Anisotropic Gauge/Gravity Dualities

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Outline

Sketch:Gauge/Gravity correspondence

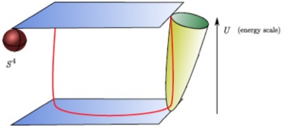
- In this talk we use the gauge/gravity correspondence! A way to map unsolved questions of the quantum field theories to questions that may be solved in gravity backgrounds (geometry and fluxes).
- In the correspondence there exist a map between gauge invariant operators in field theory and states in string theory.
- One may think is as a field theory living in the 4-dimensional boundary of at least a 5-dimensional space. The additional direction is the holographic one. Examples to follow...
- Quick example: The expectation value of the Wilson loop may be found by a minimal surface computation where the actual Wilson loop is placed on the boundary where the field theory lives, and the surface extends in the holographic direction.

Introduction-Deformations of AdS/CFT

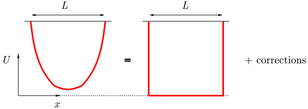
- The initial AdS/CFT correspondence between the $\mathcal{N}=4$ supersymmetric Yang-Mills and type IIB superstring theory on $AdS_5 \times S^5$ is the harmonic oscillator of the Gauge/Gravity dualities.
- First thing we did, was to understand better "the harmonic oscillator".
- Another way for research is to construct gauge/gravity dualities that can be though as toy models to describe realistic systems and theories, with the hope of some universal behaviors.

✓ Less Supersymmetry. Examples:
a)Sasaki-Einstein solutions $AdS_5 \times Y^{p,q}$, $N=1$ supersymmetry.
b)TsT deformation on every background with global $U(1) \times U(1)$ symmetry. eg. β deformation: $AdS_5 \times \tilde{S}^5$.
Experts side remark: The TsT deformation may be used to obtain top-down the Schrodinger $z=2$ space, and the non-commutative spaces.

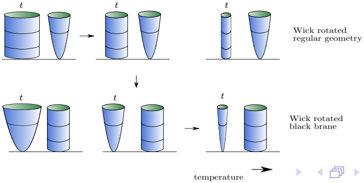
✓ Broken conformal symmetry, confinement.
Example: $D4$ Witten model.



The potential of the static heavy meson is linear:
$V_{Q\bar{Q}} = \sigma L + \mathcal{O}(e^{-L})$, with $\sigma(u_k)$ is the string tension related to the tip of the geometry u_k.



✓ Finite temperature. Example: Presence of Black hole.



✓ Inclusion of dynamical quarks. Example: D7 flavor branes in AdS.
In this case exist two limits: Quenched and Unquenched.

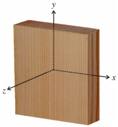
- In Quenched Limit the branes are treated as probes. The meson spectrum may be found.
- But to see how the gluons are affected by the mesons, for example to observe the screening in the static heavy meson $Q\bar{Q}$, the backreaction of the flavor branes need to be taken into account: unquenched limit.

✓ Inclusion of Anisotropy (for our purposes)

...

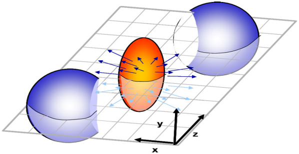
Motivation

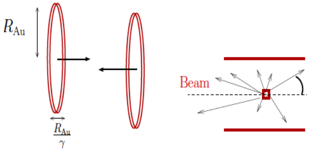
- The existence of strongly coupled anisotropic systems. Example:
  The expansion of the plasma along the longitudinal beam axis at the earliest times after the collision results to momentum anisotropic plasmas.
  Anisotropic low dimensional materials in condensed matter.
  Everyday,naturally anisotropic material: Wood. (not strongly coupled...)



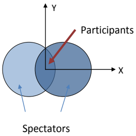
- There exist several results for observables in weakly coupled anisotropic plasmas. Is there any relevance with the strongly coupled limit models?
- Theoretical interest for the anisotropic theory, since it is a consistent top-down model. Properties of the supergravity solutions, that are dual to the anisotropic theories.
- New Features. Several Universality Relations predicted for the isotropic theories are violated!

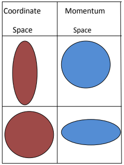
Example:Elliptic flow

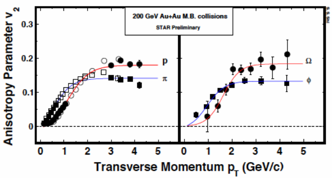




Pressure gradients for non-central collisions along the short axis of the elliptic flow are higher than the long axis. Therefore the expansion along the short axis is more rapid leading to anisotropic momentum distribution.







