Models with strong GW signature

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eLISA cosmology working group workshop

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Outline

Phase transition

Gravitational waves

Models

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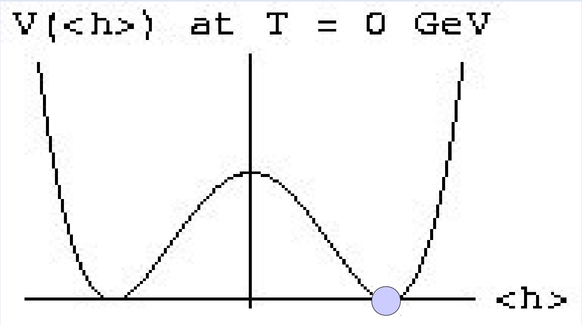
Electroweak symmetry breaking

The **Mexican hat** potential is designed to lead to a finite Higgs vacuum expectation value (VEV) and break the electroweak symmetry

$$V(h) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2$$

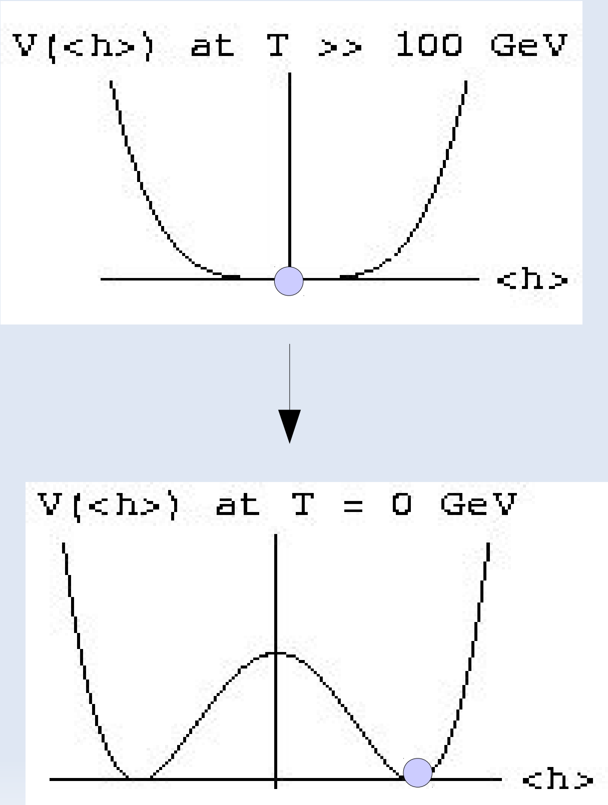


Electroweak symmetry breaking

[Weinberg '74]

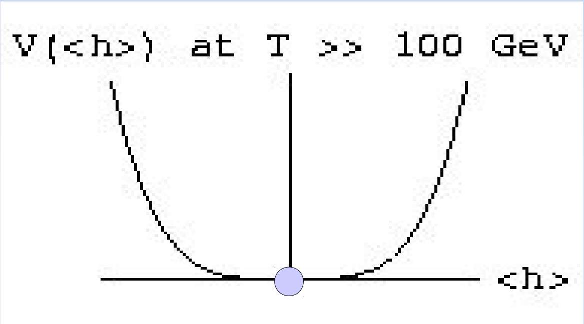
At large temperatures the symmetry is restored

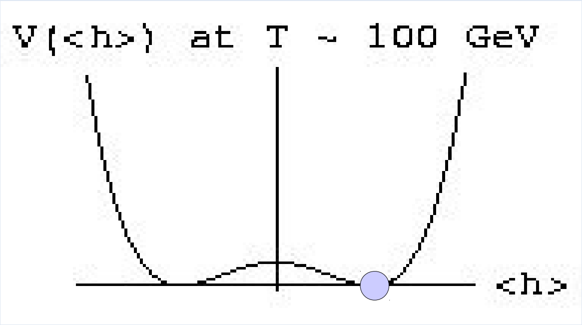
$$V(h,T) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2 + \text{const} \times h^2 T^2 + \text{details}$$

