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Testing gravity theories with gravitational waves and
compact objects: the case of Lorentz violating gravity

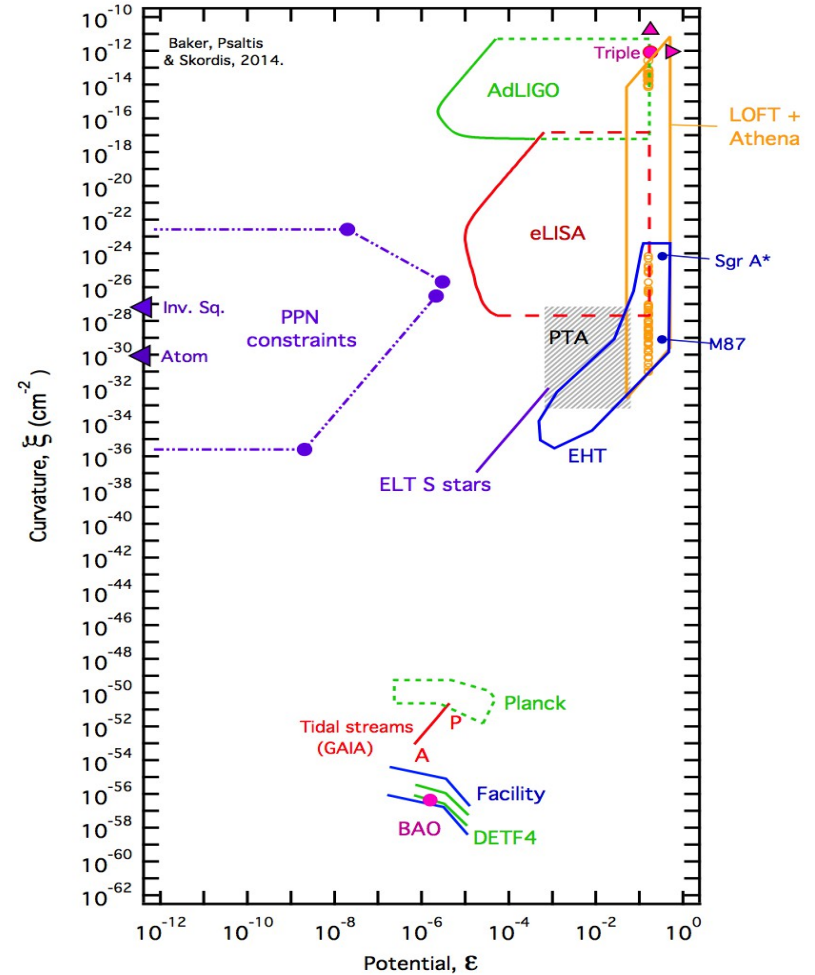
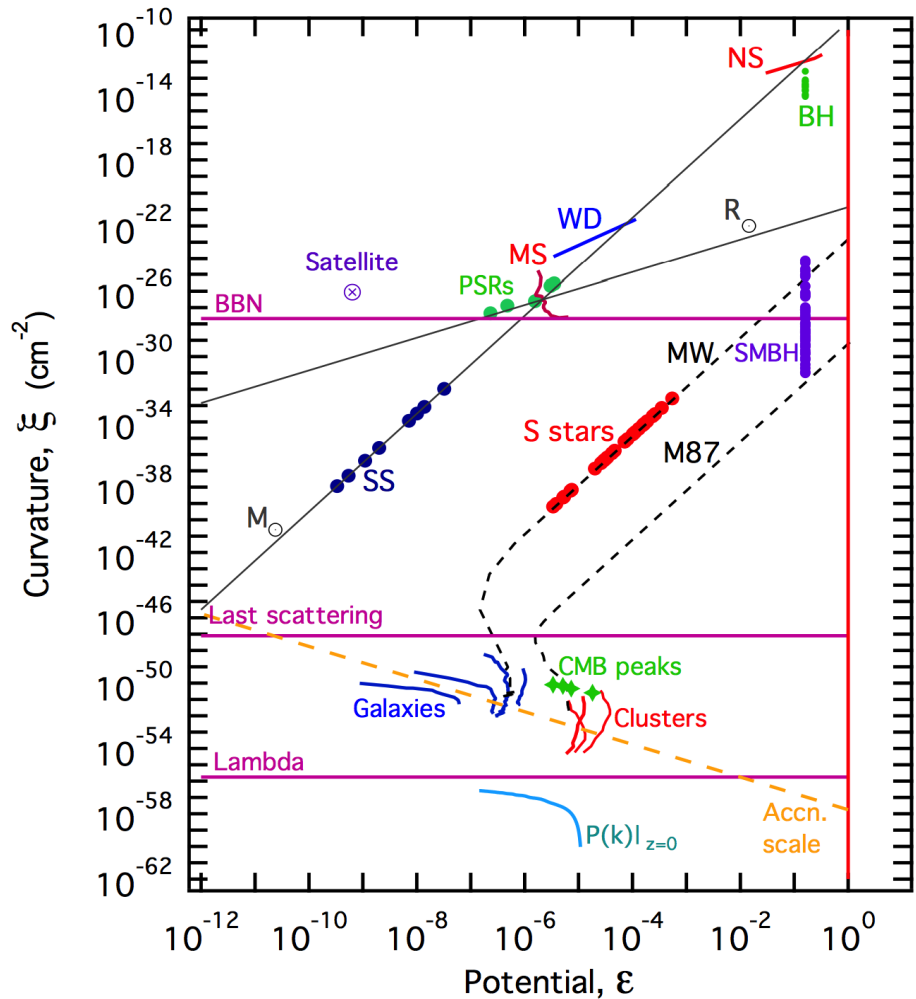


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Outline

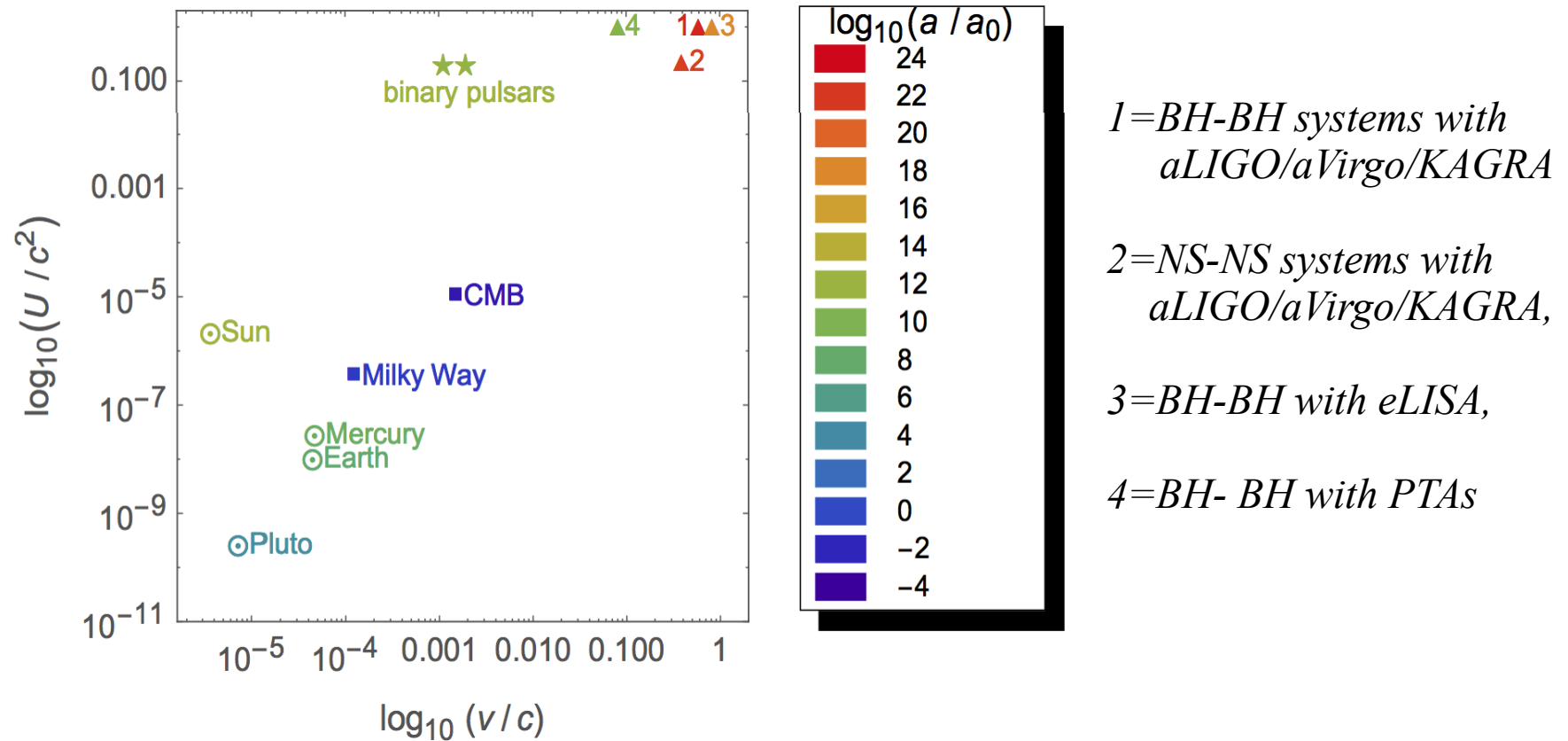
- Why and how to modify GR?
Theoretical and experimental requirements
- Why Lorentz invariance?
- There is more to life (and to testing gravity!) than cosmology:
 - Binary pulsars in Lorentz-violating gravity
 - Black-hole solutions in Lorentz-violating gravity
- Lorentz violations in gravity as substitute to Dark Matter?

Why explore corrections to GR?



Figures from Baker, Psaltis & Skordis 2014

Dark Matter/Energy or Dark Gravity?



Evidence for Dark Sector from accelerations lower than $a_0 \sim 10^{-10} m/s^2$

How to modify GR?

Lovelock's theorem

In a 4-dimensional spacetime, the only divergence-free symmetric rank-2 tensor constructed only from the metric $g_{\mu\nu}$ and its derivatives up to second differential order, and preserving diffeomorphism invariance, is the Einstein tensor plus a cosmological term, i.e. $G_{\mu\nu} + \Lambda g_{\mu\nu}$