The nuclear medium expands preferably along he short axis of the ellipse. The elliptic flow parameter

$$v_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

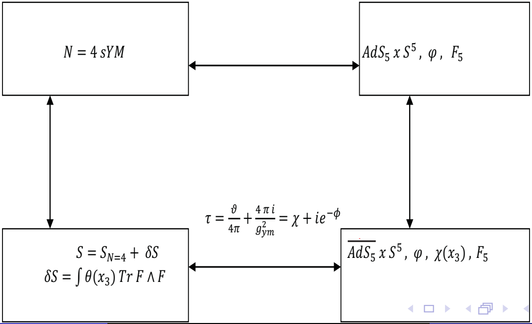
can be measured experimentally through the particle distributions.

Introducing the anisotropy. An example:

- Introduction of additional branes: Lifshitz-like Supergravity solutions with metric $ds^2 = r^{2z}(dt^2 + dx_i^2) + r^2 dx_3^2 + \frac{dr^2}{r^2}$. *(Azeyanagi, Li, Takayanagi, 2009)*

	x_0	x_1	x_2	x_3	u	S^5
D3	x	x	x	x		
D7	x	x	x			x

- Which equivalently leads to the following deformation diagram.



An anisotropic background

The metric in string frame *(Mateos, Trancanelli, 2011)*

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}\mathcal{B}\,dx_0^2 + dx_1^2 + dx_2^2 + \mathcal{H}d{x_3}^2 + \frac{du^2}{\mathcal{F}}\right) + \mathcal{Z}\,d\Omega_{S^5}^2 \,.$$

The functions $\mathcal{F}, \mathcal{B}, \mathcal{H}$ depend on the radial direction u and the anisotropy. The anisotropic parameter is α with units of inverse length ($\chi = \alpha x_3$).

In sufficiently high temperatures, $T \gg \alpha$, and imposed boundary conditions the Einstein equations can be solved analytically:

$$\mathcal{F}(u) = 1 - \frac{u^4}{u_h^4} + \alpha^2 \frac{1}{24 u_h^2}\left[8u^2(u_h^2 - u^2) - 10u^4 \log 2 + (3u_h^4 + 7u^4)\log\left(1 + \frac{u^2}{u_h^2}\right)\right]$$

$$\mathcal{B}(u) = 1 - \alpha^2 \frac{u_h^2}{24}\left[\frac{10u^2}{u_h^2 + u^2} + \log\left(1 + \frac{u^2}{u_h^2}\right)\right], \quad \mathcal{H}(u) = \left(1 + \frac{u^2}{u_h^2}\right)^{\frac{\alpha^2 u_h^2}{4}}$$

The isotropic limit $\alpha \to 0$ reproduce the well know result of the isotropic D3-brane solution (dual to $\mathcal{N} = 4$ finite sYM solution).

The metric can be expressed in α, T parameters through

$$u_h = \frac{1}{\pi T} + \alpha^2 \frac{5log2 - 2}{48\pi^3 T^3} \ .$$

The pressures in the low anisotropic limit satisfy the following inequality

$$P_{x_3} < P_{x_1 x_2} \ .$$

Experts Side Remark: The linear dependence of the axion with the anisotropic direction x_3 is crucial in order to find the supergravity solution!

Critique and Answers

Question: Is this the background that describes the anisotropic strongly coupled QGP?
Possibly not!

Question: Why do we bother then?
There exist Universal Results! The exact details of the metric do not matter in several observables. It matters only the shape of the geometry.

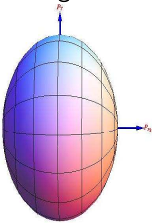
Question: What are the universal relations?
Oversimplified example: Suppose for the different observables $Q_{1,2}$ we have among the different finite temperature theories at some limit $Q_1 = c_1 T^d, Q_2 = c_2 T^d$ then $Q_1/Q_2 = c_1/c_2$.

Generic setup:

We can work in full generality by renaming for example the anisotropic metric as

$$ds^2 = g_{00}(u)dx_0^2 + g_{11}(u)dx_1^2 + g_{22}(u)dx_2^2 + g_{33}(u)dx_3^2 + g_{uu}(u)du^2 + \text{internal space}$$

Then x_p is a chosen direction. May be:
$x_p = x_1 =: x_\perp$ transverse direction to anisotropy,
$x_p = x_3 =: x_\parallel$ parallel direction to anisotropy.
If $g_{11} = g_{22} > g_{33}$ we have oblate geometries.
If $g_{11} = g_{22} < g_{33}$ we have prolate geometries.



Universality Relations

Shear Viscosity over Entropy density ratio universal low value prediction

$$\frac{\eta}{s} \gtrsim \frac{1}{4\pi} \ .$$

In anisotropic theories has been found to be clearly violated! (Rebhan, Steineder 2011)

Reason:

$$\frac{\eta}{s} \propto \frac{g_{11}(u_h)}{g_{33}(u_h)} \frac{1}{4\pi} \ .$$

All prolate geometries, violate the bound!

