On Supergravity, Superstrings and the Unification Programme

Jean-Pierre Derendinger

Albert Einstein Center for Fundamental Physics, ITP, Bern University

HEP2015 - Conference of Recent Developments in High-Energy Physics and Cosmology

Athens University, April 15–18, 2015

b
UNIVERSITÄT
BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

The Standard Model and the unification programme

The discovery of the Brout-Englert-Higgs particle, the higgs boson,

- experimentally confirms and completes the Glashow, Salam, Weinberg model as *the* description of strong and electroweak interactions
- is a triumph for (renormalizable) quantum field theory, as the appropriate theoretical framework for this description
- is an outstanding performance of experiment, accelerator and theory teams

The Standard Model: a theoretical creation, 1960's until 1973

(Glashow, Salam, Weinberg, Brout, Englert, Higgs, Schwinger, Cabibbo, Iliopoulos, Maiani, Gross, Coleman, Politzer, Wilczek, ...)

and $\sim \mathbf{50}$ years of experiments, new facilities and technologies, analysis, phenomenology and theory developments.

(> 1 *generation of physicists . . .*)

Dates and names

The unification programme, “beyond the Standard Model":

- 1974: $SU(5)$, $SO(10)$, grand unified theories
  (Georgi, Glashow, Fritzsch, Minkowski, ...)
- 1974-76: Supersymmetric field theories
  (Wess, Zumino, Fayet, Ferrara, Iliopoulos, Salam, Strathdee, ...)
- Early 80’s: the link with cosmology, “astroparticle physics", baryogenesis
  (Sakharov 67, Nanopoulos, Weinberg 79, ...)
- 1976: Simple supergravity – 1982: matter and gauge fields coupled to supergravity
  (Ferrara, Freedman, Van Nieuwenhuizen, Deser, Zumino, ...)
- 1984 – 85: anomaly-free, finite superstrings, heterotic strings, classes of compactifications
  (Green, Schwarz, Gross, Harvey, Martinec, Rohm, ...)
- ......

Waiting for observations

On the Standard Model

Extraordinary successful, gauge sector tested to impressive precision in a very large number of processes.

Simplicity wins over naturalness.

As a theory, an extraordinary simple formulation:

- The rules of renormalizable quantum field theory;
- The gauge group $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$;
- The G_{SM} representation of Weyl fermions;
- The G_{SM} representation of scalars.

Then: essentially algebraic formulation (symmetries and representations).

In essence: parameters of the theory linked to symmetry invariants which, by definition, take arbitrary values.

(More symmetry may lead to more relations)

On the Standard Model

Questions of interest (in the unification programme perspective):

- Values of all dimensionless parameters (α, α_s, $\sin^2\theta_W$, Yukawa couplings generating fermion masses and mixings)
- This includes the question of the very large hierarchy of fermion masses, from ν's, to e^-, u, d, to t).
- Value of the mass scale characterising weak interactions (M_W or G_F)
- Physics needs further scales vastly different from M_W:
  M_{Planck}, M_{GUT} (?), M_{B-L} (?), Λ.
- A scale at the origin of the neutrino masses ? $B-L$ scale ?
- Stability, phase transitions, all non-perturbative aspects ...
- Deeper questions: various why's ? (gauge group, representation, gauge theory, ...)

Very mysterious even after ~ 40 years ...

Standard Model: $B-L$, another scale ?

A curious aspect of the Standard Model is the role of $B-L$:

- All dimension six, four-fermion operators conserve $B-L$ except those involving right-handed/sterile gauge-singlet neutrinos:

$$\psi_L^1\psi_L^2\psi_L^3\psi_L^4 \qquad \text{(scalar exchange)}$$

$$\psi_L^1\psi_L^2\psi_R^3\psi_R^4 \qquad \text{(scalar or vector exchange)}$$

  $B-L$ (and L) violation in $N_L^cN_L^cN_L^cN_L^c$.

  Or in Majorana mass terms $M\,N_L^cN_L^c$, M: scale of $B-L$ violation.

