Gauss-Bonnet Inflation

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Introduction

The usual starting point is the following action functional

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa^2} + \mathcal{L}_X^i \right),$$

where R is the Ricci scalar, and $\mathcal{L}_X^i$ the source terms for all the "ingredients" present in the universe: $(\rho_r,\ \rho_m,\ \rho_{dm},\ \rho_{de})$, as dictated by the current observations

We also assume the Friedmann-Robertson-Walker form for the metric tensor of a homogeneous and isotropic universe

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1-kr^2} - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2 \right) \right],$$

with $a(t)$ the scale factor and $k = 0, \pm 1$

Introduction to Cosmology

In addition, the complete theory should account for the presence of a fifth ingredient, the inflaton field

$$\mathcal{L}_\phi = \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + V(\phi)$$

with an as-yet-unspecified potential, that cures the problems of the 'old' Cosmological Model (horizon, flatness, monopoles and density perturbations problems)

There are a lot of open questions in Cosmology today: the nature of dark matter and dark energy, the coincidence problem...

Is it our inability to find the correct answers to well-posed questions, or is it perhaps time to change the gravitational framework?

Introduction

The generalised theories of gravity have attracted a lot of attention

$$S = \int d^4x \sqrt{-g} \left[f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}) \right]$$

- as part of the string effective action at low energies
- as part of a modified scalar-tensor (Horndeski) theory

In higher dimensions, a natural generalization of Einstein's theory of gravity seems to be Lovelock's theory, whose action is a homogeneous polynomial of degree N of the Riemann curvature:

- for $N = 1$, we obtain Einstein's term, the Ricci scalar
- for $N = 2$, we obtain the Gauss-Bonnet term

$$R^2_{GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

- ...

Introduction

Lovelock's action leads to field equations that are PDE's of second order, satisfying Bianchi identities and are ghost-free

But, there is more... In 3 dimensions, Einstein's gravity is kinematic, i.e. when $R_{\mu\nu} = 0$ (vacuum) then $R_{\mu\nu\rho\sigma} = 0$ (flatness). For $d > 3$, this does not hold – the Schwarzschild solution is a vaccum solution but it is not flat...

Pure Lovelock's theory - involving only single Nth order terms in the action - can be made kinematic by appropriately defining Nth order Riemann and Ricci tensors

Does that mean that pure Lovelock's theory is simpler and perhaps more fundamental than Einstein's theory?

The Einstein-Scalar-Gauss-Bonnet Theory

Let us consider the more conventional, 4-dimensional theory

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} + \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi + \frac{1}{8} f(\phi) \, R^2_{GB} \right]$$

where $f(\phi)$ a coupling function. The field equations read

$$\frac{1}{\sqrt{-g}} \partial_\mu \left[\sqrt{-g} \, \partial^\mu \phi \right] = \frac{1}{8} \frac{df}{d\phi} R^2_{GB}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\partial_\mu \phi \, \partial_\nu \phi + \frac{1}{2} g_{\mu\nu} \, \partial_\rho \phi \, \partial^\rho \phi + K_{\mu\nu} \,,$$

with

$$K_{\mu\nu} = \frac{1}{8} \left(g_{\mu\rho} g_{\nu\lambda} + g_{\mu\lambda} g_{\nu\rho} \right) \eta^{\kappa\lambda\alpha\beta} \, D_\gamma \left(\tilde{R}^{\rho\gamma}{}_{\alpha\beta} D_\kappa f \right)$$

In 4D, the GB is a total derivative and, if $f = const.$, it drops out. On the other hand, R^2_{GB} provides a potential for the scalar field

The Einstein-Scalar-Gauss-Bonnet Theory

For the Friedmann-Robertson-Walker line-element, the equations take the explicit form

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\,\dot{\phi} = \frac{df}{d\phi}\,\frac{3\ddot{a}}{a^3}\,(k + \dot{a}^2)$$

$$\frac{3\,(k + \dot{a}^2)}{a^2}\,(\underline{1} + \dot{f}\,\frac{\dot{a}}{a}) - \frac{\dot{\phi}^2}{2} = 0$$

$$\frac{(k + \dot{a}^2)}{a^2}\,(\underline{1} + \ddot{f}) + \frac{2\ddot{a}}{a}\,(\underline{1} + \dot{f}\,\frac{\dot{a}}{a}) + \frac{\dot{\phi}^2}{2} = 0$$

The scalar field seems unaffected as it doesn't couple to R....
But the gravitational field equations do change...