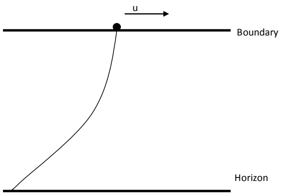
• Another universality relation which is violated is the inequality between the Langevin coefficients: $\kappa_L > \kappa_T$.

(Gursoy, Kiritsis, Mazzanti, Nitti 2010;D.G, Soltanpanahi, 2013)

The dynamics and the interactions of the heavy quark can be described by a diffusion treatment. The thermal momentum of the quark is $p_{th}^2 \simeq 3m_Q T \gg T^2$. The momentum transfer of the medium is $Q^2 \simeq T^2$. Therefore: Brownian motion of the heavy quark in a light particle fluid.

$$\frac{dp}{dt} = F_{drag} + F(t) \ .$$

In AdS/CFT the drag force of a single quark moving in the anisotropic plasma can be represented by a trailing string from the boundary where the probe quark moves with the constant speed, to the horizon of the black hole.



The drag force of a quark moving along the p direction, for any background is given by the momentum flowing from the boundary to the bulk

$$F_{drag,p} = -\sqrt{\lambda}\frac{\sqrt{-g_{00}g_{pp}}}{(2\pi)}\Big|_{u=u_0}$$

where u_0 is given by

$$\left(g_{uu}(g_{00} + g_{pp}v^2)\right) |_{u=u_0} = 0 \ .$$

At $u = u_0$ there is horizon of the induced worldsheet metric. The 'effective world-sheet temperature' is

$$T_{ws}^2 = \left|\frac{1}{16\pi^2}\frac{1}{g_{00}g_{uu}}(g_{00}\, g_{pp})'\left(\frac{g_{00}}{g_{pp}}\right)'\right|\Big|_{u=u_0} \ .$$

In near horizon Dp black brane geometries $T_{ws} < T$. (*Nakamura, Ooguri 2013*)

Momentum Broadening

The $F(t)$ is the factor that causes the momentum broadening, which leads to

$$\frac{\left\langle p_{L,T}^2 \right\rangle}{\mathcal{T}} = 2\kappa_{L,T}$$

$\kappa =$ Mean Squared Momentum Transfer per Time.

• The index L refers to the direction along the motion of quark, the index T is the direction transverse to the velocity of quark.

• In strong coupling limit for a quark moving along p direction, these fluctuations are introduced to the Wilson line

$$t = \tau, \quad u = \sigma, \quad x_p = v\ t + \xi(\sigma) + \delta x_p(\tau,\sigma), \quad x_k = \delta x_k(\tau,\sigma)\ .$$

$\delta x_p(\tau,\sigma)$: Longitudinal fluctuation.
$\delta x_k(\tau,\sigma)$: Transverse fluctuation.

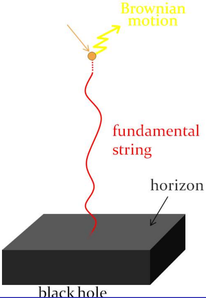
The Nambu-Goto action in fluctuations around the solution to quadratic order becomes

$$S_2 = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma \, \frac{\tilde{G}^{\alpha\beta}}{2}\left[N(u)\,\partial_\alpha\delta x_p\,\partial_\beta\delta x_p + \sum_{i=2,3} g_{ii}\partial_\alpha\delta x_i\,\partial_\beta\delta x_i\right]$$

where

$$\tilde{G}^{\alpha\beta} = \sqrt{-\tilde{g}}\tilde{g}^{\alpha\beta}, \qquad N(u) = \frac{g_{00}\,g_{pp} + C^2}{g_{00} + g_{pp}\,v^2},$$

$\tilde{g}^{\alpha\beta}$ is the world-sheet metric, expressed in metric elements of the background.



The Langevin coefficients for a quark moving along the p direction and the transverse direction is taken to be k:

$$\kappa_k = \frac{1}{\pi\alpha'}\,g_{kk}\Big|_{u=u_0} T_{ws}\,,$$

$$\kappa_p = \frac{16\,\pi}{\alpha'}\,\frac{|g_{00}|\,g_{uu}}{g_{pp}\left(\frac{g_{00}}{g_{pp}}\right)'^2}\Bigg|_{u=u_0} T_{ws}^3\,.$$

Their ratio can be simplified to

$$\frac{\kappa_p}{\kappa_k} = \frac{1}{g_{pp} g_{kk}} \frac{(g_{00} g_{pp})'}{(g_{00}/g_{pp})'}\Bigg|_{u=u_0}$$

Example: $p = 3$ and $k = 1$: Quark moves along the anisotropic direction x_3, and the transverse direction to motion is x_1.