- See-saw neutrino mass $m_\nu \sim m_D^2/M$. For $m_\nu \sim 1$ eV, $m_D \sim 1$ MeV – 1 GeV, "intermediate" $B-L$ scale: $M \sim 10^{3 \text{ to } 9}$ GeV
- Realized as a scale of spontaneous breaking in left-right symmetric models, electroweak symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, or in $SO(10)$ (or E_6) grand unified theories.
- Ratio M/M_W can be large (how? why?).

On the Standard Model

However, the Standard Model does not suffer from:

- A hierarchy problem: a one-scale renormalizable field theory then no other scale to destabilize the order parameter $\langle H \rangle$ ...
- A cosmological constant problem, since no observable in quantum field theory is related to Λ ...

... as long as the Standard Model is not embedded in a

quantum theory of gravitation (and then why and how the ratio $M_{Planck}/M_W \sim 10^{17}$?),

or in a GUT theory (with the ratio $M_{GUT}/M_W \sim 10^{10-14}$).

The hierarchy problem has two sides:

- Conceptual (and elusive): why such large ratios ?
- Technical: can one stabilize a very large ratio under quantum corrections? QFT favours relatively close symmetry breaking scales.
- Supersymmetry eliminates the technical problem.

Cosmological constant *vs* vacuum energy

A silly classical consideration:

Two Higgs potentials with identical physics: (μ^2, λ positive)

$$V_1 = -\mu^2\overline{\Phi}\Phi + \frac{\lambda}{2}(\overline{\Phi}\Phi)^2 \qquad\qquad V_2 = \frac{\lambda}{2}\left(\overline{\Phi}\Phi - \frac{\mu^2}{\lambda}\right)^2$$

In both cases: $\langle\overline{\Phi}\Phi\rangle = \mu^2/\lambda$: relevant order parameter.
One scalar with mass $4\mu^2$, one massless Goldstone scalar.

Of course, since $V_1 - V_2 = -\mu^4/2\lambda$: same dynamical equations.

But: canonical energy momentum tensor:

$$T_{\mu\nu} = \frac{\partial\mathcal{L}}{\partial\partial^\mu\Phi}\partial_\nu\Phi + \frac{\partial\mathcal{L}}{\partial\partial^\mu\overline{\Phi}}\partial_\nu\overline{\Phi} - \eta_{\mu\nu}\mathcal{L} \quad\Longrightarrow\quad \langle T_{\mu\nu}\rangle = \langle V\rangle\eta_{\mu\nu}$$

at the ground state. The energy density of the ground state $\langle T_{00}\rangle$ is well-defined, but the total energy of the ground state (the vacuum state) $\int d^3x\, T_{00}$, or the generator of time translation, is well-defined only with V_2, chosen such that $\langle T_{00}\rangle = 0$.

Cosmological constant *vs* vacuum energy

The point is that classical or quantum physics (without gravitation) only knows about energy differences (or energy exchanges).

There is no observable associated to the vacuum energy, which is without physical significance and without a priori relation to a cosmological constant which appears when gravitation switches on.

- The lowest electron state in hydrogen is 13.6 eV below the continuum.
- Rest energy mc^2 and kinetic energy.
- Casimir force instead of Casimir energy (the effect is anyway $\mathcal{O}(\alpha)$, unrelated to a supposed vacuum energy effect).
- Schrödinger: $H \to H + \epsilon \qquad |\psi\rangle \to e^{-i\epsilon t/\hbar}|\psi\rangle$,
  $\to$ physics unchanged
- Phase transitions.

How could the completely arbitrary number $\langle T_{00}\rangle$ play a role in physics ?
Anyway not a problem of the Standard Model . . .

Super – symmetry/gravity/strings

The unification programme with gravitation.

At present we do not have a better scheme than (or a concrete alternative to):

1. Superstrings as a quantum UV completion, including all four interactions, taking effect at energies $> M_{Planck} \sim \kappa^{-1}$.
2. Supergravity (classical) to organize the transition to field theory, as an effective description of the states lighter than M_{Planck}, to describe string solutions/ground states, and the decoupling of gravitation, $E/M_{Planck} \ll 1$.
3. A quantum field theory at low energies, including the Standard Model, and plausibly remnants of supersymmetry breaking generating soft breaking terms.

The last point is at the origin of the various extensions of the SM, MSSM, NMSSM (the best one!), . . .

