbf{1})$

Example: Decomposition 248^6 in E_8

```
In[102]:= Timing[Irrep[E8][248]^6]
Out[102]:=
{90.112, 79(1) + 421(248) + 575(3875) + 675(27000) + 924(30380) + 775(147250) + 1386(779247) +
 415(1763125) + 1011(2450240) + 1240(4096000) + 405(4881384) + 765(6696000) +
 1144(26411008) + 895(70680000) + 1125(76271625) + 115(79143000) + 554(146325270) +
 410(203205000) + 510(281545875) + 456(301694976) + 855(344452500) + 315(820260000) +
 605(1094951000) + 405(2172667860) + 470(2275896000) + 15(2642777280) + 15(2903770000) +
 195(3929713760) + 45(4076399250) + 325(4825673125) + 125(6899079264) + 480(8634368000) +
 80(8634368000′) + 80(12692520960) + 115(17535336000) + 216(20288765952) +
 165(21039669000) + 180(23592339045) + 144(45329752170) + 45(63513702720) +
 45(66393847000) + 69176971200 + 90(83080364250) + 90(85424220000) + 40(110977024000) +
 40(124436480000) + 15(152883490500) + 15(220778105625) + 80(234550030000) +
 267413986840 + 30(355647996000) + 5(417933862500) + 5(492957660000) +
 45(508731738750) + 45(574197082368) + 5(627099023250) + 9(841900509450) +
 5(919045960000) + 10(1041872676000) + 9(1283242632840) + 10(1349926375875) +
 16(1813461073920)}
```

Takes 90 sec on 4 year old iMac.

Check: A179663 in "Online Encyclopedia of Integral Sequences," Power of the adjoint representation of E8.

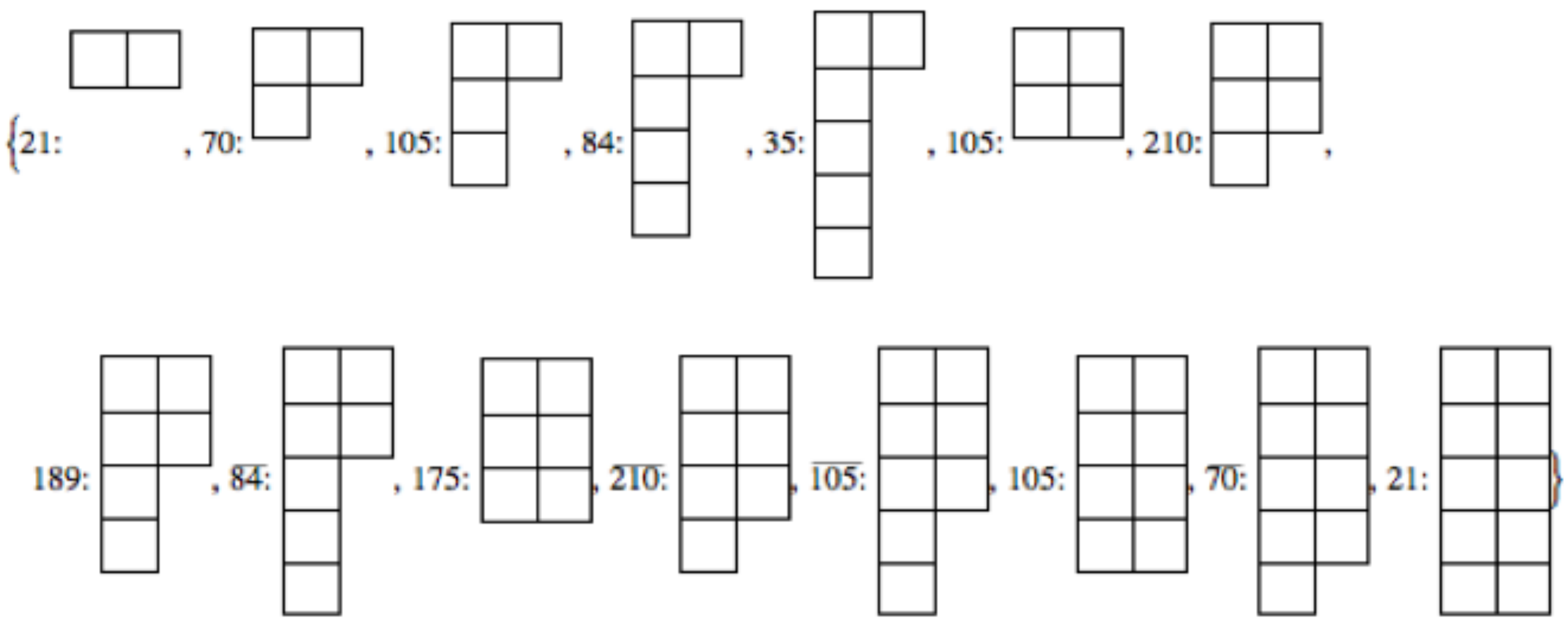
1, 1, 5, 16, 79, **421**, 2674, 19244, 156612, 1423028, 14320350, …

LieART Applications

Example:Three family SU(N) models with no chiral exotic fermions, made without using any totally antisymmetric irreps (i.e., without using any single column tableaux)

Robert Feger and TWK 1505.03403

R. Fonseca has obtained some related results with Susyno see: 1504.03695



All two column tableaux for SU(6).

SU(6) example with three families and no light exotics

$$6(\overline{21}) + 9(70) + 6(84) + 9(\overline{105}) + 3(\overline{105}') + 3(210),$$

$$= 3(10+\bar{5})+a(5+\bar{5})+b(15+\overline{15})+c(40+\overline{40})+\ldots$$

All irreps are two column tableaux

$$2(\overline{28}) + 112 + 4(140) + 5(\overline{224}) + 490 + 2(\overline{490}') + 2(588);$$

$$\overline{28} + 112 + 5(140) + \overline{196} + 210 + 5(\overline{224}) + 490 + \overline{490}' + 588;$$

$$112 + 6(140) + 2(\overline{196}) + 2(210) + 5(\overline{224}) + 490;$$

$$7(\overline{28}) + 4(112) + 140 + 196 + 3(\overline{490}) + 2(490') + 588;$$

$$6(\overline{28}) + 4(112) + 2(140) + 210 + 3(\overline{490}) + 3(490');$$

$$3(\overline{28}) + \overline{112} + 140 + 3(196) + 2(210) + 3(\overline{224}) + 3(\overline{490}) + 3(588);$$

$$2(\overline{28}) + \overline{112} + 2(140) + 2(196) + 3(210) + 3(\overline{224}) + 3(\overline{490}) + 490' + 2(588);$$

$$\overline{28} + \overline{112} + 3(140) + 196 + 4(210) + 3(\overline{224}) + 3(\overline{490}) + 2(490') + 588;$$

$$\overline{112} + 4(140) + 5(210) + 3(\overline{224}) + 3(\overline{490}) + 3(490').$$

Three family solutions in SU(7)
made from only two column tableaux.