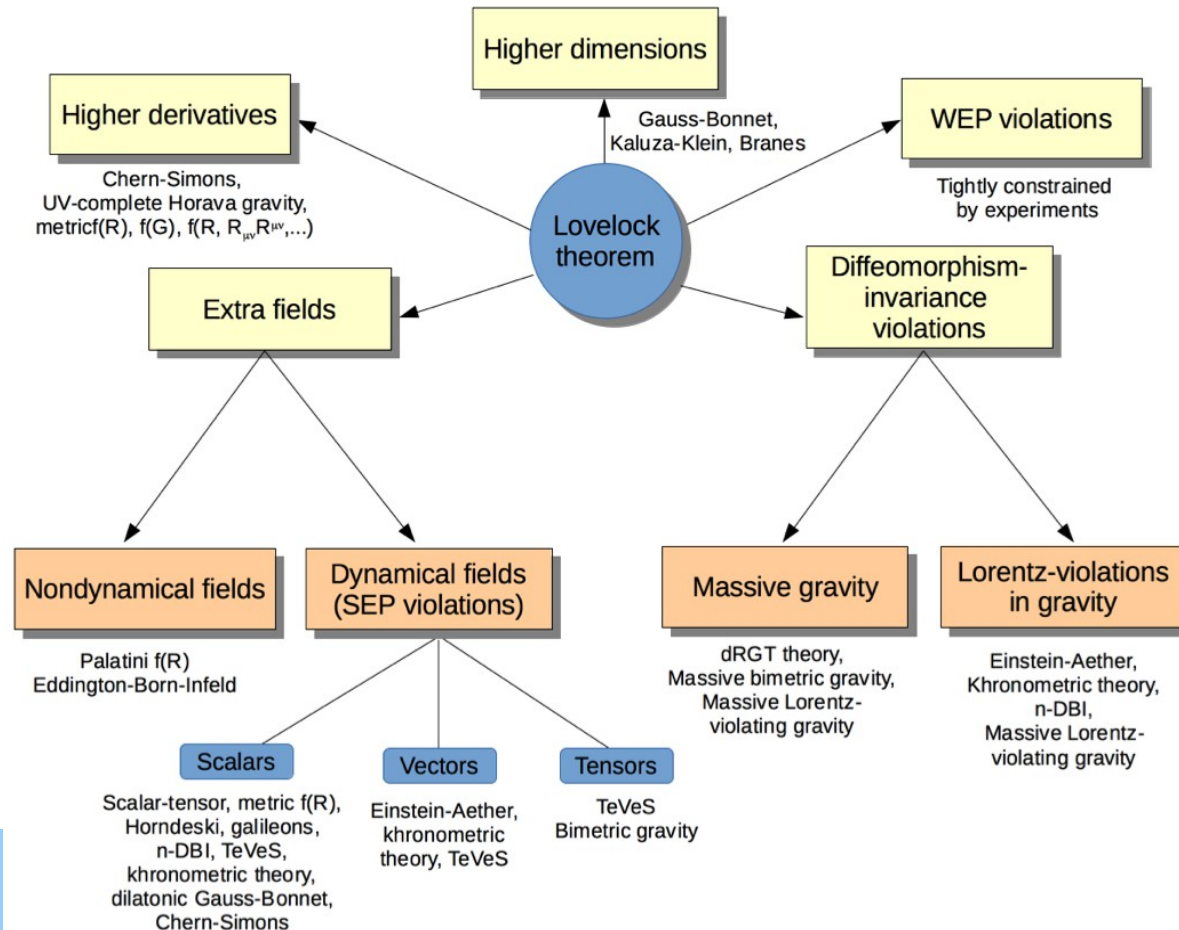


Figure from Berti, EB et al 2015

There is more to life than cosmology!

Theory's properties

Theory	Field content	Strong EP	Massless graviton	Lorentz symmetry	Linear $T_{\mu\nu}$	Weak EP	Well-posed?	Weak-field constraints
Extra scalar field								
Scalar-tensor	S	✗	✓	✓	✓	✓	✓ [30]	[31-33]
Multiscalar	S	✗	✓	✓	✓	✓	✓?	[34]
Metric $f(R)$	S	✗	✓	✓	✓	✓	✓ [35,36]	[37]
Quadratic gravity								
Gauss-Bonnet	S	✗	✓	✓	✓	✓	✓?	[38]
Chern-Simons	P	✗	✓	✓	✓	✓	✗✓? [39]	[40]
Generic	S/P	✗	✓	✓	✓	✓	?	
Horndeski	S	✗	✓	✓	✓	✓	✓?	
Lorentz-violating								
Æ-gravity	SV	✗	✓	✗	✓	✓	✓?	[41-44]
Khronometric/ Hořava-Lifshitz	S	✗	✓	✗	✓	✓	✓?	[43-46]
n-DBI	S	✗	✓	✗	✓	✓	?	none ([47])
Massive gravity								
dRGT/Bimetric	SVT	✗	✗	✓	✓	✓	?	[16]
Galileon	S	✗	✓	✓	✓	✓	✓?	[16,48]
Nondynamical fields								
Palatini $f(R)$	-	✓	✓	✓	✗	✓	✓	none
Eddington-Born-Infeld	-	✓	✓	✓	✗	✓	?	none
Others, not covered here								
TeVes	SVT	✗	✓	✓	✓	✓	?	[33]
$f(R)\mathcal{L}_m$?	?	✓	✓	✓	✗	?	
$f(T)$?	✗	✓	✗	✓	✓	?	[49]

Table 1. Catalog of several theories of gravity and their relation with the assumptions of Lovelock's theorem. Each theory violates at least one assumption (see also Figure 2.1), and can be seen as a proxy for testing a specific principle underlying GR. See text for details of the entries. Key to abbreviations: S: scalar; P: pseudoscalar; V: vector; T: tensor; ?: unknown; ✓?: not explored in detail or not rigorously proven, but there exist arguments to expect ✓. The occurrence of ✗✓? means that there exist arguments in favor of well-posedness within the EFT formulation, and against well-posedness for the full theory. Weak-field constraints (as opposed to strong-field constraints, which are the main topic of this review) refer to Solar System and binary pulsar tests. Entries below the last horizontal line are not covered in this review.

Table
from
Berti, EB
et al 2015

There is more to life than cosmology!

BH properties

Theory	Solutions	Stability	Geodesics	Quadrupole
Extra scalar field				
Scalar-tensor	\equiv GR [50, 55]	[56, 62]	–	–
Multiscalar/Complex scalar	\supset GR [51, 63, 64]	?	?	[63, 64]
Metric $f(R)$	\supset GR [53, 54]	[65, 66]	?	?
Quadratic gravity				
Gauss-Bonnet	NR [67, 69]; SR [70, 71]; FR [72]	[73, 74]	SR [70, 75, 76]; FR [72]	[71, 77]
Chern-Simons	SR [78, 80]; FR [81]	NR [82, 85]; SR [74]	[69, 86]	[80]
Generic	SR [75]	?	[75]	Eq. (3.12)
Horndeski	[87, 89]	? [90, 91]	?	?
Lorentz-violating				
Æ-gravity	NR [92, 94]	?	[93, 94]	?
Khronometric/ Hořava-Lifshitz	NR, SR [93, 96]	? [97]	[93, 94]	?
n-DBI	NR [98, 99]	?	?	?
Massive gravity				
dRGT/Bimetric	\supset GR, NR [100, 103]	[104, 107]	?	?
Galileon	[108]	?	?	?
Nondynamical fields				
Palatini $f(R)$	\equiv GR	–	–	–
Eddington-Born-Infeld	\equiv GR	–	–	–

Table
from
Berti, EB
et al 2015

Table 2. Catalogue of BH properties in several theories of gravity. The column “Solutions” refers to asymptotically-flat, regular solutions. Legend: ST=“Scalar-Tensor,” \equiv GR=“Same solutions as in GR,” \supset GR=“GR solutions are also solutions of the theory,” NR=“Non rotating,” SR=“Slowly rotating,” FR=“Fast rotating/Generic rotation,” ?=unknown or uncertain.

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NS properties

Theory	NR	Structure SR	FR	Collapse	Sensitivities	Stability	Geodesics
Extra scalar field							
Scalar-Tensor	[109, 114]	[112, 115, 116]	[117, 119]	[120, 127]	[128]	[129, 139]	[118, 140]
Multiscalar	?	?	?	?	?	?	?
Metric $f(R)$	[141, 153]	[154]	[155]	[156, 157]	?	[158, 159]	?
Quadratic gravity							
Gauss-Bonnet	[160]	[160]	[77]	?	?	?	?
Chern-Simons	\equiv GR	[25, 40, 161, 163]	?	?	[162]	?	?
Horndeski	?	?	?	?	?	?	?
Lorentz-violating							
Æ-gravity	[164, 165]	?	?	[166]	[43, 44]	[158]	?
Khronometric/ Hořava-Lifshitz	[167]	?	?	?	[43, 44]	?	?
n-DBI	?	?	?	?	?	?	?
Massive gravity							
dRGT/Bimetric	[168, 169]	?	?	?	?	?	?
Galileon	[170]	[170]	?	[171, 172]	?	?	?
Nondynamical fields							
Palatini $f(R)$	[173, 177]	?	?	?	–	?	?
Eddington-Born-Infeld	[178, 184]	[178, 179]	?	[179]	–	[185, 186]	?


Table
from
Berti, EB
et al 2015

Table 3. Catalog of NS properties in several theories of gravity. Symbols and abbreviations are the same as in Table 2

The power of astrophysical probes: the example of Lorentz-violating gravity

- LV may give better UV behavior (Horava), quantum-gravity completions generally lead to LV
- Strong constraints in matter sector, weaker ones in gravity sector (caveat: constraints expected to percolate from gravity to matter sector)
- LV allows MOND-like (Bekenstein, Ferreira, Blanchet & Marsat, Bonetti & EB) or dark-energy-like phenomenology
- Solar system/isolated & binary pulsar experiments historically used to constrain LV in weak field (1 PN) regimes (“preferred-frame parameters”: Nordvedt, Kramer, Wex, Freire, Shao, Damour, Esposito Faresse...), but **surprises may happen in strong-field regimes**

Einstein-aether theory

- We want to specify a (local) preferred time “direction”  timelike aether field U_μ with unit norm
- Most generic action (in 4D) quadratic in derivatives is given (up to total derivatives) by

$$S = \frac{1}{16\pi G} \int (R + \frac{1}{3}c_\theta\theta^2 + c_\sigma\sigma^2 + c_\omega\omega^2 + c_a a^2) \sqrt{-g} d^4x$$

$$\omega_{\mu\nu} = \nabla_{[\nu}U_{\mu]} + a_{[\mu}U_{\nu]} = \partial_{[\nu}U_{\mu]} + a_{[\mu}U_{\nu]} \quad a^\mu = U^\nu \nabla_\nu U^\mu$$

$$\sigma_{\mu\nu} = \nabla_{(\nu}U_{\mu)} + a_{(\mu}U_{\nu)} - \frac{1}{3}\theta\gamma_{\mu\nu} \quad \theta = \nabla_\mu U^\mu$$

- To satisfy weak equivalence principle, matter fields couple minimally to metric (and not directly to aether)

$$S = S_{\text{ae}} + S_{\text{matter}}(\psi, g_{\mu\nu})$$

Chronometric gravity

- To specify a global time, U must be hypersurface orthogonal (“chronometric” theory)

$$U_\mu = -\frac{\partial_\mu T}{\sqrt{-g^{\alpha\beta}\partial_\alpha T\partial_\beta T}} \quad S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R + \frac{1}{3}(\beta + 3\lambda)\theta^2 + \beta\sigma_{\mu\nu}\sigma^{\mu\nu} + \alpha a_\mu a^\mu \right]$$

- Because U is timelike, T can be used to as time coordinate

$$U_\mu = -\delta_\mu^T (-g^{TT})^{-1/2} = -N\delta_\mu^T \quad a_i = \partial_i \ln N$$

$$S_H = \frac{1-\beta}{16\pi G} \int dT d^3x N \sqrt{\gamma} \left(K_{ij}K^{ij} - \frac{1+\lambda}{1-\beta} K^2 + \frac{1}{1-\beta} {}^{(3)}R + \frac{\alpha}{1-\beta} a_i a^i \right)$$


3 free parameters vs 4 of AE theory (because aether is hypersurface orthogonal)

Chronometric vs Horava gravity

$$S_H = \frac{1 - \beta}{16\pi G} \int dT d^3x N \sqrt{\gamma} \left(L_{\text{kh}} + \frac{L_4}{M_\star^2} + \frac{L_6}{M_\star^4} \right)$$
$$L_{\text{kh}} = K_{ij} K^{ij} - \frac{1 + \lambda}{1 - \beta} K^2 + \frac{1}{1 - \beta} {}^{(3)}R + \frac{\alpha}{1 - \beta} a_i a^i$$

- L_4 and L_6 contain 4th- and 6th-order terms in the spatial derivatives
- Lower bound on M_\star depends on details of percolation of Lorentz violations from gravity to matter: from Lorentz violations in gravity alone, $M_\star \gtrsim 10^{-3}$ eV, but precise bounds depend on percolation
- Theory remains perturbative at all scales if $M_\star \lesssim 10^{16}$ GeV
- Terms crucial in the UV, but unimportant astrophysically, ie error scales as $\sim M_{\text{Planck}}^4 / (M M_\star)^2 \sim 10^{-14} (M_\odot / M)^2$