Super – symmetry/gravity/strings

The strong points:

- Consistent string theories may teach us how to build a quantum gravity theory.
- The Standard Model can be found in superstrings.
- The intrication of low-energy supersymmetry and supergravity. Supergravity as a source of susy breaking and as mediator to the Standard Model: can be tested, if susy breaking is associated with the weak scale (in a reasonable way . . .).
- An elegant mechanism to generate the weak interaction by radiative corrections only, producing a stable hierarchy of scales, a prediction.
- Allows to concretely investigate exotic ideas, large extra dimensions, non supersymmetric strings (stability in trouble) . . .
- Access to cosmology (inflation models) and dark matter (susy particle candidates).

Super – symmetry/gravity/strings

The weak points:

- Lack of experimental results in support of low-energy supersymmetry, LHC should/will speak.
- Susy very hard to establish experimentally at LHC. The supersymmetric Standard Model can live without superstrings.
- String theory has far too many solutions. Does not discriminate between the various low-energy options. Weak predictivity power, contrary to initial hopes.
- Far too many induced parameters, for instance in the geometry of compact dimensions.
- (Actually, slow theoretical progress in these directions during the last 30 years).

In the following:

I: Some concepts of supergravity theories

(in four space-time dimensions only).

II: An example of superstring compactification with fluxes,

to display the multiple induced parameters of string solutions, and the large volatility of the resulting properties

Supergravity I

Pure $N = 1$ supergravity is *very simple*:

Einstein-Hilbert + Rarita-Schwinger

$$\mathcal{S}_{ERS}[e^a_\mu, \psi_\mu, \omega_{\mu\, ab}] = \frac{1}{2\kappa_D^2} \int d^D x\, e \left(R + \overline{\psi}_\mu \gamma^{\mu\nu\rho} \tilde{D}_\nu \psi_\rho \right)$$

But: local symmetries imply covariant derivatives

$$\begin{aligned}
\tilde{D}_\mu \psi_\nu &= \partial_\mu \psi_\nu + \tfrac{1}{2} \omega_\mu{}^{ab} \sigma_{ab} \psi_\nu && \text{(spin connection)} \\
\omega_{\mu\, ab} &= \omega_{\mu\, ab}(e) + \kappa_{\mu\, ab} && \text{(contorsion tensor)} \\
\tilde{D}_\mu \psi_\nu - \tilde{D}_\nu \psi_\mu &= D_\mu \psi_\nu - D_\nu \psi_\mu + 2\, S^\lambda_{\mu\nu} \psi_\lambda && \text{(torsion tensor)}
\end{aligned}$$

with gravitino torsion (for a $D = 4$ Majorana gravitino)

$$\begin{aligned}
S^\lambda_{\mu\nu} &= -\tfrac{1}{4} \overline{\psi}_\mu \gamma^\lambda \psi_\nu \\
\kappa_{\mu\, ab} &= -\tfrac{1}{4} \left[\overline{\psi}_\mu \gamma_a \psi_b - \overline{\psi}_\mu \gamma_b \psi_a + \overline{\psi}_a \gamma_\mu \psi_b \right]
\end{aligned}$$

Supergravity II

Covariantization $\Longrightarrow$ four-gravitino interaction, and then:

Four-dimensional pure $N = 1$ supergravity is *not so simple*:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2\kappa_4^2} eR\left(\omega(e)\right) + \frac{1}{2\kappa_4^2} e\overline{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu(\omega(e))\psi_\rho \\ &+ \frac{e}{32\kappa_4^2}\Big[4(\overline{\psi}^\mu\gamma_\mu\psi_\rho)(\overline{\psi}^\nu\gamma_\nu\psi^\rho) - (\overline{\psi}_\mu\gamma_\nu\psi_\rho)(\overline{\psi}^\mu\gamma^\nu\psi^\rho) \\ &-2(\overline{\psi}_\mu\gamma_\nu\psi_\rho)(\overline{\psi}^\mu\gamma^\rho\psi^\nu)\Big]\end{aligned}$$

with now $\tilde{D}_\nu\psi_\rho = \partial_\nu\psi_\rho + \frac{1}{2}\omega_{\nu\,ab}(e)\sigma^{ab}\psi_\rho$, the usual spin connection of pure gravitation theory.