28	112	140	196	210	224	490	490′	588
−2	1	4	0	0	−5	1	−2	2
−1	1	5	−1	1	−5	1	−1	1
0	1	6	−2	2	−5	1	0	0
−7	4	1	1	0	0	−3	2	1
−6	4	2	0	1	0	−3	3	0
−3	−1	1	3	2	−3	−3	0	3
−2	−1	2	2	3	−3	−3	1	2
−1	−1	3	1	4	−3	−3	2	1
0	−1	4	0	5	−3	−3	3	0

SU(7) examples in tabular form.
(minus signs for barred irreps)

36	168	216	336	378	420	504	1008	1176	1344	1512	$2352'$
−2	1	2	−1	1	−1	−1	0	1	1	0	−1
1	−1	3	0	2	0	−4	0	−1	−1	2	0
0	−1	1	−1	1	−1	2	2	−1	2	−4	2
−2	1	3	0	−2	−3	−2	2	−2	1	1	0
2	−3	0	−1	2	−1	−3	2	0	3	−1	−1
3	−3	3	1	2	−2	2	−1	0	0	−1	1

Two column three family SU(8) solutions

45	240	315	540	630	720	1008	1050	1890	2520	2700	3402	3780	5292	6048	7560
0	−1	−1	−1	0	1	−2	−3	1	1	0	0	0	−3	0	2
−1	1	1	0	0	1	2	2	0	−1	1	0	−4	−1	1	2
1	−1	0	1	1	1	−2	−2	−1	−1	2	3	−2	0	0	0

The reason the number of solutions is dropping with N is that the anomaly coefficients are growing and it is getting more difficult to find anomaly free sets of two column tableau to

Two column three family SU(9) solutions

Another example:

Analysis of Inert Higgs models:
Where do the
doublets of SU(5) live?

T.C.Yuan and TWK, in progress

$SU(5)$ irreps of dimension less than 1000 that contain $SU(3) \times SU(2) \times U(1)$ doublets.

$SU(5)$ irrep	doublet	$SU(5)$ irrep	doublet
5	$(1,2)_{-3}$	280	$(1,2)_{-3}$
40	$(1,2)_{9}$	450	$(1,2)_{9}$
45	$(1,2)_{-3}$	$450'$	$(1,2)_{-3}$
70	$(1,2)_{-3}$	480	$(1,2)_{-3}$
$175'$	$(1,2)_{15}$	$560'$	$(1,2)_{-21}$
210	$(1,2)_{9}$	$700'$	$(1,2)_{15}$

Minimal Flavor Symmetry

Work in collaboration with

Carl Albright, NIU/Fermilab

and

Robert Feger, Würzburg University

Is there a flavor symmetry (FLASY)?

Anarchy/Landscape

Discrete flavor symmetry

Continuous flavor symmetry

If anarchy or landscape,
then masses and mixings are random numbers
and we are done--values cannot be determined.

Flavor symmetries are more
interesting and they are predictive.

Large spectrum of models has been studied:

Discrete flavor symmetry X Standard Model(SM)

Continuous flavor symmetry X SM

or

Discrete flavor symmetry X Beyond the SM (BSM)

Continuous flavor symmetry X BSM

Better results with increasing discrete group size--can we reduce it/minimize it?

Discrete FLASY X SM has been the most studied.

For reviews see:

Altarelli and Feruglio, Rev Mod Phys (2010)

Ishimori, Kobayashi, Ohki, Shimizu,
Okada and Tanimoto, Prog Theo Phys Sup (2010)

King and Luhn
Published in Rept. Prog. Phys. (2013)

Some problems

Discrete flavor symmetry problems

Origin of flavor symmetry?

Global discrete FLASY--violated by quantum gravity and fails to give desired results at EW scale

Gauge discrete FLASY--must break continuous symmetry to discrete--check for discrete anomaly cancelation

Luhn and Ramond, JHEP (2008);
Luhn, PLB(2009).

Recent progress on breaking continuous to discrete

SU(3) example

see:

Ludl, J Phys A (2010)

Grimus and Ludl, J Phys A (2011)

Merle and Zwicky, JHEP (2012)

Discrete flavor symmetry problems

Most severe: ``FLASY irrep problem''
How do SM fermions irreps get into Discrete FLASY irreps?

Requires initial gauge group G where contains FLASY X SM and irreps

$R \longrightarrow (R_{Flav}, R_{SM})$
not $R \longrightarrow (R_{Flav}, 1) + (1, R_{SM})$

Domain walls at EW scale from discrete FLASY breaking and other cosmic defects

Is there a flavor symmetry?

Masses and mixings from textures with
no or small discrete flavor group?

SU(7) and SU(8) models (S. Barr, PRD 2008)
High scale M, SU(5) scale v_5, EW scale v_{SM}

Small parameter
$\epsilon = v_5/M$

Is there a flavor symmetry?

top mass

$m_t \sim v_{SM}$

b-quark mass

$m_b \sim \epsilon\, v_{SM}$

etc.

Equalities with O(1) Yukawa couplings. All done with choice of SU(N) irreps—no discrete symmetry—but no mixings yet.

Is there a flavor symmetry?

Extend Barr's results to include mixings:

Difficult without discrete group

Keep SU(N) and include small discrete group

Example in SU(9) X Z_2 X Z_2 X Z_2 X Z_3
Model found by hand (computer assisted).
J. Dent, R. Feger, TWK, and S. Nandi, PLB (2011)

Needs full model scan (LieART).

Reducing the flavor symmetry

Strategy:

Increase gauge group G size,
Decrease discrete group D size:

Typically G = SM and D = S_3, A_4, S_4,
T ′, Delta(27), PSL(2, 7) … X Z_n's.

