Supersymmetry, Gravity and Branes: Remembering Dick Arnowitt

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Dick Arnowitt's many contributions to physics have illuminated the path to a deeper understanding in the theory of gravitation, the unification of particle physics, the phenomenology of supergravity and supermatter theories, the phenomenology of string theory, cosmology, current algebras and many other important topics in theoretical physics.

The relations between Dick's work and my own started out with the series of key papers he wrote together with Ali Chamseddine and Pran Nath laying the foundations of supersymmetric unification phenomenology, starting with the highly cited paper

A.H. Chamseddine, R.L. Arnowitt and P. Nath,

"Locally Supersymmetric Grand Unification," Phys. Rev. Lett. 49, 970 (1982)

and then continuing on in a major series of other key papers.

This used the general scheme of coupling between supergravity and matter derived using the $N = 1$ supergravity tensor calculus

K. S. Stelle and P. C. West,

Phys. Lett. B74 (1978) 330; Phys. Lett. B 77, 376 (1978); Nucl. Phys. B 145, 175 (1978)

S. Ferrara and P. van Nieuwenhuizen,

Phys. Lett. B74 (1978) 333; Phys. Lett. B 76, 404 (1978); Phys. Lett. B 78, 573 (1978)

Dick and I collaborated directly in just one paper, but the topic was illustrative of his general approach to the implementation of mathematical structure taken from more formal work, then applied to important topics of physical concern. In this case, it was one of my infrequent forrays into phenomenology: the CP problem,

R.L. Arnowitt, M.J. Duff and K.S. Stelle,

"Supersymmetry and the Neutron Electric Dipole Moment," Phys. Rev. D43 (1991) 3085

which focused on CP violation following from nonminimal super Yang-Mills kinetic terms involving spontaneous supersymmetry breaking. This then to a more detailed consideration of the neutron electric dipole moment in

R.L. Arnowitt, J.L. Lopez and D.V. Nanopoulos,

"Keeping the Demon of SUSY at Bay," Phys. Rev. D42 (1990) 2423.

typically retitled in rather prissy fashion by Phys. Rev. D as

"Electric Dipole Moment of the Neutron in Supersymmetric Theories."

The universe as a membrane

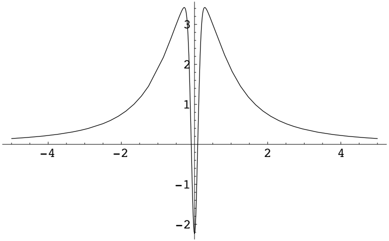
Dick's interests in cosmology, developed through something like 35-40 papers, were a natural extension of his pioneering work in the dynamics of general relativity and in particle physics phenomenology. The idea of the universe as a braneworld, dates back to the work of Rubakov and Shaposhnikov in 1983. Dick's work in this direction, often together with Pran Nath or Bhaskar Dutta, is another topic on which Dick's interests and mine have had parallels.

Now let's switch to a somewhat more detailed investigation of a key problem in this approach: how to get gravity to localise its effects on a subsurface of a higher-dimensional spacetime.

B. Crampton, C.N. Pope and K.S. Stelle,

"Braneworld localisation in hyperbolic spacetime," JHEP **1412** (2014) 035.

Attempts in a supergravity context to achieve a localization of gravity on a brane embedded in an infinite transverse space were made by Randall and Sundrum (RS II) Phys. Rev. Lett. 83 (1999) 4690 and by Karch and Randall JHEP 0105 (2001) 008 using patched-together sections of AdS_5 space with a delta-function source at the joining surface. This produced a "volcano potential" for the effective Schrödinger problem in the direction transverse to the brane, giving rise to a bound state concentrating gravity in the 4D directions.



Another approach: Salam-Sezgin theory and its embedding

Abdus Salam and Ergin Sezgin constructed in 1984 a version of 6D minimal (chiral, *i.e.* (1,0)) supergravity coupled to a 6D 2-form tensor multiplet and a 6D super-Maxwell multiplet which gauges the U(1) R-symmetry of the theory. Phys.Lett. B147 (1984) 47 This Einstein-tensor-Maxwell system has the bosonic Lagrangian

$$\begin{aligned} \mathcal{L}_{\mathrm{SS}} &= \tfrac{1}{2}R - \tfrac{1}{4g^2}e^{\sigma}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{6}e^{-2\sigma}G_{\mu\nu\rho}G^{\mu\nu\rho} - \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - g^2e^{-\sigma} \\ G_{\mu\nu\rho} &= 3\partial_{[\mu}B_{\nu\rho]} + 3F_{[\mu\nu}A_{\rho]} \end{aligned}$$

Note the *positive* potential term for the scalar field σ. This is a key feature of all R-symmetry gauged models generalizing the Salam-Sezgin model, leading to models with noncompact symmetries. For example, upon coupling to yet more vector multiplets, the sigma-model target space can have a structure $SO(p,q)/(SO(p)\times SO(q))$.

