UNIVERSITY OF BIRMINGHAM

Prospects of testing general relativity with gravitational waves detectors

Walter Del Pozzo

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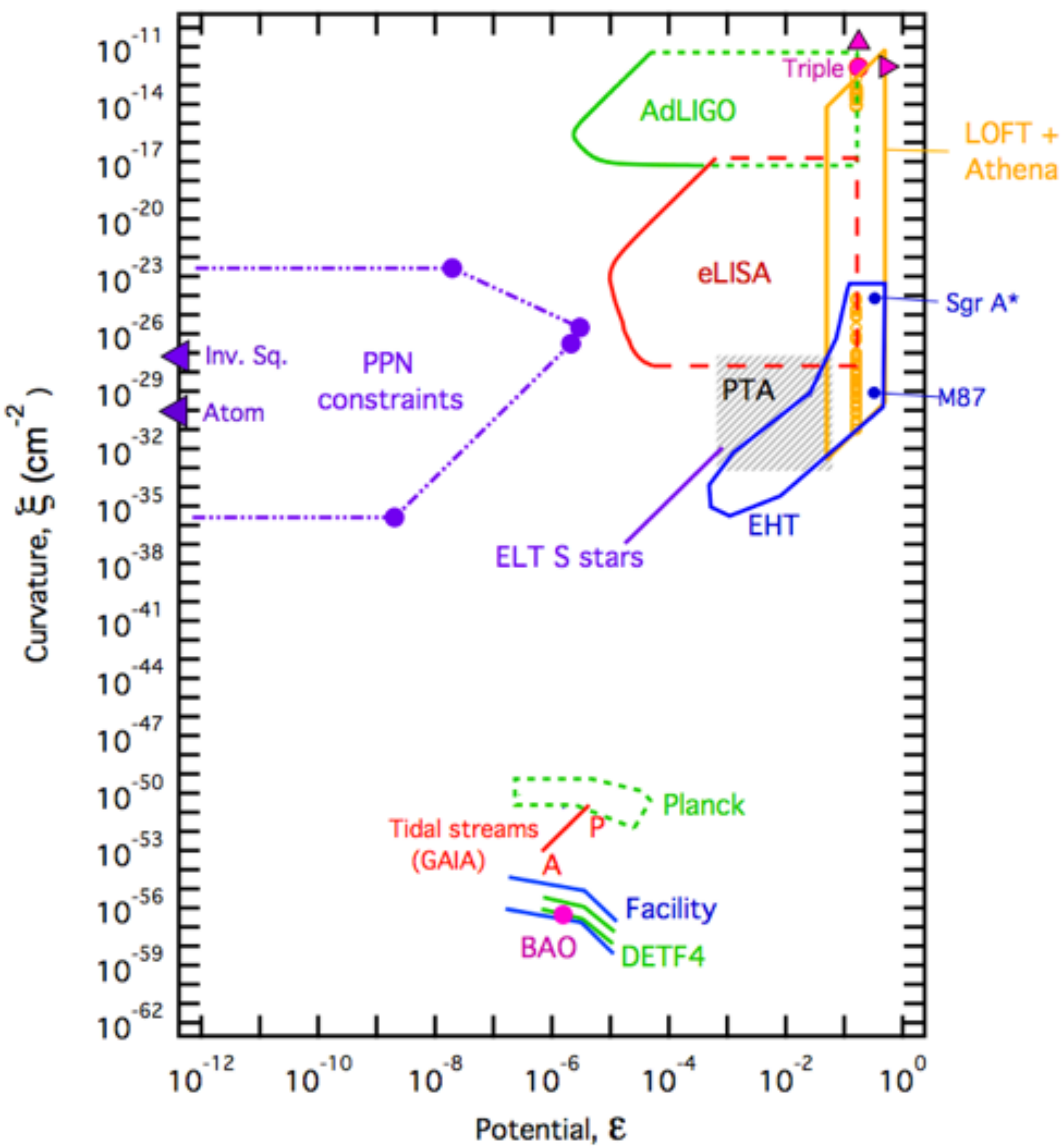
Outline

- Motivation
- GW Waveforms
- Data analysis
- Method to detect generic violations
- Example for LIGO/Virgo
- Outlook

Motivation

- Gravitational waves observations will open a new window on the dynamics of space-time in extreme curvature

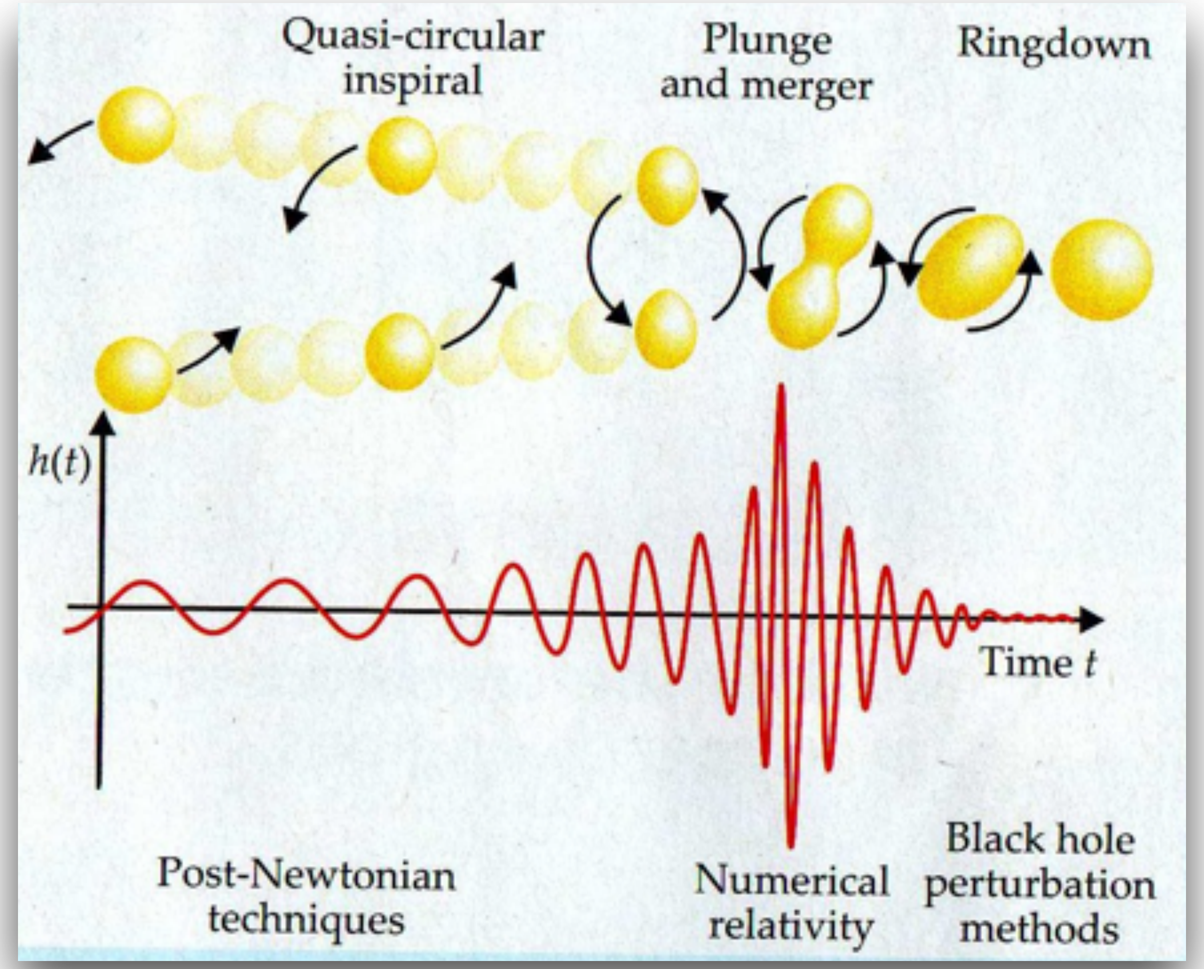
- Clean system
  - Contamination from absorptions/ scattering negligible



Baker et al, 2015

Motivation

- GR signal well understood
  - inspiral
  - merger
  - ringdown



Inspiral waveform

- The inspiral waveform in the post-Newtonian approximation

$$h(t) = A(t)\cos(\Phi(t))$$

$$\Phi(t) = v(t)^{-5}\sum_{n=0}^{7}(\phi_n + \phi_n^l \log(v(t)))v^n(t)$$

- The ϕ_n (post-Newtonian coefficients) encode the physical predictions from the theory of gravity