Similar to SU(9) example, try
G = SU(N) and D = Z_{n1} X Z_{n2} X …

Reducing the flavor symmetry

Is there an N where D = 1?

Scan hundreds of models to find:
Yes! Examples for N=12

C. Albright, R. Feger and TWK, PRD (2012)

Example of a model with no discrete flavor symmetry:

$SU(12)$ model of masses and mixings

- Anomaly free fermion set: (totally antisym irreps, so no exotics)

$$6(\mathbf{495}) + 4(\overline{\mathbf{792}}) + 4(\overline{\mathbf{220}}) + (\overline{\mathbf{66}}) + 4(\overline{\mathbf{12}}) \to 3(\mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}) + Real$$

$$\text{Real} = (\mathbf{5} + \overline{\mathbf{5}})s + (\mathbf{10} + \overline{\mathbf{10}})s + \text{singlets}$$

- The $SU(5)$ singlet VEV $v_5 = \langle 1 \rangle_5$ and $SU(12)$ unification scale M appear in the ratio:

$$\epsilon = \frac{v_5}{M} \sim \frac{1}{50}.$$

$SU(12)$ **model of masses and mixings**

- Yukawa interactions of dimension $4+n$ give rise to mass matrix elements of the form:

$$h_{ij}\epsilon^n v_{SM}\, u_{iL}^T u_{jL}^c\,,$$

- h_{ij} are Yukawa couplings
- $v_{SM} = 174$ GeV is the EW scale.

$SU(12)$ model of masses and mixings

- Fermion Assignments: Assign family components to $SU(12)$ irreps.

First Family	$(\mathbf{10})\mathbf{495}_1$	$\to u_L, u^c_L, d_L, e^c_L$
	$(\overline{\mathbf{5}})\overline{\mathbf{66}}_1$	$\to d^c_L, e_L, \nu_{1,L}$
	$(\mathbf{1})\overline{\mathbf{792}}_1$	$\to N^c_{1,L}$
Second Family	$(\mathbf{10})\overline{\mathbf{792}}_2$	$\to c_L, c^c_L, s_L, \mu^c_L$
	$(\overline{\mathbf{5}})\overline{\mathbf{792}}_2$	$\to s^c_L, \mu_L, \nu_{2,L}$
	$(\mathbf{1})\overline{\mathbf{220}}_2$	$\to N^c_{2,L}$
Third Family	$(\mathbf{10})\overline{\mathbf{220}}_3$	$\to t_L, t^c_L, b_L, \tau^c_L$
	$(\overline{\mathbf{5}})\overline{\mathbf{792}}_3$	$\to b^c_L, \tau_L, \nu_{3,L}$
	$(\mathbf{1})\overline{\mathbf{12}}_3$	$\to N^c_{3,L}$

$SU(12)$ model of masses and mixings

- Higgs bosons:

  $(\mathbf{5})\mathbf{924}_{\mathrm{H}},(\bar{\mathbf{5}})\mathbf{924}_{\mathrm{H}}$

  $(\mathbf{1})\mathbf{66}_{\mathrm{H}},\ (\mathbf{1})\overline{\mathbf{66}}_{\mathrm{H}}$

  $(\mathbf{1})\mathbf{220}_{\mathrm{H}},\ (\mathbf{1})\overline{\mathbf{220}}_{\mathrm{H}}$

  $(\mathbf{24})\mathbf{143}_{\mathrm{H}}$

  Massive fermion:

  $\mathbf{220}\times\overline{\mathbf{220}}$

  $\mathbf{792}\times\overline{\mathbf{792}}$

Typical Model: raw output 10/13/2014

Model 1: SU(12), 7 Higgs, 10 massive fermions,	
Anom. Free Set:	$(66) + (495) + 2\,(\overline{220}) + 2\,(\overline{12})$
SU(5) Level:	$21\,(\overline{10}) + 24\,(10) + 45\,(\overline{5}) + 42\,(5) + 140\,(1)$
Fermions:	$(10)495_1,\ (10)\overline{220}_2,\ (10)66_3,\ (\overline{5})\overline{12}_1,\ (\overline{5})\overline{220}_2,\ (\overline{5})\overline{12}_3,\ (1)1_1,\ (1)1_2,\ (1)1_3$
Higgs:	$(5)\overline{495},\ (\overline{5})495,\ (1)66,\ (1)\overline{66},\ (1)792,\ (1)\overline{792},\ (24)143$
Massive Fermions:	$12,\ \overline{12},\ 66,\ \overline{66},\ 220,\ \overline{220},\ 495,\ \overline{495},\ 792,\ \overline{792}$

Mass Matrices:

$$M_U=\begin{pmatrix} h^u_{11}\left(2\zeta^*\epsilon^2+(\zeta^*)^2\right) & h^u_{12}\left(\epsilon^3+2(\zeta^*)^2\epsilon-\frac{2\kappa\zeta^*\epsilon}{3}+\zeta^*\epsilon\right) & h^u_{13}\left(\epsilon^2+\zeta^*\right) \\ h^u_{12}\left(\epsilon^3+2(\zeta^*)^2\epsilon+\frac{2\kappa\zeta^*\epsilon}{3}+\zeta^*\epsilon\right) & h^u_{22}\left(4\zeta^*\epsilon^2+\epsilon^2\right) & h^u_{23}\left(\frac{2\kappa\epsilon}{3}+2\zeta^*\epsilon+\epsilon\right) \\ h^u_{13}\left(\epsilon^2+\zeta^*\right) & h^u_{23}\left(-\frac{2\kappa\epsilon}{3}+2\zeta^*\epsilon+\epsilon\right) & h^u_{33} \end{pmatrix} v_u$$

$$M_D=\begin{pmatrix} h^d_{11}\left(\zeta^*\epsilon+\epsilon\right) & h^d_{12}\left(\epsilon^3+\zeta\epsilon+\zeta\zeta^*\epsilon\right) & h^d_{13}\left(\zeta^*\epsilon+\epsilon\right) \\ h^d_{21} & h^d_{22}\left(\epsilon^2+\zeta\right) & h^d_{23} \\ h^d_{31}\left(\frac{2\kappa\epsilon}{3}+2\zeta\epsilon+\epsilon\right) & h^d_{32}\left(\epsilon^3+2\zeta^2\epsilon+\frac{2\kappa\zeta\epsilon}{3}+\zeta\epsilon\right) & h^d_{33}\left(\frac{2\kappa\epsilon}{3}+2\zeta\epsilon+\epsilon\right) \end{pmatrix} v_d$$