$\mathcal{H}^{(2,2)}$ embedding of the Salam-Sezgin theory

A way to obtain the Salam-Sezgin theory from M theory was given by Cvetič, Gibbons & Pope. Nucl. Phys. B677 (2004) 164 This employed a reduction from 10D type IIA supergravity on the space $\mathcal{H}^{(2,2)}$, or, equivalently, from 11D supergravity on $S^1 \times \mathcal{H}^{(2,2)}$. The $\mathcal{H}^{(2,2)}$ space is a cohomogeneity-one 3D hyperbolic space which can be obtained by embedding into $\mathbb{R}^4$ via the condition $\mu_1^2 + \mu_2^2 - \mu_3^2 - \mu_4^2 = 1$. This embedding condition is SO(2, 2) invariant, but the embedding $\mathbb{R}^4$ space has SO(4) symmetry, so the isometries of this space are just $\mathrm{SO}(2,2) \cap \mathrm{SO}(4) = \mathrm{SO}(2) \times \mathrm{SO}(2)$. The cohomogeneity-one $\mathcal{H}^{(2,2)}$ metric is $ds_3^2 = \cosh 2\rho d\rho^2 + \cosh^2\rho d\alpha^2 + \sinh^2\rho d\beta^2$.

Since $\mathcal{H}^{(2,2)}$ admits a natural $SO(2,2)$ group action, the resulting 7D supergravity theory has maximal (32 supercharge) supersymmetry and a gauged $SO(2,2)$ symmetry, linearly realized on $SO(2) \times SO(2)$. Note how this fits neatly into the general scheme of extended Salam-Sezgin gauged models.

The Kaluza-Klein spectrum

Reduction on the non-compact $\mathcal{H}^{(2,2)}$ space from ten to seven dimensions, despite its mathematical consistency, does not provide a full physical basis for compactification to 4D, however. The chief problem is that the truncated Kaluza-Klein modes form a continuum instead of a discrete set with mass gaps. Moreover, the wavefunction of “reduced” 4D states when viewed from 10D or 11D includes a non-normalizable factor owing to the infinite $\mathcal{H}^{(2,2)}$ directions. This infinite transverse volume also has the consequence that the resulting 4D Newton constant vanishes. Accordingly, the higher-dimensional supergravity theory does not naturally localize gravity in the lower-dimensional subspace when handled by ordinary Kaluza-Klein methods.

Bound states and mass gaps

Crampton, Pope & K.S.S., JHEP 1412 (2014) 035; 1408.7072

An approach to solving the non-localization problem of gravity on the 4D subspace of the ground-state Salam-Sezgin (SS) solution is to look for a *normalizable* transverse-space wavefunction with a mass gap before the onset of the continuous massive Kaluza-Klein spectrum. This could be viewed as analogous to an effective field theory for a system confined to a metal by a nonzero work function.

General study of the fluctuation spectra about brane solutions shows that the mass spectrum of the spin-two fluctuations about a brane background is given by the spectrum of the scalar Laplacian in the transverse embedding space of the brane

Csaki, Erlich, Hollowood & Shirman, Nucl.Phys. B581 (2000) 309; Bachas & Estes, JHEP 1106 (2011) 005

$$\begin{aligned}
\Box_{(10)}F &= \frac{1}{\sqrt{-\det g_{(10)}}}\partial_M\left(\sqrt{-\det g_{(10)}}\,g_{(10)}^{MN}\partial_N F\right)\\
&= H_{\rm SS}^{\frac{1}{4}}(\Box_{(4)} + g^2\triangle_{\theta,\phi,y,\psi,\chi} + g^2\triangle_{\rm rad})\\
H_{\rm SS} &= (\cosh 2\rho)^{-1}\ \text{warp factor;}\quad \triangle_{\rm rad} = \frac{\partial^2}{\partial\rho^2} + \frac{2}{\tanh(2\rho)}\frac{\partial}{\partial\rho}
\end{aligned}$$

The directions θ, ϕ, y, ψ & χ are all compact, and one can employ ordinary Kaluza-Klein methods for reduction on them by truncating to the invariant sector for these coordinates, *i.e.* making an S-wave reduction.

To handle the noncompact radial direction ρ, one needs to expand in eigenmodes of $\triangle_{\rm rad}$. The ansatz for 4D metric fluctuations simply replaces $\eta_{\mu\nu}$ in the 10D metric by $\eta_{\mu\nu} + h_{\mu\nu}(x,\rho)$, where one may take $\partial^\mu h_{\mu\nu} = \eta^{\mu\nu} h_{\mu\nu} = 0$

$$h_{\mu\nu}(x,\rho) = \sum_i h^{\lambda_i}_{\mu\nu}(x)\xi_{\lambda_i}(\rho) + \int_{\Lambda_{\rm edge}}^{\infty} d\lambda\, h^{\lambda}_{\mu\nu}(x)\xi_\lambda(\rho)$$

in which the ξ_{λ_i} are discrete eigenmodes and the ξ_λ are continuous Kaluza-Klein eigenmodes of the scalar Laplacian $\triangle_{\rm rad}$; their eigenvalues give the Kaluza-Klein masses $m^2 = g^2\lambda$ in 4D from $\square_{(10)} h^\lambda_{\mu\nu} = 0$ using $\triangle_{\theta,\phi,y,\psi,\chi} h^\lambda_{\mu\nu}(x,\rho) = 0$:

$$\begin{aligned} \triangle_{\rm rad}\xi_\lambda(\rho) &= -\lambda\xi_\lambda(\rho) \\ \square_{(4)} h^\lambda_{\mu\nu}(x) &= (g^2\lambda) h^\lambda_{\mu\nu}(x) \end{aligned}$$

