Extensions of Nonlinear Massive Gravity

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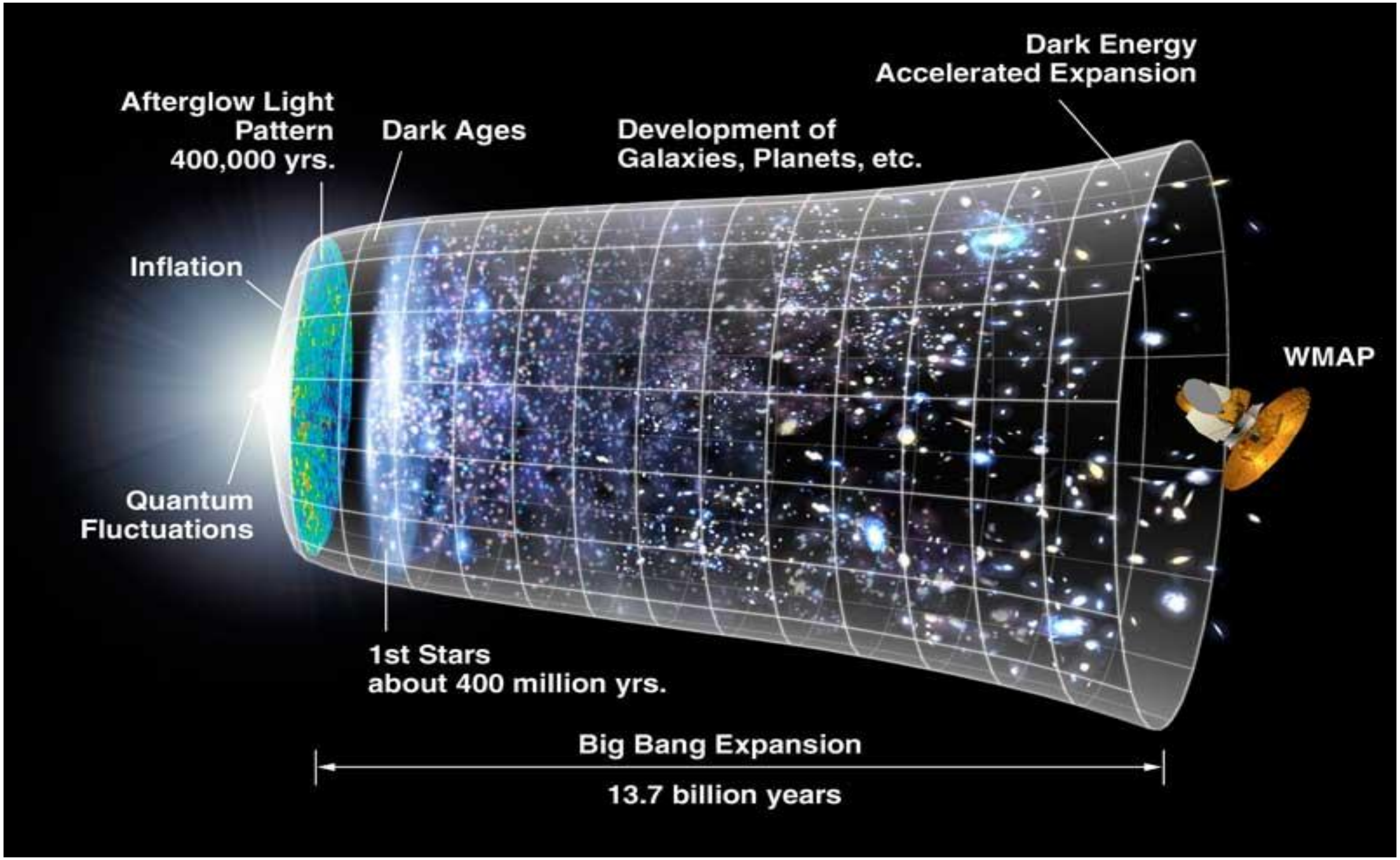
Goal

- We investigate various versions of nonlinear massive gravity and their cosmological implications
- Note:
- A consistent or interesting cosmology is not a proof for the consistency of the underlying gravitational theory
- A consistent gravity does not guarantee a consistent or interesting cosmology.

Talk Plan

- 1) Introduction: motivation
- 2) Simplest linear version has the vDVZ discontinuity
- 3) Non-linearities cure it but bring the Boulware-Deser ghost
- 4) New nonlinear massive gravity: free of BD ghosts and vDVZ discontinuity
- 5) FRW cosmology is impossible (instabilities). Need anisotropic geometry.
- 6) Extensions: Varying mass MG, quasi-dilaton MG etc.
- 7) F(R) nonlinear massive gravity. Free of BD ghost, vDVZ discontinuity. Good and rich cosmology free of instabilities.
- 8) Conclusions-Prospects

Why Modified Gravity?



Introduction

- Massive Gravity, i.e adding mass to a spin-2 particle, goes back to 1939
- Motivation: i) Theoretical (we know the answer for scalars and vectors)

  ii) Cosmological (explain acceleration)
- Indeed it is the most reasonable modified gravity

  (not the simplest one, since you add 3 dof's)
- It is promising, but…

  [Hinterbichler, Rev.Mod.Phys.84]

Introduction

- 1939: Fierz and Pauli add a linear mass-term to GR $\propto m^2(h_{\mu\nu} - h^2)$
- 1970: van Dam, Veltman, Zakharov: When the linear theory couples to a source, the limit $\boxed{m \to 0}$ does not give GR

  (vDVZ discontinuity)

- 1972: Vainstein: The non-linearities become stronger and stronger as m decreases. They must be taken into account and they do cure vDVZ discontinuity
- 1972: Boulware, Deser: Nonlinearities bring a ghost!

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- 1972: Boulware, Deser: Nonlinearities bring a ghost!
- 2010: de Rham, Gabadadze, Tolley: Adding higher-order graviton self-interaction systematically removes the BD ghost
- 2011 and on: The cosmology has severe problems.

Fierz-Pauli linear theory

- Linear massive gravity around flat background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \ \ |h| << 1$

$$S = \int d^4x \ \ \underbrace{[-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h}$$

Linearized Einstein-Hilbert action
(all possible2-powers of h and up to 2-derivatives):
massless spin-2 graviton

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Linearized Einstein-Hilbert action
(all possible2-powers of h and up to 2-derivatives):
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a=-b (Fierz-Pauli tuning)
NOT enforced by symmetry

$$m_{ghost}=\frac{m^2}{a+b}$$

[Fierz, Pauli, PRLS 1939]

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[Fierz, Pauli, PRLS 1939]

- The m=0 part has gauge symmetry $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

  This symmetry fixes the coefficients.
- The mass term violates it!