$$M_L=\begin{pmatrix} h^l_{11}\left(\zeta^*\epsilon+\epsilon\right) & h^l_{12} & h^l_{13}\left(\kappa\epsilon+2\zeta\epsilon+\epsilon\right) \\ h^l_{21}\left(\epsilon^3+\zeta\epsilon+\zeta\zeta^*\epsilon\right) & h^l_{22}\left(\epsilon^2+\zeta\right) & h^l_{23}\left(\epsilon^3+2\zeta^2\epsilon+\kappa\zeta\epsilon+\zeta\epsilon\right) \\ h^l_{31}\left(\zeta^*\epsilon+\epsilon\right) & h^l_{32} & h^l_{33}\left(\kappa\epsilon+2\zeta\epsilon+\epsilon\right) \end{pmatrix} v_d$$

$$M_{DN}=\begin{pmatrix} h^{dn}_{11}\left(-3\kappa\epsilon+4\zeta^*\epsilon+2\epsilon\right) & h^{dn}_{12}\left(-3\kappa\epsilon+4\zeta^*\epsilon+2\epsilon\right) & h^{dn}_{13}\left(-3\kappa\epsilon+4\zeta^*\epsilon+2\epsilon\right) \\ h^{dn}_{21}\left(-\kappa\epsilon+4\zeta\epsilon+2\epsilon\right) & h^{dn}_{22}\left(-\kappa\epsilon+4\zeta\epsilon+2\epsilon\right) & h^{dn}_{23}\left(-\kappa\epsilon+4\zeta\epsilon+2\epsilon\right) \\ h^{dn}_{31}\left(-3\kappa\epsilon+4\zeta^*\epsilon+2\epsilon\right) & h^{dn}_{32}\left(-3\kappa\epsilon+4\zeta^*\epsilon+2\epsilon\right) & h^{dn}_{33}\left(-3\kappa\epsilon+4\zeta^*\epsilon+2\epsilon\right) \end{pmatrix} v_u$$

$$M_{MN}=\begin{pmatrix} h^{mn}_{11}\left(3\zeta\epsilon^2+3\zeta^*\epsilon^2+2\epsilon^2+2\zeta\zeta^*\right) & h^{mn}_{12}\left(3\zeta\epsilon^2+3\zeta^*\epsilon^2+2\epsilon^2+2\zeta\zeta^*\right) & h^{mn}_{13}\left(3\zeta\epsilon^2+3\zeta^*\epsilon^2+2\epsilon^2+2\zeta\zeta^*\right) \\ h^{mn}_{12}\left(3\zeta\epsilon^2+3\zeta^*\epsilon^2+2\epsilon^2+2\zeta\zeta^*\right) & h^{mn}_{22}\left(3\zeta\epsilon^2+3\zeta^*\epsilon^2+2\epsilon^2+2\zeta\zeta^*\right) & h^{mn}_{23}\left(3\zeta\epsilon^2+3\zeta^*\epsilon^2+2\epsilon^2+2\zeta\zeta^*\right) \\ h^{mn}_{13}\left(3\zeta\epsilon^2+3\zeta^*\epsilon^2+2\epsilon^2+2\zeta\zeta^*\right) & h^{mn}_{23}\left(3\zeta\epsilon^2+3\zeta^*\epsilon^2+2\epsilon^2+2\zeta\zeta^*\right) & h^{mn}_{33}\left(3\zeta\epsilon^2+3\zeta^*\epsilon^2+2\epsilon^2+2\zeta\zeta^*\right) \end{pmatrix} \Lambda_R$$

Quark Pheno fit:

ϵ=0.0139093

κ=1.48907

ζ=0.01391 $e^{167.8°i}$

v_u=173.951 GeV

v_d=4.10861 GeV

χ^2_q= 1.32×10^{-7}

Up−type masses	Down−type masses	Charged Lepton Masses
m_u = 0.4877 MeV	m_d = 0.5093 MeV	m_e = 0.206 MeV
m_c = 237. MeV	m_s = 10. MeV	m_μ = 43.5 MeV
m_t = 94.7 GeV	m_b = 0.61 GeV	m_τ = 0.7734 GeV

CKM Angles	CKM Phase	CKM Matrix
θ_{12} = 13.04° θ_{23} = 2.38° θ_{13} = 0.2007°	δ = 68.76°	$\begin{pmatrix} 0.97421 & 0.22563 & 0.003503e^{-68.76°i} \\ 0.2255e^{-180.°i} & 0.9734e^{-0.001803°i} & 0.041527 \\ 0.008733e^{-21.34°i} & 0.04075e^{-179.°i} & 0.99913 \end{pmatrix}$

measured:

CKM Angles	CKM Phase	CKM Matrix
$\theta_{12} = 13.04°$ $\theta_{23} = 2.38°$ $\theta_{13} = 0.201°$	$\delta = 68.75°$	$\begin{pmatrix} 0.97421 & 0.22563 & 0.003508e^{-68.75° i} \\ 0.2255e^{-180.° i} & 0.9734e^{-0.001803° i} & 0.041527 \\ 0.008733e^{-21.37° i} & 0.04075e^{-179.° i} & 0.99913 \end{pmatrix}$

Neutrino Pheno:
fitted:
$\Lambda_R = 5.2 \times 10^{14}$GeV,
$\chi^2_\nu = 1.3 \times 10^{-6}$

Heavy Masses	Light Masses	$0\nu\beta\beta$
$M_1 = 1.05 \times 10^{12}$ GeV $M_2 = 1.071 \times 10^{12}$ GeV $M_3 = 1.356 \times 10^{12}$ GeV	$m_1 = 48.92$ meV $m_2 = 49.68$ meV $m_3 = 0.2527$ meV	$m_{ee} = 50.72$ meV