- In four space-time dimensions:
  - Gravitino: $4 \times 4 - 4 = 12_F$ off-shell. 2_F with helicities $\pm 3/2$ on-shell.
  - Graviton: $10 - 4 = 6_B$ off-shell. 2_B with helicities ± 2 on-shell.

Supergravity III

More complications: auxiliary fields of $N = 1$ supergravity

The $N = 1$ supergravity action is invariant under local susy variations

$$\begin{array}{rclcrcl} \delta e_\mu^a & = & -\frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu & \qquad & \delta e_a^\mu & = & \frac{1}{2}\bar{\epsilon}\gamma^\mu\psi_a \\ \delta\psi_\mu & = & D_\mu\epsilon & & \delta\overline{\psi}_\mu & = & D_\mu\bar{\epsilon} \end{array}$$

With standard covariant derivative $D_\mu\epsilon = \partial_\mu\epsilon + \frac{1}{2}\omega_{\mu\,ab}\sigma^{ab}\epsilon$

But it is not an off-shell representation of the supersymmetry algebra:
$[\delta_1, \delta_2]$ is a diffeomorphism only for fields solving the field equations

Another sign is the number of off-shell field components: $12_F \neq 6_B$.

More (auxiliary) fields needed for a linear off-shell representation, with
$N_B^{aux} - N_F^{aux} = 6$

Vanish for pure supergravity, produce interactions when coupled to matter or gauge multiplets.

Supergravity IV

Four-dimensional supersymmetry (linear) representations

SUSY	Supergravity	$\vert$Hel.$\vert\leq 1$	$\vert$Hel.$\vert\leq 1/2$	Chirality	
$N=1$	2_B+2_F	$\checkmark$	$\checkmark$ *	$\checkmark$	
$N=2$	4_B+4_F	$\checkmark$ *	$\checkmark$ *	-	$D=6$
$N=3$	8_B+8_F	$\checkmark$ *	-	-	
$N=4$	16_B+16_F *	$\checkmark$ *	-	-	$D=10$
$N=5$	32_B+32_F *	-	-	-	
$N=6$	64_B+64_F *	-	-	-	
$N=8$	128_B+128_F *	-	-	-	$D=11$

- *: scalar fields in supermultiplet
- Number of supercharges is $4N$
- 16 supercharges ($N=4$): type I, heterotic strings
- 32 supercharges ($N=8$): type IIA, IIB strings, M–“theory"
- $N=7$ does not exist (it is the $N=8$ theory)
- $N=0,1$ only for realistic models, or nonlinear, (or truncated. . .)

$N = 1$ supergravity and matter couplings

$N = 1$ supergravity couples to:

all gauge groups (gauge superfield A_μ, λ, helicities $\pm 1, \pm 1/2$)

all representations for chiral multiplets (ψ and z, helicities $\pm 1/2, 0, 0$)

and allows chirality of fermion representations.

The idea is then:

- Couple the SSM to supergravity, add a “hidden" sector to break supersymmetry.
- Generate a susy breaking scale $m_{3/2}$ and scalar vev's in the hidden sector $\langle \phi \rangle$
- Decouple gravity: expand, take $M_P \longrightarrow \infty$, keep $m_{3/2}$ fixed, ...
- The result is a global $N = 1$ theory with soft breaking terms.
- However: $N = 1$ Poincaré only if the cosmological constant at the breaking point is zero. In general, AdS global $N = 1$ ...
- A severe constraint on the hidden sector ...

$N = 1$ supergravity and matter couplings

The complete Lagrangian has been obtained by Cremmer, Ferrara, Girardello and Van Proeyen (1982). It needs 1.5 pages in Nucl. Phys. B212.

