

# Mass Insertion vs. Mass Eigenstate calculations in flavour physics. The Flavour Expansion Theorem.

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Present an **expansion theorem for Hermitian Matrix functions**,  
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At the **theoretical** level:

- Provide with a connection to a more compact description of flavour basis QFT - matrix description.
- Provide explicit correspondence between the building blocks of a flavour-mass basis QFT , *i.e.*, propagators.

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At the **practical** level:

- Algebraic derivation of the flavour basis transition amplitudes, directly from the mass eigenstate expressions.
- Replace standard flavour basis diagrammatic treatments - Mass Insertion Approximation (MIA).

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1 Scalar amplitudes calculations

2 FET- Formulation

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# A scalar toy model

$\sum \equiv \text{sum over repeated indices everywhere!}$

Consider the toy model (**not realistic!**)

*Particle content:*  $N$ -generations of complex scalars  $\Phi_I$   
 One real scalar  $\eta$ .

Lagrangian density in flavour basis

$$\begin{aligned}\mathcal{L}_{\text{FLAV}} \supset & \sum (\partial^\mu \Phi_I^\dagger) (\partial_\mu \Phi_I) - M_{IJ}^2 \Phi_I^\dagger \Phi_J + (\partial^\mu \eta) (\partial_\mu \eta) - \frac{1}{2} m_\eta^2 \eta^2 \\ & - Y_{IJ} \eta \Phi_I^\dagger \Phi_J\end{aligned}$$

# The mass eigenstates calculation

$$\mathcal{L}_{\text{FLAV}} \leftrightarrow \mathcal{L}_{\text{MASS}}$$

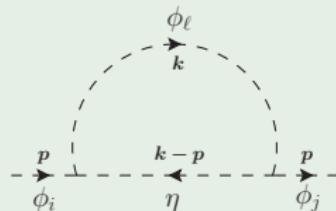
$$\Phi_I = \sum U_{Ii} \phi_i \quad (\text{Mass Eigenstates } \phi_i)$$

$$\mathbf{U}^\dagger \mathbf{M^2} \mathbf{U} = \mathbf{m^2} = \text{diag}(m_1^2, \dots, m_N^2) \quad (\text{Mass Diagonalization})$$

$$\mathbf{U}^\dagger \mathbf{Y} \mathbf{U} = \mathbf{y} \quad (\text{Non-Diagonal})$$

The mass basis self-energy.

$$\begin{aligned} \mathcal{L}_{\text{MASS}} &= \sum (\partial^\mu \phi_i^\dagger) (\partial_\mu \phi_i) - m_i^2 \phi_i^\dagger \phi_i + (\partial^\mu \eta) (\partial_\mu \eta) - \frac{1}{2} m_\eta^2 \eta^2 \\ &- y_{ij} \eta \phi_i^\dagger \phi_j \end{aligned}$$



$$-i\Sigma^{ji}(p) = \sum \int \frac{d^4 k}{(2\pi)^4} \frac{i^4 y_{j\ell} y_{\ell i}}{(k^2 - m_\ell^2)((k-p)^2 - m_\eta^2)} = \sum y_{j\ell} B_0[p; m_\ell^2, m_\eta^2] y_{\ell i}$$

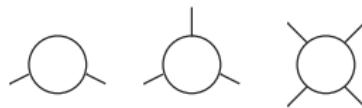
## PV-functions

- n-point scalar one-loop functions - Passarino-Veltman (PV)

$$n=2 \quad : \quad B_0[p; m_1^2, m_2^2] \equiv \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m_1^2)((k+p)^2 - m_2^2)} \sim \frac{1}{k^4}$$

$$n=3 \quad : \quad C_0[p_1, p_2; m_1^2, m_2^2, m_3^2] \equiv \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m_1^2)((k+p_1)^2 - m_2^2)((k+p_1+p_2)^2 - m_3^2)} \sim \frac{1}{k^6}$$

$$n=4 \quad : \quad D_0[p_1, p_2, p_3; m_1^2, m_2^2, m_3^2, m_4^2] \equiv \int \frac{d^4 k}{(2\pi)^4} \cdots \sim \frac{1}{k^8}$$



Can we obtain the flavour basis amplitudes from mass eigenstates result?

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## The Flavour Expansion Theorem

$$\mathcal{M}_{ji} = f(\mathbf{y}, \mathbf{m}) \xrightarrow{\text{FET}} \hat{\mathcal{M}}_{JI} = \hat{f}(\mathbf{Y}, \mathbf{M}) = \dots$$

Can we obtain the flavour basis amplitudes from mass eigenstates result?