Mass Differences	Mixing Angles	Phase	MNS Matrix
$\Delta_{21} = 7.5 \times 10^{-5}\mathrm{eV}^2$ $\Delta_{31} = -2.4 \times 10^{-3}\mathrm{eV}^2$ $\Delta_{32} = -2.5 \times 10^{-3}\mathrm{eV}^2$	$\sin^2(\theta_{12}) = 0.306$ $\sin^2(\theta_{23}) = 0.4119$ $\sin^2(\theta_{13}) = 0.021$	$\delta = 145.1°$	$\begin{pmatrix} 0.8243 & 0.5473 & 0.1449e^{-145.1° i} \\ 0.4978e^{-18.12° i} & 0.5907e^{166.6° i} & 0.635 \\ 0.2697e^{171.6° i} & 0.5929e^{-17.11° i} & 0.7588 \end{pmatrix}$

Neutrino Pheno
measured:

Mass Differences	Mixing Angles	Phase	MNS Matrix
$\vert\Delta_{21}\vert = 7.6 \times 10^{-5}\mathrm{eV}^2$ $\vert\Delta_{31}\vert = 2.4 \times 10^{-3}\mathrm{eV}^2$ $\vert\Delta_{32}\vert = 2.4 \times 10^{-3}\mathrm{eV}^2$	$\sin^2(\theta_{12}) = 0.306$ $\sin^2(\theta_{23}) = 0.42$ $\sin^2(\theta_{13}) = 0.021$	$\delta = 0.°$	$\begin{pmatrix} 0.8243 & 0.5473 & 0.1449 \\ -0.4995 & 0.5825 & 0.6412 \\ 0.2666 & -0.6009 & 0.7535 \end{pmatrix}$

Model: SU(12), 7 Higgs, 10 massive fermions, 55 mass terms

Anom. Free Set: $(66) + (495) + 2\,(\overline{220}) + 2\,(\overline{12})$

SU(5) Level: $21\,(\overline{10}) + 24\,(10) + 45\,(\overline{5}) + 42\,(5) + 140\,(1)$

Fermions: $(10)495_1$, $(10)\overline{220}_2$, $(10)66_3$, $(\overline{5})\overline{12}_1$, $(\overline{5})\overline{220}_2$, $(\overline{5})\overline{12}_3$, $(1)1_1$, $(1)1_2$, $(1)1_3$

Higgs: $(5)\overline{495}$, $(\overline{5})495$, $(1)66$, $(1)\overline{66}$, $(1)792$, $(1)\overline{792}$, $(24)143$

M. Fermions: 12, $\overline{12}$, 66, $\overline{66}$, 220, $\overline{220}$, 495, $\overline{495}$, 792, $\overline{792}$

Leading UpType Diagrams:

Dim 4: **U33:** $(10)66_3.(5)\overline{495}.(10)66_3$

Dim 5: **U13:** $(10)495_1.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66_3$

U31: $(10)66_3.(5)\overline{495}.(10)66\times(\overline{10})\overline{66}.(1)\overline{66}.(10)495_1$

U23: $(10)\overline{220}_2.(1)792.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66_3$

U32: $(10)66_3.(5)\overline{495}.(10)66\times(\overline{10})\overline{66}.(1)792.(10)\overline{220}_2$

Dim 6: **U11:** $(10)495_1.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66\times(\overline{10})\overline{66}.(1)\overline{66}.(10)495_1$

U12: $(10)495_1.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66\times(\overline{10})\overline{66}.(1)792.(10)\overline{220}_2$

U21: $(10)\overline{220}_2.(1)792.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66\times(\overline{10})\overline{66}.(1)\overline{66}.(10)495_1$

U22: $(10)\overline{220}_2.(1)792.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66\times(\overline{10})\overline{66}.(1)792.(10)\overline{220}_2$

Subleading UpType Diagrams:

Dim 5: **U33:** $(10)66_3.(5)\overline{495}.(10)66\times(\overline{10})\overline{66}.(24)143.(10)66_3$

U33: $(10)66_3.(24)143.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66_3$

Dim 6: **U13:** $(10)495_1.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66\times(\overline{10})\overline{66}.(24)143.(10)66_3$

U13: $(10)495_1.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(24)143.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66_3$

U13: $(10)495_1.(1)792.(\overline{10})220\times(10)\overline{220}.(1)792.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66_3$

U31: $(10)66_3.(5)\overline{495}.(10)66\times(\overline{10})\overline{66}.(24)143.(10)66\times(\overline{10})\overline{66}.(1)\overline{66}.(10)495_1$

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Spontaneous symmetry breaking

Take SUSY scale to be

$M_{SUSY} \sim 10^{10}$ GeV

then

$M >> v_5 >> M_{SUSY} >> v_{EW}$

SSB of SU(12) —> SU(5)

via superpotential.

SUSY remains unbroken if sum of the Dynkin weight of all the VEVs vanishes.

Spontaneous symmetry breaking

Route 1:
Adjoints of SU(12) to break:
$SU(12) \longrightarrow SU(5) \times U(1)^7$
above the SUSY scale.

One more adjoint to break
$SU(5) \times U(1)^7 \longrightarrow SU(3) \times SU(2) \times U(1)^8$
at v_5, then VEVs for SU(5) singlets with U(1) charges below SUSY scale to complete breaking to
$SU_C(3) \times SU_L(2) \times U_Y(1)$

Route 1 can be automated.

Spontaneous symmetry breaking

Route 2:
VEVs for totally antisymmetric irreps of SU(12) to break directly to SU(5), then one adjoint to SM.

Example below

(VEVs can be in SU(7) sector of scalar partners of chiral fermions.)

Route 2 is more model dependent, so each model needs attention.