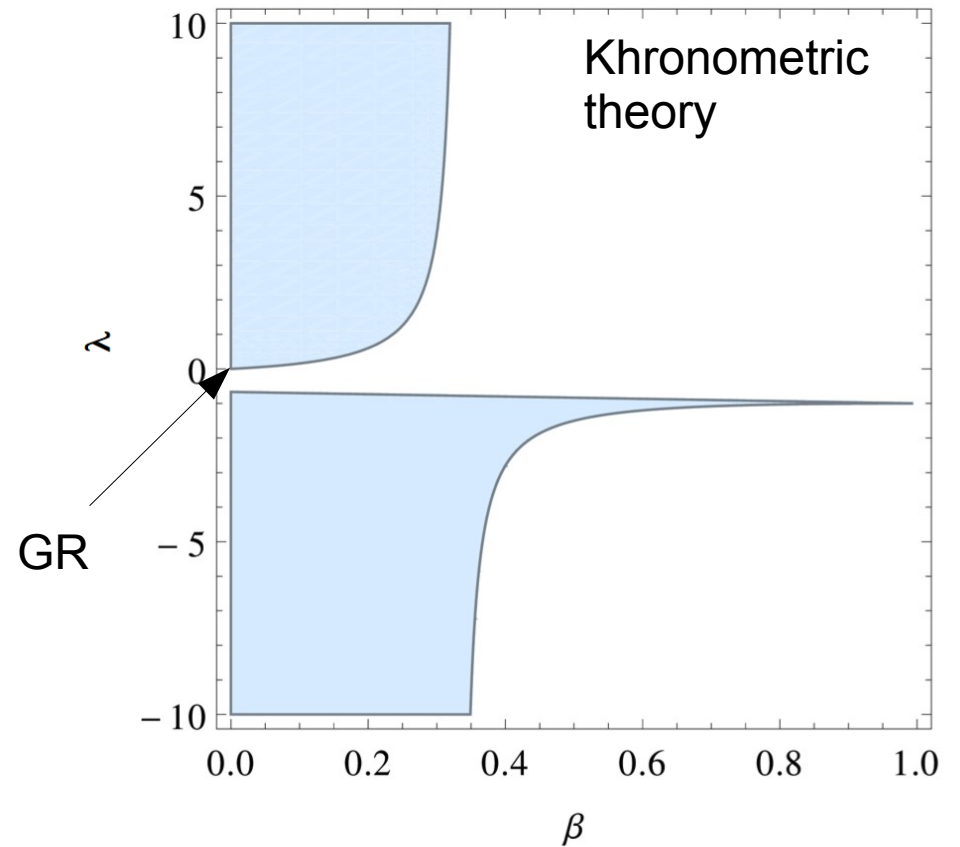
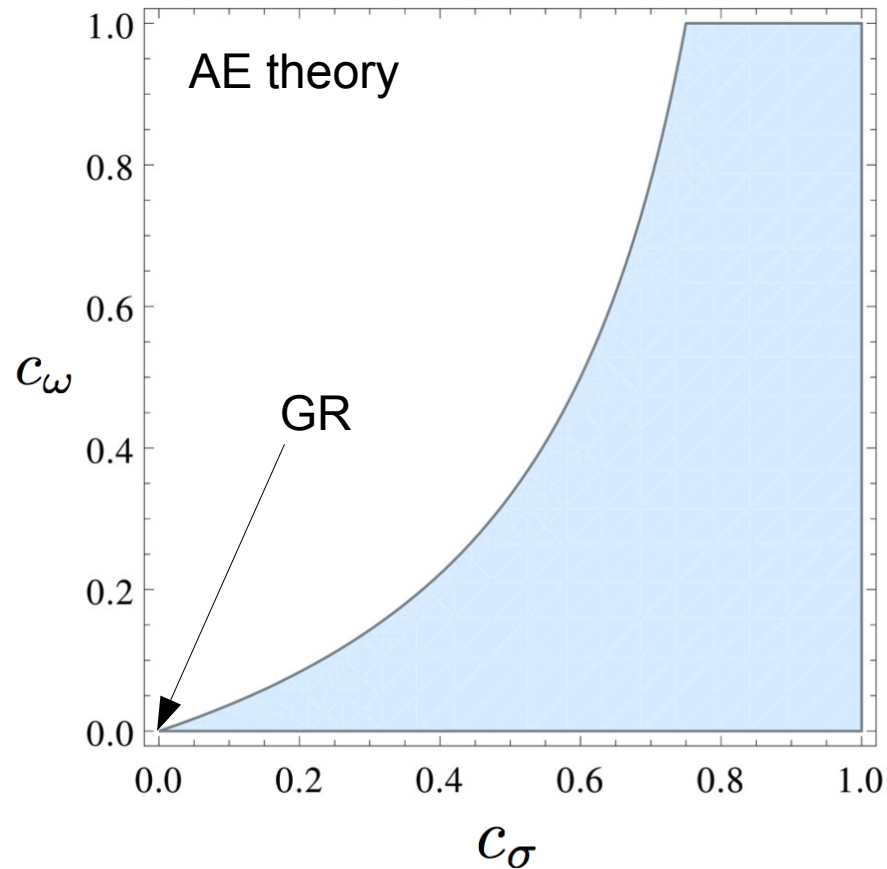
Constraints: stability and solar-system tests

- Solar-system experiments set combinations of the couplings essentially to zero 

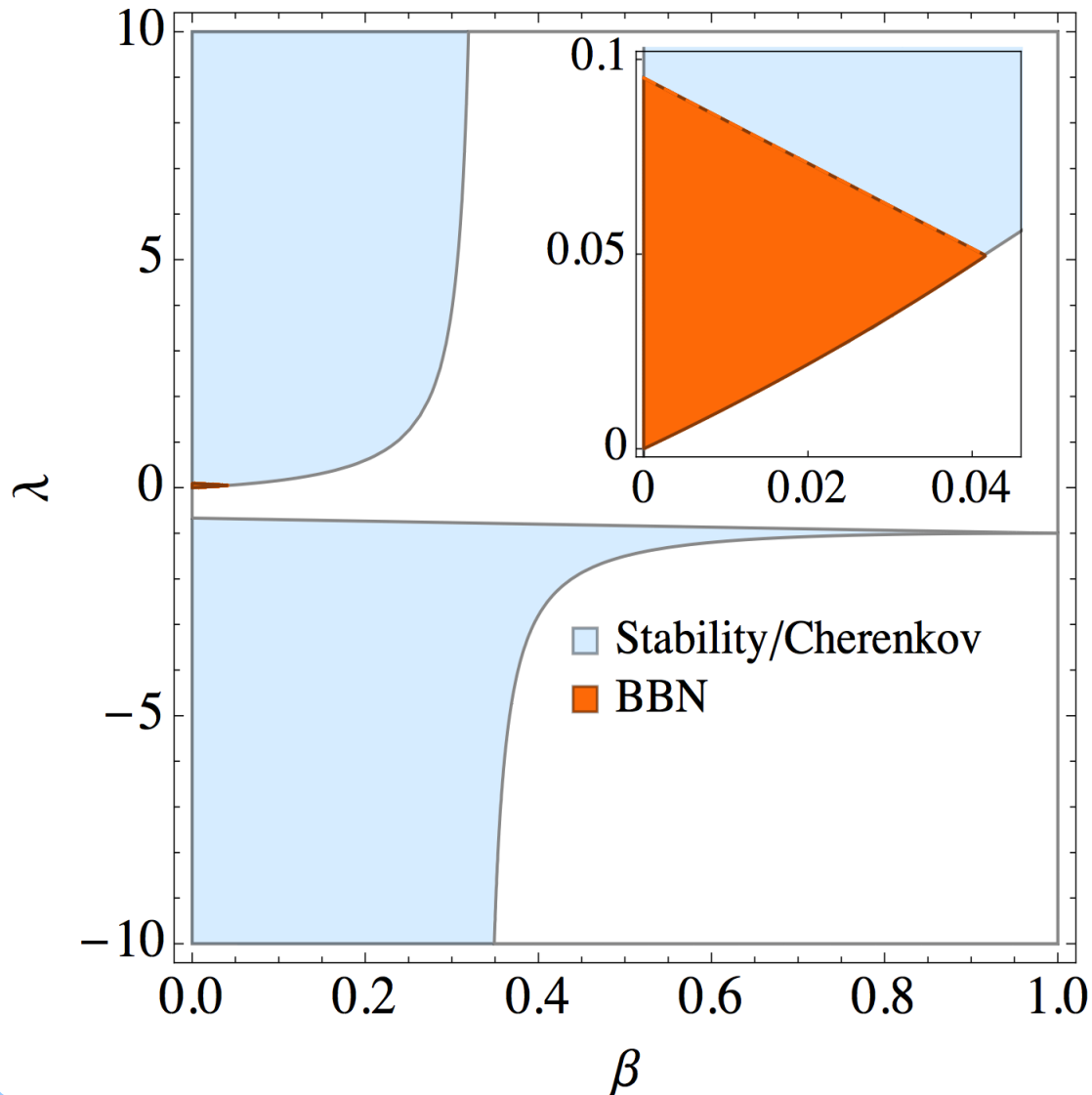
Both AE and khrometric theory have only two unconstrained couplings, c_σ , c_ω (AE) and λ , β (khronometric)

- AE theory has propagating spin-0, spin-1 and spin-2 gravitons; khronometric theory has spin-0, spin-2 gravitons:
 - Classical stability (real propagation speeds) and quantum stability (positive energies)
 - Propagation speed larger than speed of light to avoid gravitational Cherenkov radiation
- Well posedness proved in flat space and in spherical symmetry

Stability+Solar System+Cherenkov constraints



How about cosmological constraints?



- Weak for AE theory

- For khronometric theory,

$$\frac{G_N}{G_C} = \frac{2 + \beta + 3\lambda}{2(1 - \beta)}$$

and BBN requires

$$|G_N/G_C - 1| < 1/8$$

- No constraints from CMB in khronometric theory yet

Why are astrophysical effects expected?

- Matter couples minimally to metric, but metric couples non-minimally to aether \longrightarrow effective matter-aether coupling in strong-field regimes
- For strongly gravitating body (e.g. neutron star), binding energy depends on velocity relative to the aether $\gamma = U_\mu u^\mu$ (i.e. structure depends on motion relative to preferred frame, as expected from Lorentz violation!)
- Gravitational mass depends on velocity relative to the aether

$$\longrightarrow S_{matter} = \sum_i \int m_i(\gamma) d\tau_i \longrightarrow u_a^\mu \nabla_\mu (m_a u^\nu) = -\frac{d m_a}{d \gamma} u^\mu \nabla^\nu U_\mu$$

Violations of strong equivalence principle (aka Nordtvedt effect in Brans Dicke theory, scalar tensor theories, etc)

Why are astrophysical effects expected?

