Prospects of testing general relativity with gravitational waves detectors

Walter Del Pozzo

eLISA Cosmology Working Group Workshop
16th April 2015
CERN
Outline

• Motivation
• GW Waveforms
• Data analysis
• Method to detect generic violations
• Example for LIGO/Virgo
• Outlook
Motivation

- Gravitational waves observations will open a new window on the dynamics of space-time in extreme curvature
- Clean system
  - Contamination from absorptions/scattering negligible

Baker et al, 2015
Motivation

• GR signal well understood
  • inspiral
  • merger
  • ringdown
Inspiral waveform

- The inspiral waveform in the post-Newtonian approximation

\[ h(t) = A(t) \cos(\Phi(t)) \]

\[ \Phi(t) = v(t)^{-5} \sum_{n=0}^{7} (\phi_n + \phi_n^l \log(v(t)))v^n(t) \]

- The \( \phi_n \) (post-Newtonian coefficients) encode the physical predictions from the theory of gravity
Inspiral waveform

• In GR the $\phi_n$ are unique functions of the component masses and their spins
  
  • $\phi_3$ lowest-order “tail” effects and spin-orbit interaction
  
  • $\phi_4$ spin-spin coupling
  
  • $\phi_5^{(l)}$ lowest order logarithmic coefficient
non-GR effects on the waveform

- In GR the $\phi_n$ are unique functions of the component masses and their spins

- Alternative theories of gravity modify the waveform
  - change the $\phi_n$ coefficients by introducing additional parameters
  - add extra orders not present in the GR waveform
The theory space
The theory space
The theory space
The theory space

Brans-Dicke

-1PN

-0.5PN 0PN 0.5PN

-1PN

-1.5PN

-2PN

-2.5PN

-3PN

-3.5PN

-4PN

1PN

1.5PN

2PN

2.5PN

3PN

3.5PN

4PN

GR

Walter Del Pozzo
The theory space

-0.5PN  0PN  0.5PN
-1PN    1PN
-1.5PN   1.5PN
-2PN     2PN
-2.5PN   2.5PN
-3PN     3PN
-3.5PN   3.5PN
-4PN     4PN

GR

2PN Chern-Simons
The theory space
The theory space

“Massive Gravity”

1PN
Data analysis

- Given a model of gravity $H$ and some data $d$, we want to
  - infer the parameters $\theta$
  - estimate the “goodness of fit” of the model
- Bayes’ theorem
  $$p(\theta|d, H) = \frac{p(d|\theta, H)p(\theta|H)}{p(d|H)}$$
  \noindent posterior \hspace{1cm} prior \hspace{1cm} likelihood

- Evidence
  $$Z \equiv p(d|H) = \int d\theta p(\theta|H)p(d|\theta, H)$$
Data analysis

• Consider two alternative models $H_1$ and $H_2$

• Given some data $d$ the odds ratio

$$O_{1,2} \equiv \frac{p(H_1)}{p(H_2)} \frac{Z_1}{Z_2} \equiv \frac{p(H_1)}{p(H_2)} B_{1,2}$$

• Bayesian figure of merit for the relative “goodness of fit”
Data analysis

- The odds ratio *accumulates* across multiple statistically independent events

- Given some data \( d_1, \ldots, d_n \) and two competing hypotheses \( H_1 \) and \( H_2 \):

\[
O_{1,2} = \frac{p(H_1)}{p(H_2)} \prod_j B_{1,2}^{(j)}
\]
The noise

• Signals are embedded in noise

\[ d = n + h(\theta) \]

• The likelihood for \( h(\theta) \) is defined by the expected noise distribution \( f(n) \)

\[ p(d|\theta, H) = f(d - h(\theta)) \]

• mis-modelling the noise leads to biases and may mimic GR violations
Detecting GR violations

• We detected a GW event, how do we then test GR?