Fierz-Pauli linear theory

- Put source $T^{\mu\nu}$ with coupling $\kappa\, h_{\mu\nu} T^{\mu\nu}$. Eoms':

$$\Diamond h_{\mu\nu} - \partial_\lambda \partial_\mu h_\nu^\lambda - \partial_\lambda \partial_\nu h_\mu^\lambda + \eta_{\mu\nu} \partial_\lambda \partial_\sigma h^{\lambda\sigma} + \partial_\mu \partial_\nu h - \eta_{\mu\nu} \Diamond h \boxed{- m^2 \left(h_{\mu\nu} - \eta_{\mu\nu} h^2 \right)} = -\kappa\, T_{\mu\nu}$$

- Note: For $m = 0 \Rightarrow \partial^\mu T_{\mu\nu} = 0$ (conservation)

  For $m \neq 0$ no such condition (but we assume it, otherwise obvious discontinuity)

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- Point source $T^{\mu\nu}(\vec{x}) = M\delta^\mu_0\delta^\nu_0\delta^3(\vec{x})$. Solution:

$$h_{00}(\vec{x}) = \frac{2M}{3M_p}\frac{1}{4\pi}\frac{e^{-mr}}{r}$$

Yukawa suppression

$$h_{0i}(\vec{x}) = 0$$

$$h_{ij}(\vec{x}) = \frac{M}{3M_p}\frac{1}{4\pi}\frac{e^{-mr}}{r}\left[\frac{1+mr+m^2r^2}{m^2r^2}\delta_{ij} - \frac{1}{m^2r^4}\left(3+3mr+m^2r^2\right)x_i x_j\right]$$

- GR result:

$$h_{00}(\vec{x}) = \frac{M}{2M_p}\frac{1}{4\pi r}$$

$$h_{0i}(\vec{x}) = 0$$

$$h_{ij}(\vec{x}) = \frac{M}{2M_p}\frac{1}{4\pi r}\delta_{ij}$$

Fierz-Pauli linear theory

- Thus, for massless $\boxed{\varphi = -\frac{GM}{r}, \quad \gamma = 1}$ (PPN)
- For massive: $\boxed{\varphi = -\frac{4}{3}\frac{GM}{r}, \quad \gamma = \frac{1}{2}}$
- If rescale $G \to \frac{3}{4}G$ then bending of light 25% larger than GR
- GR is NOT recovered in the massless limit (vDVZ discontinuity)

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Massless gravity: 2 spin states

2 helicity states of a massless graviton

Massive gravity: 5 spin states

2 helicity states of a massless graviton

2 helicity states of a massless vector

1 single massive scalar

no 6th dof since the time components h_{00} appear as Lagr. multiplier

- The scalar (longitudinal graviton) maintains a coupling to T even in the massless limit
- I.e, the massless limit does not describe a massless graviton, but a massless graviton plus a coupled scalar
- The gauge symmetry of GR, that kills the extra dof appears ONLY for $m = 0$ and NOT for $m \rightarrow 0$

[van Dam, Veltman 1970], [Zakharov 1970]

Nonlinear theory and the BD ghost

- Nonlinearities become stronger as $m \to 0$, need to be taken into account.

$$S=\frac{1}{2\kappa^2}\underbrace{\int d^4x\ \ [\sqrt{-g}R}_{\substack{\text{Full nonlinear}\\ \text{EH action}}}\boxed{\underbrace{-\sqrt{-g^{(0)}}\,\frac{1}{4}m^2 g^{(0)\mu\alpha}g^{(0)\nu\beta}\left(h_{\mu\nu}h_{\alpha\beta}-h_{\mu\alpha}h_{\nu\beta}\right)\ ]}_{\text{Fierz-Pauli mass term}}}$$

$g_{\mu\nu}^{(0)}$ the fixed metric on which the massive graviton propagates

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$g^{(0)}_{\mu\nu}$ the fixed metric on which the massive graviton propagates

- The nonlinearities re-bring the 6th dof (no Lagrange multiplier anymore)
- The Hamiltonian constraint analysis shows that it is a ghost!

[Boulware,Deser 1972]

- But this ghost cures the vDVZ discontinuity! (it provides a repulsive force that counteracts the attractive force of the longitudinal scalar mode) [Vainstein 1972]

- But it could still make sense, if quantum effects push the ghost above a cutoff Λ, and see the whole story as an effective theory [Arkani-Hamed, Georgi, Schwartz 2002]

Stückelberg fields trick

- The $m \to 0$ is not smooth (you kill immediately the new dof's). Not good form for studying: fundamental discontinuity.
- Idea: Introduce new fields (new dof's) and restore gauge symmetries, without altering the theory. Then study the limit you want.
- E.g: Massive EM: $\boxed{S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A_\mu J^\mu - \frac{1}{2} m^2 A_\mu A^\mu \right]}$ not necessarily $\partial_\mu J^\mu = 0$

Massless EM: 2 dof's

2 helicity states of a massless spin-1 particle

Massive EM: 3 dof's

3 dof's of a massive spin-1 particle

- The mass term breaks the would-be gauge invariance $\delta A_\mu = \partial_\mu \Lambda$

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- Introduce ϕ through $A_\mu \to A_\mu + \partial_\mu \phi$

NOT change of field variables, NOT gauge transf. (massive action is not g.inv.), NOT decomposition to transverse and longitudinal (not $\partial_\mu A^\mu = 0$)

$$\boxed{S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A_\mu J^\mu - \frac{1}{2} m^2 A_\mu A^\mu - m A_\mu \partial^\mu \varphi - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{m} \varphi \partial_\mu J^\mu \right]}$$

$$\varphi = m\phi$$

- I restored the gauge symmetry $\delta A_\mu = \partial_\mu \Lambda, \ \delta\varphi = -m\Lambda$
- Now massless limit is smooth: Number of dof's is preserved. φ decouples.

$$\boxed{L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A_\mu J^\mu - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi}$$

dRGT nonlinear massive gravity

- The 6th dof (ghost) survives since the lapse function N is not a Lagrange multiplier in the nonlinear case, as it was in the linear one.
- Idea: Specially design nonlinear terms, so that N becomes again a Lagrange multiplier

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- Toy example:

physical: $ds^2 = -N^2dt^2 + \gamma_{ij}(dx^i + \cancelto{0}{N^i}dt)(dx^j + \cancelto{0}{N^j}dt)$

reference: $ds_f{}^2 = -dt^2 + dx_i dx^i$

- Define $K^\mu_\nu \equiv \delta^\mu_\nu - \left(\sqrt{g^{-1}f}\right)^\mu_\nu = \begin{bmatrix} 1-1/N & 0 \\ 0 & \delta^i_j - \sqrt{\gamma^{ij}\delta_{kj}} \end{bmatrix}$

- Lagrangian: $L = L_{EH} - m_g^2 M_p{}^2 \sqrt{-g}\,\det(\delta^\mu_\nu + \beta K^\mu_\nu)$

$$\Rightarrow L = L_{EH} - m_g^2 M_p{}^2 \sqrt{\gamma}\,[\mathrm{N}(1+\beta) - \beta]\det\left[(1+\beta)\delta^i_j - \beta\sqrt{\gamma^{ik}\delta_{kj}}\right]$$

