Integrability and Exact results in $\mathcal{N} = 2$ gauge theories

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DESY Theory

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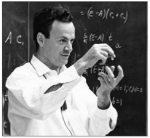
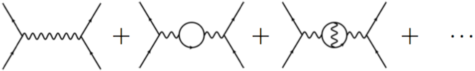
arXiv:1406.3629 with Vladimir Mitev
work in progress

Motivation: Can we go beyond perturbation theory?





$$= c_1 \lambda + c_2 \lambda^2 + c_3 \lambda^3 + \cdots$$

$$\lambda << 1$$

Noether: *Symmetry* ➜ *Conservation law*

The more symmetry the easier it is to solve the problem.

- Yes! If we add **more symmetry** to the problem: SUSY

The **most symmetric gauge theory** in 4D is $\mathcal{N} = 4$ super Yang Mills.

Motivation: The success story for $\mathcal{N} = 4$ SYM

Possible to compute observables in the **strong coupling regime** and in some cases to even obtain **Exact results** (for any value of the coupling).

- **AdS/CFT** (gravity/sigma model description)
- **Integrability** (The spectral problem is solved) *at large N_c*
- **Localization** (Exact results: e.x. Circular WL) for *any N_c*

Which of these properties/techniques are transferable to **more realistic gauge theories** in 4D with less SUSY?

The statement

1. $\forall$ conformal $\mathcal{N}=2$ gauge theory there is a **purely gluonic** subset of local operators $SU(2,1|2)$ **integrable in the planar limit**

$$\gamma_{\mathcal{N}=2}\left(g\right)=\gamma_{\mathcal{N}=4}\left(\mathbf{g}\right)$$

2. The **Exact Effective coupling** (relative **finite renormalization** of g)

$$\mathbf{g}^2=f(g^2)=g^2+g^2\left(Z_{\mathcal{N}=2}-Z_{\mathcal{N}=4}\right)$$

we compute using localization

$$W_{\mathcal{N}=2}\left(g^2\right)=W_{\mathcal{N}=4}\left(\mathbf{g}^2\right)$$

$\mathcal{N} = 4$ Super Yang Mills (SYM)

$\boxed{SU(4) \to U(1) \times SU(2)_R \times SU(2)_L}$

The $\mathcal{N} = 4$ vector multiplet in the **adjoint** of $SU(N)$:

the **gluon** and its **SUSY partners**:

- $\mathcal{N} = 2$ vector multiplet **adjoint** in $SU(N)$:

$$\begin{array}{ccc} & A_\mu & \\ \lambda^1_\alpha & & \lambda^2_\alpha \\ & \phi_1 & \end{array} \quad , \qquad \lambda^{\mathcal{I}} = \begin{pmatrix} \lambda^1 \\ \lambda^2 \end{pmatrix} , \qquad \mathcal{I} = 1, 2$$

- $\mathcal{N} = 2$ hyper multiplet in the **adjoint** of $SU(N)$:

$$\begin{array}{ccc} & \lambda^3_\alpha & \\ \phi_2 & & \phi_3 \\ & \lambda^4_\alpha & \end{array} \quad , \qquad \Phi^{\mathcal{I}} = \begin{pmatrix} \phi_2 \\ \phi_3 \end{pmatrix}$$

It is conformal $\beta = 0$ and has an **exactly marginal coupling**!

$\mathcal{N} = 4$ SYM has no quarks!

$\mathcal{N} = 2$ SuperConformal QCD (SCQCD)

$\boxed{U(1) \times SU(2)_R}$

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- $\mathcal{N} = 2$ hyper multiplet **fundamental** in $SU(N)$ and $U(N_f)$:

$$\begin{array}{ccc} & \psi_{\alpha\ i} & \\ q_i & & (\tilde{q})^*_i \\ & \left(\tilde{\psi}_\alpha\right)^\dagger_i & \end{array} \,, \quad Q^{\mathcal{I}} = \begin{pmatrix} q \\ \tilde{q}^* \end{pmatrix} \,, \quad i = 1, \dots N_f$$

When $N_f = 2N$: $\beta = \frac{g^3_{YM}}{16\pi^2}\left(N_f - 2N\right) = 0$, **exactly marginal coupling**!