A Toy Model

Let's impose in the theory the constraint $R^2_{GB} = 0$. Then:

$$R^2_{GB} = \frac{24\,\ddot{a}}{a^3}\,(k + \dot{a}^2) \equiv 0 \;\Rightarrow\; a(t) = At + B$$

and

$$\ddot{\phi} + 3\dot{\phi}\,\frac{\dot{a}}{a} = 0 \;\Rightarrow\; \dot{\phi}(t) = \frac{C}{a^3(t)}$$

If R is present, the system of equations closes only for

$$f(\phi) = f_1\,\phi + \frac{f_2}{\phi} + f_3$$

where A, B, C, and f_i are integration constants. If R is absent, then $f(\phi)$ is as above but with $f_2 = 0$.

A more general case

We now restore the GB term and assume that $f(\phi) = \lambda\,\phi$, where λ a constant. Then, the dilaton equation can be written as

$$\frac{d(\dot{\phi}\,a^3)}{dt} = 3\lambda\ddot{a}\,(k + \dot{a}^2) \Rightarrow \dot{\phi} = \frac{C}{a^3} + \frac{\lambda\dot{a}\,(3k + \dot{a}^2)}{a^3}$$

Keeping only the second GB-related term, the Einstein's equations can be rearranged to eventually give the solution (only for $k = 0$)

$$a(t)\,F\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}; \frac{3a^4(t)}{c_1}\right] = \left(\frac{2c_1}{5\lambda^2}\right)^{1/4} (t + t_0)$$

with $F(a, b, c; x)$ the hypergeometric function. For $a \to 0$, we find the asymptotic solution $a(t) \sim At + B$

A more general case

Now, we ignore the presence of the Ricci term in the theory. Then, the gravitational equations are easily re-written to give the constraint

$$(k + 5\dot{a}^2)\,\ddot{a} = 0$$

that leads again (much-much easier and for all values of k!) to the linear solution $a(t) = At + B$ derived previously.

One may thus conclude that, in the context of a scalar-GB theory, ignoring the Ricci scalar:

- does not significantly modify the dynamics of either the scalar field or the scale factor
- makes the analytic treatment much easier

The Scalar-Gauss-Bonnet Theory

We thus forget henceforth the Ricci scalar and assume that $f(\phi) = \lambda\,\phi^2$. Then, Einstein's equations give the constraint

$$(k + 5\dot{a}^2)\,\ddot{a} + 24\lambda\,\frac{\dot{a}^2}{a^3}\,(k + \dot{a}^2) = 0$$

which can be integrated once to give

$$\frac{12\lambda}{a^2} = -\frac{1}{k}\,\ln\left(\frac{\sqrt{k + \dot{a}^2}}{\dot{a}}\right) - \frac{2}{(k + \dot{a}^2)} + C_1$$

In order to proceed, we have to set $k = 0$, and then find

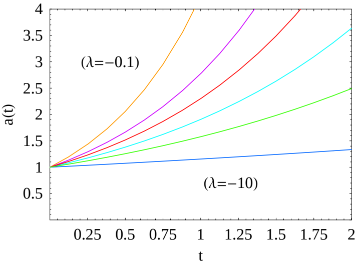
$$\frac{5}{2\dot{a}^2} = C_1 - \frac{12\lambda}{a^2}$$

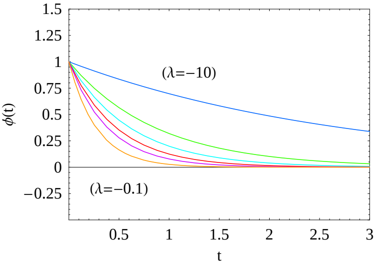
where C_1 is an arbitrary integration constant

The Scalar-Gauss-Bonnet Theory

- If $C_1 = 0$ and $\lambda < 0$, we easily obtain a pure de Sitter solution for the scale factor and an exponentially decaying solution for ϕ

$$a(t) = a_0 \exp\left(\sqrt{\frac{5}{24|\lambda|}}\, t\right), \quad \phi = \phi_0 \exp\left(-\frac{5}{4}\sqrt{\frac{5}{6|\lambda|}}\, t\right)$$