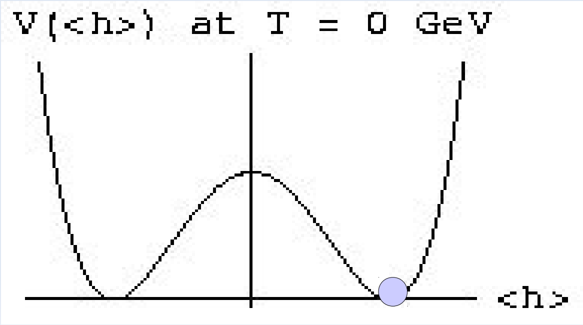


Electroweak symmetry breaking

Depending on the details, the phase transition can be very weak or even a crossover



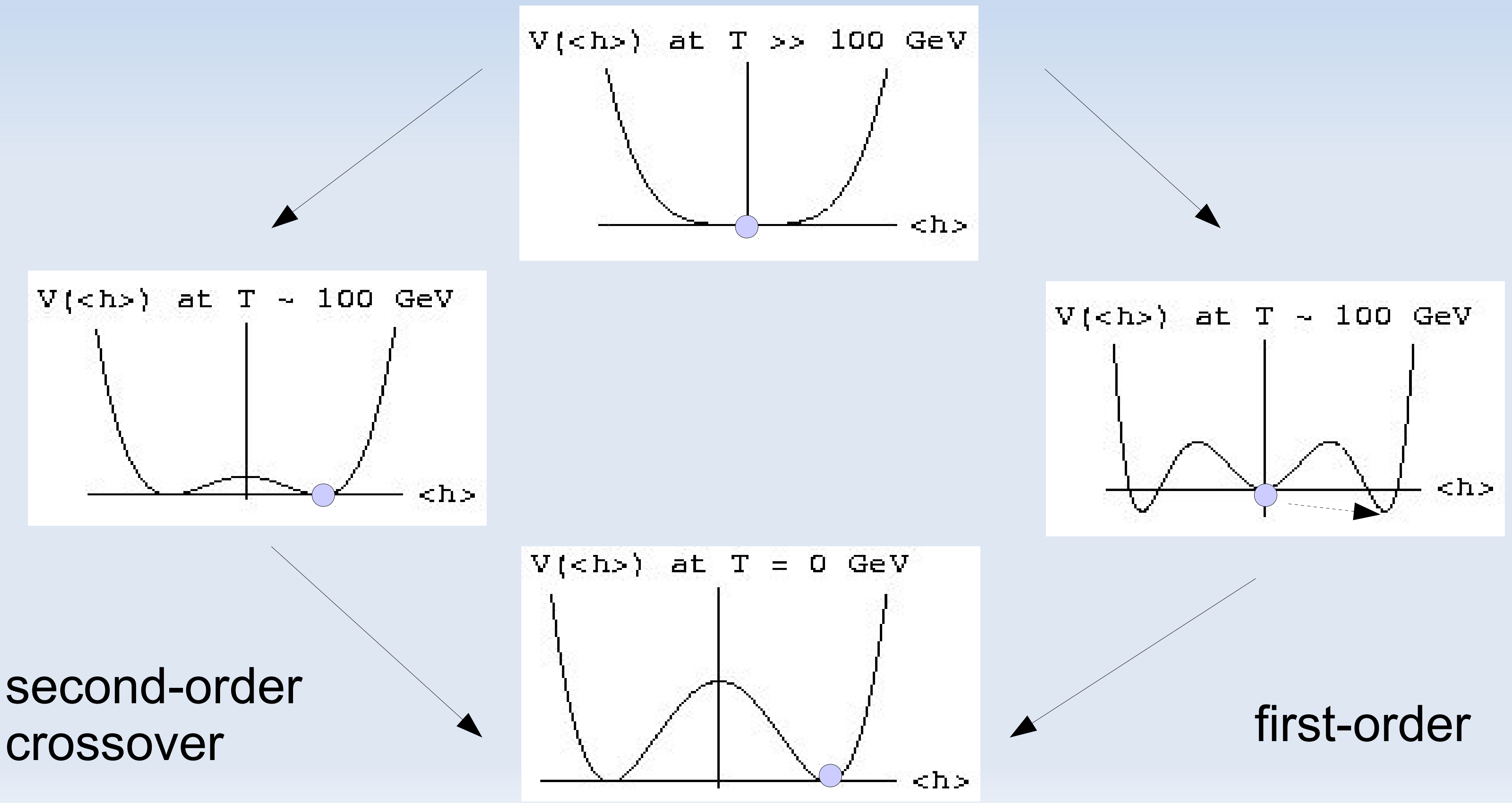




second-order
crossover

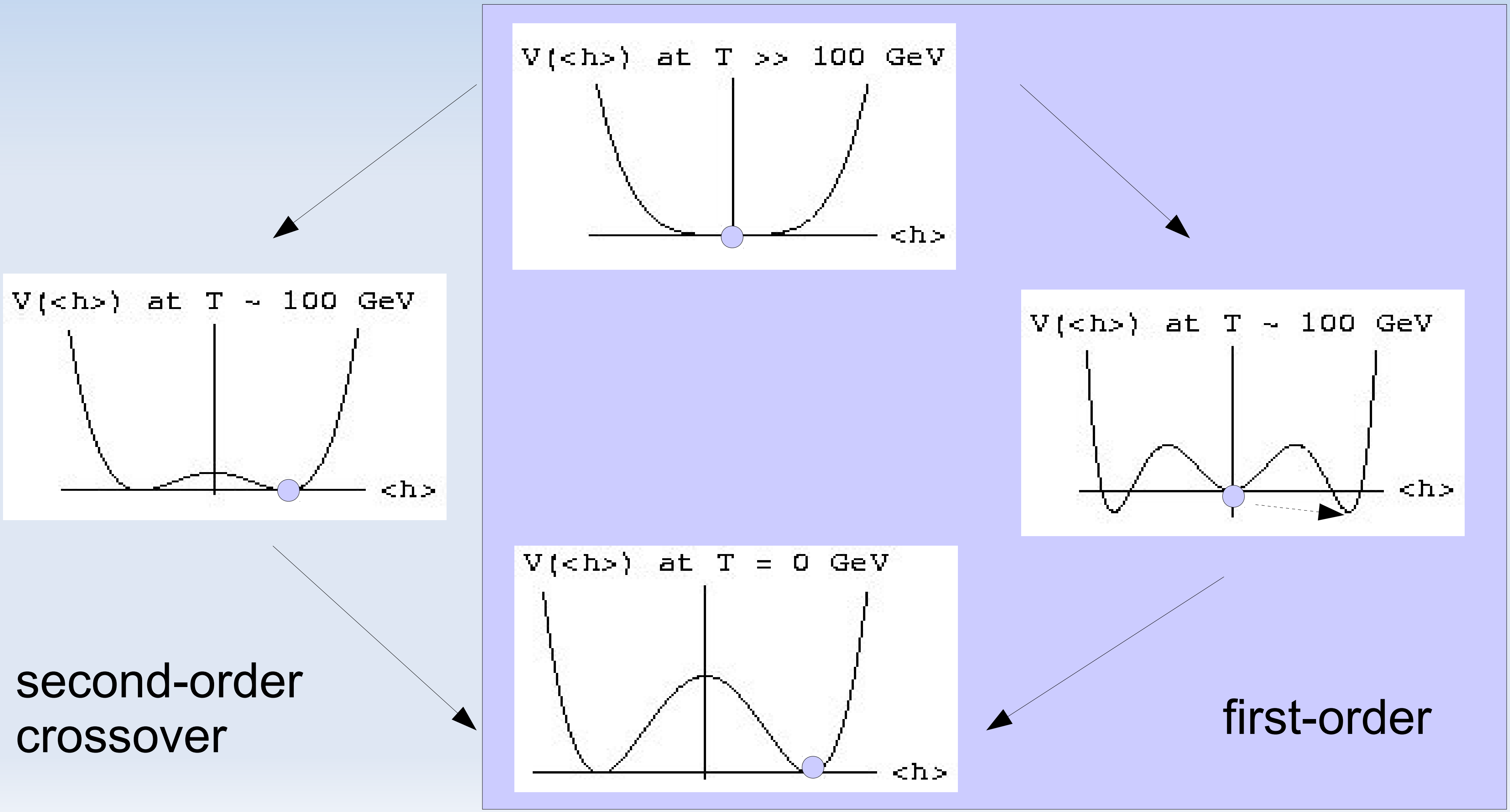
Electroweak symmetry breaking

It can also be a strong phase transition if a **potential barrier** seperates the new phase from the old phase



Electroweak symmetry breaking

It can also be a strong phase transition if a **potential barrier** seperates the new phase from the old phase



First-order phase transitions

• first-order phase transitions proceed by bubble nucleations

• in case of the electroweak phase transition, the "Higgs bubble wall" separates the symmetric from the broken phase