Inspiral waveform

- In GR the ϕ_n are unique functions of the component masses and their spins
  - ϕ_3 lowest-order “tail” effects and spin-orbit interaction
  - ϕ_4 spin-spin coupling
  - $\phi_5^{(l)}$ lowest order logarithmic coefficient

non-GR effects on the waveform

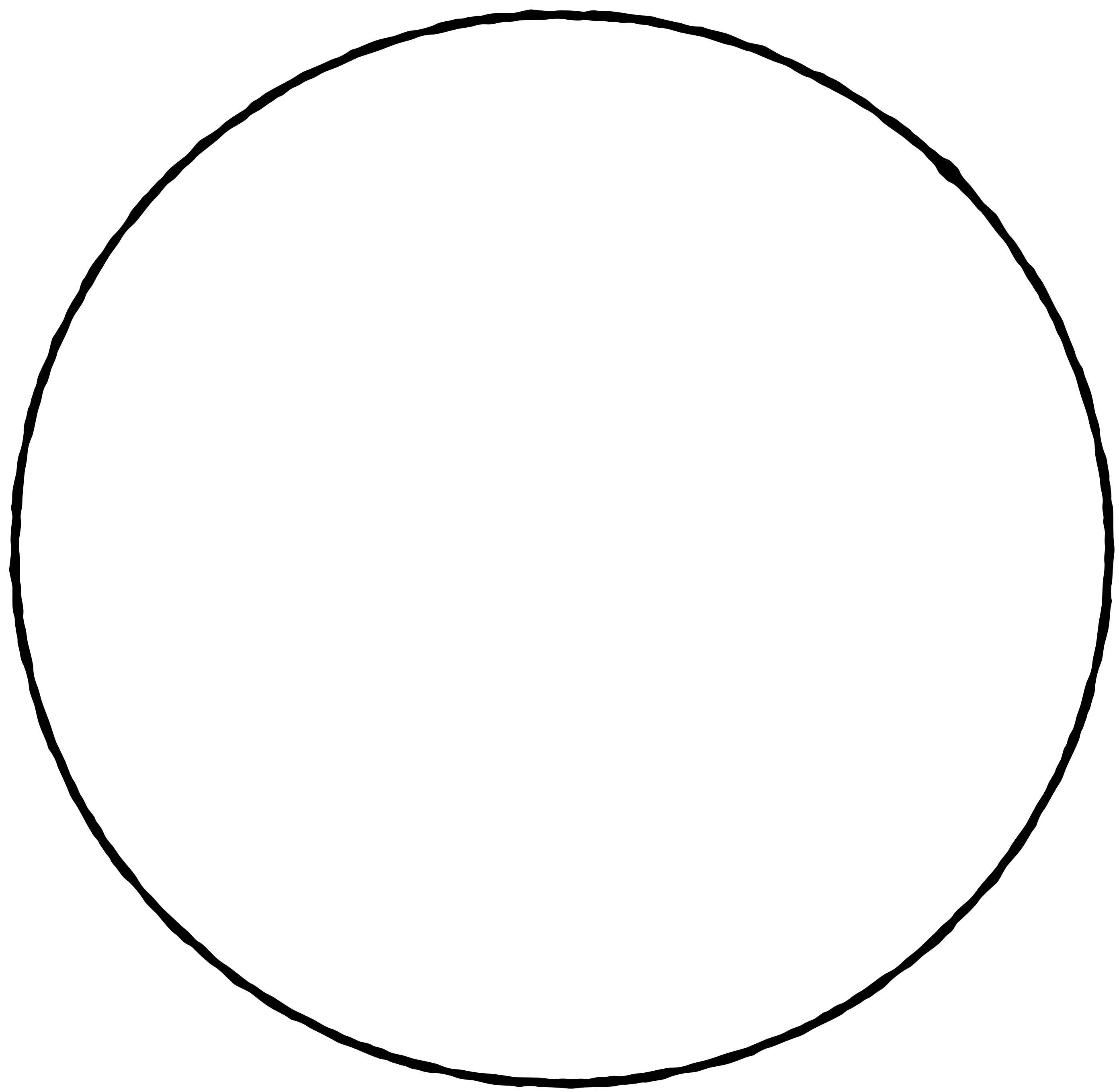
- In GR the ϕ_n are unique functions of the component masses and their spins

- Alternative theories of gravity modify the waveform

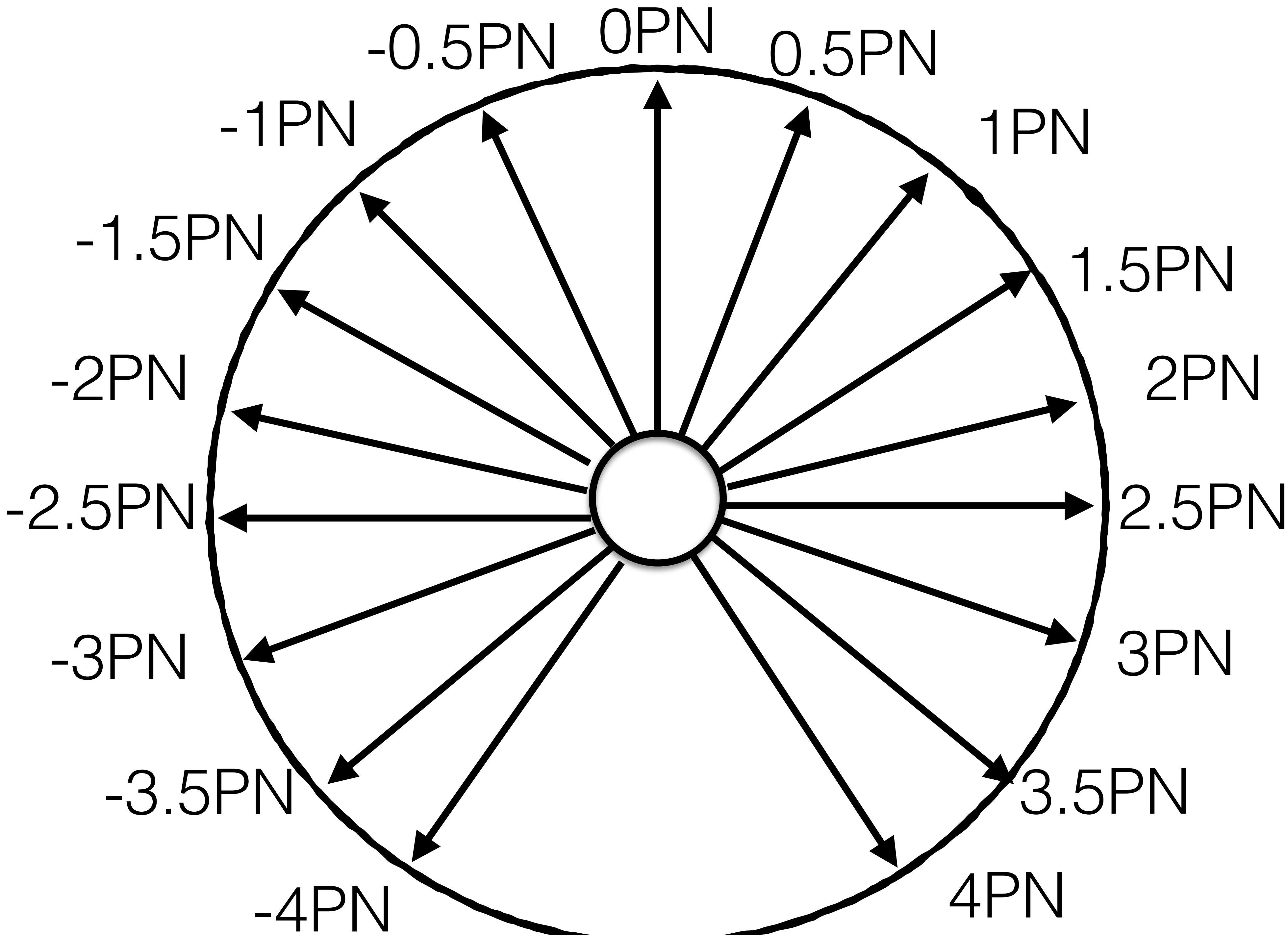
  - change the ϕ_n coefficients by introducing additional parameters