- Mass term linear in N: Lagrange multiplier
- Recover the Hamiltonian constraint, remove the 6th (ghost) dof:

$$\Rightarrow \frac{\partial L}{\partial N} = H - m_g^2 M_p{}^2 \sqrt{\gamma}(1+\beta)\det\left[(1+\beta)\delta^i_j - \beta\sqrt{\gamma^{ik}\delta_{kj}}\right] = 0$$

- Similar for the general case $N_i \neq 0$

[de Rham, Gabadadze, PRD 82],
[de Rham, Gabadadze, Tolley PRL 106]

dRGT nonlinear massive gravity

- Finally:

$$\boxed{S_{MG}=M_p^2\int d^4x\sqrt{-g}\left[\frac{R}{2}+m_g^2\left(L_2+\alpha_3L_3+\alpha_4L_4\right)\right]}$$

where

$$L_2=\frac{1}{2}\left([K]^2-[K^2]\right)$$

$$L_3=\frac{1}{6}\left([K]^3-3[K][K^2]+2[K^3]\right)$$

$$L_4=\frac{1}{24}\left([K]^4-6[K]^2[K^2]+3[K^2]^2+8[K][K^3]-6[K^4]\right) \qquad [K]=tr\left(K^\nu_\mu\right)$$

$$K^\mu_\nu\equiv\delta^\mu_\nu-\sqrt{g^{\mu\sigma}f_{ab}(\phi)\partial_\nu\phi^a\partial_\sigma\phi^b}$$

fiducial metric

Stückelberg fields

[de Rham, Gabadadze, PRD 82],
[de Rham, Gabadadze, Tolley PRL 106]

- Free of BD ghost! Free of vDVZ discontinuity!
- Vainstein mechanism: extra dof's are suppressed at small scales due to non-linearities

Cosmological applications

- Simplest Example: Physical metric: flat FRW: $ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j$

  Fiducial metric: Minkowski: $f_{ab} = \eta_{ab}$

  Stückelberg scalars: $\phi^0 = b(t), \ \phi^i = x^i$

Variation wrt ϕ : $m^2\partial_0(a^3 - a^2) = 0 \Rightarrow \dot{a} = 0$ NO nontrivial solution (same for closed)

[dRGT et al, PRD 84]

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- Next: Physical metric open FRW: $ds^2 = -N^2dt^2 + a^2(t)\left[dx^2 + dy^2 + dz^2 - \frac{|K|\left(xdx + ydy + zdz\right)^2}{1+|K|\left(x^2+y^2+z^2\right)}\right]$

  Fiducial metric: Minkowski: $f_{ab} = \eta_{ab}$

  Stückelberg scalars: $\phi^0 = b(t)\sqrt{1+|K|(x^2+y^2+z^2)},\ \phi^i = \sqrt{|K|}b(t)x^i$

Variation wrt ϕ gives a constraint for b(t): $\frac{b(t)}{a(t)} = \frac{X_\pm}{\sqrt{|K|}} = const.,\ X_\pm = \frac{1+2\alpha_3+\alpha_4 \pm \sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}}{\alpha_3+\alpha_4}$

- Friedmann equations:

$$3H^2 - 3\frac{|K|}{a^2} = \rho_m + m_g^2 c_\pm$$

$$-2\dot{H} - 2\frac{|K|}{a^2} = \rho_m + p_m$$

We get an Effective Cosmological Constant: $\boxed{\Lambda_\pm = m_g^2 c_\pm(\alpha_3, \alpha_4)}$

Self-acceleration for $c_\pm(\alpha_3,\alpha_4) > 0$

[Gumrukcuoglu, Lin, Mukohyama, JCAP1111]

Cosmological applications

- Next Example: Physical metric: open FRW

  Fiducial metric: open FRW

$$\boxed{\Rightarrow \Lambda_{\pm} = m_g^2 c_{\pm}(\alpha_3, \alpha_4)}$$

- Next: Physical metric: open FRW:

  Fiducial metric: de Sitter:

$$\boxed{\Rightarrow \Lambda_{\pm} = m_g^2 c_{\pm}(\alpha_3, \alpha_4)}$$

as before

plus a new branch: $3H^2 - 3\frac{|K|}{a^2} = \rho_m + \rho_{MG}$

$$\boxed{\rho_{MG}(t) = -m_g^2\left(1-\frac{H}{H_C}\right)\left[6+4\alpha_3+\alpha_4-(3+5\alpha_3+2\alpha_4)\frac{H}{H_C}+(\alpha_3+\alpha_4)\frac{H^2}{H_C^2}\right]}$$

[Langlois, Naruko CQG 29]

Perturbations

- Let’s see the perturbations of all the above solutions.
- Unfortunately, there is ALWAYS a ghost instability (it’s frequency tends to vanish at low scales so it always remain in the low-energy effective theory)
- The linear kinetic term vanishes, so the leading kinetic term is cubic
- This instability is related to the FRW structure of the physical metric, and in particular from the high symmetries (isotropy).

  [Gumrukcuoglu, Lin, Mukohyama, JCAP1203], [De Felice, Gumrukcuoglu, Mukohyama, PRL 109]

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- In order to construct a healthy model we must insert anisotropies:

  Physical metric: axisymmetric Bianchi I: $ds^2 = -N^2 dt^2 + a(t)^2 \left(e^{4\sigma(t)} dx^2 + e^{-2\sigma(t)} dy^2 + e^{-2\sigma(t)} dz^2 \right)$

  Fiducial metric: FRW: as before

  Stückelberg scalars: as before

  $$\boxed{\Rightarrow \rho_{MG}(t) = \cdots}$$

  [Gumrukcuoglu, Lin, Mukohyama, PLB717]

- The only healthy model. Disadvantage: There is NO isotropic limit!

Extension 1: Varying mass massive gravity

- Need to find extensions of nonlinear massive gravity where FRW solutions are stable.

$$\boxed{S_{MG}=M_p^2\int d^4x\sqrt{-g}\left[\frac{R}{2}+V(\psi)\left(L_2+\alpha_3L_3+\alpha_4L_4\right)-\frac{1}{2}\partial_\mu\psi\partial^\mu\psi-W(\psi)\right]}$$

[Huang, Piao, Zhou PRD86]

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  Fiducial metric: Minkowski: $f_{ab}=\eta_{ab}$