• this is a violent process ($v_{bubble} \simeq O(c)$) that drives the plasma out-of-equilibrium

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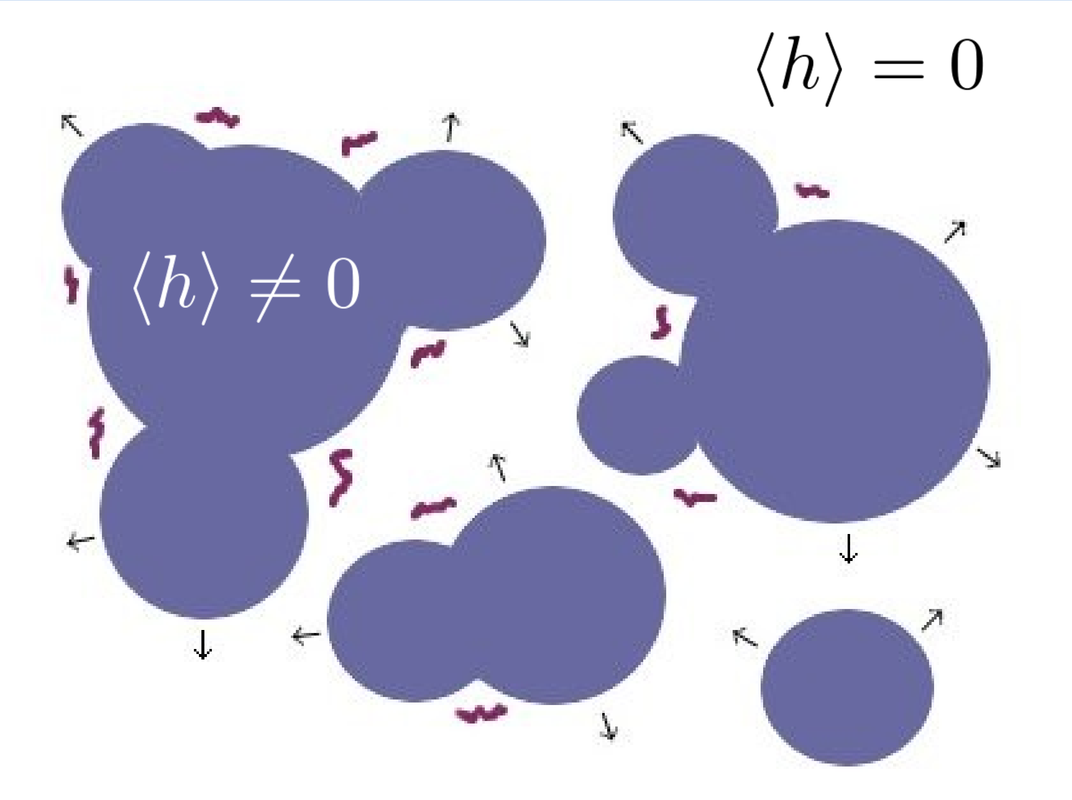
Gravitational waves



During the first-order phase transitions, the nucleated bubbles expand. Finally, the colliding bubbles generate **stochastic gravitational waves**.

Weinberg's master formula

Using linearized GR and the wave zone approximation, the total energy radiated into a direction $\hat{\mathbf{k}}$ is given by

$$\frac{dE_{GW}}{d\omega d\Omega} = 2G\omega^2 \Lambda_{ij,lm}(\hat{\mathbf{k}}) T^*_{ij}(\hat{\mathbf{k}},\omega) T_{lm}(\hat{\mathbf{k}},\omega),$$

where $T_{ij}(\hat{\mathbf{k}},\omega)$ denotes the stress-energy tensor in Fourier space and Λ is the projection tensor for the transverse-traceless part.

Spherical symmetric configurations do hence **not** contribute and the bubbles produce GWs only when they collide.