In the superconformal formulation, it is symbolically

$$\mathcal{L} = -\frac{3}{2}\Big[S_0\overline{S}_0\, e^{-\mathcal{K}/3}\Big]_D + \Big[S_0^3 W + \frac{1}{4} f(\Phi)\, \mathrm{Tr}\, \mathcal{W}\mathcal{W}\Big]_F$$

where S_0 is the chiral compensating multiplet of the old minimal formalism. Similar to the global superspace Lagrangian

$$\mathcal{L} = \int d^2\theta d^2\overline{\theta}\, \mathcal{K}(\overline{\Phi}^{\mathcal{A}}, \Phi) + \int d^2\theta \left[W(\Phi) + \frac{1}{4} f(\Phi)\, \mathrm{Tr}\, \mathcal{W}\mathcal{W}\right] + \mathrm{h.c.}$$

Superconformal and superspace calculus turn these symbolic expression into Lagrangians . . . .

Curiously, most applications use only the scalar potential and some fermion mass terms.

The scalar potential

$$V = \frac{1}{\kappa^4}\Big[e^{\mathcal{K}}\mathcal{K}^{-1}{}_j^i(W_i+\mathcal{K}_iW)(W^j+\mathcal{K}^jW)-3\,e^{\mathcal{K}}\,\overline{W}W\Big] + \frac{1}{2}f(\Phi)^{-1}\mathcal{K}_i(T^Az)^i\mathcal{K}_j(T^Az)^j$$

- Blue terms are positive or zero. Susy breaks if they are not zero.
- The red term is negative or zero. Unbroken susy: Anti-de Sitter or Minkowski ($W = 0$).
- Supergravity and supersymmetry are actually extensions of Anti-de Sitter symmetry: $SO(2,3) \sim Sp(4,\mathbb{R}) \longrightarrow OSp(n|4)$
  Poincaré is obtained in the large AdS radius limit only.
- A de Sitter ground state can only be generated with broken supersymmetry.
- (Good point for AdS/CFT)

Supergravity scalar potentials again

- The gauge potential plays a fundamental role in the vacuum structure of supergravities, and in superstring compactifications.
- Produced by gauging a symmetry of the theory: abelian (for instance, R–symmetry in $N = 1$) or non-abelian. Compact or non-compact.

$(\ldots)$: number of abelian, ungauged, gauge fields in the supermultiplet:

SUSY	Supergravity	|Hel.|≤ 1	|Hel.|$\leq 1/2$	
$N = 1$	$2_B + 2_F \quad (0)$	$\checkmark \quad (N)$	$\checkmark$	
$N = 2$	$4_B + 4_F \quad (1)$	$\checkmark \quad (N)$	$\checkmark$	$D = 6$
$N = 3$	$8_B + 8_F \quad (3)$	$\checkmark \quad (N)$	-	
$N = 4$	$16_B + 16_F \quad (6)$	$\checkmark \quad (N)$	-	$D = 10$
$N = 5$	$32_B + 32_F \quad (10)$	-	-	
$N = 6$	$64_B + 64_F \quad (15)$	-	-	
$N = 8$	$128_B + 128_F \quad (28)$	-	-	$D = 11$

Relation to superstring compactifications

String compactifications ($\mathbf{16}$ or $\mathbf{32}$ supercharges) include various background quantities in their compact geometry. These include background values of:

- tensor fields (in the supergravity multiplet)
- dual tensors (of the tensor hierarchy) and branes
- geometric fluxes (spin connection fluxes)
- various “non-geometric" fluxes (condensates for instance) . . .

These background quantities can be associated with the generalized structure constants of a gauged supergravity, and the vacuum structure of the string compactification can be studied directly in the supergravity (numerical methods help).

This holds for large classes of string compactifications (but not for all) with a large variety of breaking patterns and low-energy structure.

The idea is to develop a bottom-up approach to the vast problem of string compactifications with fluxes.

String flux compactification, beginnings

1985: two sources of induced low-energy parameters identified in $E_8 \times E_8$ heterotic strings on Calabi-Yau spaces:

- $\langle H_{ijk} \rangle$: the Calabi-Yau flux of the three-index tensor (universal)
- Strong gauge dynamics in “hidden sector" creates gaugino condensates $\langle \lambda\lambda \rangle$

$N = 1$ *effective supergravity* defined by:

Kähler potential: $K = -\ln(S + \overline{S}) - 3\ln(T + \overline{T})$

Superpotential: $W = Ae^{bS} + C$

Scalar potential: $V = e^{K} \left| Ae^{bS} b(S + \overline{S} - 1) - C \right|^2$

Supersymmetry broken in T sector, zero cosmological constant

The prototype and first example of a string compactification with fluxes.