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$\mathcal{N} = 2$ SCFT with $SU(N) \times SU(N)$ gauge group: **two exactly marginal** g and $\check{g}$:

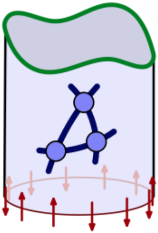
- For $g = \check{g}$ we get the $\mathcal{N} = 4$ result (for the observables we consider)
- In the limit $\check{g} \to 0$ obtain $\mathcal{N} = 2$ SCQCD with $N_f = 2N$

Integrability of the purely gluonic $SU(2,1|2)$ Sector

$\mathcal{N} = 4$ Integrability

$\mathcal{N} = 4$ SYM is integrable in the planar limit for **any coupling**

- ***Perturbation theory***: mapped to an integrable spin chain
- ***Strong coupling***: integrable 2D theory on the string world-sheet



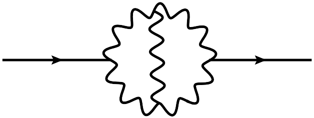
Powerful integrability toolkit

- The spectral problem is solved **exactly**: for **any coupling**

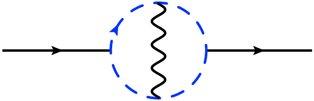
Integrability now is applied to **other observables.**

Next step $\mathcal{N} = 2$: A diagrammatic observation

The **only possible way** to make diagrams with **external fields in the vector mult.** different from the $\mathcal{N} = 4$ ones is to make a loop with hyper's and then in this loop let a **checked vector** propagate! (EP-Christoph Sieg)



The same with $\mathcal{N} = 4$ SYM



Different from $\mathcal{N} = 4$ SYM
but **finite !!**

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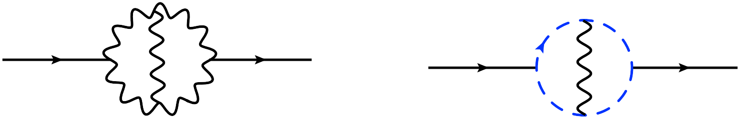

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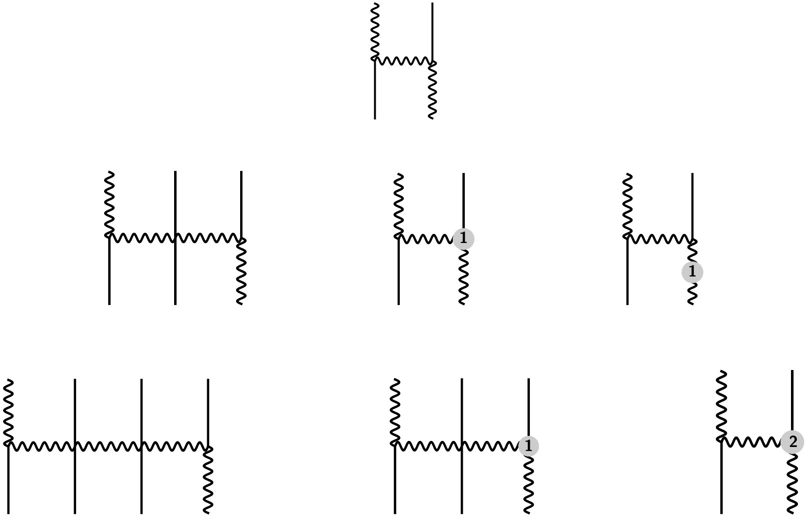
Novel Regularization prescription:

For every individual $\mathcal{N} = 2$ diagram subtract its $\mathcal{N} = 4$ counterpart.



$$H^{(3)}_{\mathcal{N}=2}(\lambda) - H^{(3)}_{\mathcal{N}=4}(\lambda) \sim H^{(1)}_{\mathcal{N}=4}(\lambda) \quad \Rightarrow \quad H^{(3)}_{\mathcal{N}=2}(\lambda) = H^{(3)}_{\mathcal{N}=4}(f(\lambda))$$

with $f(\lambda) = \lambda + c\lambda^3$

Operator renormalization in the Background Field Gauge

Background Field Method: $\varphi \to A + Q$

where A the classical background and Q the quantum fluctuation

$$g_{bare} = Z_g\, g_{ren}\,,\ A_{bare} = \sqrt{Z_A}\, A_{ren}\,,\ Q_{bare} = \sqrt{Z_Q}\, Q_{ren}\,,\ \xi_{bare} = Z_\xi\, \xi_{ren}$$

In the Background Field Gauge $\boxed{Z_g \sqrt{Z_A} = 1}$ and $\boxed{Z_Q = Z_\xi}$

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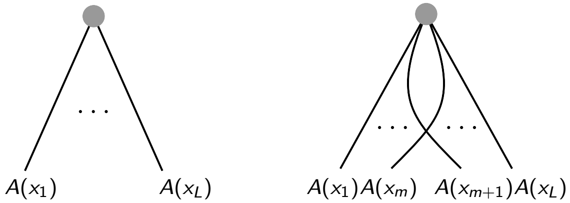
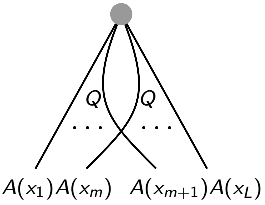
- Compute $\langle \mathcal{O}(y) A(x_1) \cdots A(x_L) \rangle$ for $\mathcal{O} \sim \mathrm{tr}\left(\varphi^L\right)$.