  - add extra orders not present in the GR waveform

The theory space



The theory space



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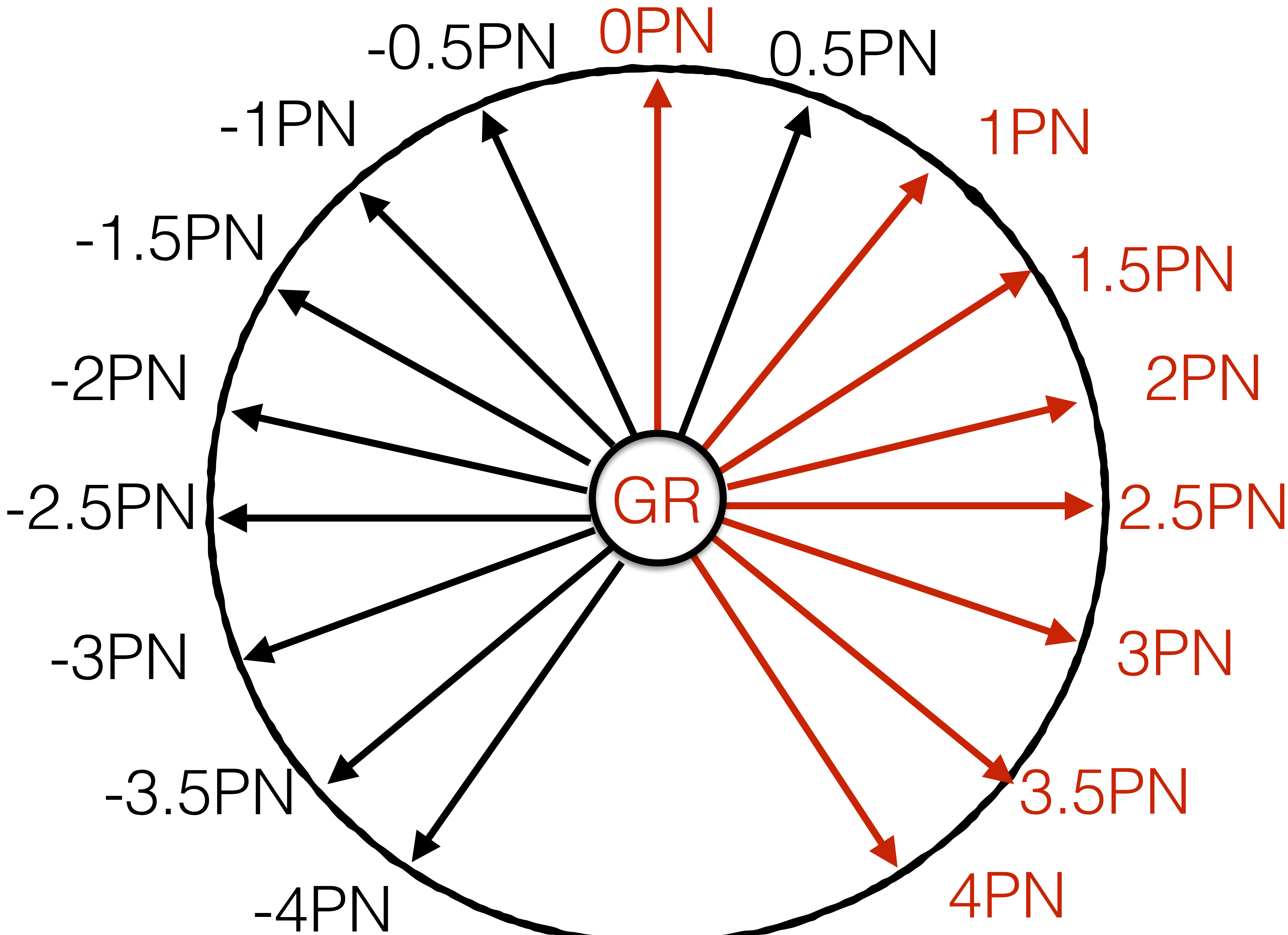

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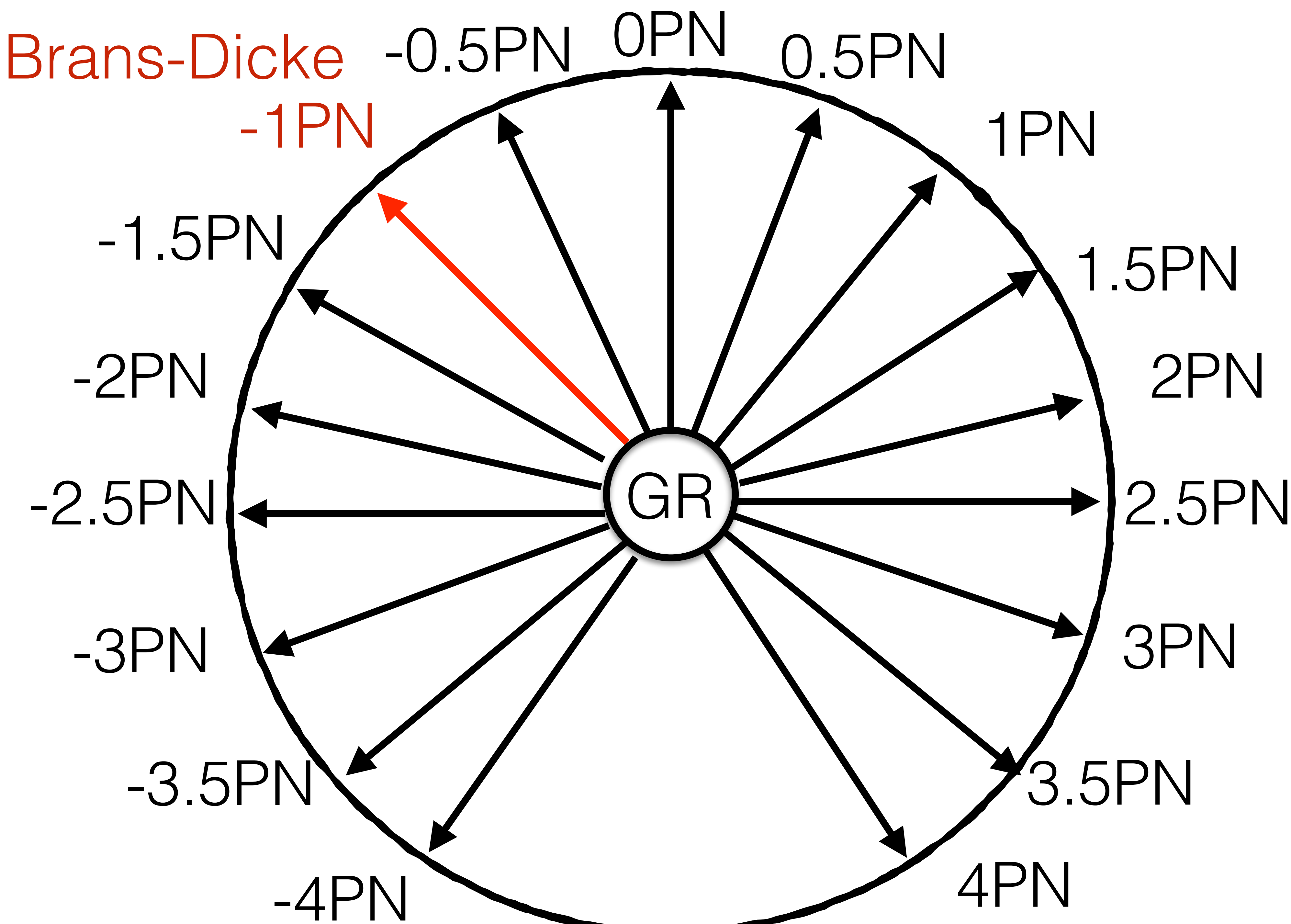