[Nilles, Ibanez, J.-P. D.; Dine, Rohm, Seiberg, Witten]

String flux compactifications, a recent example

A “simple" (because based on a Scherk-Schwarz, orbifold-like, reduction into four dimensions) example to display the complexity of superstring compactifications with fluxes.

To compare equivalent sets of fluxes in the following situations:

I: G_2 compactifications of M–theory (32 supercharges):

II: IIA (related) orientifold (32 supercharges)

III: Effective, four-dimensional, supergravity description in terms of gauged $N = 4$ supergravity (16 supercharges) or its $N = 8$ extension (32 supercharges)

Cases I and II: flux parameters with consistency constraints from compactification. *(Top-down)*

Case III: gauging parameters (structure constants) with algebraic constraints ($\sim$ closure of the gauged algebra). *(Bottom-up)*

An example, M–theory version

Based on Guarino, J.-P. D. (1406.6930), Dall'Agata, Prezas (0509052), and others.

Framework: Scherk-Schwarz reduction of M-theory on G_2 manifold with fluxes.

G_2 is the natural framework to reduce $N = 8$ (32 supercharges) to $N = 1$.

All fluxes have an interpretation in 11 dimensions (either geometric fluxes (from spin connections), or G_4 fluxes, or G_7 fluxes).

- Seven complex moduli with Kähler potential

$$K = -\sum_{A=1}^{7} \log\left[-i(T_A - \bar{T}_A)\right]$$

- Scalar potential from $N = 1$ supergravity

$$V = e^K \left[K^{A\bar{B}}\, D_A W\, D_{\bar{B}} \bar{W} - 3\, W\, \bar{W}\right]$$

Superpotential W generated by flux parameters.

An example, M–theory version

- Flux-generated superpotential, symbolic expression:

$$W_{\text{M-theory}} = \frac{1}{4}\int_{X_7}\left[G_{(7)} + (A_{(3)} + i\Phi_{(3)}) \wedge \left(G_{(4)} + \frac{1}{2}d(A_{(3)} + i\Phi_{(3)})\right)\right]$$

$\Phi_{(3)}$: the G_2–invariant three-form characterizing the compactification.

- Flux-generated superpotential, explicit expression:

$$\begin{aligned} W_{\text{M-theory}} &= a_0 - b_0\, S + \sum_{K=1}^{3} c_0^{(K)} T_K - \sum_{K=1}^{3} a_1^{(K)}\, U_K \\ &+ \sum_{K=1}^{3} a_2^{(K)} \frac{U_1U_2U_3}{U_K} + \sum_{I,J=1}^{3} U_I\, \mathcal{C}_1^{(IJ)}\, T_J + S\sum_{K=1}^{3} b_1^{(K)}\, U_K \\ &- \sum_{K=1}^{3} c_3'^{(K)} \frac{T_1T_2T_3}{T_K} - S\sum_{K=1}^{3} d_0^{(K)}\, T_K \end{aligned}$$

In terms of the flux parameters . . .