The Schrödinger problem

One can rewrite the $\triangle_{\text{rad}}$ eigenvalue problem as a Schrödinger equation by making the substitution

$$\Psi_\lambda = \sqrt{\sinh(2\rho)}\xi_\lambda$$

after which the first derivative term is eliminated and the eigenfunction equation takes the Schrödinger equation form

$$-\frac{d^2\Psi_\lambda}{d\rho^2} + V(\rho)\Psi_\lambda = \lambda\Psi_\lambda$$

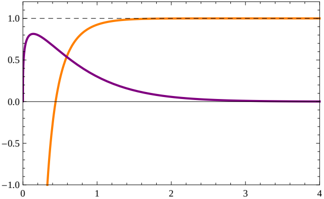
where the potential is

$$V(\rho) = 2 - \frac{1}{\tanh^2(2\rho)}$$

The general behavior of candidate Ψ_λ eigenfunctions cannot be given in terms of simple functions, but for $\lambda = 0$ the Schrödinger equation luckily can be solved exactly. In this case, one has a clear link between behaviors near the origin and at infinity, and one can also then implement the requirement of L^2 normalizability at infinity. The exact result is

$$\Psi_0(\rho) = \sqrt{\sinh(2\rho)}\xi_0(\rho) = \frac{2\sqrt{3}}{\pi}\sqrt{\sinh(2\rho)}\log(\tanh\rho)$$

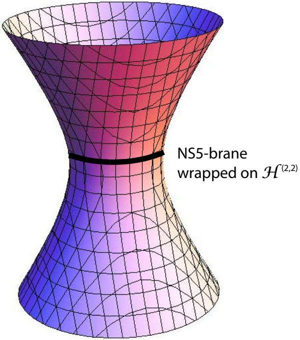
Note, however, that $\xi_0 = \frac{2\sqrt{3}}{\pi}\log(\tanh\rho)$ is logarithmically singular as $\rho \to 0$.



$\mathcal{H}^{(2,2)}$ Schrödinger equation potential (orange) and zero-mode Ψ_0 (purple)

The Salam-Sezgin background with an NS5-brane inclusion

Justifying the singularity of the $\xi(\rho)$ bound state as $\rho \to 0$ requires introduction of some other element into the solution. It turns out that what can be included nicely is an NS5-brane.



$\mathcal{H}^{(2,2)}$ space with an NS5-brane source wrapped around its 'waist' and smeared on a transverse S^2

The braneworld Newton constant

Reducing to 4D on the NS-5 modified SS solution, gravity has an effective action

$$\frac{g^3}{16\pi G_{(10)}} V_{(5)} \int d\rho \sqrt{g_{\rm EH}} H \int d^4x (\partial_\mu h_{\sigma\tau}(x) \partial^\mu h^{\sigma\tau}(x) |\xi(\rho)|^2 + \ldots)$$

where $V_{(5)} = g^{-4}\pi^2 \ell_y$ is the volume of the 5 compact directions.

In a conventional Kaluza-Klein reduction where $\xi(\rho) = \text{const}$, the ρ integral diverges and one consequently finds $G_{(4)} = 0$ for the 4D Newton constant. For the $\xi_0(\rho) \propto \log\tanh\rho$ bound state in the SS + NS5 geometry, however, the integral now converges and one obtains instead a finite 4D Newton constant. The corresponding gravitational coupling constant $\kappa_{(4)} = \sqrt{32\pi G_{(4)}}$ is

$$\begin{aligned}
\kappa_{(4)} &= \sqrt{32\pi g} \sqrt{\frac{G_{(10)}}{V_{(5)}}} \frac{\int d\rho \sinh 2\rho (1 - k\cosh 2\rho \log\tanh\rho) \xi^3}{(\int d\rho \sinh 2\rho (1 - k\cosh 2\rho \log\tanh\rho) \xi^2)^{\frac{3}{2}}} \\
&= 144\sqrt{6}\zeta(3) \left(\frac{G_{(10)} g^5}{\pi^7 \ell_y} \right)^{\frac{1}{2}} \frac{(1+2k)}{(2+3k)^{\frac{3}{2}}} .
\end{aligned}$$

This brings us back to Dick, who, among his many investigations in cosmology, considered precisely the same question:

R.L. Arnowitt and J. Dent,

"Gravitational forces in the brane world," Phys. Rev. D 71 (2005) 124024.

He is greatly missed, but his memory will persist for all the important things he taught us and for the enlightenment he gave us in our attempts to understand the physical world.