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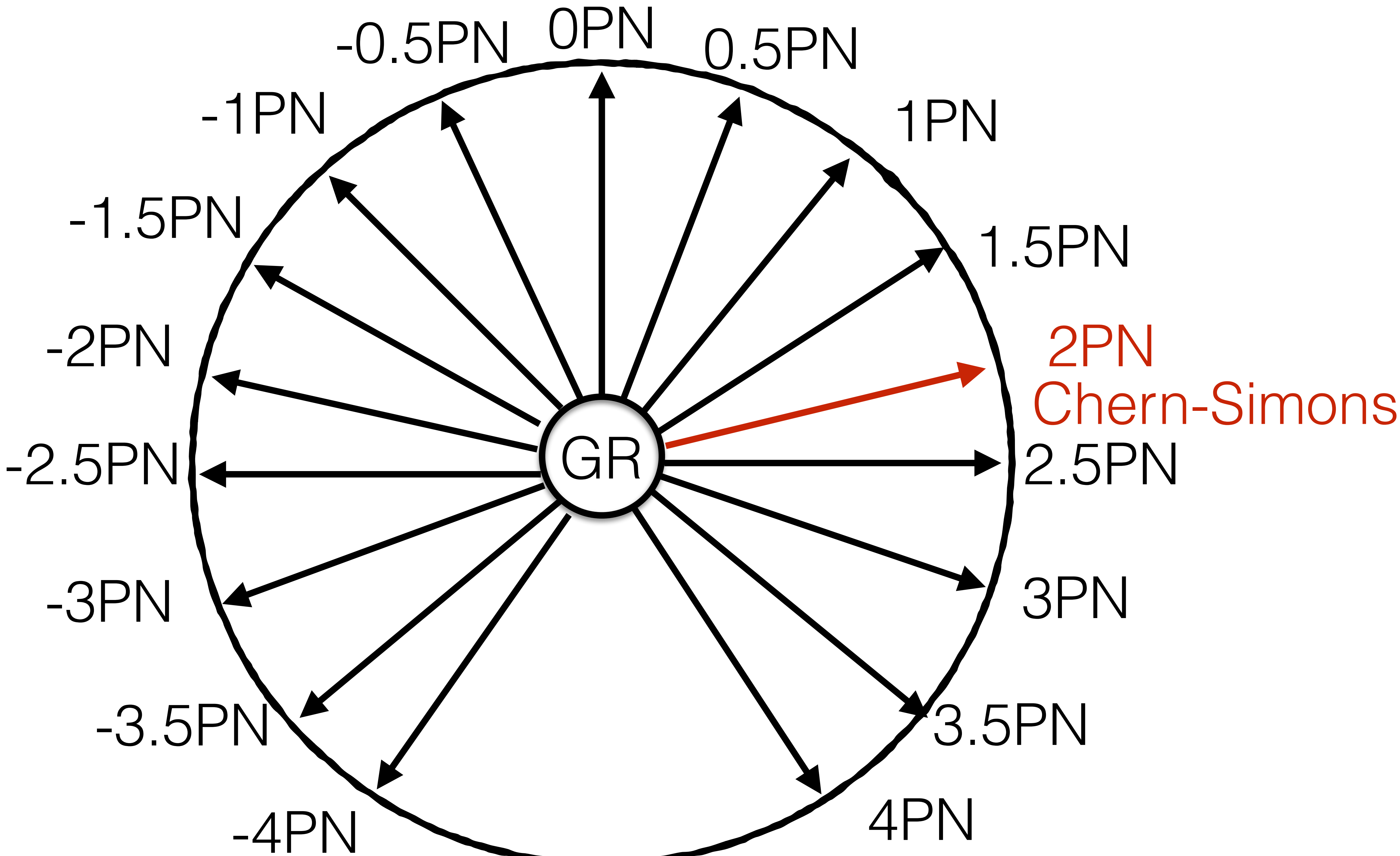

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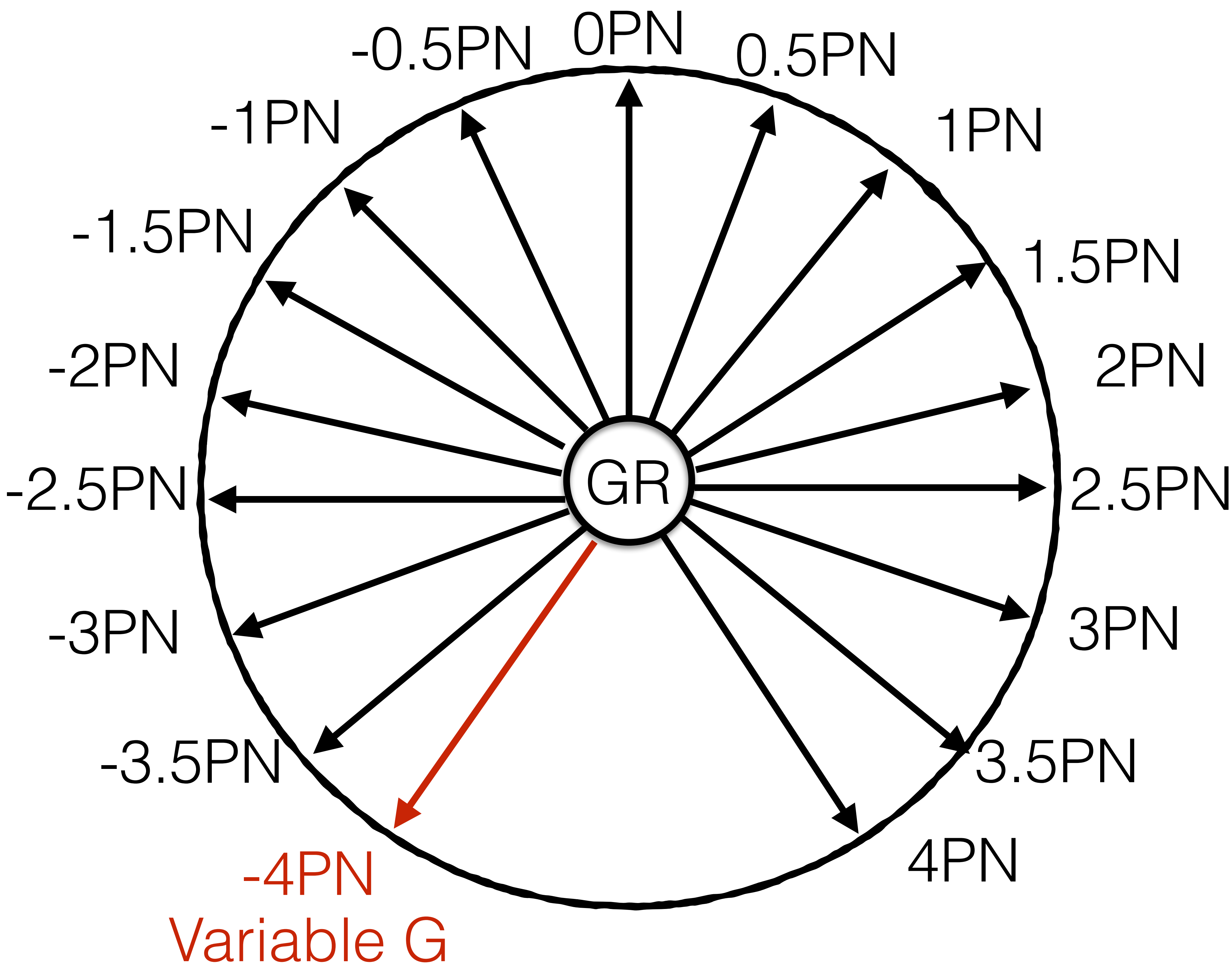

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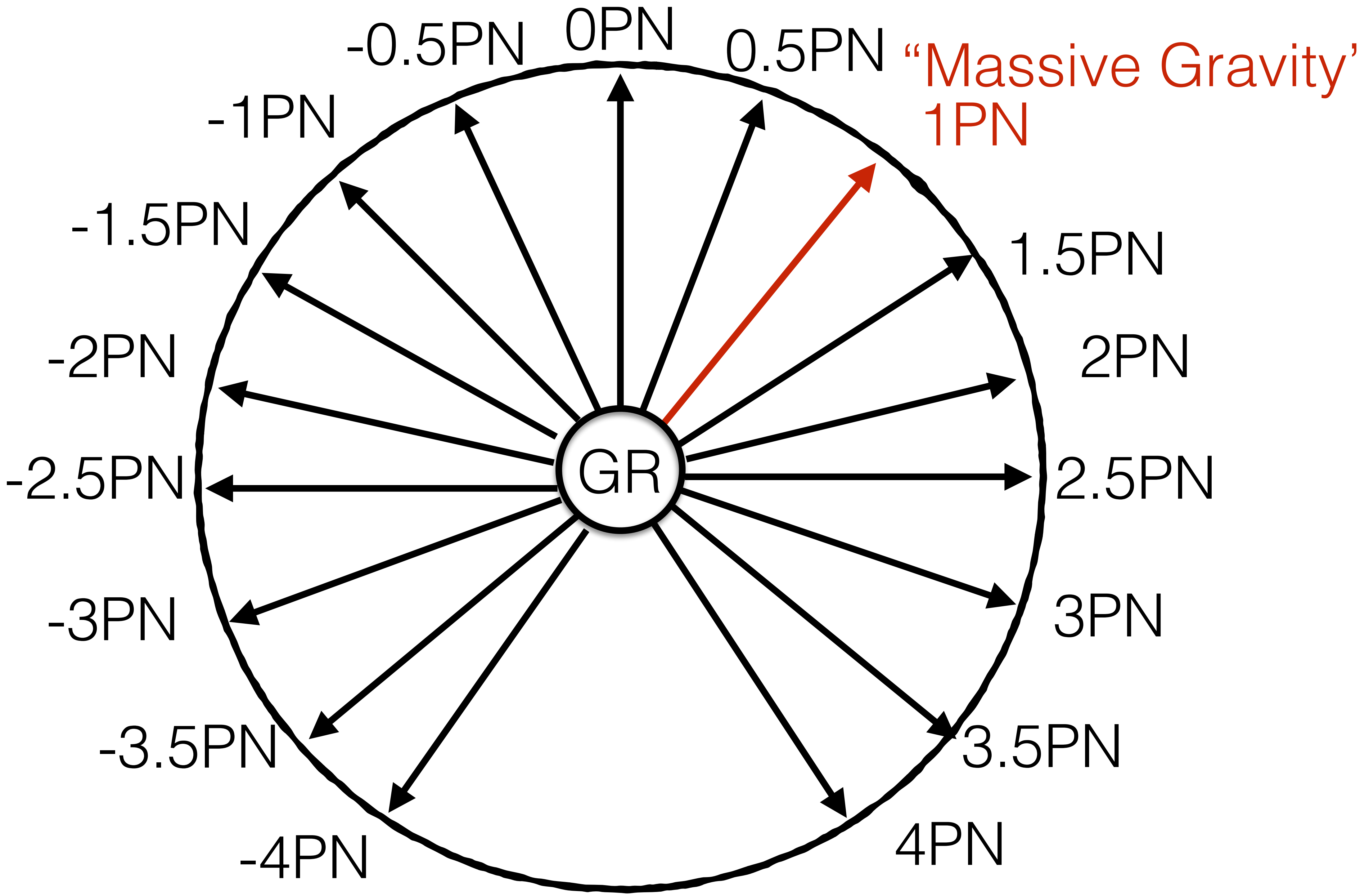