An example, M–theory version, flux parameters

M-theory origin	Components	Fluxes
$\omega_{bc}{}^{a}$	$\omega_{35}{}^{1}\,,\ \omega_{51}{}^{3}\,,\ \omega_{13}{}^{5}$	$\tilde{c}_1^{(1)}\,,\ \tilde{c}_1^{(2)}\,,\ \tilde{c}_1^{(3)}$
$\omega_{aj}{}^{k}$	$\omega_{14}{}^{6}\,,\ \omega_{36}{}^{2}\,,\ \omega_{52}{}^{4}$	$\hat{c}_1^{(1)}\,,\ \hat{c}_1^{(2)}\,,\ \hat{c}_1^{(3)}$
$\omega_{ka}{}^{j}$	$\omega_{61}{}^{4}\,,\ \omega_{23}{}^{6}\,,\ \omega_{45}{}^{2}$	$\check{c}_1^{(1)}\,,\ \check{c}_1^{(2)}\,,\ \check{c}_1^{(3)}$
$\omega_{jk}{}^{a}$	$\omega_{46}{}^{1}\,,\ \omega_{62}{}^{3}\,,\ \omega_{24}{}^{5}$	$b_1^{(1)}\,,\ b_1^{(2)}\,,\ b_1^{(3)}$
$-\omega_{ai}{}^{7}$	$-\omega_{12}{}^{7}\,,\ -\omega_{34}{}^{7}\,,\ -\omega_{56}{}^{7}$	$a_2^{(1)}\,,\ a_2^{(2)}\,,\ a_2^{(3)}$
$-\omega_{7i}{}^{a}$	$-\omega_{72}{}^{1}\,,\ -\omega_{74}{}^{3}\,,\ -\omega_{76}{}^{5}$	$d_0^{(1)}\,,\ d_0^{(2)}\,,\ d_0^{(3)}$
$-\omega_{a7}{}^{i}$	$-\omega_{17}{}^{2}\,,\ -\omega_{37}{}^{4}\,,\ -\omega_{57}{}^{6}$	$c_3'^{(1)}\,,\ c_3'^{(2)}\,,\ c_3'^{(3)}$
$-\frac{1}{2}\,G_{aibj}$	$-\frac{1}{2}\,G_{3456}\,,\ -\frac{1}{2}\,G_{1256}\,,\ -\frac{1}{2}\,G_{1234}$	$a_1^{(1)}\,,\ a_1^{(2)}\,,\ a_1^{(3)}$
$\frac{1}{2}\,G_{ijk7}$	$\frac{1}{2}\,G_{2467}$	b_0
$\frac{1}{2}\,G_{ibc7}$	$\frac{1}{2}\,G_{2357}\,,\ \frac{1}{2}\,G_{4517}\,,\ \frac{1}{2}\,G_{6137}$	$c_0^{(1)}\,,\ c_0^{(2)}\,,\ c_0^{(3)}$
$\frac{1}{4}\,G_{aibjck7}$	$\frac{1}{4}\,G_{1234567}$	a_0

$\Longrightarrow$ 29 parameters, before consistency constraints

An example, IIA orientifold version

The corresponding $[T_6/Z_2 \times Z_2|] \times \mathcal{O}_7$ reduction of IIA superstrings leads:

$$\begin{aligned} W_{\text{M-theory}} &= W_{\text{IIA}}^{(a_3=0)} + W_{\text{non-geom}} \\ &= W_{\text{IIA}}^{(a_3=0)} - \sum_{K=1}^{3} c_3'^{(K)} \frac{T_1 T_2 T_3}{T_K} - S \sum_{K=1}^{3} d_0^{(K)} T_K \end{aligned}$$

- Non-geometric: geometric in eleven dimensions, not seen as geometric in ten.
- a_3: the Romans mass parameter absent in the M–theory case.

An example, IIA orientifold version, flux parameters

Type IIA origin	Components	Fluxes (M–th.)
$\omega_{bc}{}^{a}$	$\omega_{35}{}^{1}\,,\ \omega_{51}{}^{3}\,,\ \omega_{13}{}^{5}$	$\tilde{c}_1^{(1)}\,,\ \tilde{c}_1^{(2)}\,,\ \tilde{c}_1^{(3)}$
$\omega_{aj}{}^{k}$	$\omega_{14}{}^{6}\,,\ \omega_{36}{}^{2}\,,\ \omega_{52}{}^{4}$	$\hat{c}_1^{(1)}\,,\ \hat{c}_1^{(2)}\,,\ \hat{c}_1^{(3)}$
$\omega_{ka}{}^{j}$	$\omega_{61}{}^{4}\,,\ \omega_{23}{}^{6}\,,\ \omega_{45}{}^{2}$	$\check{c}_1^{(1)}\,,\ \check{c}_1^{(2)}\,,\ \check{c}_1^{(3)}$
$\omega_{jk}{}^{a}$	$\omega_{46}{}^{1}\,,\ \omega_{62}{}^{3}\,,\ \omega_{24}{}^{5}$	$b_1^{(1)}\,,\ b_1^{(2)}\,,\ b_1^{(3)}$
F_{ai}	$F_{12}\,,\ F_{34}\,,\ F_{56}$	$a_2^{(1)}\,,\ a_2^{(2)}\,,\ a_2^{(3)}$
non-geometric		$d_0^{(1)}\,,\ d_0^{(2)}\,,\ d_0^{(3)}$
non-geometric		$c_3'^{(1)}\,,\ c_3'^{(2)}\,,\ c_3'^{(3)}$
$-F_{aibj}$	$-F_{3456}\,,\ -F_{1256}\,,\ -F_{1234}$	$a_1^{(1)}\,,\ a_1^{(2)}\,,\ a_1^{(3)}$
H_{ijk}	H_{246}	b_0
H_{ibc}	$H_{235}\,,\ H_{451}\,,\ H_{613}$	$c_0^{(1)}\,,\ c_0^{(2)}\,,\ c_0^{(3)}$
F_{aibjck}	F_{123456}	a_0
$-F_{(0)}$ (Romans mass)		a_3