  Stückelberg scalars: $\phi^0=b(t),\ \phi^i=a_{ref}x^i$

$$\Rightarrow 3M_p^2H^2=\rho_m+\rho_{MG}$$

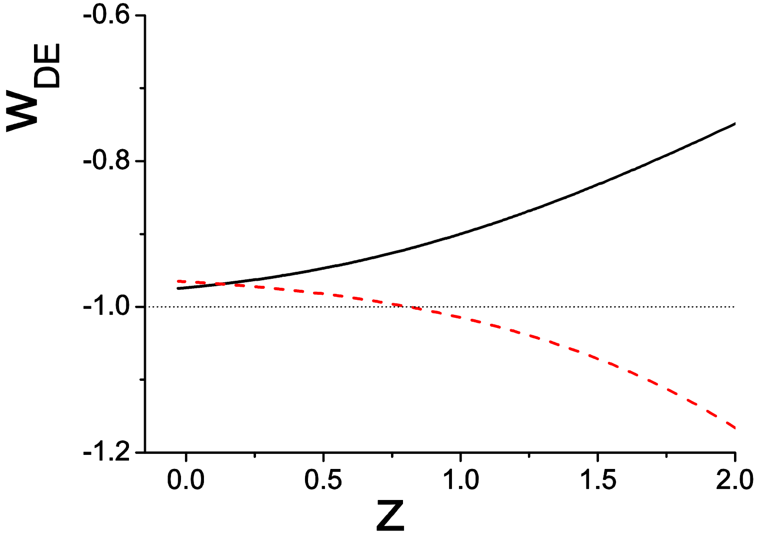
$$-2M_p^2\dot{H}=\rho_m+p_m+\rho_{MG}+p_{MG}$$

$$\rho_{MG}=\frac{1}{2}\dot{\psi}^2+W(\psi)+V(\psi)\left(\frac{a_{ref}}{a}-1\right)\left[f_3(a)+f_1(a)\right]$$

$$p_{MG}=\frac{1}{2}\dot{\psi}^2-W(\psi)-V(\psi)\left[f_4(a)+\dot{b}f_1(a)\right]$$

$$w_{DE}=\frac{p_{MG}}{\rho_{MG}}$$

$$\rho_{MG}+p_{MG}=\dot{\psi}^2-V(\psi)\left(\dot{b}-\frac{a_{ref}}{a}\right)f_1(a)$$



[Saridakis CQG 30]

Extension 1: Varying mass massive gravity

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  Stückelberg scalars: $\phi^0=b(t)\sqrt{1+|K|(x^2+y^2+z^2)},\ \phi^i=\sqrt{|K|}b(t)x^i$

- Variation wrt b provides the constraint equation: $V(\psi)\left(H-\frac{\sqrt{|K|}}{a}-1\right)f_1\left(\frac{b}{a}\right)+\dot{V}(\psi)\,f_2\left(\frac{b}{a}\right)=0$

  Variation wrt ψ: $\ddot{\psi}+3\mathrm{H}\,\dot{\psi}+\frac{dW}{d\psi}+\frac{dV}{d\psi}\left\{\left(\frac{\sqrt{|K|}b}{a}-1\right)\left[f_3\left(\frac{b}{a}\right)+f_1\left(\frac{b}{a}\right)\right]+3\dot{b}f_2\left(\frac{b}{a}\right)\right\}=0$

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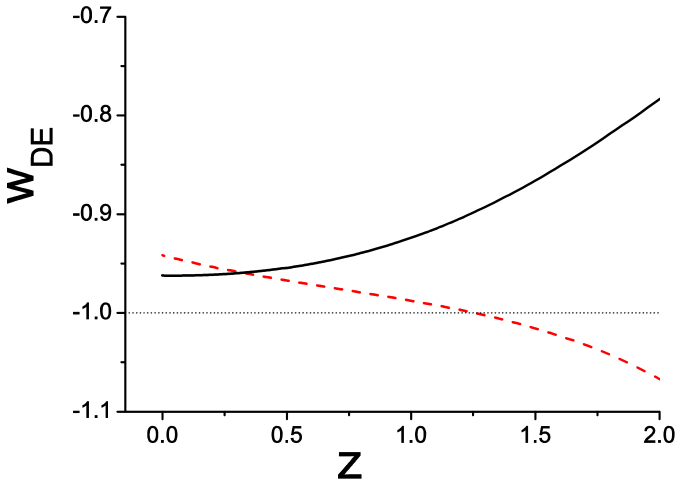
$$-2M_p^2\left(\dot{H}+\frac{|K|}{a^2}\right)=\rho_m+p_m+\rho_{MG}+p_{MG}$$

$$\rho_{MG}=\frac{1}{2}\dot{\psi}^2+W(\psi)+V(\psi)\left(\frac{\sqrt{|K|}b}{a}-1\right)\left[f_3\left(\frac{b}{a}\right)+f_1\left(\frac{b}{a}\right)\right]$$

$$p_{MG}=\frac{1}{2}\dot{\psi}^2-W(\psi)-V(\psi)\left[f_4\left(\frac{b}{a}\right)+\dot{b}f_1\left(\frac{b}{a}\right)\right]$$

$$w_{DE}=\frac{p_{MG}}{\rho_{MG}}$$



[Saridakis CQG 30]

Bounce and Cyclic behavior in varying mass massive gravity

- Contracting ($H<0$), bounce ($H=0$), expanding ($H>0$)
  near and at the bounce $\dot{H}>0$

- Expanding ($H>0$), turnaround ($H=0$), contracting $H<0$
  near and at the turnaround $\dot{H}<0$

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$$3M_p^2\left(H^2-\frac{|K|}{a^2}\right)=\rho_m+\rho_{MG}$$

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$$\rho_{MG}=\frac{1}{2}\dot{\psi}^2+W(\psi)+V(\psi)\left(\frac{\sqrt{|K|}b}{a}-1\right)\left[f_3\left(\frac{b}{a}\right)+f_1\left(\frac{b}{a}\right)\right]$$

$$p_{MG}=\frac{1}{2}\dot{\psi}^2-W(\psi)-V(\psi)\left[f_4\left(\frac{b}{a}\right)+\dot{b}f_1\left(\frac{b}{a}\right)\right]$$

- Bounce and cyclicity can be easily obtained

  [Cai, Gao, Saridakis JCAP1210]

Bounce and Cyclic behavior in varying mass massive gravity

- Input: $a(t)$ oscillatory, $b(t)$ at will

- Output: $$\psi(t)=\int^{t} dt'\left\{-2M_p^2\dot{H}-\rho_m(a(t'))-p_m(a(t'))+V(t')\left(\dot{b}(t')-\frac{a_{ref}}{a(t')}\right)f_1(a(t'))\right\}^{\frac{1}{2}}$$