Whenever strong equivalence principle (SEP) is violated, dipolar gravitational-wave emission may be produced

- In GR, dipolar emission not present because of SEP + conservation of linear momentum

$$M_1 \equiv \int \rho x_i d^3x \quad h \sim \frac{G}{c^3} \frac{d}{dt} \frac{M_1}{r} \sim \frac{G}{c^3} \frac{P}{r} \quad \text{not a wave!}$$

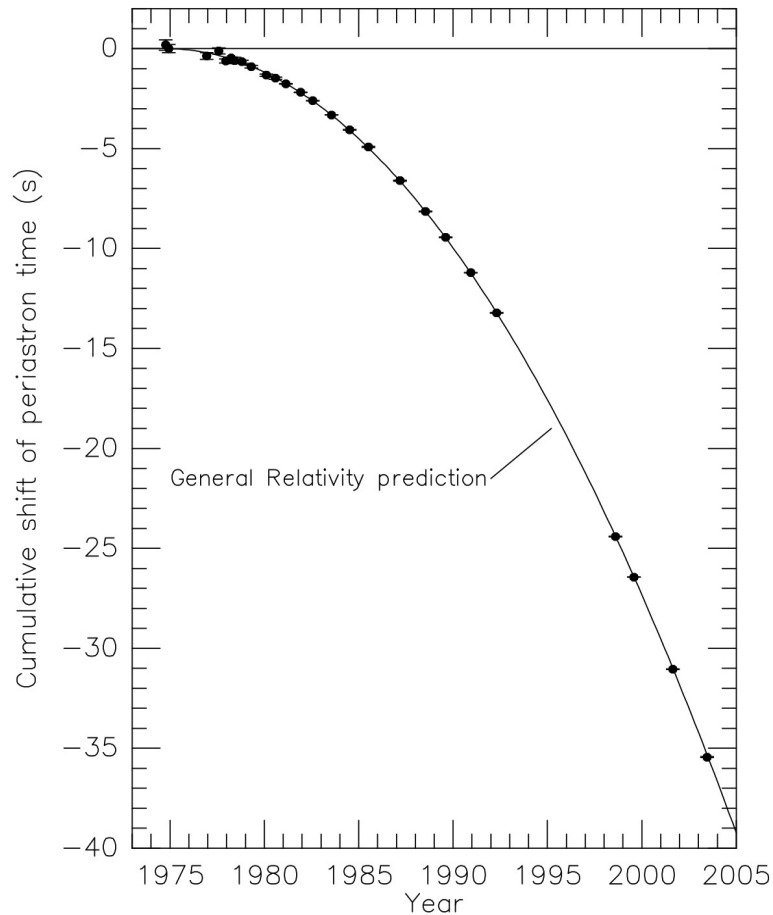
- If SEP is violated,

$$h \sim \frac{1}{R} \frac{d}{dt} [m_1(\gamma) x_1 + m_2(\gamma) x_2] \propto \left(\frac{d \log m_1}{d \log \gamma} - \frac{d \log m_2}{d \log \gamma} \right)$$

- Dipolar mode might be observable directly by interferometers, or indirectly via its backreaction on a binary's evolution

Why is this interesting?

Binary pulsars are the strongest test of GR to date!



To calculate rate of change of orbital period we need sensitivities

$$\sigma = - \left. \frac{\partial \log M}{\partial \log \gamma} \right|_{v=0} = -2 \left. \frac{\partial \log M}{\partial (v^2)} \right|_{v=0}$$

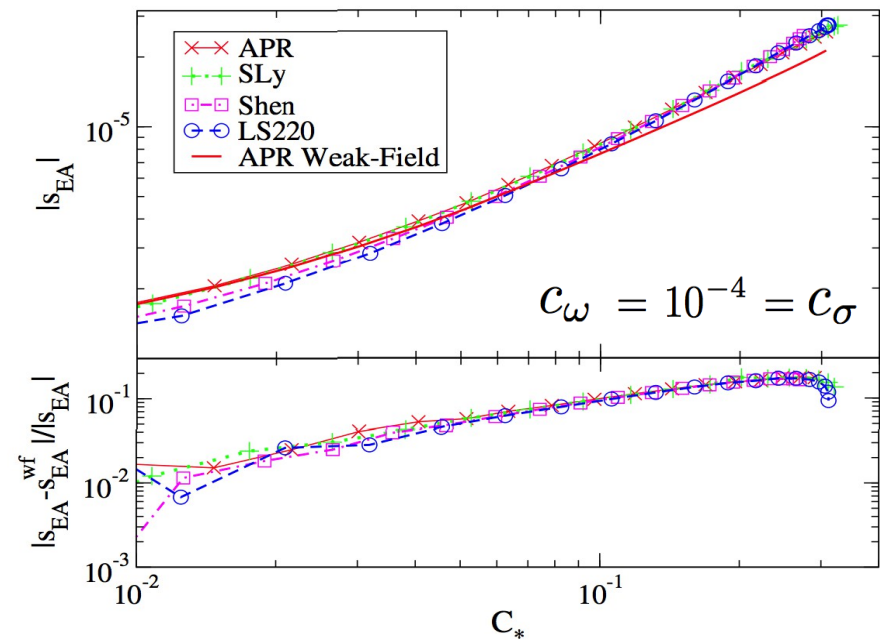
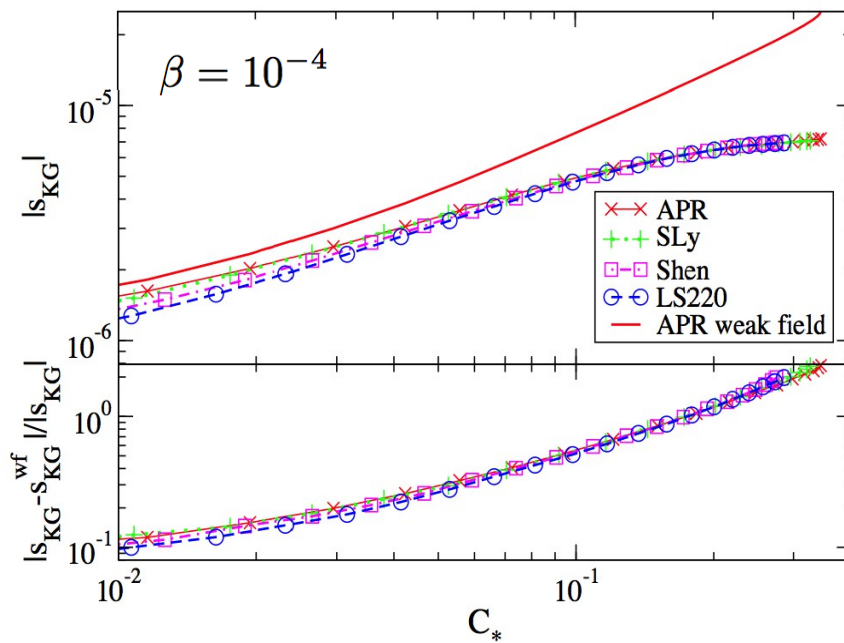
PSR B1913+16
(Weisberg & Taylor 2004)

The sensitivity of neutron stars

(Yagi, Blas, Yunes, EB 2013; Yagi, Blas, EB, Yunes 2013)

Calculation is non trivial!

Requires solving numerically for stars in motion relative to aether, to first order in velocity (thanks to Gauss theorem)



$$\alpha_1 = 10^{-4}$$

$$\alpha_2 = 4 \times 10^{-7}$$

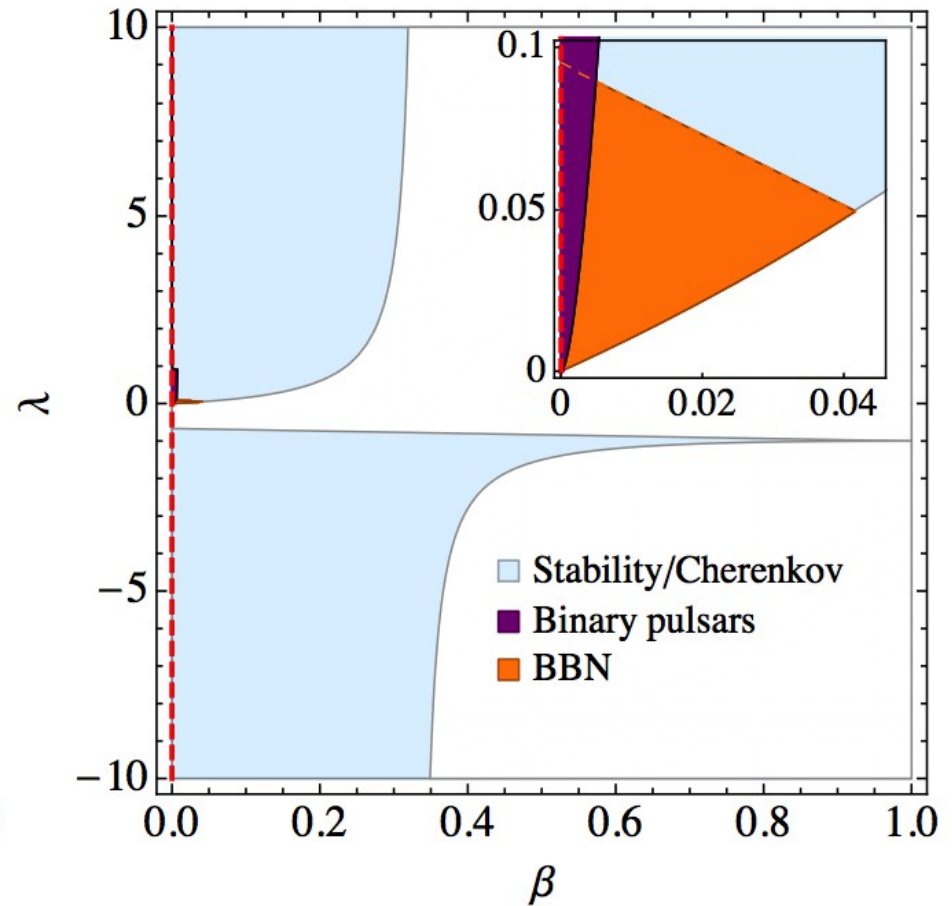
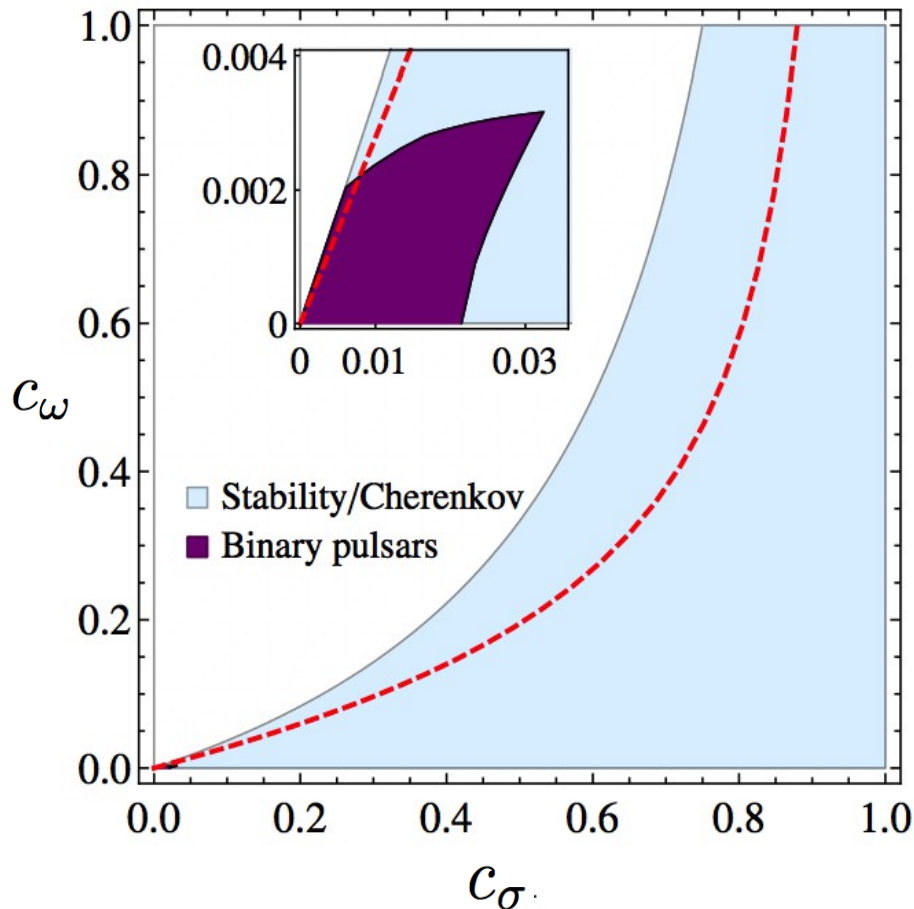
$$C_* = M_* / R_*$$

Red = weak field prediction
(Foster 2007)

$$s_{wf} = \left(\alpha_1 - \frac{2}{3} \alpha_2 \right) \frac{\Omega}{M_*} + \mathcal{O} \left(\frac{\Omega^2}{M_*^2} \right)$$





Constraints on Lorentz violation in gravity

(Yagi, Blas, Yunes, EB 2013; Yagi, Blas, EB, Yunes 2013)