Simplest anomaly-free 3 family set in SU(12)

$$66 + 495 + 2\overline{(220)} + 2(\overline{12})$$

with VEVs of equal strength v along Dynkin weights

[-1 0 1 0 0 0 0 0 0 0 0] [0 0 -1 0 0 0 1 0 0 0 0] [0 0 0 0 0 -1 0 1 0 0 0]

[1 0 0 -1 0 0 0 0 0 0 0] [0 0 0 1 -1 0 0 0 0 0 0] [0 0 0 0 1 0 0 -1 0 0 0]

which correspond to

$$v^{12,11} \quad v^{10,9,8,7} \quad v^{7,6}$$

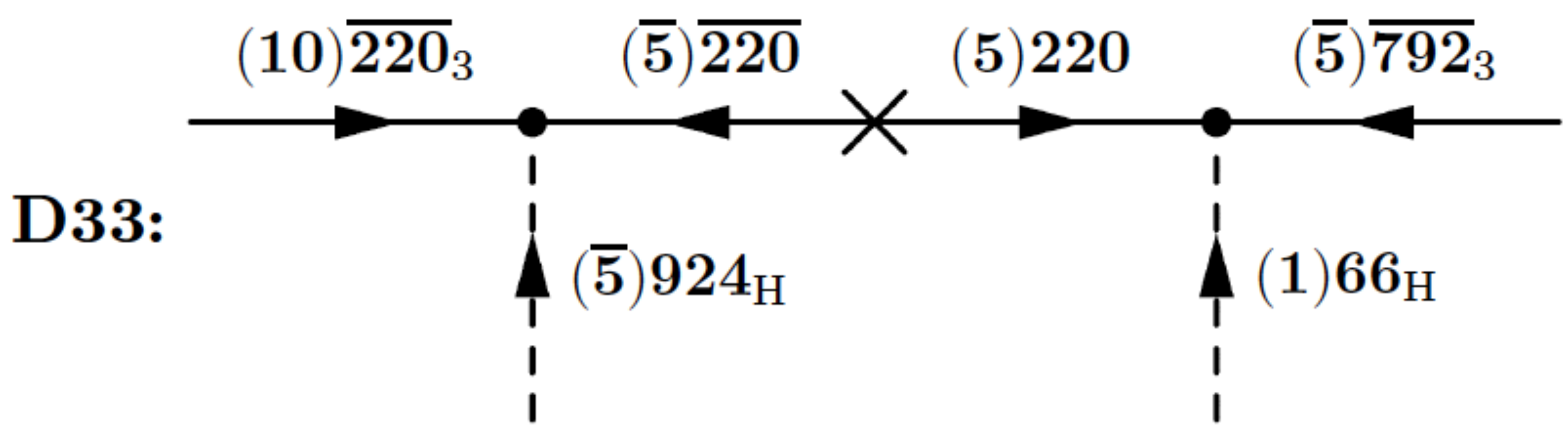
$$v_{12,11,10} \quad v_9 \quad v_{8,7,6}$$

breaks $SU(12)$ directly to $SU(5)$.

$SU(12)$ model of masses and mixings

- Yukawa Interactions:
  Froggatt-Nielsen type diagrams. (SUSY suppresses loops)

- Typical diagram:



Up terms

- Leading Up Terms:

Up-Type Quark Mass-Term Diagrams

Dim 4: **U33**: $(10)\overline{220}_3.(5)924_{\mathrm{H}}.(10)\overline{220}_3$

Dim 5: **U23**: $(10)\overline{792}_2.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}_3$

U32: $(10)\overline{220}_3.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220.(1)66_{\mathrm{H}}.(10)\overline{792}_2$

Dim 6: **U13**: $(10)495_1.(1)220_{\mathrm{H}}.(\overline{10})792\times(10)\overline{792}.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}_3$

U31: $(10)\overline{220}_3.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220.(1)66_{\mathrm{H}}.(10)\overline{792}\times(\overline{10})792.(1)220_{\mathrm{H}}.(10)495_1$

U22: $(10)\overline{792}_2.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220.(1)66_{\mathrm{H}}.(10)\overline{792}_2$

Dim 7: **U12**: $(10)495_1.(1)220_{\mathrm{H}}.(\overline{10})792\times(10)\overline{792}.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220.(1)66_{\mathrm{H}}.(10)\overline{792}_2$

U21: $(10)\overline{792}_2.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220.(1)66_{\mathrm{H}}.(10)\overline{792}\times(\overline{10})792.(1)220_{\mathrm{H}}.(10)495_1$

Dim 8: **U11**: $(10)495_1.(1)220_{\mathrm{H}}.(\overline{10})792\times(10)\overline{792}.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220$
$.(1)66_{\mathrm{H}}.(10)\overline{792}\times(\overline{10})792.(1)220_{\mathrm{H}}.(10)495_1$

Down terms

- Leading Down Terms:

Down-Type Quark Mass-Term Diagrams

Dim 5: **D32**: $(10)\overline{220}_3.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792}_2$

D33: $(10)\overline{220}_3.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792}_3$

Dim 6: **D31**: $(10)\overline{220}_3.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792}\times(5)792.(1)\overline{220}_{\rm H}.(\overline{5})\overline{66}_1$

D22: $(10)\overline{792}_2.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792}_2$

D23: $(10)\overline{792}_2.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792}_3$

Dim 7: **D12**: $(10)495_1.(1)220_{\rm H}.(\overline{10})792\times(10)\overline{792}.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792}_2$

D21: $(10)\overline{792}_2.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792}\times(5)792.(1)\overline{220}_{\rm H}.(\overline{5})\overline{66}_1$

D13: $(10)495_1.(1)220_{\rm H}.(\overline{10})792\times(10)\overline{792}.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792}_3$

Dim 8: **D11**: $(10)495_1.(1)220_{\rm H}.(\overline{10})792\times(10)\overline{792}.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220$
$.(1)66_{\rm H}.(\overline{5})\overline{792}\times(5)792.(1)\overline{220}_{\rm H}.(\overline{5})\overline{66}_1$

Mass matrices

$$M_{\mathrm{U}} = \begin{pmatrix} h^{\mathrm{u}}_{11}\epsilon^4 & h^{\mathrm{u}}_{12}\epsilon^3 & h^{\mathrm{u}}_{13}\epsilon^2 \\ h^{\mathrm{u}}_{12}\epsilon^3 & h^{\mathrm{u}}_{22}\epsilon^2 & h^{\mathrm{u}}_{23}\epsilon \\ h^{\mathrm{u}}_{13}\epsilon^2 & h^{\mathrm{u}}_{23}\epsilon & h^{\mathrm{u}}_{33} \end{pmatrix} v \,,$$

$$M_{\mathrm{D}} = \begin{pmatrix} h^{\mathrm{d}}_{11}\epsilon^4 & h^{\mathrm{d}}_{12}\epsilon^3 & h^{\mathrm{d}}_{13}\epsilon^3 \\ h^{\mathrm{d}}_{21}\epsilon^3 & h^{\mathrm{d}}_{22}\epsilon^2 & h^{\mathrm{d}}_{23}\epsilon^2 \\ h^{\mathrm{d}}_{31}\epsilon^2 & h^{\mathrm{d}}_{32}\epsilon & h^{\mathrm{d}}_{33}\epsilon \end{pmatrix} v \,,$$

$$M_{\mathrm{L}} = \begin{pmatrix} h^{\ell}_{11}\epsilon^4 & h^{\ell}_{12}\epsilon^3 & h^{\ell}_{13}\epsilon^2 \\ h^{\ell}_{21}\epsilon^3 & h^{\ell}_{22}\epsilon^2 & h^{\ell}_{23}\epsilon \\ h^{\ell}_{31}\epsilon^3 & h^{\ell}_{32}\epsilon^2 & h^{\ell}_{33}\epsilon \end{pmatrix} v = M_{\mathrm{D}}^T \,.$$

(Here $v = v_{SM}$ and below we use $\Lambda_R = M$.)