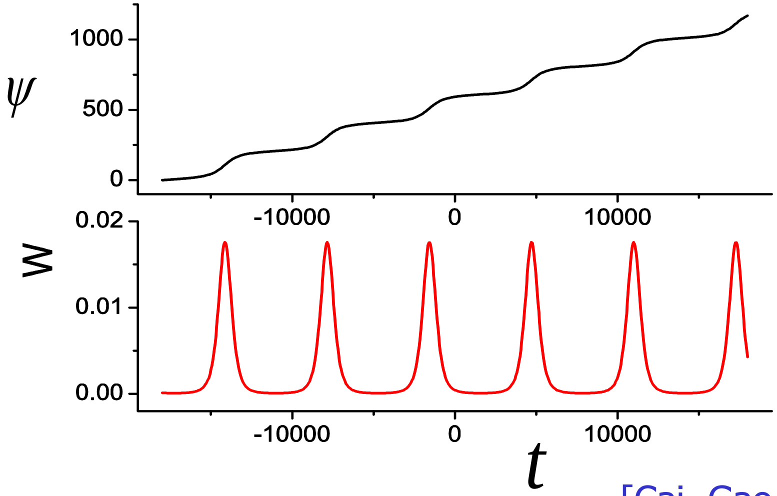
$$W(t)=M_p^2\left(3H^2+\dot{H}\right)+\frac{p_m(a(t'))}{2}-\frac{\rho_m(a(t'))}{2}-V(t')\left\{f_4(a(t'))+\left(\dot{b}(t')+\frac{a_{ref}}{a(t')}\right)\frac{f_1(a(t'))}{2}\right\}$$

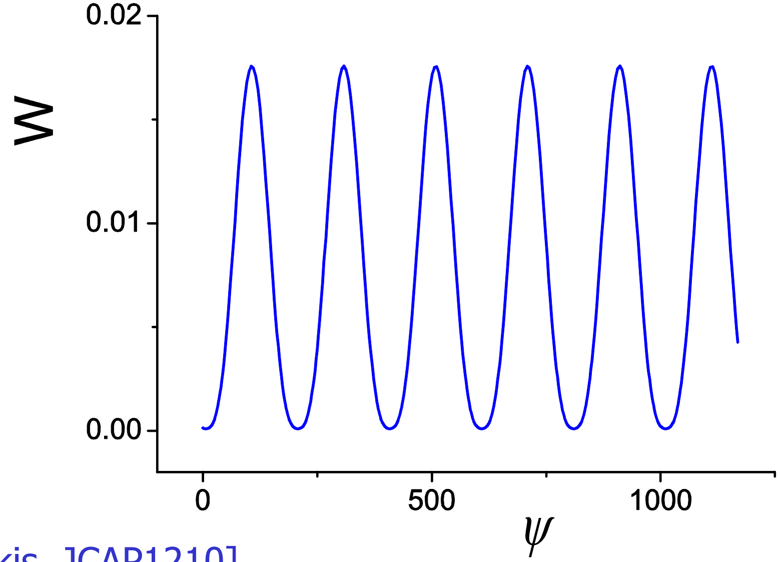
- Reconstruct $W(t)$

[Cai, Gao, Saridakis JCAP1210]

Bounce and Cyclic behavior in varying mass massive gravity

- Input: $\boxed{a(t) = A\sin(\omega t) + a_c}$, $b(t) = t$
- Output





[Cai, Gao, Saridakis JCAP1210]

- Important: Processing of perturbations [Brandenberger, PRD 80]

- Black Hole analysis also very interesting [Cai, Easson, Gao, Saridakis PRD 87]

Extension 2: Quasi-dilaton massive gravity

$$\boxed{S_{MG} = M_p^2 \int d^4x \sqrt{-g}\left[\frac{R}{2} + m_g^2\left(L_2+\alpha_3 L_3+\alpha_4 L_4\right) - \frac{\omega}{2M_p^2} g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma\right]}$$

where

$$L_2 = \frac{1}{2}\left([K]^2 - [K^2]\right)$$

$$L_3 = \frac{1}{6}\left([K]^3 - 3[K][K^2] + 2[K^3]\right)$$

$$L_4 = \frac{1}{24}\left([K]^4 - 6[K]^2[K^2] + 3[K^2]^2 + 8[K][K^3] - 6[K^4]\right) \qquad [K] = tr\left(K_\mu^\nu\right)$$

$$K^\mu_\nu \equiv \delta^\mu_\nu - e^{\sigma/M_p}\sqrt{g^{\mu\sigma}\eta_{ab}(\phi)\partial_\nu\phi^a\partial_\sigma\phi^b}$$

quasi-dilaton   fiducial metric   Stückelberg fields

[D'Amico, Gabadadze, Hui, Pirtskhalava PRD 87]

Extension 2: Quasi-dilaton massive gravity

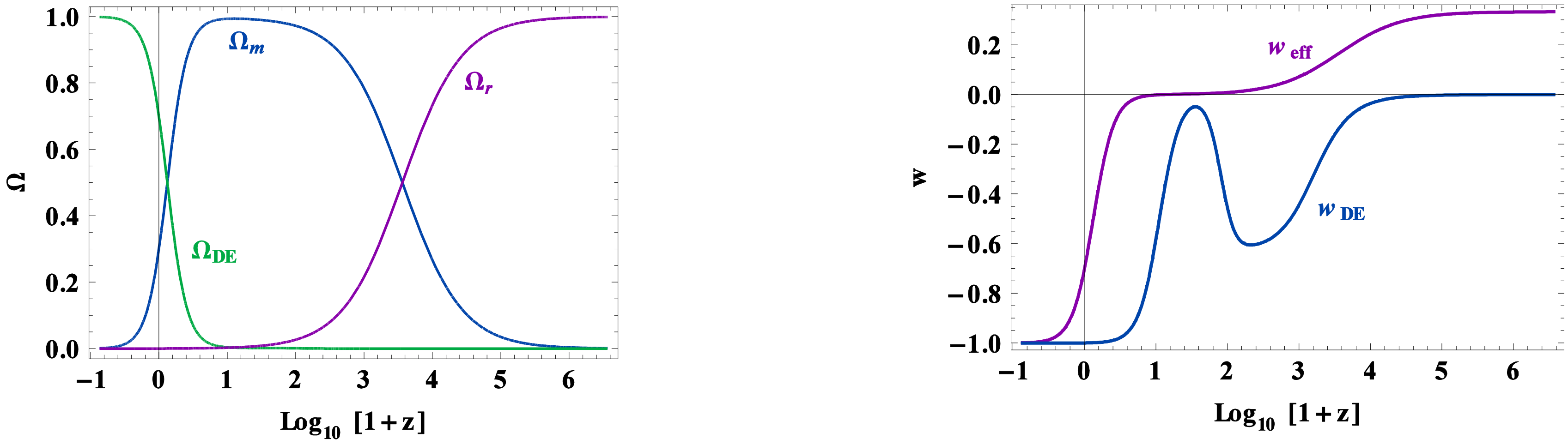
- Physical metric: flat FRW: $ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j$