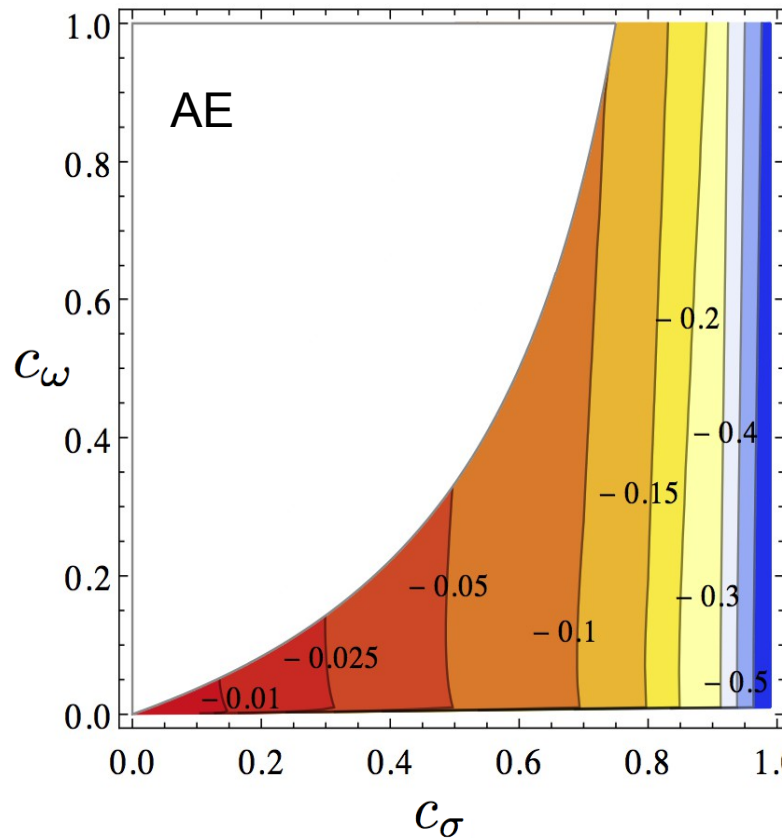
- Red = weak field prediction for $\alpha_1 = \alpha_2 = 0$ (by requiring exactly same fluxes as GR)
- Combined constraints from almost-circular WD-pulsar and pulsar-pulsar systems (PSR J1141-6545, PSR J0348+0432, PSR J0737-3039, PSR J1738+0333)
- Includes observational uncertainties (masses, spins, eccentricity, EOS)

Are BHs possible in LV gravity?

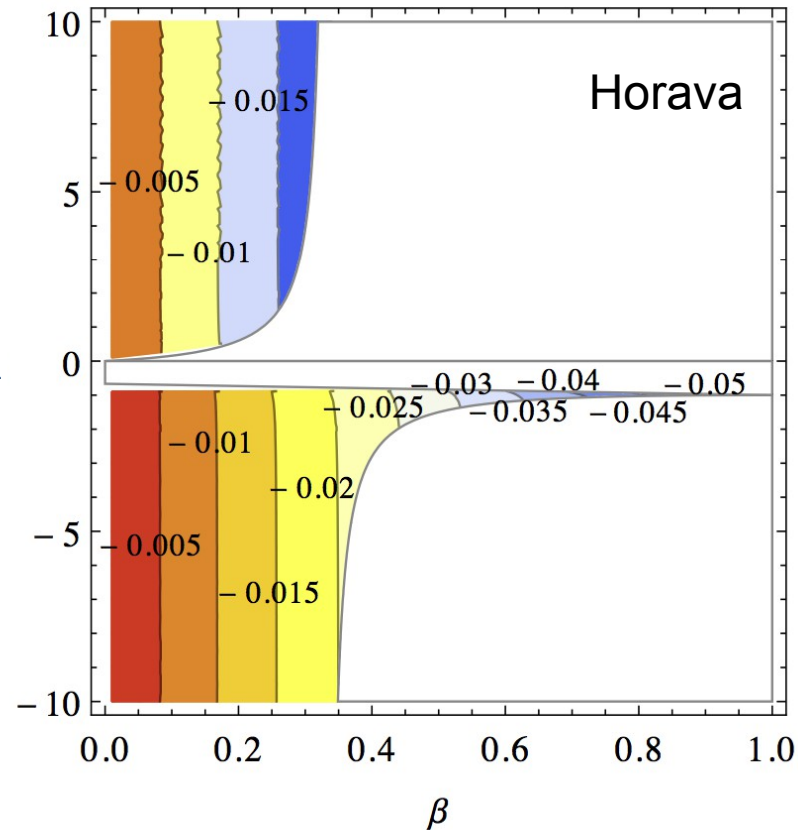
- BHs in GR defined in terms of spacetime causal structure
eg in static spherical spacetime, horizon lies where light cones “tilt inwards” (cf Eddington Finkelstein coordinates).
- In GR, matter (photons) and gravitons have same speed c
- In LV gravity, photon, spin-2, spin-1 and spin-0 gravitons have different propagation speeds 
different propagation cones  multiple horizons
- If higher-order terms included in the action, non-linear dispersion relations for gravitons $\omega^2 = k^2 + \alpha k^4 + \dots$ 
infinite speed in the UV limit  do BHs exist at all?

BH exterior structure

Outside metric horizons, BHs similar to Schwarzschild



EB, Jacobson & Sotiriou 2011

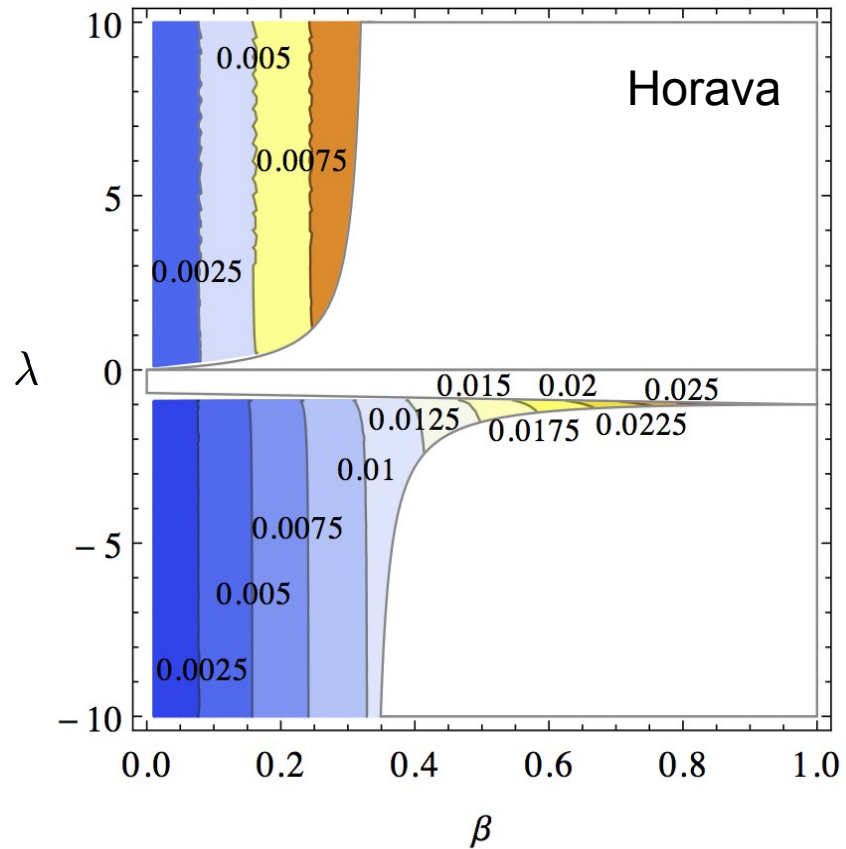
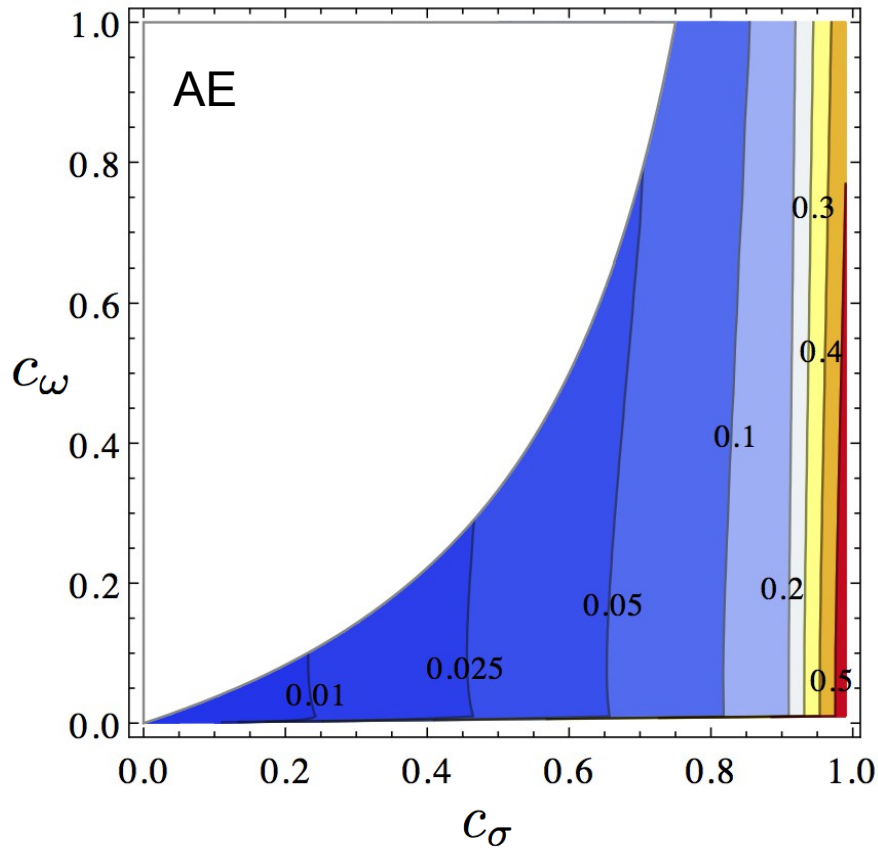


Measurable with eLISA?

$$\frac{\Delta(\Omega_{ISCO} r_g)}{\Omega_{ISCO} r_g}$$

$$f(r) = 1 - \frac{r_g}{r} + \dots$$

BH exterior structure



$$\frac{\Delta(b_{ph}/r_g)}{b_{ph}/r_g} = \frac{\Delta(\Omega_{ph} r_g)}{\Omega r_g}$$

Measurable with eLISA?