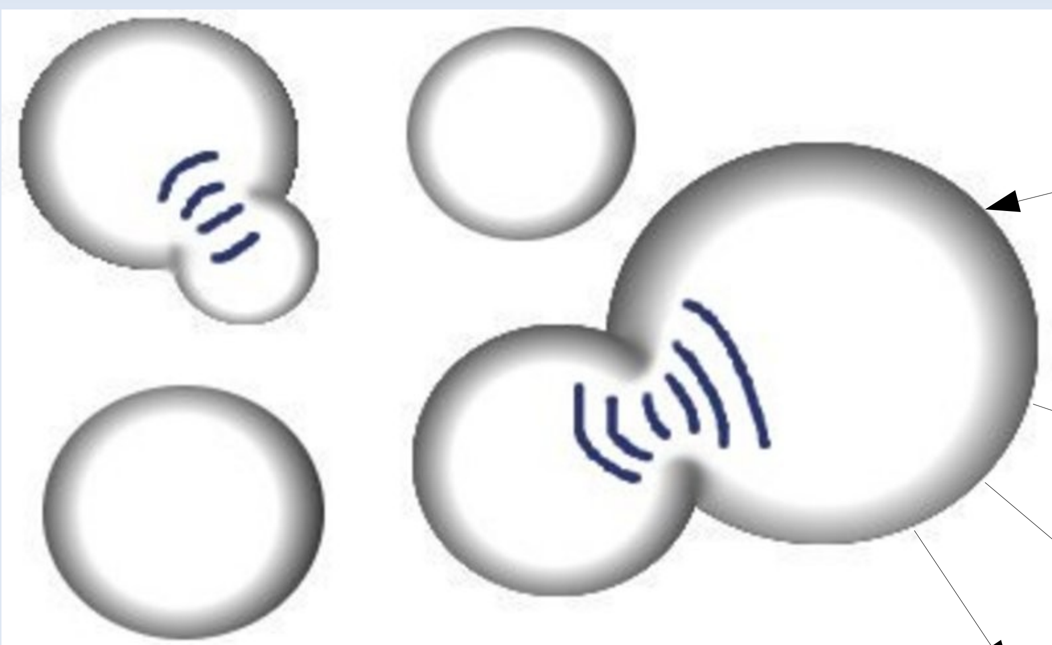
PT characteristics

Tunnel probability $P \propto \exp(-S_3/T) \propto \exp(\beta t)$

And typically $\tau \sim 1/\beta, \quad R \sim v_b/\beta$

β

α



$T_{\mu\nu} \propto \ \rho_{vac}$

$\alpha = \rho_{vac}/\rho_{rad}$

v_b

[wall velocity: see talk by A. Megevand]

Back on the envelope

1) The time scale of the PT is set by the tunnel probability

$$P \propto \exp(-S_3/T) \quad \text{and typically} \quad \beta/H = O(100).$$
$$\propto \exp \beta\, t$$

$$\beta$$

2) The vacuum energy ρ_{vac} is with efficiency κ transformed into the bulk motion of the fluid

$$\alpha$$

$$T_{\mu\nu} \propto \rho_{vac}, \quad G \propto H^2/(\rho_{vac} + \rho_{rad}).$$

$$v_b$$

For dimensional reasons one obtains for the energy fraction in GWs per frequency octave

$$\frac{d\Omega_{GW}}{d\log\omega} = \omega \frac{dE_{GW}}{d\omega} \frac{1}{E_{tot}} = \left(\frac{H}{\beta}\right)^2 \left(\frac{\alpha}{\alpha+1}\right)^2 \Delta(\omega/\beta, v_b, \alpha),$$