• Enumerate all possible alternatives \( \{H_i\}_{i=1,...,n} \)

• Given a GW event, for all \( H_i \) compute the odds ratio against GR

• Select the theory with the highest odds as the “correct” one

• There is an infinite number of potential alternatives to GR
Detecting GR violations

• Look at the GR waveform

\[ h(t) = A(t) \cos(\Phi(t)) \]

\[ \Phi(t) = v(t)^{-5} \sum_{n=0}^{7} (\phi_n + \phi^l_n \log(v(t)))v^n(t) \]

• The GR model is a set of very definite propositions

\[ H_{GR} = (\phi_1 = \phi^R_1) \land (\phi_2 = \phi^R_2) \land \ldots \]

• Define \( H_{modGR} \) as the hypothesis that one or more of the \( \phi_i \) is not as predicted by GR, but not specifying which
Detecting GR violations

- $H_{modGR}$ has no waveform model associated to it

- Decompose into mutually exclusive sub-hypotheses

- $H_{i_1,i_2,...,i_n}$ is the hypothesis that $\phi_{i_1}, \phi_{i_2}, \ldots, \phi_{i_n}$ do not have the dependence on masses and spins as GR, but all the other $\phi_j, j \not\in i_1, i_2, \ldots, i_n$ do

- Let $\theta = \{m_1, m_2, s_1, s_2, \ldots\}$, then $H_{i_1,i_2,...,i_n}$ is tested by waveforms with parameters $\{\theta, \phi_{i_1}, \phi_{i_2}, \ldots, \phi_{i_n}\}$
Detecting GR violations

- \( H_{modGR} \) is the logical union of all the \( H_{i_1,i_2,...,i_n} \)

- Example: 2 PN coefficients, \( \phi_1, \phi_2 \):

\[
H_{modGR} = H_1 \lor H_2 \lor H_{12}
\]

- Tested by three waveforms with parameters

\[
H_1 : \{ m_1, m_2, s_1, s_2, \ldots, \phi_1 \} \\
H_2 : \{ m_1, m_2, s_1, s_2, \ldots, \phi_2 \} \\
H_{12} : \{ m_1, m_2, s_1, s_2, \ldots, \phi_1, \phi_2 \}
\]
Detecting GR violations

• The odds ratio is given by

\[ O_{H_{modGR},H_{GR}} = \frac{p(H_1)}{p(H_{GR})} \frac{p(d|H_1)}{p(d|H_{GR})} + \frac{p(H_2)}{p(H_{GR})} \frac{p(d|H_2)}{p(d|H_{GR})} + \frac{p(H_{12})}{p(H_{GR})} \frac{p(d|H_{12})}{p(d|H_{GR})} \]

• It can be generalised to \( N_T \) hypotheses and \( N \) events:

\[
(N_T) O_{GR}^{modGR} = \frac{\alpha}{2^{N_T} - 1} \sum_{k=1}^{N_T} \sum_{i_1 < i_2 < ... < i_k} \prod_{A=1}^{N} (A) B_{GR}^{i_1 i_2 ... i_k}
\]

\[
\alpha = \frac{p(H_{modGR})}{p(H_{GR})}
\]
The noise - again

- Given a GW detection, the odds ratio alone is not sufficient
  - mis-modelling of the noise and/or different noise realisations lead to a different odds for the same signal
- Compute the expected distribution of odds from simulated GR signals in many different stretches of data
  - false alarm probability
  - significance of the detection
TIGER

• We implemented the Test Infrastructure for GEneral Relativity (TIGER) for LIGO/Virgo data and binary neutron stars (BNS) systems

• 10% level violation in the 1.5PN (tail) term

Li, DP et al 2012
• Capture “non-PN” deviations, e.g. a “1.25PN”

Li, DP et al 2012
Prospects for O1

- TIGER is the testing GR pipeline for the LVC
- Robustness against systematics proved in Agathos, DP et al 2014
- TIGER is robust in real (non-Gaussian) data
Outlook

• The ability to test GR depends on
  • the understanding of GR
    • faithful WF models
  • the understanding of the instrument
    • noise distribution, non-Gaussiananities
  • Combining information across sources is a powerful tool to increase sensitivity to small GR violations