  Fiducial metric: Minkowski: $f_{ab} = \eta_{ab}$

  Stückelberg scalars: $\phi^0 = b(t),\ \ \phi^i = x^i$

$$\Rightarrow 3M_p^2 H^2 = \rho_m + \rho_r + \rho_{DE}$$

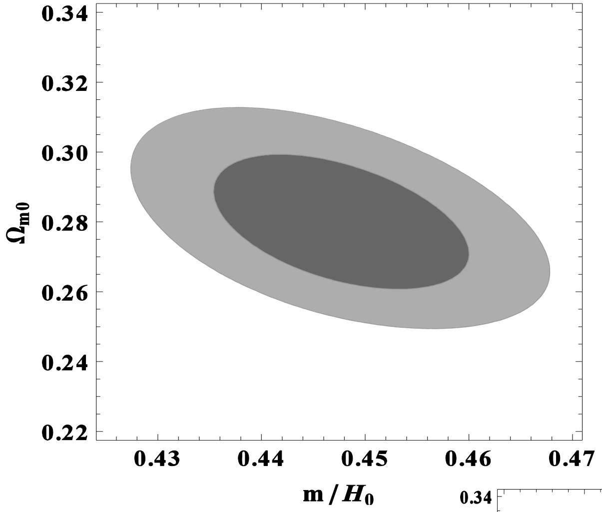
$$\rho_{DE} = \frac{\omega}{2}\dot{\psi}^2 - 3M_p^2 m_g^2\left[(2+\alpha_3+\alpha_4) - \left(3+\frac{9}{4}\alpha_3+3\alpha_4\right)\frac{e^{\frac{\sigma}{M_p}}}{a} + \left(1+\frac{3}{2}\alpha_3+3\alpha_4\right)\frac{e^{\frac{2\sigma}{M_p}}}{a^2} - \frac{1}{4}(\alpha_3+4\alpha_4)\frac{e^{\frac{3\sigma}{M_p}}}{a^3}\right]$$

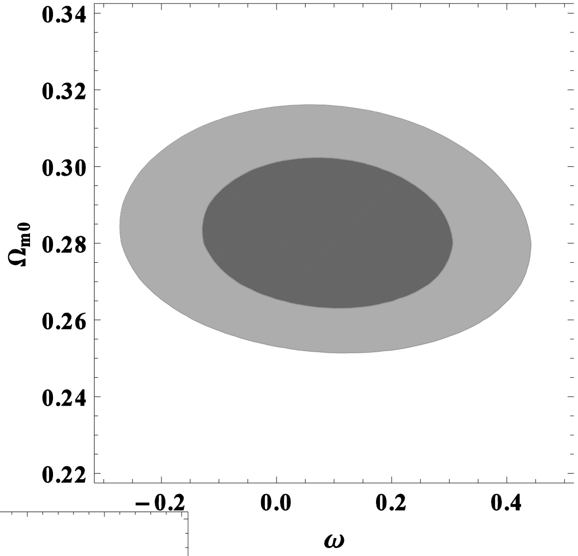


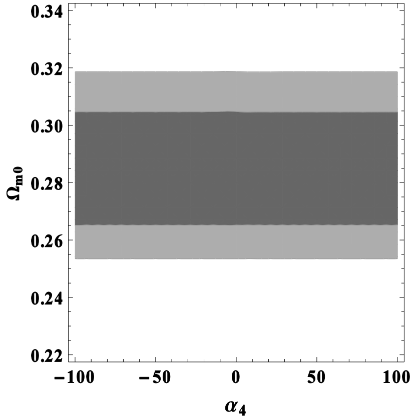
[Gannouji, Hossain, Sami, Saridakis PRD 87]

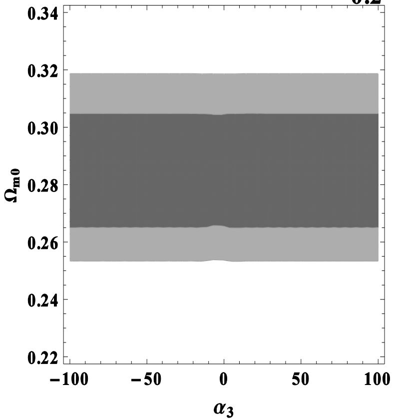
Observational constraints on quasi-dilaton massive gravity

- Use observational data (SNIa, BAO, CMB) to constrain the parameters of the theory. We fit $\Omega_{M0}, \Omega_{DE0}, m_g, \omega, \alpha_3, \alpha_4$









[Gannouji, Hossain, Sami, Saridakis PRD 87]

Extension 3: F(R) nonlinear massive gravity

$$\boxed{S = M_p^2 \int d^4x \sqrt{-g}\left[\frac{F(R)}{2} + m_g^2\left(L_2+\alpha_3 L_3+\alpha_4 L_4\right)\right]}$$

↑ UV modification

↑ IR modification

where

$$L_2 = \frac{1}{2}\left([K]^2 - [K^2]\right)$$

$$L_3 = \frac{1}{6}\left([K]^3 - 3[K][K^2] + 2[K^3]\right)$$

$$L_4 = \frac{1}{24}\left([K]^4 - 6[K]^2[K^2] + 3[K^2]^2 + 8[K][K^3] - 6[K^4]\right)$$

$$[K] = tr\left(K_\mu^\nu\right)$$

$$K_\nu^\mu \equiv \delta_\nu^\mu - \sqrt{g^{\mu\sigma} f_{ab}(\phi)\partial_\nu \phi^a \partial_\sigma \phi^b}$$

[Cai, Duplessis, Saridakis PRD 90a]

[Cai, Saridakis PRD 90b]

Extension 3: F(R) nonlinear massive gravity

- Einstein frame: $g_{\mu\nu}\rightarrow\tilde{g}_{\mu\nu}=\Omega^2 g_{\mu\nu}$ with $\Omega^2=F_{,R}=\exp\left(\sqrt{\frac{2}{3}}\frac{\varphi}{M_p}\right)$

$$\boxed{S=\int d^4x\sqrt{-g}\left[M_p^2\frac{\tilde{R}}{2}+M_p^2 m_g^2\left(\tilde{L}_2+\alpha_3\tilde{L}_3+\alpha_4\tilde{L}_4\right)-\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-U(\varphi)\right]}$$

with $U(\varphi)=M_p^2\frac{RF_{,R}-F}{2F_{,R}^2}$

- Hamiltonian constraint analysis: the BD ghost is removed similar to usual nonlinear massive gravity

- Much more general than other massive gravity extensions.