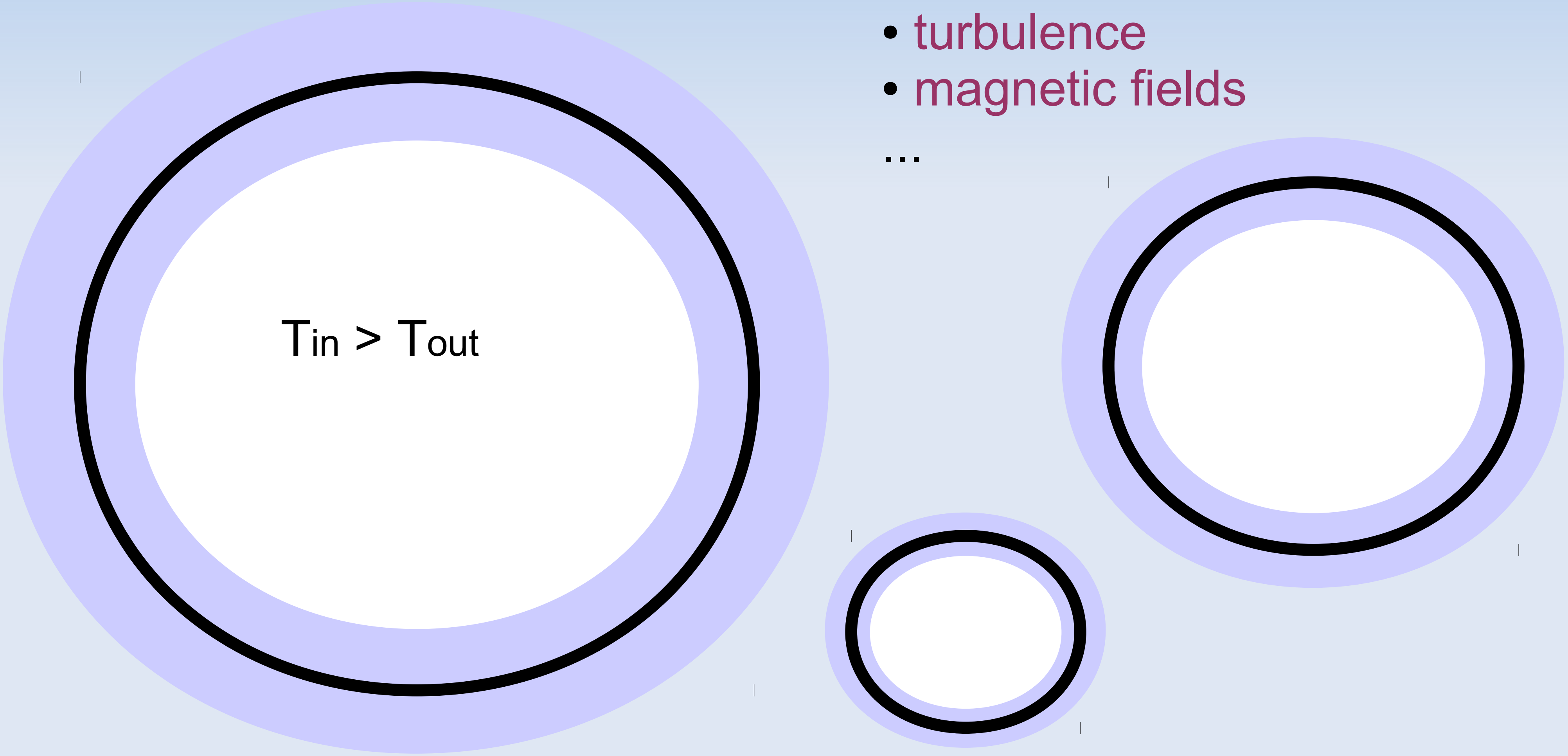
with Δ some dimensionless function and $\alpha = \rho_{vac}/\rho_{rad}$

Energy budget and production mechanisms

vacuum energy of the Higgs →

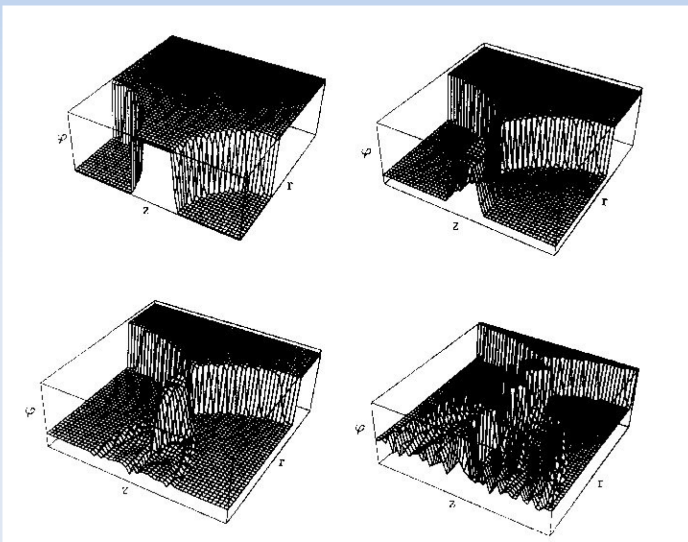
- kinetic energy of the Higgs
- bulk motion
- heat
- turbulence
- magnetic fields