Dirac neutrino terms

- Leading Dirac Neutrino Terms:

Dirac-Neutrino Mass-Term Diagrams

Dim 4: **DN23**: $(\bar{5})\overline{792}_2.(5)924_{\rm H}.(1)\overline{12}_3$

DN33: $(\bar{5})\overline{792}_3.(5)924_{\rm H}.(1)\overline{12}_3$

Dim 5: **DN13**: $(\bar{5})\overline{66}_1.(1)\overline{220}_{\rm H}.(5)792\times(\bar{5})\overline{792}.(5)924_{\rm H}.(1)\overline{12}_3$

DN22: $(\bar{5})\overline{792}_2.(1)66_{\rm H}.(5)220\times(\bar{5})\overline{220}.(5)924_{\rm H}.(1)\overline{220}_2$

DN32: $(\bar{5})\overline{792}_3.(1)66_{\rm H}.(5)220\times(\bar{5})\overline{220}.(5)924_{\rm H}.(1)\overline{220}_2$

Dim 6: **DN12**: $(\bar{5})\overline{66}_1.(1)\overline{220}_{\rm H}.(5)792\times(\bar{5})\overline{792}.(1)66_{\rm H}.(5)220\times(\bar{5})\overline{220}.(5)924_{\rm H}.(1)\overline{220}_2$

DN21: $(\bar{5})\overline{792}_2.(1)66_{\rm H}.(5)220\times(\bar{5})\overline{220}.(5)924_{\rm H}.(1)\overline{220}\times(1)220.(1)66_{\rm H}.(1)\overline{792}_1$

DN31: $(\bar{5})\overline{792}_3.(1)66_{\rm H}.(5)220\times(\bar{5})\overline{220}.(5)924_{\rm H}.(1)\overline{220}\times(1)220.(1)66_{\rm H}.(1)\overline{792}_1$

Dim 7: **DN11**: $(\bar{5})\overline{66}_1.(1)\overline{220}_{\rm H}.(5)792\times(\bar{5})\overline{792}.(1)66_{\rm H}.(5)220\times(\bar{5})\overline{220}.(5)924_{\rm H}.(1)\overline{220}\times(1)220.(1)66_{\rm H}.(1)\overline{792}_1$

Majorana neutrino terms

- Leading Majorana Neutrino Terms::

Majorana-Neutrino Mass-Term Diagrams

Dim 4: **MN11**: $(1)\overline{792}_1.(1)\overline{66}_\mathrm{H}.(1)\overline{792}_1$

MN33: $(1)\overline{12}_3.(1)66_\mathrm{H}.(1)\overline{12}_3$

Dim 5: **MN12**: $(1)\overline{792}_1.(1)\overline{66}_\mathrm{H}.(1)\overline{792}\times(1)792.(1)\overline{66}_\mathrm{H}.(1)\overline{220}_2$

MN21: $(1)\overline{220}_2.(1)\overline{66}_\mathrm{H}.(1)792\times(1)\overline{792}.(1)\overline{66}_\mathrm{H}.(1)\overline{792}_1$

Dim 6: **MN13**: $(1)\overline{792}_1.(1)\overline{66}_\mathrm{H}.(1)\overline{792}\times(1)792.(1)\overline{66}_\mathrm{H}.(1)\overline{220}\times(1)220.(1)\overline{66}_\mathrm{H}.(1)\overline{12}_3$

MN31: $(1)\overline{12}_3.(1)\overline{66}_\mathrm{H}.(1)220\times(1)\overline{220}.(1)\overline{66}_\mathrm{H}.(1)792\times(1)\overline{792}.(1)\overline{66}_\mathrm{H}.(1)\overline{792}_1$

MN22: $(1)\overline{220}_2.(1)\overline{66}_\mathrm{H}.(1)792\times(1)\overline{792}.(1)\overline{66}_\mathrm{H}.(1)\overline{792}\times(1)792.(1)\overline{66}_\mathrm{H}.(1)\overline{220}_2$

Dim 7: **MN23**: $(1)\overline{220}_2.(1)\overline{66}_\mathrm{H}.(1)792\times(1)\overline{792}.(1)\overline{66}_\mathrm{H}.(1)\overline{792}\times(1)792.(1)\overline{66}_\mathrm{H}.(1)\overline{220}\times(1)220.(1)\overline{66}_\mathrm{H}.(1)\overline{12}_3$

MN32: $(1)\overline{12}_3.(1)\overline{66}_\mathrm{H}.(1)220\times(1)\overline{220}.(1)\overline{66}_\mathrm{H}.(1)792\times(1)\overline{792}.(1)\overline{66}_\mathrm{H}.(1)\overline{792}\times(1)792.(1)\overline{66}_\mathrm{H}.(1)\overline{220}_2$

Dirac and Majorana neutrino mass matrices

- Dirac and Majorana neutrino mass matrices

$$M_{\mathrm{DN}} = \begin{pmatrix} h_{11}^{\mathrm{dn}}\epsilon^3 & h_{12}^{\mathrm{dn}}\epsilon^2 & h_{13}^{\mathrm{dn}}\epsilon \\ h_{21}^{\mathrm{dn}}\epsilon^2 & h_{22}^{\mathrm{dn}}\epsilon & h_{23}^{\mathrm{dn}} \\ h_{31}^{\mathrm{dn}}\epsilon^2 & h_{32}^{\mathrm{dn}}\epsilon & h_{33}^{\mathrm{dn}} \end{pmatrix} v \,,$$

$$M_{\mathrm{MN}} = \begin{pmatrix} h_{11}^{\mathrm{mn}} & h_{12}^{\mathrm{mn}}\epsilon & h_{13}^{\mathrm{mn}}\epsilon^2 \\ h_{12}^{\mathrm{mn}}\epsilon & h_{22}^{\mathrm{mn}}\epsilon^2 & h_{23}^{\mathrm{mn}}\epsilon^3 \\ h_{13}^{\mathrm{mn}}\epsilon^2 & h_{23}^{\mathrm{mn}}\epsilon^3 & h_{33}^{\mathrm{mn}} \end{pmatrix} \Lambda_{\mathrm{R}} .$$

M_{D} and M_{L} and M_{DN} are all doubly lopsided.