An example, gauged supergravity version

Here, one writes the effective four-dimensional supergravity in terms of a (truncated) $N = 4$ gauged supergravity with gauging parameters

$$f_{+ABC} \qquad \text{(electric)} \qquad\qquad f_{-ABC} \qquad \text{(magnetic)}$$

- Includes all flux parameters of the M or IIA versions.
- Consistency conditions are also equivalent.
- Critical points of the scalar, gauging-induced, scalar potential produces the “string solutions".

The bottom-up approach, using gauged supergravities, proves fruitful and simpler to investigate basic properties of very large classes of string solutions

An example, gauged supergravity fluxes

M-theory origin	Type IIA origin	Fluxes	Embedding tensor
${\omega_{bc}}^a$	${\omega_{bc}}^a$	$\tilde{c}_1^{(I)}$	${f_+{}^{bc}}_a$
${\omega_{aj}}^k$	${\omega_{aj}}^k$	$\hat{c}_1^{(I)}$	${f_+{}^{aj}}_k$
${\omega_{ka}}^j$	${\omega_{ka}}^j$	$\check{c}_1^{(I)}$	${f_+{}^{ka}}_j$
${\omega_{jk}}^a$	${\omega_{jk}}^a$	$b_1^{(I)}$	$f_-{}^{ibc}$
$-{\omega_{ai}}^7$	F_{ai}	$a_2^{(I)}$	$-f_+{}^{ajk}$
$-{\omega_{7i}}^a$	non-geometric	$d_0^{(I)}$	${f_-{}^{bc}}_i$
$-{\omega_{a7}}^i$	non-geometric	$c_3'^{(I)}$	${f_{+jk}}^a$
$-\frac{1}{2}\,G_{aibj}$	$-F_{aibj}$	$a_1^{(I)}$	$f_+{}^{abk}$
$\frac{1}{2}\,G_{ijk7}$	H_{ijk}	b_0	$-f_-{}^{abc}$
$\frac{1}{2}\,G_{ibc7}$	H_{ibc}	$c_0^{(I)}$	${f_+{}^{bc}}_i$
$\frac{1}{4}\,G_{aibjck7}$	F_{aibjck}	a_0	$-f_+{}^{abc}$
non-geometric	$-F_{(0)}$ (Romans mass)	a_3	$f_+{}^{ijk}$

String flux compactifications, a recent example

The analysis of the constraints reveals:

- 19 vacua of family of vacua,
- With $N = 0, 1$ or 3 supersymmetries,
- 8 vacua with $N = 0$ are stable, 7 are unstable
- 5 vacua have flat directions, the others do not (moduli stabilization).
- Minkowski or anti-de Sitter

Complicated, weak predictivity, . . .

But methods have been developed to systematically analyze such string backgrounds. Already a performance.

Some final words

- Supersymmetry, supergravity and superstring theories have seen an enormous development over the last 40 years.
- They propose a scheme for the unification programme beyond the Standard Model.
- Predictivity however is not compelling.
- Data in support of low-energy supersymmetry eagerly / anxiously expected.
- And most, if not all, questions left open by the Standard Model are still with us ...
- Can we use string theories to formulate quantum gravity with economy ? Maybe recent developments in $N = 8$ maximal supergravity will prove more sugestive.
- And ... what about the QCD string ?