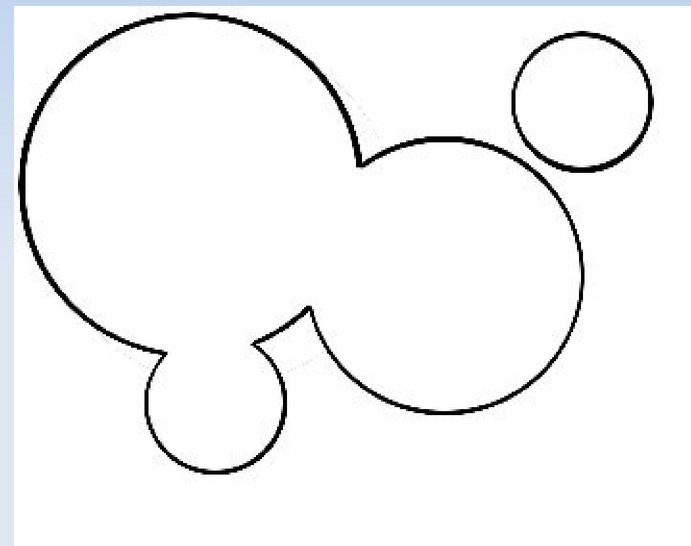
...



(A) Envelope approximation

[Kosowski, Turner, Watkins '91]



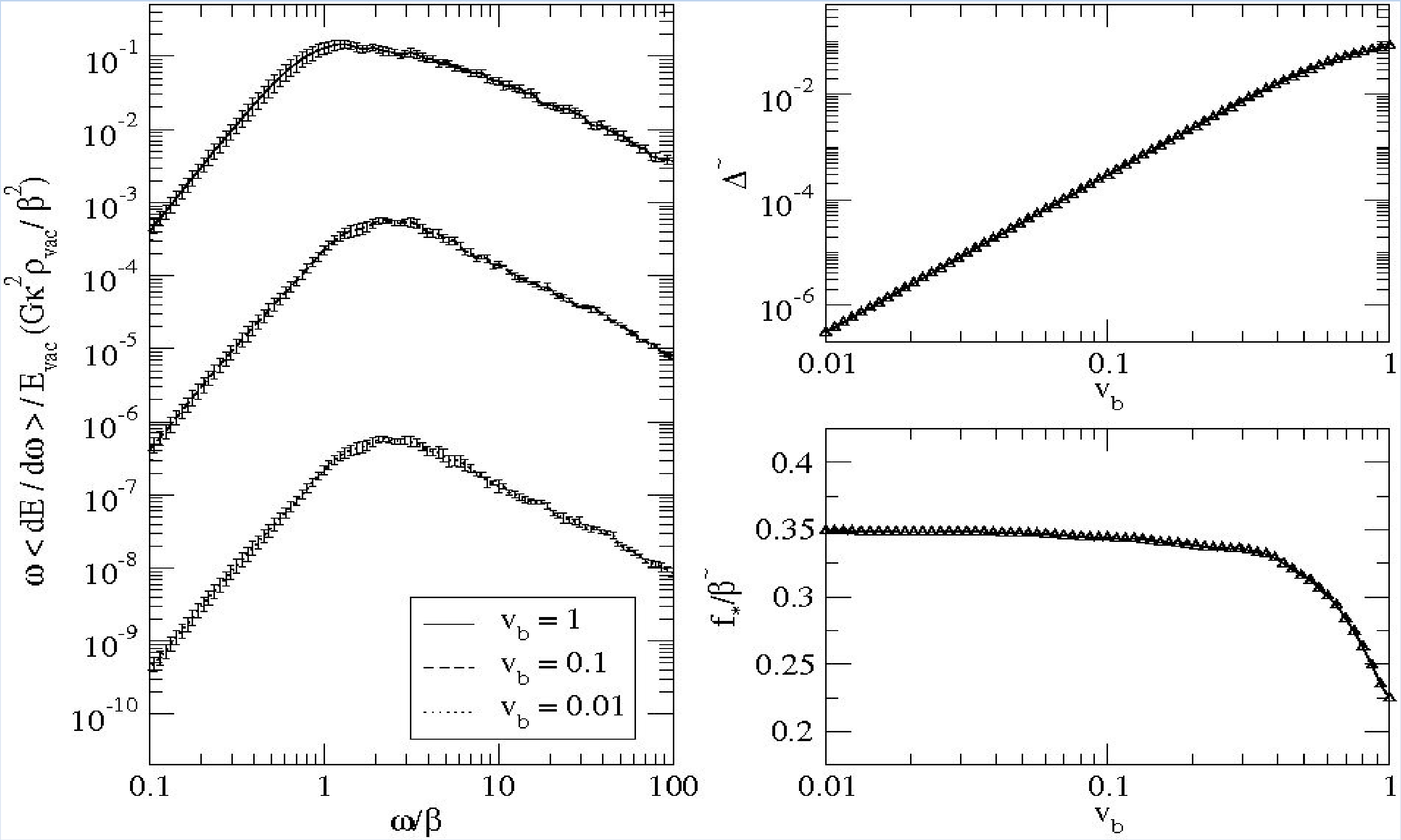


Simulations show that for the production of GWs the energy of the Higgs field can be approximated by its envelope.