## The Flavour Expansion Theorem

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Flavour basis QFT without diagrammatic treatment (MIA) !

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# The Flavour expansion Theorem

## FET formulation

Let  $\mathbf{A}$  be any Hermitian matrix and  $\mathbf{D} = \text{diag}\{\lambda_i\}$  its corresponding diagonal matrix of eigenvalues, associated through  $\mathbf{UDU}^\dagger = \mathbf{A}$ . For  $f(x) = \sum c_n x^n$  one can define a Hermitian matrix function  $f(\mathbf{A})$ , through

$$\mathbf{U}f(\mathbf{D})\mathbf{U}^\dagger = \sum_n c_n \mathbf{UD}^n \mathbf{U}^\dagger = \sum_n c_n \mathbf{A}^n \equiv f(\mathbf{A})$$

Decompose  $\mathbf{A}$  as a sum of its diagonal and non-diagonal part.

$$\begin{aligned} \mathbf{A} &= \mathbf{A}_0 + \hat{\mathbf{A}} \\ A_{IJ} &= A_I \delta_{IJ} + \hat{A}_{IJ}, \quad \hat{A}_{II} \equiv 0 \end{aligned}$$

The “Flavour Expansion Theorem”:

$$\begin{aligned} f(\mathbf{A})_{IJ} &= \delta_{IJ} f(A_I) + \hat{A}_{IJ} f^{[1]}(A_I, A_J) + \sum_{K_1} \hat{A}_{IK_1} \hat{A}_{K_1 J} f^{[2]}(A_I, A_{K_1}, A_J) \\ &\quad + \sum_{K_1, K_2} \hat{A}_{IK_1} \hat{A}_{K_1 K_2} \hat{A}_{K_2 J} f^{[3]}(A_I, A_{K_1}, A_{K_2}, A_J) + \dots \end{aligned}$$

and the divided differences  $f^{[k]}$  of  $f$ .

# The Flavour expansion Theorem

## Divided Differences

Divided differences (DDs),  $f^{[k]}(x_0, \dots, x_k)$  are multivariable functions obtained through a certain recursive division process

$$\begin{aligned} f^{[0]}(A_I) &\equiv f(A_I) \\ f^{[1]}(A_I, A_J) &\equiv \frac{f(A_I) - f(A_J)}{A_I - A_J} \\ &\cdots \\ f^{[k+1]}(A_I, \dots, A_J) &\equiv \frac{f^{[k]}(A_I, \dots) - f^{[k]}(\dots, A_J)}{A_I - A_J} \end{aligned}$$

- DDs are **symmetric** under any permutation of respective arguments and have a **well defined degeneracy limit** if  $f$  has a well-defined Taylor expansion.

$$\lim_{\{x_0, \dots, x_m\} \rightarrow \{\xi, \dots, \xi\}} f^{[k]}(x_0, \dots, x_k) = \frac{1}{m!} \frac{\partial^m}{\partial \xi^m} f^{[k-m]}(\xi, x_{m+1}, \dots, x_k)$$

## General FET treatment

$$\sum U_{Ji} f(\lambda_i) U_{iI}^\dagger = \left( \mathbf{U} f(\mathbf{D}) \mathbf{U}^\dagger \right)_{JI} = f(\mathbf{A})_{JI}$$

$$\stackrel{\text{FET}}{\equiv} \delta_{JI} f(A_J) + \hat{A}_{JI} f^{[1]}(A_J, A_I) + \dots$$

## FET on loop-functions

$$\sum U_{Ji} B_0[p; m_i^2, m_\eta^2] U_{iI}^\dagger = \left( \mathbf{U} B_0[p; \mathbf{m}^2, m_\eta^2] \mathbf{U}^\dagger \right)_{JI} = \left( B_0[p; \mathbf{M}^2, m_\eta^2] \right)_{JI}$$

$$\stackrel{\text{FET}}{\equiv} \delta_{JI} B_0[p; M_J^2, m_\eta^2] + \hat{M}_{JI}^2 \frac{\left( B_0[p; M_J^2, m_\eta^2] - B_0[p; M_I^2, m_\eta^2] \right)}{M_J^2 - M_I^2} + \dots$$

$\hat{M}_{JI}^2 \equiv \text{mass insertions}$  ,  $M_I^2 \equiv \text{flavour masses}$

$$B_0^{[1]} = C_0 , \quad B_0^{[2]} = C_0^{[1]} = D_0 \dots$$

## FET result

$$\sum U_{Ji} B_0[p; m_i^2, m_\eta^2] U_{iI}^\dagger = \delta_{JI} B_0[p; M_J^2, m_\eta^2] + \hat{M}_{JI}^2 C_0[0, p; M_J^2, M_I^2, m_\eta^2] + \dots$$