Data analysis

- Given a model of gravity H and some data d, we want to
  - infer the parameters θ
  - estimate the “goodness of fit” of the model
- Bayes’ theorem

$$p(\theta|d,\mathrm{H}) = p(\theta|\mathrm{H})\frac{p(d|\theta,\mathrm{H})}{p(d|\mathrm{H})}$$

posterior — prior — likelihood: $p(d|\theta,\mathrm{H})$ — evidence: $p(d|\mathrm{H})$

- Evidence

$$Z \equiv p(d|\mathrm{H}) = \int d\theta p(\theta|\mathrm{H})p(d|\theta,\mathrm{H})$$

Data analysis

- Consider two alternative models H_1 and H_2

- Given some data d the odds ratio

$$O_{1,2} \equiv \frac{p(\mathrm{H}_1)}{p(\mathrm{H}_2)} \frac{Z_1}{Z_2} \equiv \frac{p(\mathrm{H}_1)}{p(\mathrm{H}_2)} B_{1,2}$$

Bayes factor

- Bayesian figure of merit for the relative "goodness of fit"

Data analysis

- The odds ratio *accumulates* across multiple statistically independent events
- Given some data $d_1, \ldots, d_n$ and two competing hypotheses H_1 and H_2:

$$O_{1,2} = \frac{p(\mathrm{H}_1)}{p(\mathrm{H}_2)} \prod_j B_{1,2}^{(j)}$$

The noise

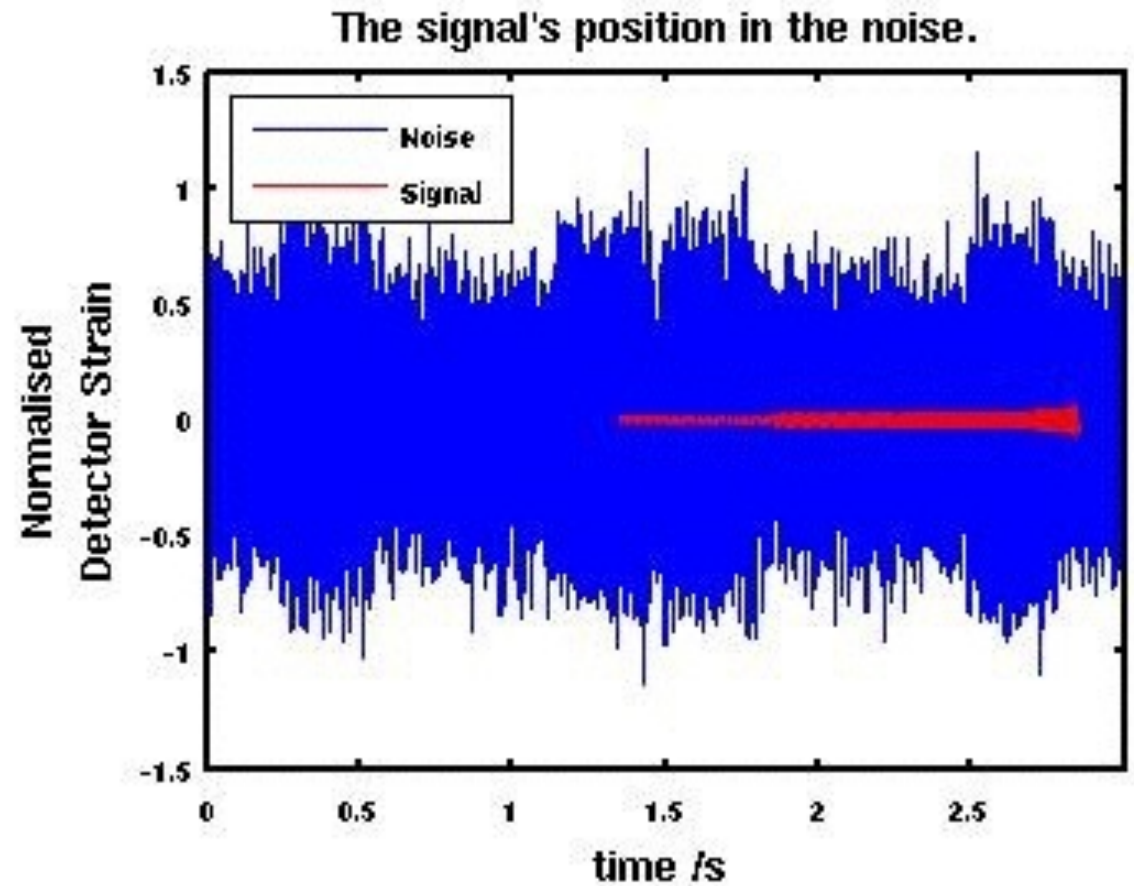
- Signals are embedded in noise

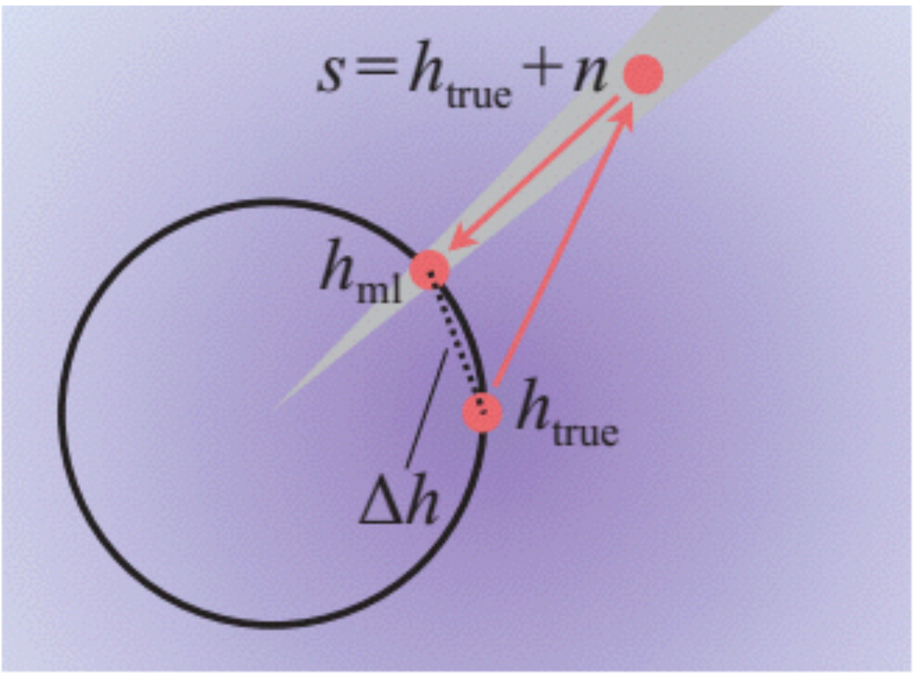
$$d = n + h(\theta)$$

- The likelihood for $h(\theta)$ is defined by the expected noise distribution $f(n)$

$$p(d|\theta, \mathrm{H}) = f(d - h(\theta))$$

- mis-modelling the noise leads to biases and may mimic GR violations





Detecting GR violations

- We detected a GW event, how do we then test GR?
- Enumerate all possible alternatives $\{\mathrm{H}_i\}_{i=1,\ldots,n}$
- Given a GW event, for all H_i compute the odds ratio against GR
- Select the theory with the highest odds as the "correct" one
- There is an infinite number of potential alternatives to GR