Light-neutrino mass matrix

- Light-neutrino mass matrix is from type I seesaw:

$$M_\nu = -M_{\mathrm{DN}} M_{\mathrm{MN}}^{-1} M_{\mathrm{DN}}^T .$$

$$M_\nu \approx \frac{v^2}{\Lambda_R} \times$$

$$\begin{pmatrix} \epsilon^2 \left(\frac{{h_{12}^{\mathrm{dn}}}^2 h_{11}^{\mathrm{mn}}}{{h_{12}^{\mathrm{mn}}}^2 - h_{11}^{\mathrm{mn}} h_{22}^{\mathrm{mn}}} - \frac{{h_{13}^{\mathrm{dn}}}^2}{h_{33}^{\mathrm{mn}}} \right) & \epsilon \left(\frac{h_{12}^{\mathrm{dn}} h_{22}^{\mathrm{dn}} h_{11}^{\mathrm{mn}}}{{h_{12}^{\mathrm{mn}}}^2 - h_{11}^{\mathrm{mn}} h_{22}^{\mathrm{mn}}} - \frac{h_{13}^{\mathrm{dn}} h_{23}^{\mathrm{dn}}}{h_{33}^{\mathrm{mn}}} \right) & \epsilon \left(\frac{h_{12}^{\mathrm{dn}} h_{32}^{\mathrm{dn}} h_{11}^{\mathrm{mn}}}{{h_{12}^{\mathrm{mn}}}^2 - h_{11}^{\mathrm{mn}} h_{22}^{\mathrm{mn}}} - \frac{h_{13}^{\mathrm{dn}} h_{33}^{\mathrm{dn}}}{h_{33}^{\mathrm{mn}}} \right) \\ \epsilon \left(\frac{h_{12}^{\mathrm{dn}} h_{22}^{\mathrm{dn}} h_{11}^{\mathrm{mn}}}{{h_{12}^{\mathrm{mn}}}^2 - h_{11}^{\mathrm{mn}} h_{22}^{\mathrm{mn}}} - \frac{h_{13}^{\mathrm{dn}} h_{23}^{\mathrm{dn}}}{h_{33}^{\mathrm{mn}}} \right) & \frac{{h_{22}^{\mathrm{dn}}}^2 h_{11}^{\mathrm{mn}}}{{h_{12}^{\mathrm{mn}}}^2 - h_{11}^{\mathrm{mn}} h_{22}^{\mathrm{mn}}} - \frac{{h_{23}^{\mathrm{dn}}}^2}{h_{33}^{\mathrm{mn}}} & \frac{h_{22}^{\mathrm{dn}} h_{32}^{\mathrm{dn}} h_{11}^{\mathrm{mn}}}{{h_{12}^{\mathrm{mn}}}^2 - h_{11}^{\mathrm{mn}} h_{22}^{\mathrm{mn}}} - \frac{h_{23}^{\mathrm{dn}} h_{33}^{\mathrm{dn}}}{h_{33}^{\mathrm{mn}}} \\ \epsilon \left(\frac{h_{12}^{\mathrm{dn}} h_{32}^{\mathrm{dn}} h_{11}^{\mathrm{mn}}}{{h_{12}^{\mathrm{mn}}}^2 - h_{11}^{\mathrm{mn}} h_{22}^{\mathrm{mn}}} - \frac{h_{13}^{\mathrm{dn}} h_{33}^{\mathrm{dn}}}{h_{33}^{\mathrm{mn}}} \right) & \frac{h_{22}^{\mathrm{dn}} h_{32}^{\mathrm{dn}} h_{11}^{\mathrm{mn}}}{{h_{12}^{\mathrm{mn}}}^2 - h_{11}^{\mathrm{mn}} h_{22}^{\mathrm{mn}}} - \frac{h_{23}^{\mathrm{dn}} h_{33}^{\mathrm{dn}}}{h_{33}^{\mathrm{mn}}} & \frac{{h_{32}^{\mathrm{dn}}}^2 h_{11}^{\mathrm{mn}}}{{h_{12}^{\mathrm{mn}}}^2 - h_{11}^{\mathrm{mn}} h_{22}^{\mathrm{mn}}} - \frac{{h_{33}^{\mathrm{dn}}}^2}{h_{33}^{\mathrm{mn}}} \end{pmatrix}$$

SU(12) model summary

SU(12) model gives prospect of
decent phenomenology:

-masses
-mixing angles
$\theta_{13} \neq 0$ naturally.

No domain wall problem
-no discrete symmetry to break.

No fine tuning except for
usual hierarchy problem
-i.e., why is v_{EW} so small?

SU(N) models with smaller N?

SU(12) is big, maybe too big
-approaching landscape.

But it seems worthwhile to explore
this small piece of landscape.

SU(N) models with smaller N?

Chiral SU(N) models for N as large as 14 have been found via orbifolded AdS/CFT (H. P. Nilles et al.).

Is there a discrete symmetry free SU(N) model for $N < 12$ with good pheno?

Best from scan for N=8 is an $SU(8) \times Z_2$ model (under investigation.)

Searching $9 < N < 11$ now.

Summary

SU(N) models of masses and mixing
with little (e.g., Z_2)
or no discrete symmetry are possible.

Better SU(12) models in hand
-simpler irrep structure

(Feger, Albright, TWK, 2015, to appear)

Some problems that arise in discrete FLASY models can be avoided or solved:

-origin of discrete symmetries
-domain walls
-FLASY irrep problem

Stay tuned!