# Applying FET on amplitudes

The flavour basis amplitude with FET - toy model

- ① The mass eigenstate result is

$$-i\Sigma_{ji}(p) = \sum y_{j\ell} B_0[p; m_\ell^2, m_\eta^2] y_{\ell i}$$

- ② Express all (contracted) couplings in flavour basis, ( $\mathbf{y} = \mathbf{U}^\dagger \mathbf{Y} \mathbf{U}$ ),

$$-i\Sigma_{ji}(p) = \sum (\mathbf{U}^\dagger \mathbf{Y} \mathbf{U})_{j\ell} B_0[p; m_\ell^2, m_\eta^2] (\mathbf{U}^\dagger \mathbf{Y} \mathbf{U})_{\ell i}$$

- ③ Consider the transformation rule for the respective amputated Green's function<sup>1</sup>,

$$\begin{aligned} -i\hat{\Sigma}_{JI}(p) &= \sum U_{Jj} \left( -i\Sigma_{ji}(p) \right) U_{iI}^\dagger \\ &= \sum Y_{JK} \left( U_{K\ell} B_0[p; m_\ell^2, m_\eta^2] U_{\ell L}^\dagger \right) Y_{LI} \end{aligned}$$

- ④ Apply FET expansion on loop functions

$$\begin{aligned} \sum U_{K\ell} B_0[p; m_\ell^2, m_\eta^2] U_{\ell L}^\dagger &= \delta_{KL} B_0[p; M_L^2, m_\eta^2] + \hat{M}_{KL}^2 C_0[0, p; M_K^2, M_L^2, m_\eta^2] \\ &\quad + \dots \end{aligned}$$

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<sup>1</sup>This can be easily obtained from the relevant T-matrix element

## FET calculation for Self-Energies

$$\begin{aligned} -i\hat{\Sigma}_{JI}(p) &= \sum Y_{JK} \times \left( \delta_{KL} B_0[p; M_K^2, m_\eta^2] + \hat{M}_{KL}^2 C_0[0, p; M_K^2, M_L^2, m_\eta^2] \right. \\ &\quad \left. + \hat{M}_{KN}^2 \hat{M}_{NL}^2 D_0[0, 0, p; M_K^2, M_N^2, M_L^2, m_\eta^2] + \dots \right) \times Y_{LI} \end{aligned}$$

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# Diagrammatic MIA.

$$\mathcal{L}_{\text{FLAV}} = \sum (\partial^\mu \Phi_I^\dagger) (\partial_\mu \Phi_I) - M_{IJ}^2 \Phi_I^\dagger \Phi_J + \dots$$

- The flavour mass matrix is non-diagonal. How do we treat?

## The diagrammatic Mass Insertion Approximation (MIA)

- Decompose the flavour (squared) mass matrix as

$$\begin{aligned} \mathbf{M}^2 &= \mathbf{M}_0^2 + \hat{\mathbf{M}}^2 \\ M_{IJ}^2 &= M_I^2 \delta_{IJ} + \hat{M}_{IJ}^2, \quad \hat{M}_{II}^2 = 0, \quad (\text{no sum } I) \end{aligned}$$

- Absorb the diagonal part  $M_I^2$  in definition of (unphysical) massive propagators.

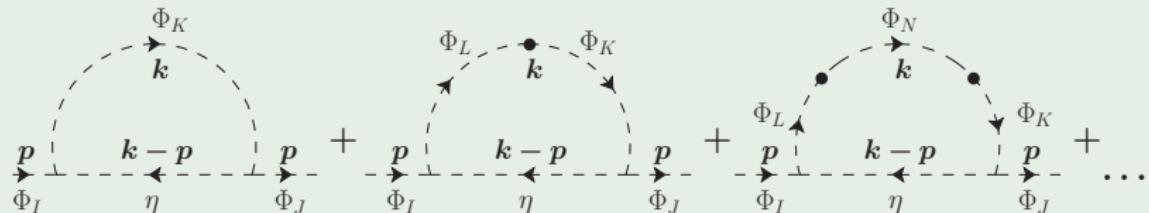
$$\text{---} \xrightarrow[\Phi_I]{\mathbf{k}} \text{---} = \frac{i}{k^2 - M_I^2}$$

- Treat the non-diagonal part  $\hat{M}_{IJ}^2$  as quadratic interactions (*mass insertions*).

$$\text{---} \xrightarrow[\Phi_I]{} \bullet \xrightarrow[\Phi_J]{} \text{---} = -i \hat{M}_{JI}^2 \quad (\hat{M}_{II}^2 = 0)$$