Detecting GR violations

- Look at the GR waveform

$$h(t) = A(t)\cos(\Phi(t))$$

$$\Phi(t) = v(t)^{-5}\sum_{n=0}^{7}(\phi_n + \phi_n^l \log(v(t)))v^n(t)$$

- The GR model is a set of very definite propositions

$$\mathrm{H}_{GR} = (\phi_1 = \phi_1^{GR}) \wedge (\phi_2 = \phi_2^{GR}) \wedge \ldots$$

- Define H_{modGR} as the hypothesis that one or more of the ϕ_i is not as predicted by GR, but not specifying which

Detecting GR violations

- H_{modGR} has no waveform model associated to it

- Decompose into mutually exclusive sub-hypotheses

    - $H_{i_1,i_2,\ldots,i_n}$ is the hypothesis that $\phi_{i_1}, \phi_{i_2}, \ldots, \phi_{i_n}$ do not have the dependence on masses and spins as GR, but all the other $\phi_j, j \notin i_1, i_2, \ldots, i_n$ do

    - Let $\theta = \{m_1, m_2, s_1, s_2, \ldots\}$, then $H_{i_1,i_2,\ldots,i_n}$ is tested by waveforms with parameters $\{\theta, \phi_{i_1}, \phi_{i_2}, \ldots, \phi_{i_n}\}$

Detecting GR violations

- H_{modGR} is the logical union of all the $H_{i_1,i_2,\ldots,i_n}$
- Example: 2 PN coefficients, ϕ_1, ϕ_2 :

$$\mathrm{H}_{modGR} = \mathrm{H}_1 \vee \mathrm{H}_2 \vee \mathrm{H}_{12}$$

- Tested by three waveforms with parameters

$$\mathrm{H}_1 : \{m_1, m_2, s_1, s_2, \ldots, \phi_1\}$$
$$\mathrm{H}_2 : \{m_1, m_2, s_1, s_2, \ldots, \phi_2\}$$
$$\mathrm{H}_{12} : \{m_1, m_2, s_1, s_2, \ldots, \phi_1, \phi_2\}$$

Detecting GR violations

- The odds ratio is given by

$$O_{\mathrm{H}_{modGR},\mathrm{H}_{GR}} = \frac{p(\mathrm{H}_1)}{p(\mathrm{H}_{GR})}\frac{p(d|\mathrm{H}_1)}{p(d|\mathrm{H}_{GR})} + \frac{p(\mathrm{H}_2)}{p(\mathrm{H}_{GR})}\frac{p(d|\mathrm{H}_2)}{p(d|\mathrm{H}_{GR})} + \frac{p(\mathrm{H}_{12})}{p(\mathrm{H}_{GR})}\frac{p(d|\mathrm{H}_{12})}{p(d|\mathrm{H}_{GR})}$$

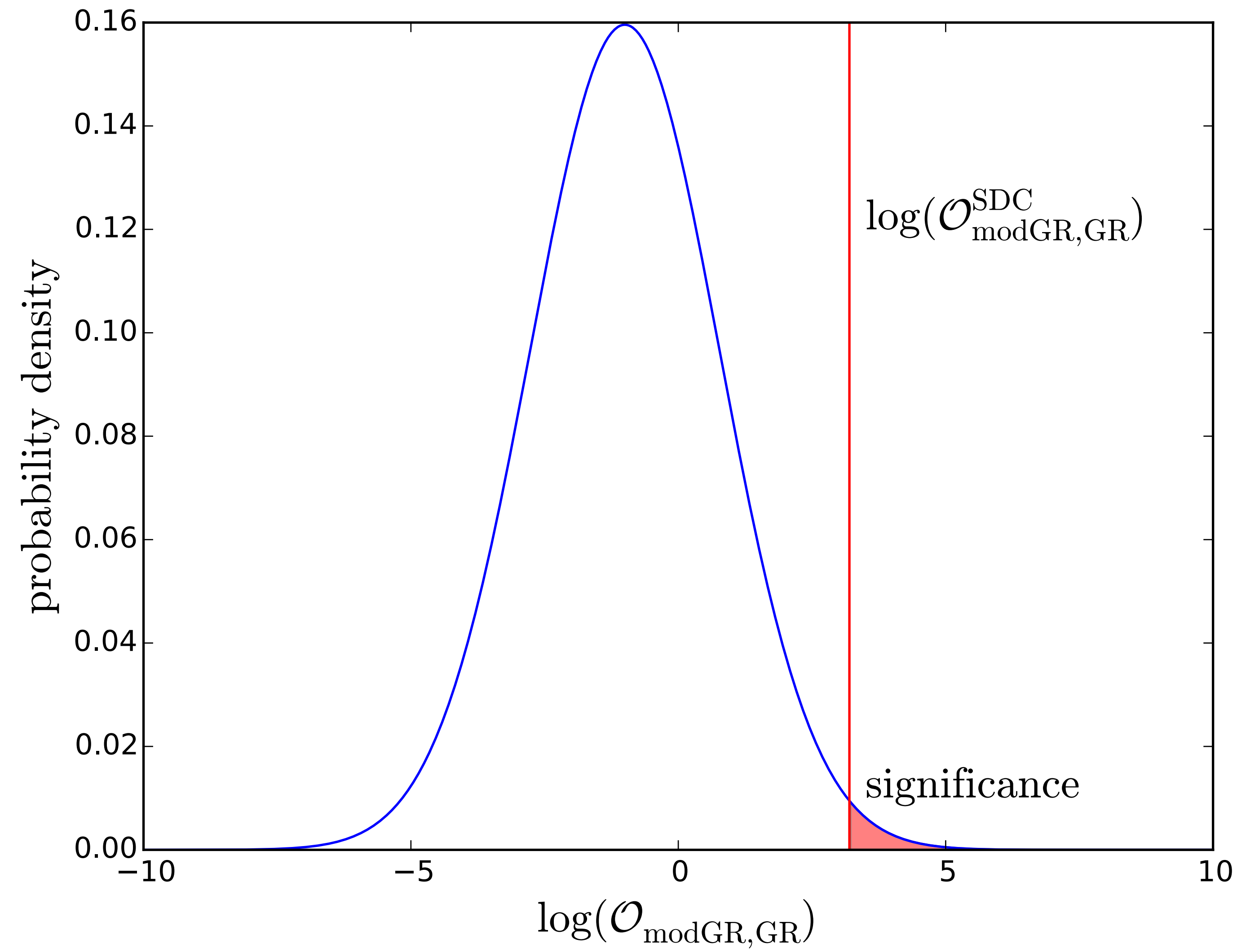
- It can be generalised to N_T hypotheses and N events:

$$^{(N_T)}\mathcal{O}_{\mathrm{GR}}^{\mathrm{modGR}} = \frac{\alpha}{2^{N_T}-1}\sum_{k=1}^{N_T}\sum_{i_1<i_2<\ldots<i_k}\prod_{A=1}^{\mathcal{N}}{}^{(A)}B_{\mathrm{GR}}^{i_1 i_2 \ldots i_k}$$