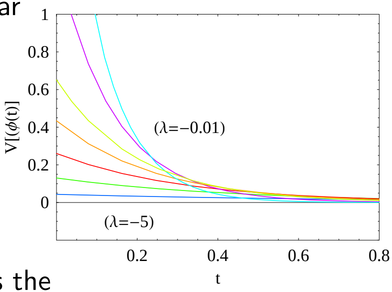


The Scalar-Gauss-Bonnet Theory

This may be an alternative for the usual inflation with the GB term providing a potential for ϕ

The effective potential of the scalar field receives contributions from both the Gauss-Bonnet term and the coupling function

$$V_{eff} = -\frac{1}{8} f(\phi) R^2_{GB} = \frac{25}{24} \frac{\phi^2}{8|\lambda|}$$



The potential remains bounded as the field evolves, from its initial value ϕ_0 to zero, but it may become arbitrarily large by appropriately choosing the coupling constant λ

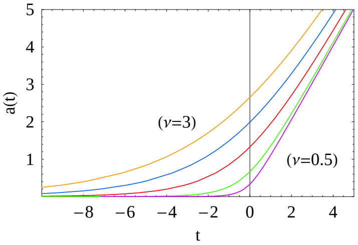
The Scalar-Gauss-Bonnet Theory

- For $C_1 > 0$ and $\lambda < 0$, we find the solution

$$\sqrt{a^2+\tilde{\nu}^2} + \tilde{\nu}\ln\left(\frac{\sqrt{a^2+\tilde{\nu}^2}-\tilde{\nu}}{a}\right) = \pm\sqrt{\frac{5}{2C_1}}\,(t+t_0)$$

where $\tilde{\nu}^2 \equiv 12|\lambda|/C_1$. In the limit $a(t) \to 0$, the above reduces to the pure de Sitter solution

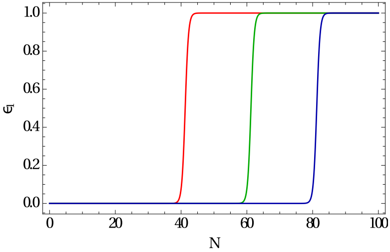
$$a(t) \simeq a_0 \exp\left(\sqrt{\frac{5}{24|\lambda|}}\,(t+t_0)\right)$$



while, for $a^2 \gg \tilde{\nu}^2$, we obtain a linearly expanding Milne-type universe. Thus, this solution describes an inflationary phase with a natural exit mechanism

The Scalar-Gauss-Bonnet Theory

If we define the slow-roll parameter $\epsilon_1 = -\dot{H}/H^2$, and use the previous solution, we find its exact behaviour



During inflation, ϵ_1 aproaches zero, while at later times moves away from this value marking the end of the inflationary era

The same behaviour is obtained for a large range of values of C_1 and λ – no fine-tuning or super-Planckian field values are required

The Scalar-Gauss-Bonnet Theory

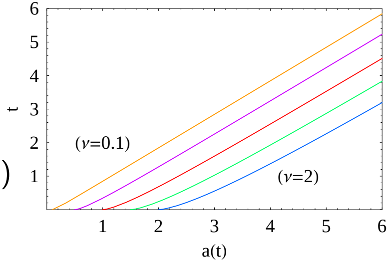
- For $C_1 > 0$ and $\lambda > 0$, after integrating, we obtain

$$\sqrt{a^2 - \nu^2} - \nu \arccos\left(\frac{\nu}{a}\right) = \pm\sqrt{\frac{5}{2C_1}}\,(t + t_0)$$

where $\nu^2 \equiv 12\lambda/C_1$. The constraint $a^2 \geq \nu^2$ should always hold, therefore the scale factor never vanishes and no singularity appears. Close to its minimum value, we find the approximate form

$$a(t) \simeq \nu\,[1 + (At + B)^{2/3}]$$

This is in agreement with previous studies (Kanti, Rizos & Tamvakis, 1999) where singularity-free solutions were found in the presence of R



Conclusions

- Even in the absence of the Ricci term, the Gauss-Bonnet term, in conjunction with a scalar field, seems to encode all important information for the existing solutions in the theory

- By ignoring R, we have found very easily analytical solutions describing de Sitter inflationary expansion as well as solutions with a de Sitter phase at early times and a transition to a Milne-type expansion at later times

- We have also found analytically non-singular cosmological solutions - for an even, quadratic coupling function, in accordance to previous suggestions

- Is this a fundamental theory or a particular limit of a more general theory? More investigation is necessary...