# The MIA calculation

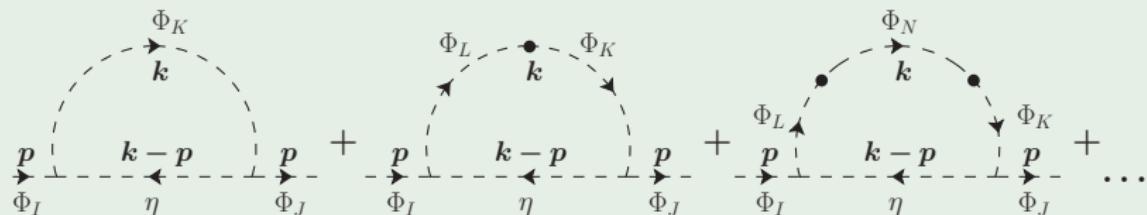
MIA for self-energies.



$$\begin{aligned}
 -i\hat{\Sigma}_{JI}(p) &= \sum Y_{JK} \times \left( \delta_{KL} B_0[p; M_K^2, m_\eta^2] + \hat{M}_{KL}^2 C_0[0, p; M_K^2, M_L^2, m_\eta^2] \right. \\
 &\quad \left. + \hat{M}_{KN}^2 \hat{M}_{NL}^2 D_0[0, 0, p; M_K^2, M_N^2, M_L^2, m_\eta^2] + \dots \right) \times Y_{LI}
 \end{aligned}$$

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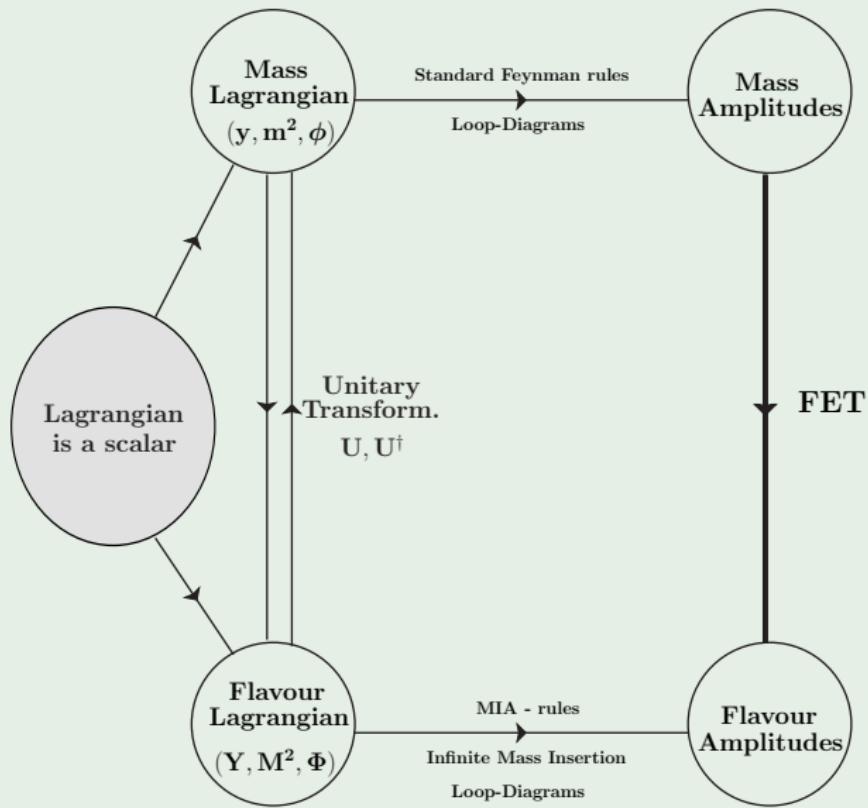
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 \end{aligned}$$

**MIA = FET  
General Result!**

## FET on amplitudes



## Physical Applications

- Applied on the dominant squark-gluino mediated decays of  $(t \rightarrow c + h)$ , explaining the suppression mechanism of the leading effects in MSSM.  
*“Rare Top-quark Decays to Higgs boson in MSSM,” JHEP **1411**, 137 (2014) [arXiv:1409.6546 [hep-ph]].*
- Applied on nEDM bounds within MSSM, giving non-trivial constraints for the soft breaking parameters, of the up-squark sector, *i.e.*,  $\text{Re}(A_U^{31}) - \text{Re}((m_{\tilde{U}}^2)^{31})$   
**“Mass Insertions vs. Mass Eigenstates calculations in Flavour Physics,”**  
**arXiv:1504.00960 [hep-ph].**
- Applies on fermionic amplitudes, as well!