[Cai, Duplessis, Saridakis PRD 90a] [Cai, Saridakis PRD 90b]

Cosmology of F(R) nonlinear massive gravity

- Physical metric: open FRW: $ds^2 = -N^2dt^2 + a^2(t)\left[dx^2+dy^2+dz^2 - \frac{|K|\left(xdx+ydy+zdz\right)^2}{1+|K|\left(x^2+y^2+z^2\right)}\right]$

  Fiducial metric: Minkowski: $f_{ab} = \eta_{ab}$

  Stückelberg scalars: $\phi^0 = b(t)\sqrt{1+|K|(x^2+y^2+z^2)},\ \phi^i = \sqrt{|K|}b(t)x^i$

- Variation wrt b provides the constraint equation with solution: $\frac{b(t)}{a(t)} = const.$

$$3M_p^2\left(H^2 - \frac{|K|}{a^2}\right) = \rho_m + \rho_{MG} + \rho_{F_R}$$

$$\left.\begin{aligned} \rho_{MG} &= m_g^2 c_\pm \\ \rho_{F_{,R}} &= M_p^2\left[\frac{RF_{,R} - F}{2} - 3H\dot{R}F_{,RR}\right] \end{aligned}\right\}$$

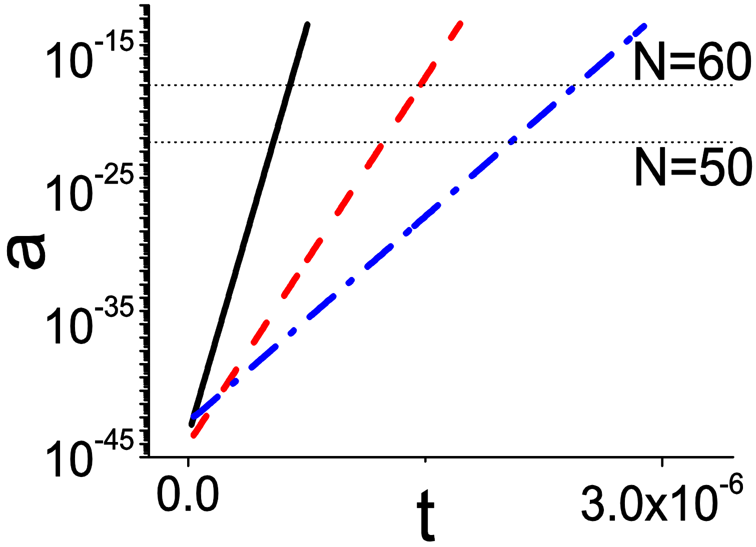
$$\boxed{\rho_{DE} \equiv \rho_{MG} + \rho_{F_R}}$$

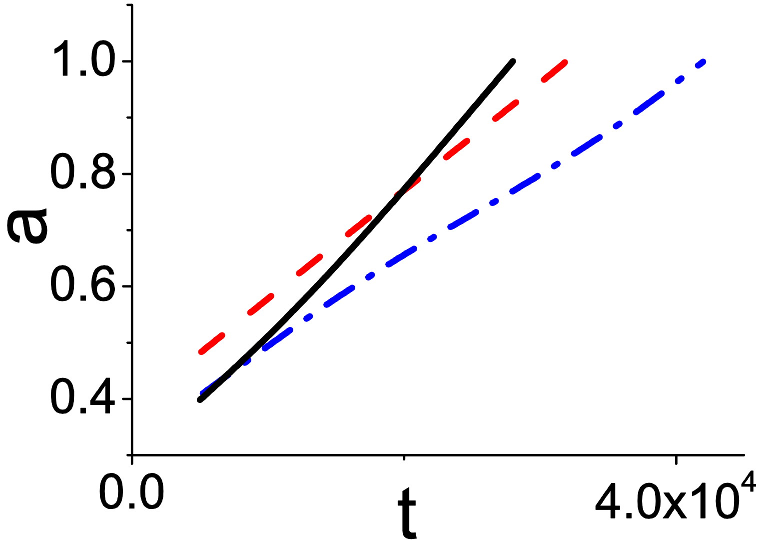
- Both IR and UV gravity modifications play a role in universe evolution.
- Huge capabilities.

[Cai, Duplessis, Saridakis PRD 90a] [Cai, Saridakis PRD 90b]

Cosmology of F(R) nonlinear massive gravity

- 1) $F(R) = R + \frac{\xi}{M_p^2} R^2$



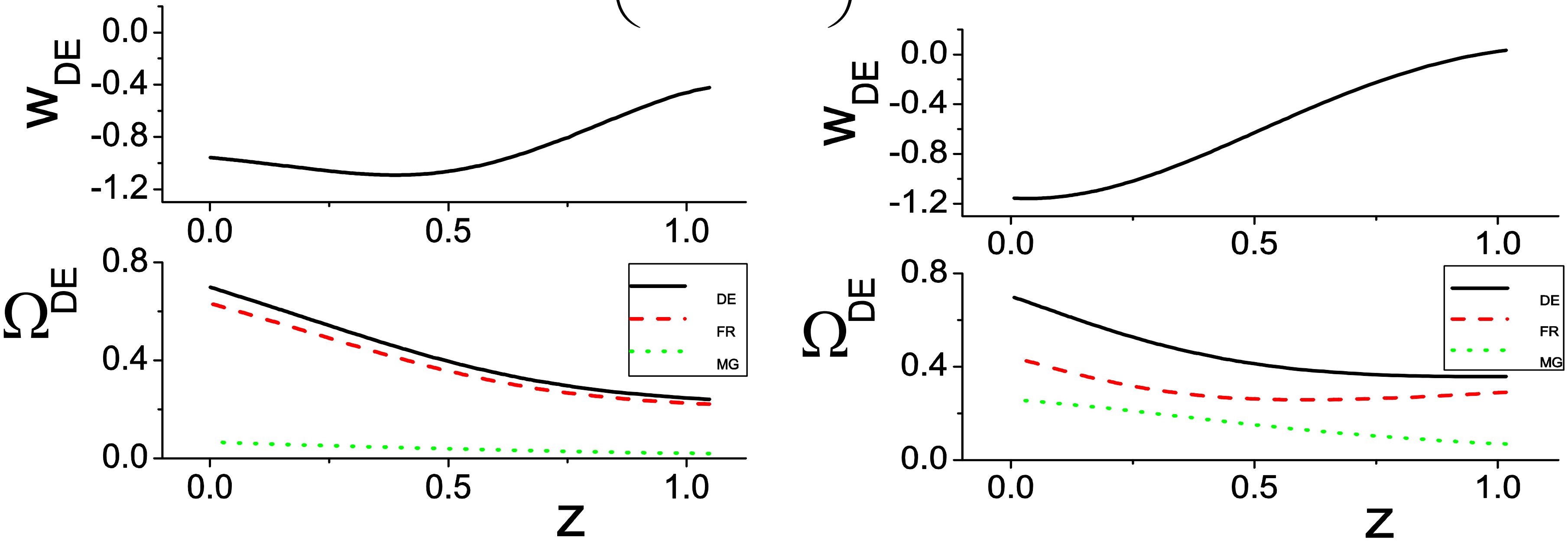


- Early times: F(R) sector drives inflation
- Late times: MG sector drives late-time acceleration

[Cai, Duplessis, Saridakis PRD 90a]