•For any isotropic theory $g_{pp} = g_{kk}$ and $g_{00} = g_{00,bh}\ g_{kk}$, we prove $\kappa_L > \kappa_T$.

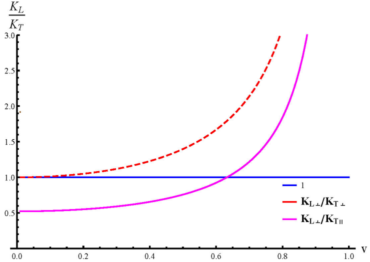
• This is a Universal Inequality independent of the background used! *(D.G, Soltanpanahi 2013)*

• The only possibility to have violation of the inequality is in the anisotropic theories!

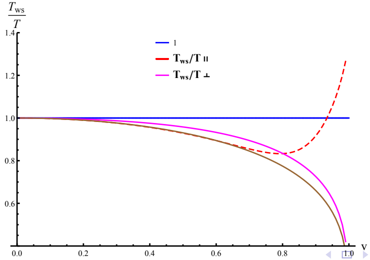
• In fact the motion of the quark in the axion deformed anisotropic theory violates the inequality!

Consider the space dependent axion anisotropic background.

- $\kappa_L < \kappa_T$ when the motion of the quark is along the transverse to the anisotropic direction.



- $T_{ws} \lesseqgtr T$ in aniso theories in contrast to iso $T_{ws} < T$.



Other Observables I:

- Static potential in the anisotropic plasma:
  - $V_{\parallel} < V_{\perp} < V_{iso}$.
  - The critical length of the string beyond the quarks are not bounded is decreased in presence of anisotropy as $L_{c\parallel} < L_{c\perp} < L_{c\ iso}$.

  (D.G ; Chernicoff, Fernandez, Mateos, Trancanelli; Rebhan, Steineder; 2012; D.G 2013)

- Analysis of the Imaginary Potential and the Thermal Width can be made by fluctuating the same string configuration.
  (Bitaghsir, D.G, Soltanpanahi, 2013)

- Experts Side Remark: k-strings (multi-quark bound states) can be written as fundamental strings (meson states) in anisotropic theories. *(D.G, 2015)*.

Other Observables II:

- Jet quenching (momentum broadening)

$$\hat{q}_{p\ (k)} = \frac{\sqrt{2}}{\pi\alpha'} \left(\int_0^{u_h} \frac{1}{g_{kk}} \sqrt{\frac{g_{uu}}{g_{--}}} \right)^{-1} .$$

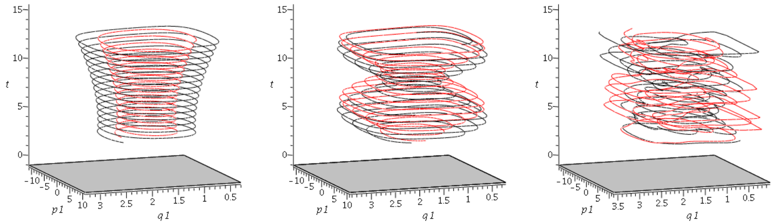
(D.G 2012)

where $g_{--} = 1/2(g_{00} + g_{pp})$. The index p denotes the direction along the motion of the quark and k the direction along which the momentum broadening happen.

- $\hat{q}_{\|(\perp)} > \hat{q}_{\perp(\|)} > \hat{q}_{\perp(\perp)} > \hat{q}_{iso}$.
  (D.G; Chernicoff, Fernandez, Mateos, Trancanelli; Rebhan, Steineder 2012)
  Note:The result appears to have similarities between different strongly coupled models and also weakly coupled calculations.
  (Janik,Witaszczyk 2008; Dumitru, Nara, Schenke, Strickland, 2007; Romatschke 2006; Baier Mehtar-Tani 2008).

Other Observables III:

- All the zero temperature anisotropic Lifshitz-like solutions are proven to be non-integrable. *(D.G, Sfetsos 2014)*.
  This is in contrast to the integrable AdS space.
- Existence of chaos in the spectrum of glueballs in anisotropic phase in finite temperature? Example of chaotic strings in other theories:



Conclusions

Working with general backgrounds we have presented properties of the anisotropic finite temperature theories.

- Several Universal Relations are violated in the Anisotropic Theories, but in a controlled way.
  The Langevin coefficients inequality $\kappa_L > \kappa_T$ proved to hold for isotropic backgrounds is violated for the anisotropic theories!

Work in progress:

- More violations to come: Speed of Sound bound. Deeper reasons?
- Anisotropic Thermalization with time dependent black hole solutions. Existence of instabilities?
  Challenging, but hope for analytical results.

Thank you