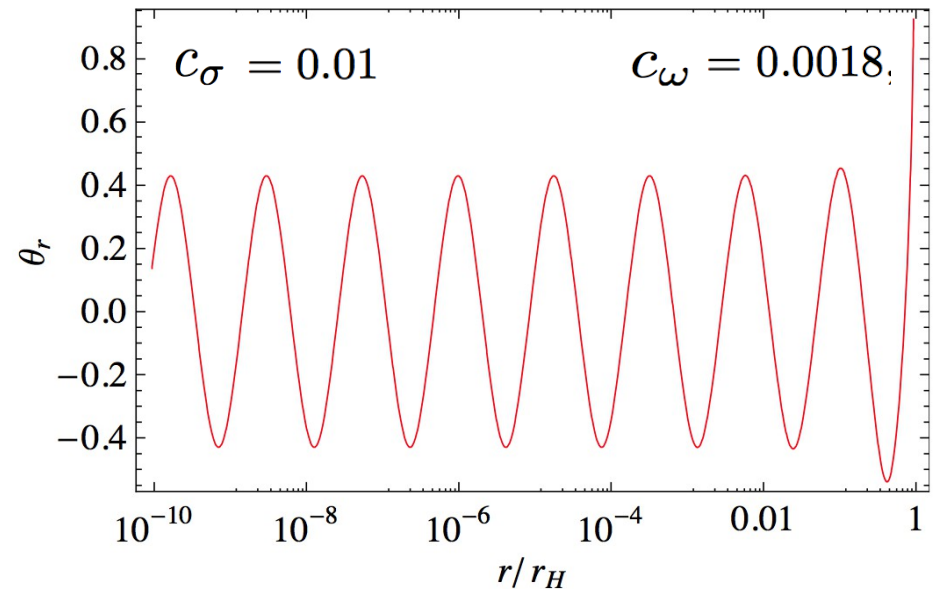
BH interior structure

Metric qualitatively similar to Schwarzschild (curvature singularity at $r=0$), aether oscillates

$$\theta_r = \operatorname{arccosh} \gamma_r$$

$$\gamma_r \equiv u_{\text{obs}}^\alpha u_\alpha = -\frac{u^r}{\sqrt{g^{rr}}}$$

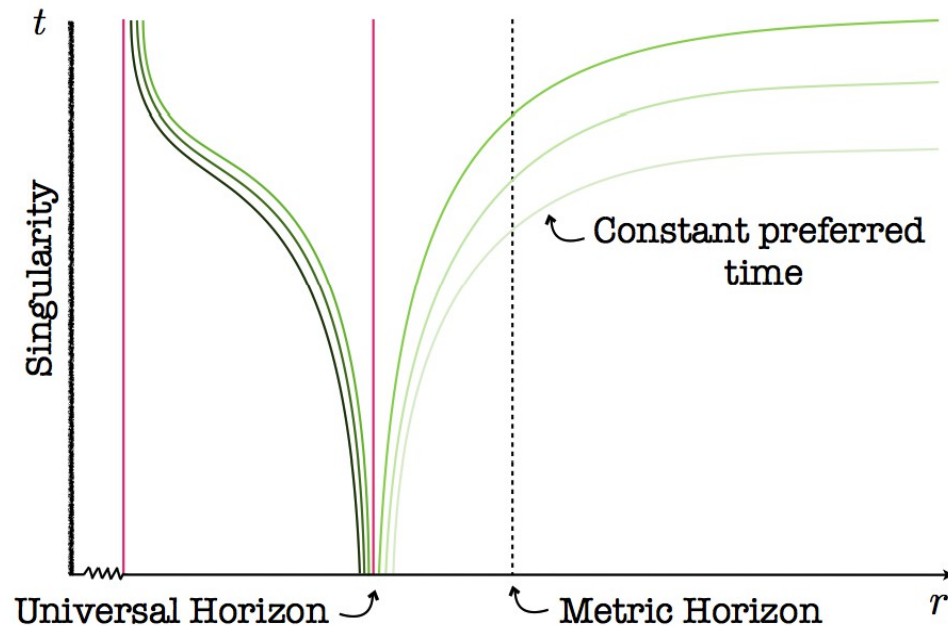
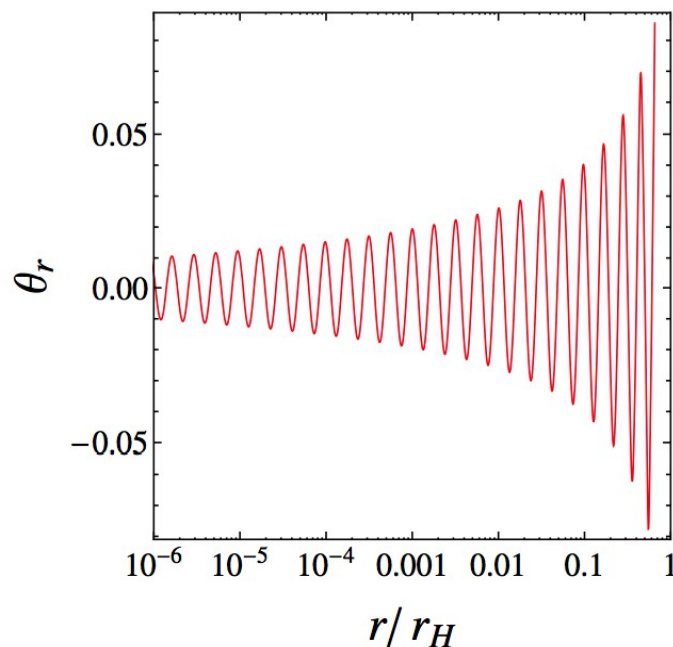
γ_r is aether's Lorentz factor relative to observer orthogonal to (spacelike) hypersurface $r = \text{const}$



EB, Jacobson & Sotiriou 2011

Implications for causal structure in BH interior

$\theta_r = 0 \implies$ aether orthogonal to (spacelike) hypersurface $r = r_u = \text{const}$



Any signal $r < r_u$ can only propagate inwards, whatever its speed, because future=inwards $\implies r = r_u$ is a **Universal Horizon**

(Blas and Sibiryakov 2011; EB, Jacobson & Sotiriou 2011)

A universal horizon for signals of infinite speed

(Blas and Sibiryakov 2011; EB, Jacobson & Sotiriou 2011)

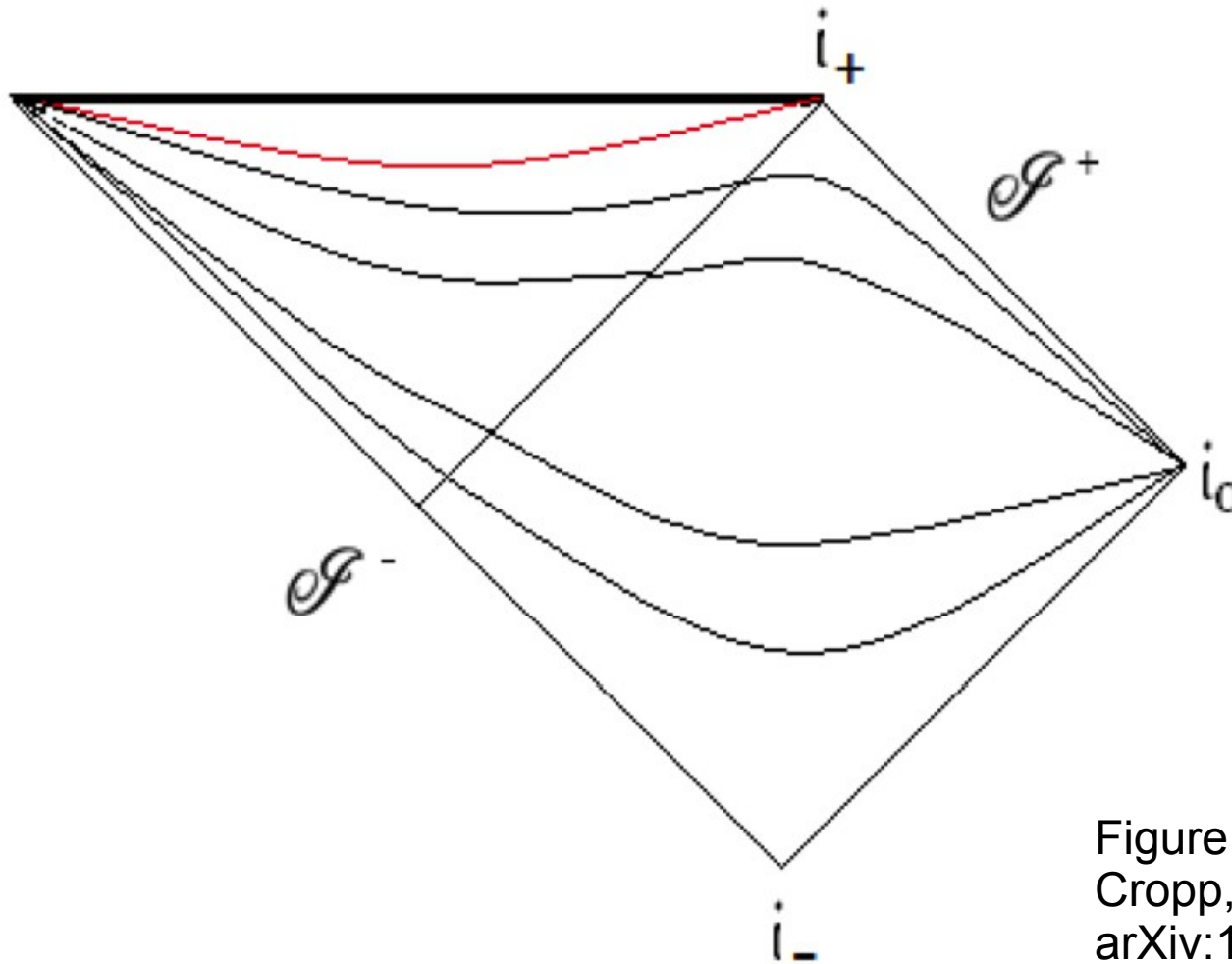


Figure adapted from
Cropp, Liberati and Mohd,
arXiv:1312.0405

A universal horizon for signals of infinite speed

(Blas and Sibiryakov 2011; EB, Jacobson & Sotiriou 2011)