Cosmology of F(R) nonlinear massive gravity

- 2) $F(R) = R - \beta R_S\left(1 - e^{-\frac{R}{R_S}}\right)$

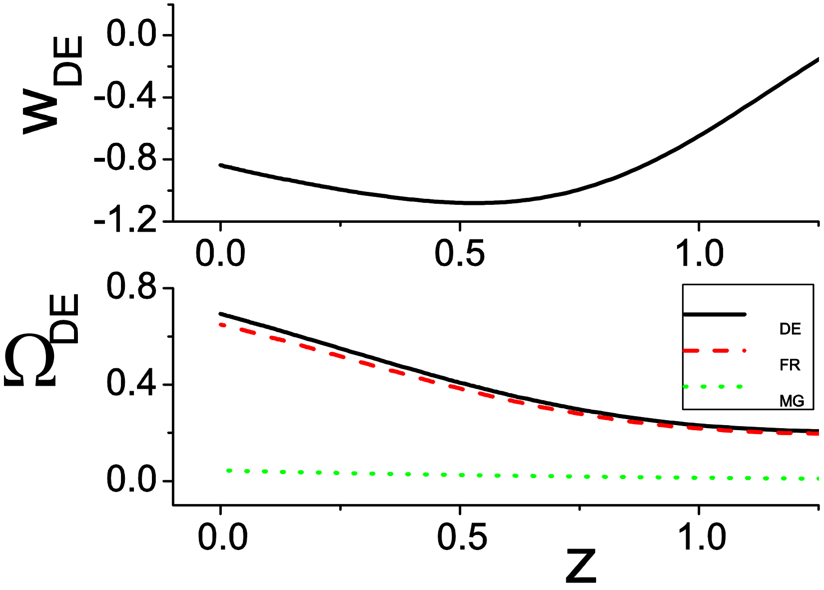


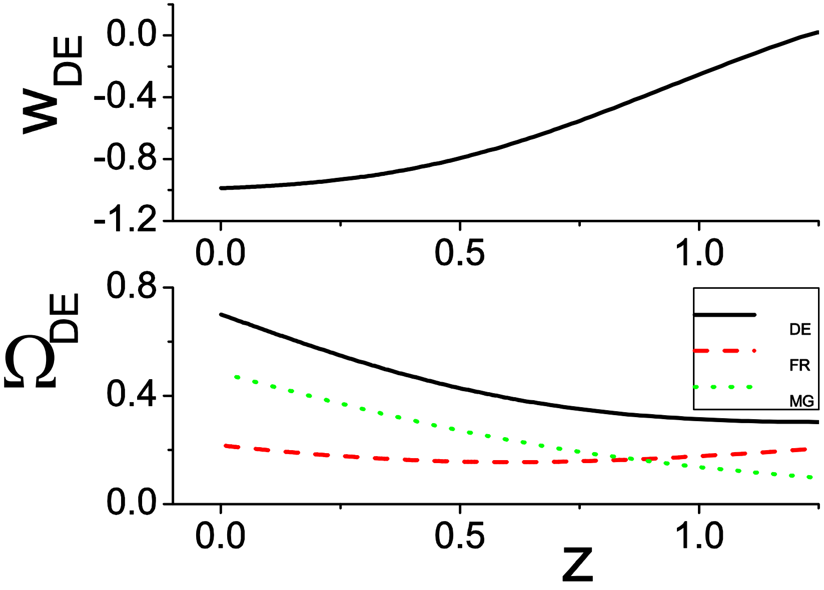
- Both F(R) sector and MG sector constitute Dark Energy $\rho_{DE} \equiv \rho_{MG} + \rho_{F_R}$
- w_{DE} can lie in the phantom regime.

[Cai, Saridakis PRD 90b]

Cosmology of F(R) nonlinear massive gravity

- 3) $F(R) = R - \lambda R_C \left[1 - \left(1 + \frac{R^2}{R_C^2} \right)^{-n} \right]$





- Both F(R) sector and MG sector constitute Dark Energy $\rho_{DE} \equiv \rho_{MG} + \rho_{F_R}$
- w_{DE} can lie in the phantom regime.

[Cai, Saridakis PRD 90b]

Cosmological Perturbations

- $$\boxed{S=\int d^4x\sqrt{-\tilde{g}}\left[M_p^2\frac{\tilde{R}}{2}+M_p^2m_g^2\left(\tilde{L}_2+\alpha_3\tilde{L}_3+\alpha_4\tilde{L}_4\right)-\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-U(\varphi)\right]}$$

$$\delta\tilde{g}_{00}=-2N^2\phi,\ \ \delta\tilde{g}_{0i}=Na\partial_iB,\ \ \delta\tilde{g}_{ij}=a^2\left[2\tilde{\gamma}_{ij}^{K}\psi+\left(\nabla_i\nabla_j-\frac{1}{3}\tilde{\gamma}_{ij}^{K}\nabla_k\nabla^k\right)\right]E,\ \ \delta\varphi$$

$$\Rightarrow\cdots\ \ \cdots$$

- Integrate out non-dynamical dof's $\phi,\ B,\ E$
- Since ϕ is non-dynamical at the linear level on the self-accelerating solution, we introduce the Bardeen potential ψ_{B} and Mukkanov-Sasaki variable

$$\boxed{Q\equiv\delta\varphi+\frac{\dot{\varphi}\psi_B}{H}}$$

$$\Rightarrow\underbrace{\ddot{Q}_k+3H\dot{Q}_k+\left[\frac{k^2}{a^2}+U_{,\varphi\varphi}-\frac{1}{M_p^2a^3}\left(\frac{a^3}{H}\dot{\varphi}^2\right)^{\bullet}\right]Q_k}_{\substack{\uparrow\\ \text{GR + scalar}}}=\underset{\substack{\uparrow\\ \text{MG contribution}}}{\frac{2m_g^2\tilde{Y}_Q}{3\Omega^4}Q_k}-2\frac{k^2}{a^2H^2}\left(\ddot{\varphi}-\frac{\dot{H}\dot{\varphi}}{H}\right)\psi_{\mathrm{B}}$$

- $\tilde{\mathrm{Y}}_Q(\alpha_3,\alpha_4)<0\Rightarrow$ Stability!

[Cai, Duplessis, Saridakis PRD 90a]   [Cai, Saridakis PRD 90b]

Conclusions

- i) Massive gravity is a reasonable modification to describe acceleration.
- ii) The simplest linear model has the vDVZ discontinuity.
- iii) Non-linearities cure it but bring the BD ghost.
- iv) New nonlinear MG uses suitable graviton self-interactions in order to be free of BD ghosts and vDVZ discontinuity.
- v) But simple FRW cosmology is impossible (cosmological instabilities).
- vi) One should go to anisotropic geometry.
- vii) Or other extensions: Varying mass massive gravity, quasi-dilaton massive gravity.
- viii) F(R) nonlinear massive gravity is the most promising. It is free of BD ghost and vDVZ discontinuity. It exhibits good and rich cosmology, free of instabilities!

Outlook

- Many subjects are open. Amongst them:
- i) The first simple idea does not work. Are we doing epicycles?
- ii) Massive gravity, partially massless gravity or bi-gravity

  (or multi-metric gravity)?
- iii) Is the initial BD ghost just hidden under the carpet and reincarnate as instability, superluminality, acausality etc!
- iv) Re-parametrization of our ignorance? (instead to explain why Λ is small, we have to explain why m_g is small).

THANK YOU!