$$\alpha = \frac{p(\mathrm{H}_{modGR})}{p(\mathrm{H}_{GR})}$$

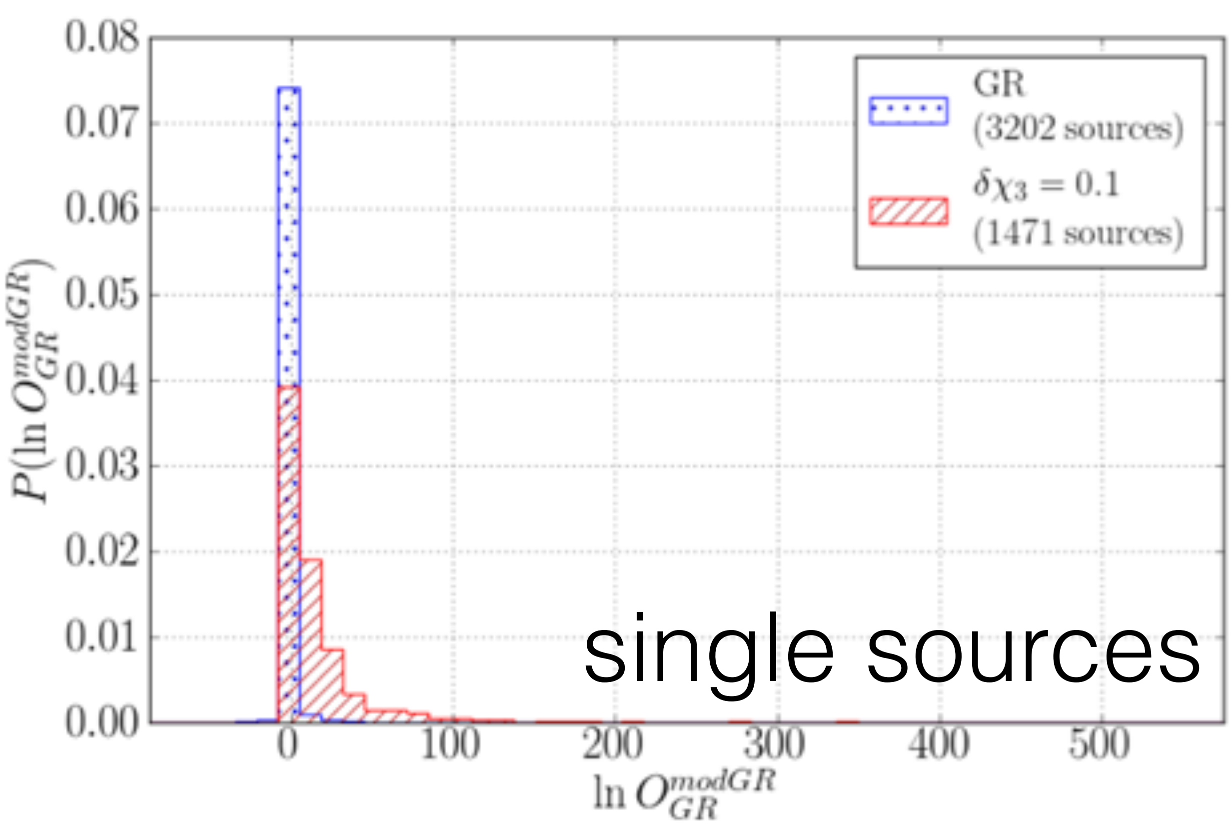
The noise - again

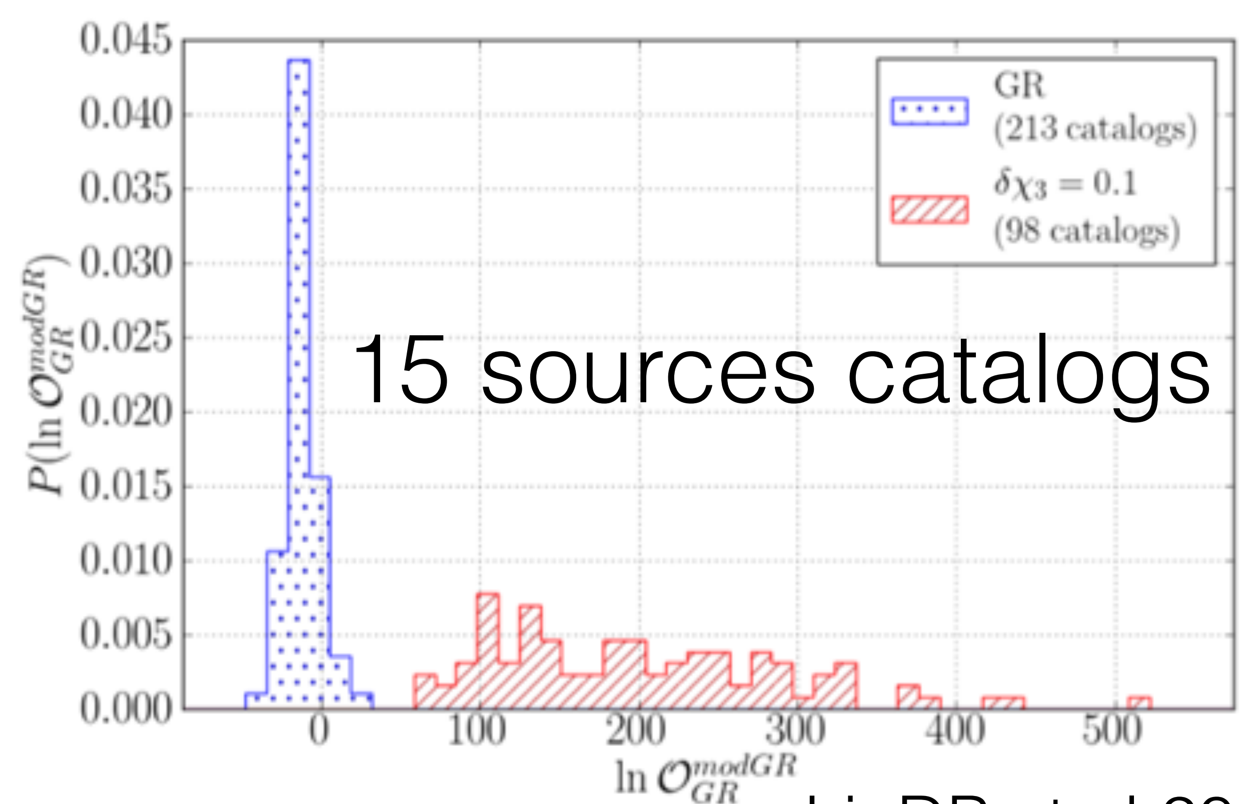
- Given a GW detection, the odds ratio alone is not sufficient
    - mis-modelling of the noise and/or different noise realisations lead to a different odds for the same signal
- Compute the expected distribution of odds from simulated GR signals in many different stretches of data
    - false alarm probability
    - significance of the detection



TIGER

- We implemented the Test Infrastructure for GEneral Relativity (TIGER) for LIGO/Virgo data and binary neutron stars (BNS) systems
- 10% level violation in the 1.5PN (tail) term