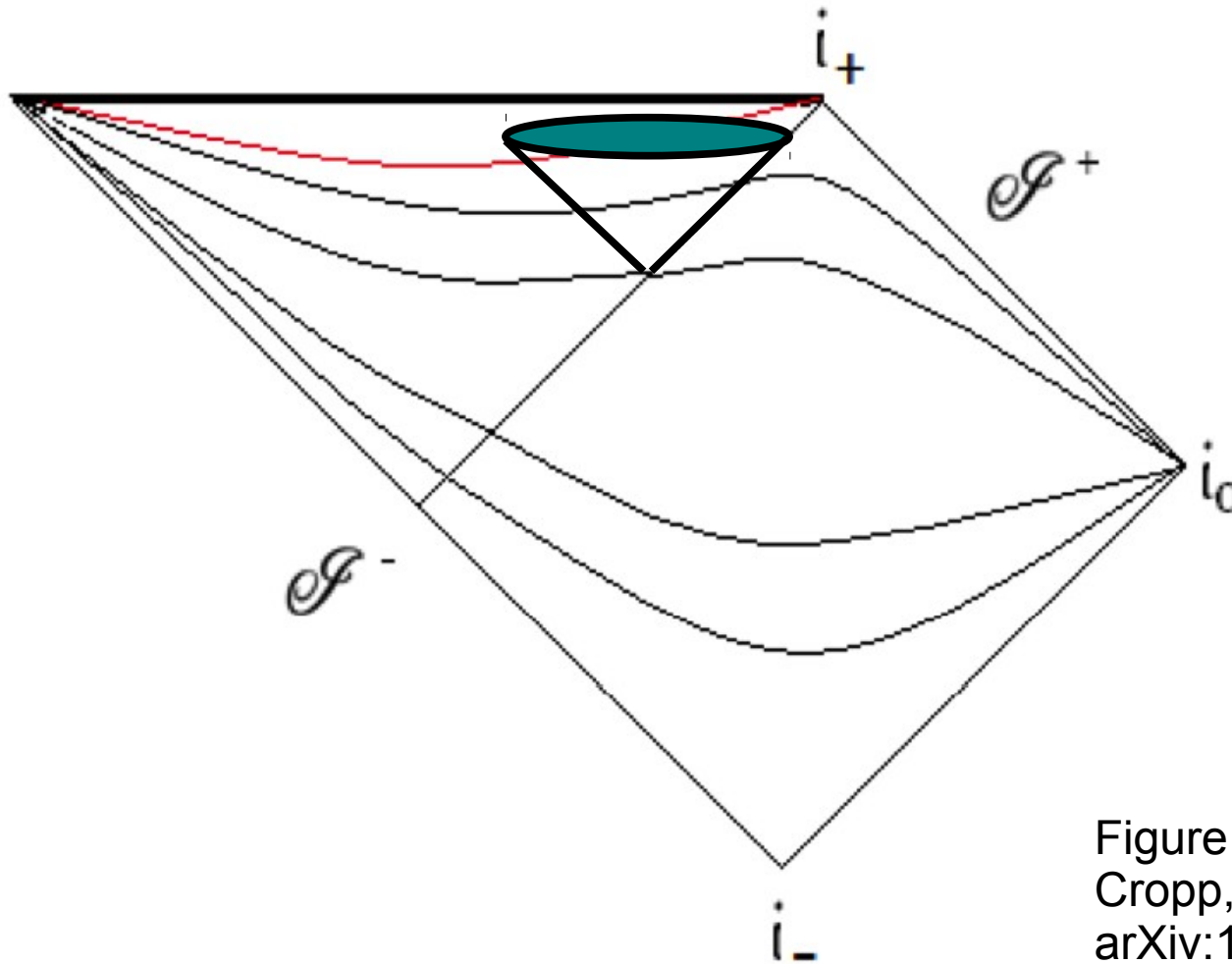


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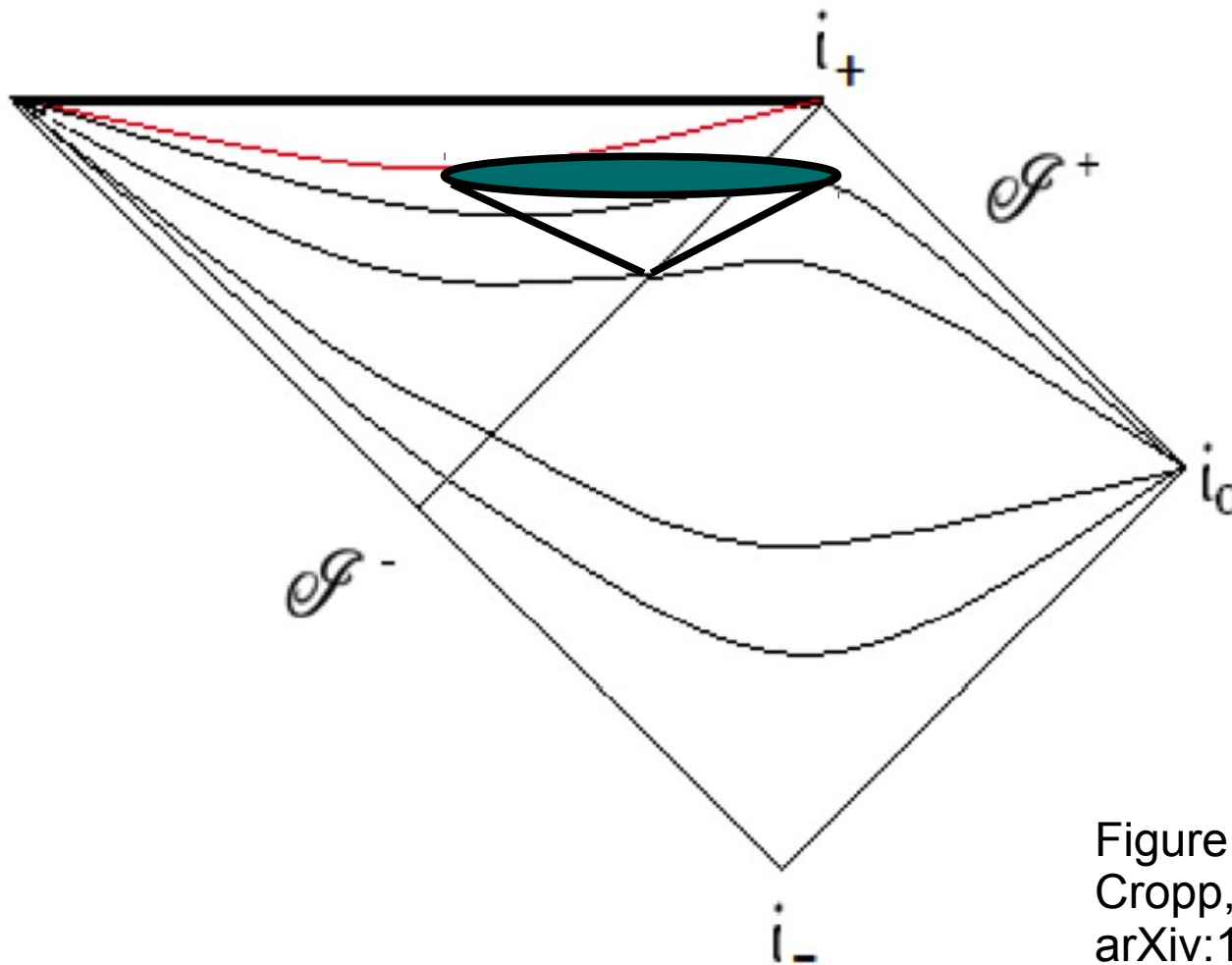


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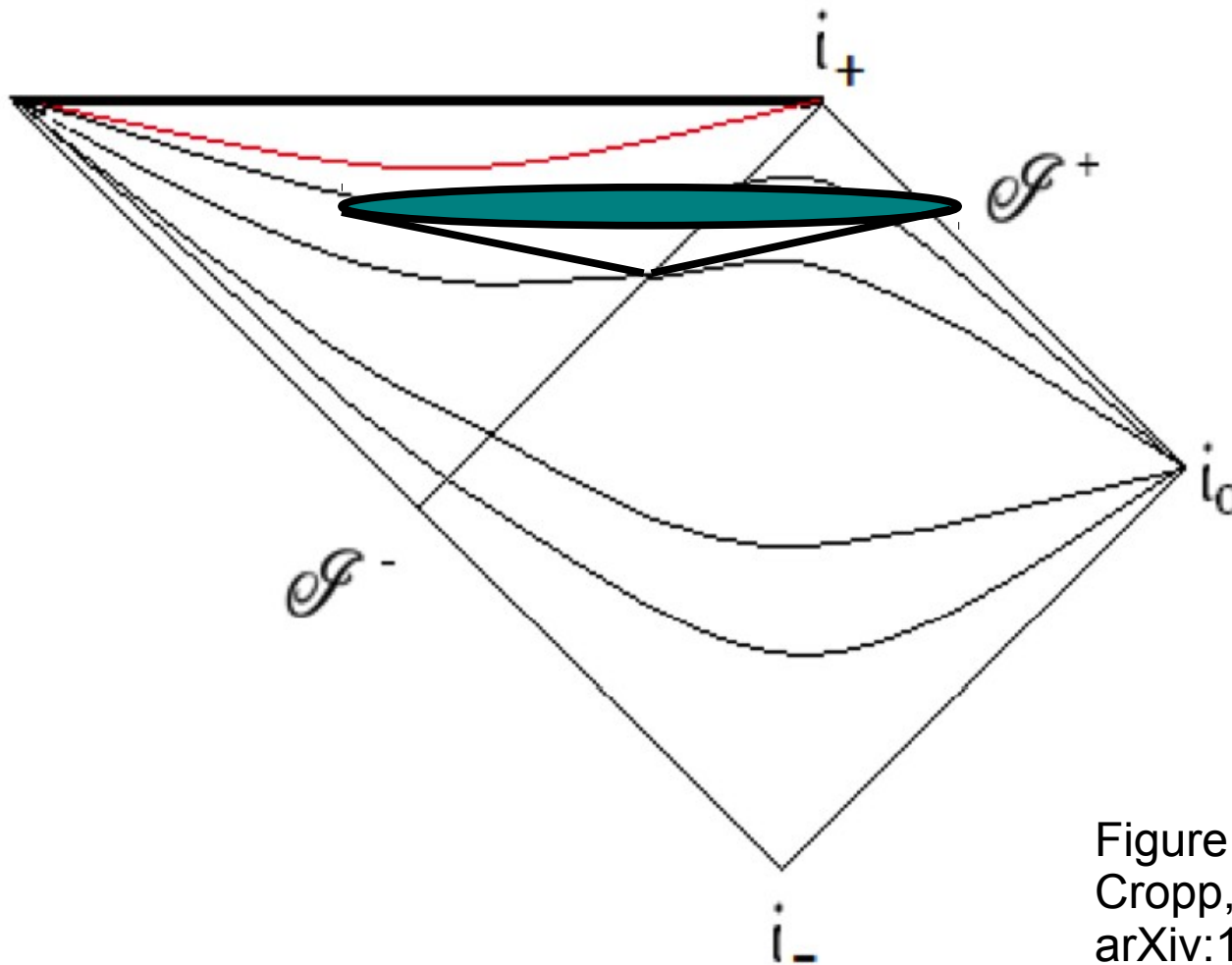


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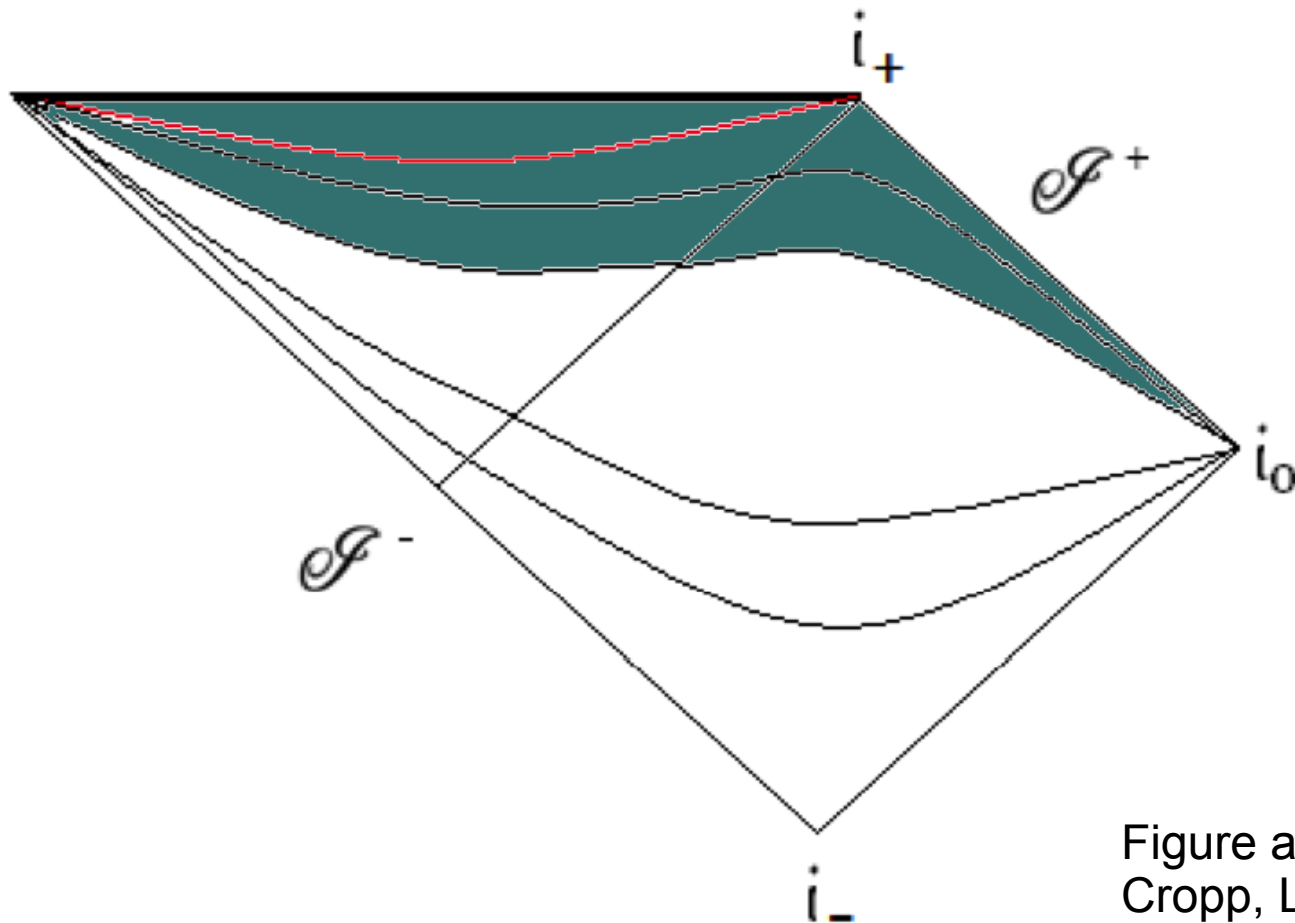