+ *more diagrams*

Wick contact $\mathcal{O}_i^{ren}(Q_{ren}\,,\, A_{ren}) = \sum_j Z_{ij} \mathcal{O}_j^{bare}\left(Z_Q^{1/2} Q\,,\, Z_A^{1/2} A\right)$

Background Field Method: No Q's outside, no A's inside!



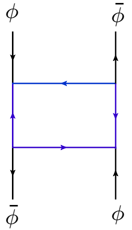
- $\langle QQAA \rangle$ renormalize as $Z_Q^{2/2} Z_A^{2/2} \langle QQAA \rangle$
- The Q propagators as Z_Q^{-1}
- the $\mathcal{O}^{ren}$ has two more $Z_Q^{1/2}$
- all Z_Q will cancel (We knew it - gauge invariance!)
- Only $Z = Z_g^2 = Z_A^{-1}$, the combinatorics the same as in $\mathcal{N} = 4$:

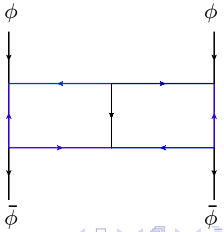
$$\boxed{\mathrm{H}_{\mathcal{N}=2}(g) = H_{\mathcal{N}=4}(\mathbf{g}) \quad \text{with} \quad \mathbf{g}^2 = f(g^2, \check{g}^2) = g^2 + g^2 (Z_{\mathcal{N}=2} - Z_{\mathcal{N}=4})}$$

New vertices cannot contribute

$$\Gamma = \Gamma_{ren.\,tree} + \Gamma_{new}$$

- $\Gamma_{ren.\,tree}$: **vertex** and **self-energy renormalization**
  all encoded in $\delta Z = Z_{\mathcal{N}=2} - Z_{\mathcal{N}=4}$
- New vertices cannot contribute due to the **non-renormalization theorem** (Fiamberti, Santambrogio, Sieg, Zanon)





Localization and Exact Effective couplings

Localization

$$Z_{S^4} = \int [D\Phi]\, e^{-S[\Phi]} = \int da\, |\mathcal{Z}(a)|^2$$

The **path integral** localizes to an **ordinary integral**
(*Cancelations due to supersymmetry*)

We can do an ordinary integral.
Compute the path integral exactly.
For **any value of the coupling constant.**

(Pestun)

$\mathcal{N}=4$ SYM:

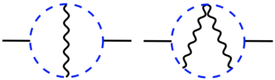
$$W_{\mathcal{N}=4}(g) = \frac{I_1(4\pi g)}{2\pi g}$$

$\mathcal{N}=2$ theories:

$$W_{\mathcal{N}=2}(g,\check{g}) = W_{\mathcal{N}=4}(f(g,\check{g}))$$

$$f(g,\check{g}) = \begin{cases} g^2 + 2\left(\check{g}^2 - g^2\right)\left[6\zeta(3)g^4 - 20\zeta(5)g^4\left(\check{g}^2 + 3g^2\right)\right] + \mathcal{O}(g^{10}) \\ \frac{2g\check{g}}{g+\check{g}} + \mathcal{O}(1) \end{cases}$$

- Checked with Feynman diagrams calculation (up to 4-loops)



- Agrees with AdS/CFT (strong coupling)

Conclusions

- $\forall$ observable in the purely gluonic $SU(2,1|2)$ sector

  take the $\mathcal{N}=4$ answer and replace $g^2 \to \mathbf{g}^2 = f(g^2)$

  We need more checks!! (EP-Mitev), (Leoni-Mauri-Santambrogio) and (Fraser)

- <u>Lesson</u>: Think of the $\mathcal{N}=4$ **SYM as a regulator !!**

  The **integrable** $\mathcal{N}=4$ **model** knows all about the **combinatorics**.

  For $\mathcal{N}=2$: **relative finite renormalization** encoded in $\mathbf{g}^2 = f(g^2)$.

- In asymptotically conformal $\mathcal{N}=2$ theories and $\mathcal{N}=1$ SCFTs

  all loop statement: purely gluonic $SU(2,1|1)$ sector (EP-Roček)