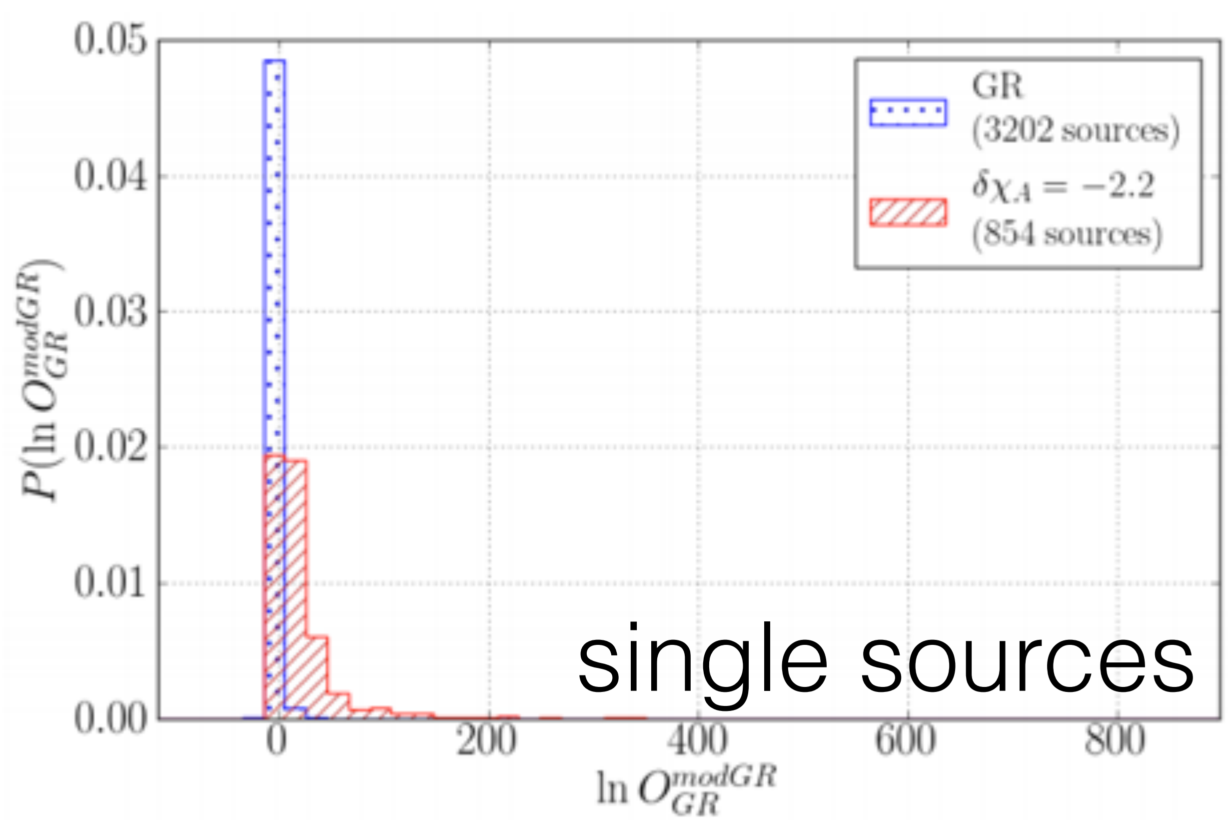


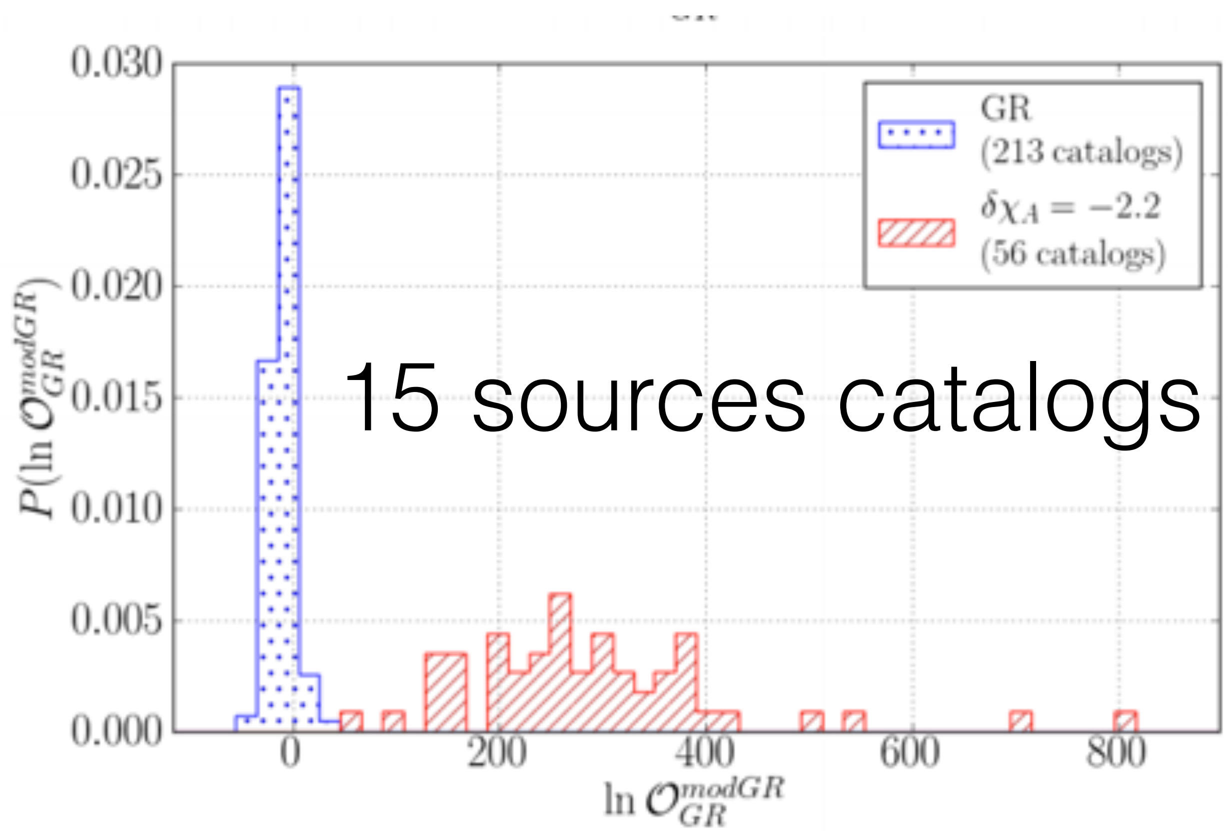


Li, DP et al 2012

TIGER

- Capture “non-PN” deviations, e.g. a “1.25PN”

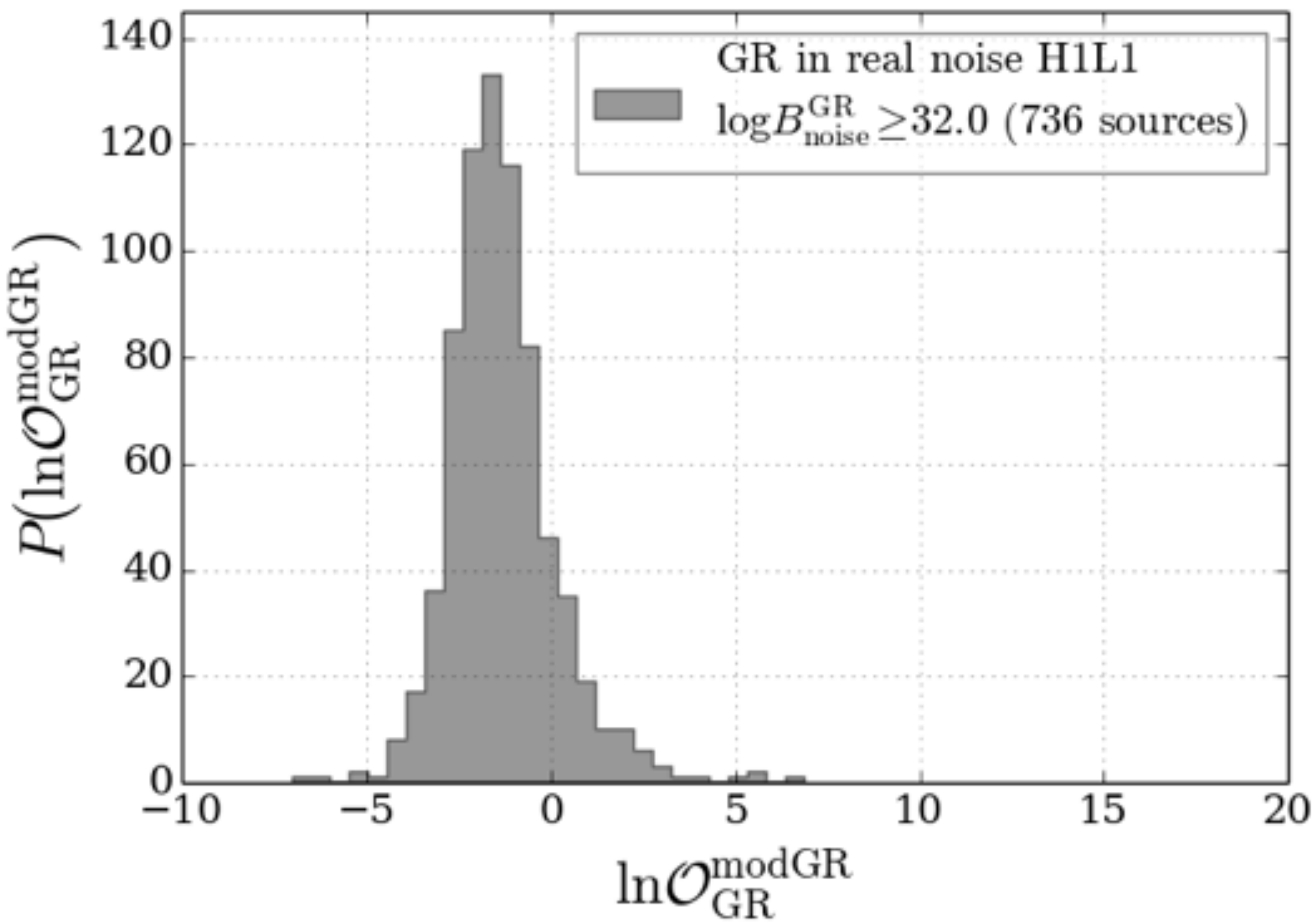




Li, DP et al 2012

Prospects for O1

- TIGER is the testing GR pipeline for the LVC
- Robustness against systematics proved in Agathos, DP et al 2014
- TIGER is robust in real (non-Gaussian) data



Outlook

- The ability to test GR depends on
  - the understanding of GR
    - faithful WF models
  - the understanding of the instrument
    - noise distribution, non-Gaussianities
- Combining information across sources is a powerful tool to increase sensitivity to small GR violations