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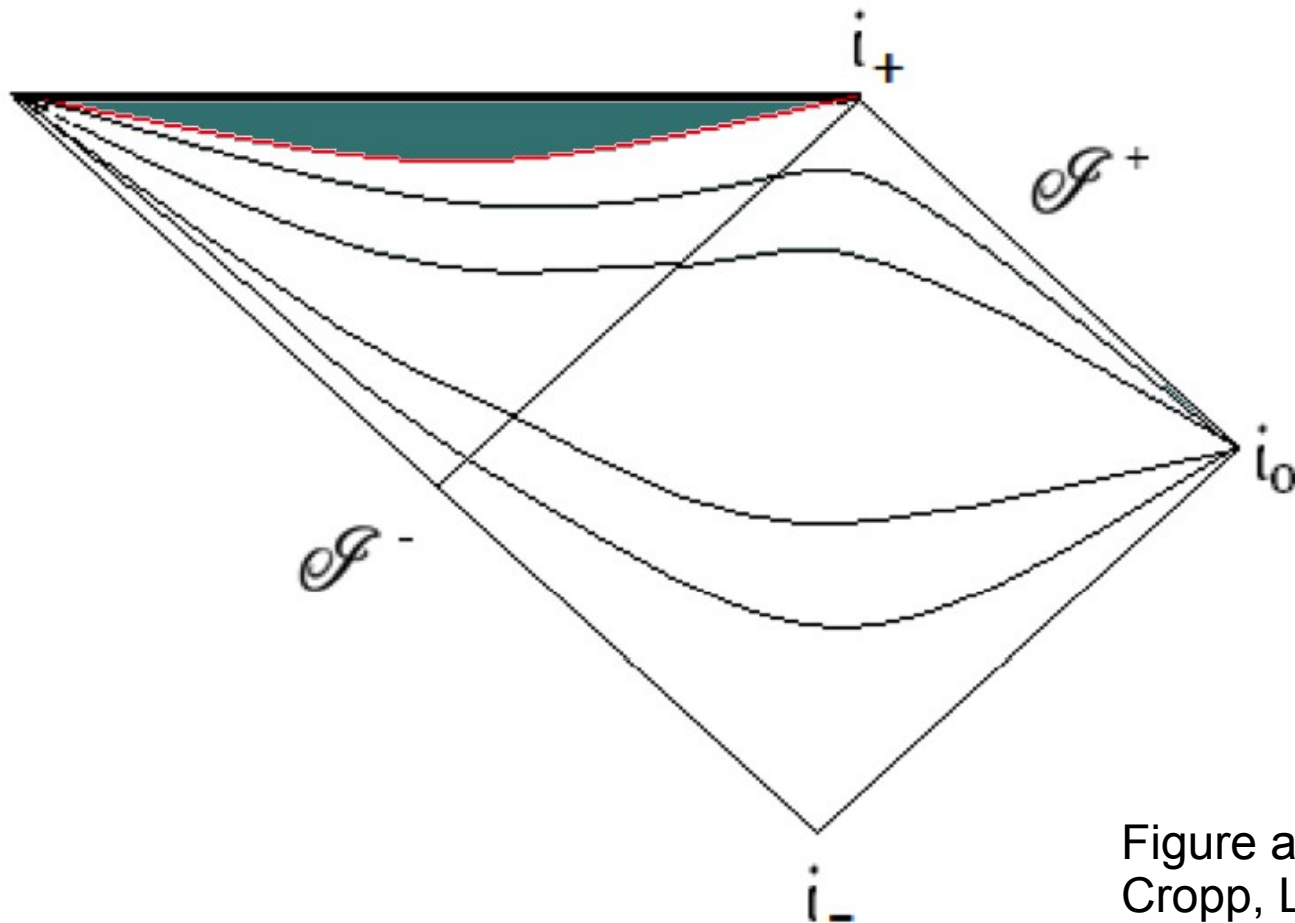


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Modified gravity as substitute for Dark Matter?

- Unorthodox way to explain Dark Matter phenomenology at galactic scales (galaxy rotation curves, Tully-Fisher & Faber-Jackson relations) is to modify Newtonian dynamics (MOND: Milgrom 1983) below acceleration $a_0 \sim \sqrt{\Lambda}$

$$\vec{\nabla} \cdot \left[\mu \left(\frac{|\vec{\nabla}\Phi|}{a_0} \right) \vec{\nabla}\Phi \right] = 4\pi G\rho \cdot \quad \begin{array}{l} a \gg a_0: \mu \sim 1 \\ a \ll a_0: \mu(x) \sim x \end{array}$$

- Advantages: naturally explains appearance of universal scale $a_0 \sim \sqrt{\Lambda}$ (no feedback)
- Open problems: predictions for larger scale cosmology need relativistic extension

A MOND Relativistic extension via Lorentz violations

(Blanchet & Marsat 2011, Bonetti & EB 2015)

- Chronometric gravity in adapted foliation

$$S = \frac{(1-\beta)c^4}{16\pi G} \int dt d^3x \sqrt{\gamma} N \left[\frac{{}^{(3)}R}{1-\beta} + K^{ij} K_{ij} - \frac{1+\lambda}{1-\beta} K^2 + \frac{\alpha}{1-\beta} a_i a^i \right] + S_{\text{mat}}(\varphi, g_{\mu\nu})$$

At Newtonian order: $\left(1 - \frac{\alpha}{2}\right) \nabla^2 \Phi = 4\pi G \rho$

- Modified chronometric gravity

$$S = \frac{(1-\beta)c^4}{16\pi G} \int dt d^3x \sqrt{\gamma} N \left[\frac{{}^{(3)}R}{1-\beta} + K^{ij} K_{ij} - \frac{1+\lambda}{1-\beta} K^2 + \frac{f(a)}{1-\beta} \right] + S_{\text{mat}}(\varphi, g_{\mu\nu})$$

At Newtonian order: $\vec{\nabla} \cdot \left[\left(1 - \frac{\chi}{2}\right) \vec{\nabla} \Phi \right] = 4\pi G \rho$

$$\chi(a) = f'(a)/(2a) \quad a \gg a_0: f(a) \simeq \alpha a^2 \quad a \ll a_0: f(a) \simeq 2a^2 - \frac{4c^2}{3a_0} a^3$$

1PN rotation curves for galaxy accreting matter

$$v_{\varphi}^{\text{0PN}} = \sqrt[4]{GMa_0} \quad r_0 = \sqrt{\frac{GM}{a_0}}$$

$$\begin{aligned}
 v_{\varphi, \text{1PN}}^2 &= \sqrt{G_N M(t) a_0} \\
 &+ \frac{1}{c^2} \left\{ -\frac{a_0 (2 + \beta + 3\lambda)^2}{144(\beta + \lambda)} \frac{\dot{M}^2}{M(t)^2} \left[4(r_0^3 - r^3) + 3r^3 \ln \left(\frac{r}{r_0} \right) \right] \right. \\
 &- \frac{\dot{M}^2}{36r(\beta + \lambda)M(t)} \sqrt{\frac{a_0 G_N}{M(t)}} \left[(2 + \beta + 3\lambda) \times \right. \\
 &\times \left. \left(4r^3 + 14r_0^3 - 3r^3 \ln \left(\frac{r}{r_0} \right) \right) + 18(2\beta - 2)r_0^3 \right] \left. \right\} \\
 &+ \mathcal{O}(\beta + \lambda)^0 + \mathcal{O}(\alpha_1, \alpha_2) + \mathcal{O}_{\text{finite}}(\dot{M}, \Lambda_{\text{obs}}) + \mathcal{O}(4).
 \end{aligned}$$

Strong coupling problem at 1PN if $\beta + \lambda$ is small
(Bonetti & EB, 2015)

How to avoid strong coupling

Choose realistic galaxy masses and accretion rate and impose
1PN terms do not dominate over Newtonian terms

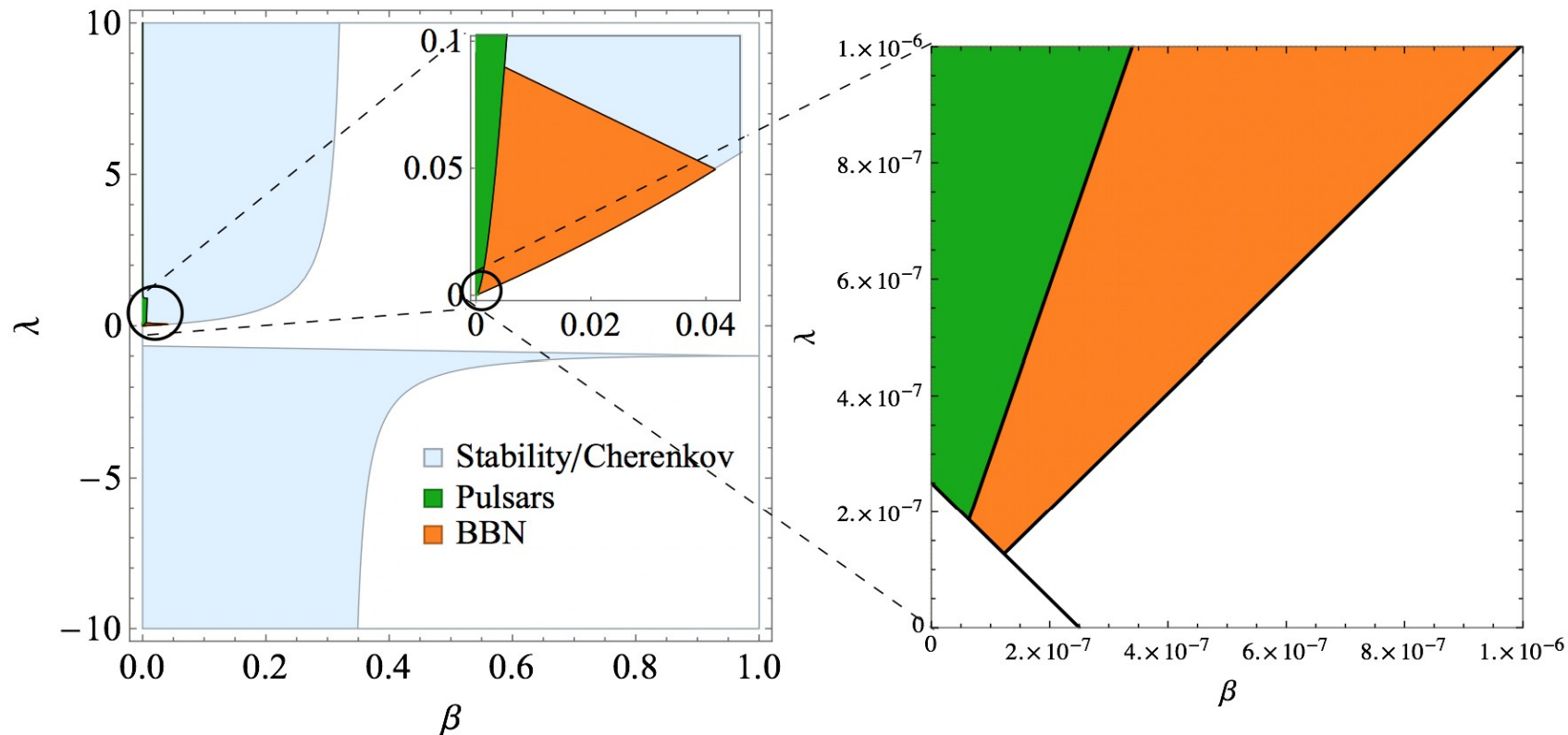


Figure from Bonetti & EB 2015

Conclusions

- Lorentz violations in gravity generically introduces violations of strong equivalence principle and thus dipole emission
- Placing precise constraints with binary pulsars requires exact values of sensitivities (non-trivial calculation)
- Resulting constraints are strong-field and \sim order of magnitude stronger than previous ones
- BH solutions very similar to GR in the “exterior”, but causal structure is very different in the “interior” (universal horizon acts as boundary for perturbations with infinite speed)
- Dark-Matter phenomenology without Dark Matter on galactic scales