Still, simulations with many bubbles and high accuracy have been too demanding in the 90s.

Simulations of bubble collisions

[Huber&TK '08]

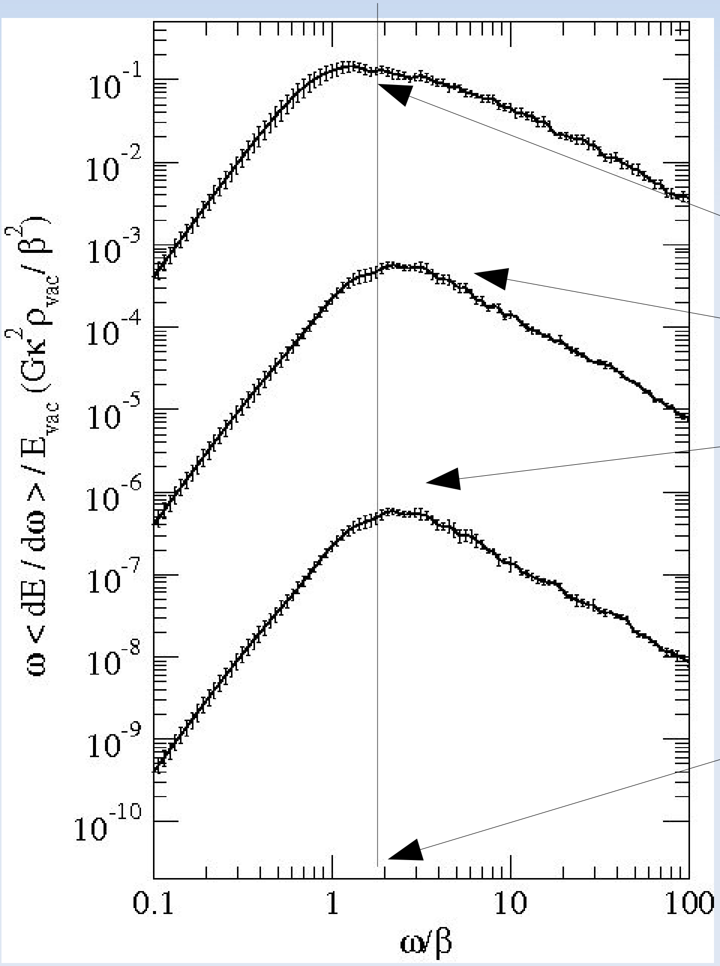


High precision results with ~130 bubbles show that the spectrum scales as ω^{-1} for large frequencies unlike earlier results.

(A) Envelope approximation

[Huber&TK '08]

For large wall velocities, the system can be descibed using the envelope approximation.



The amplitude depends mostly on the **wall velocity** and the ratio

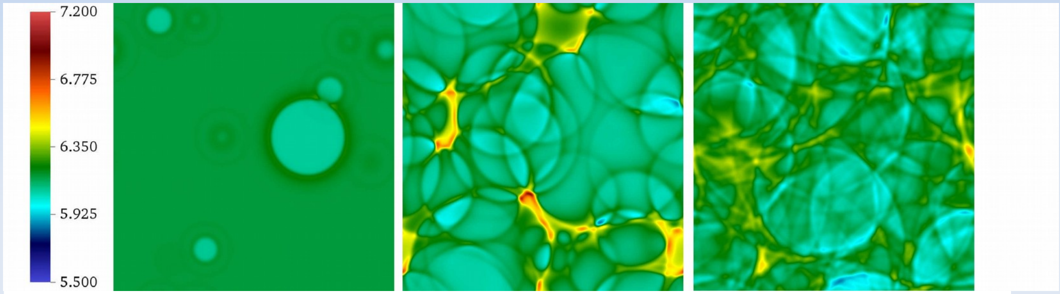
$$\alpha = \frac{\text{latent heat}}{\text{total energy}}$$

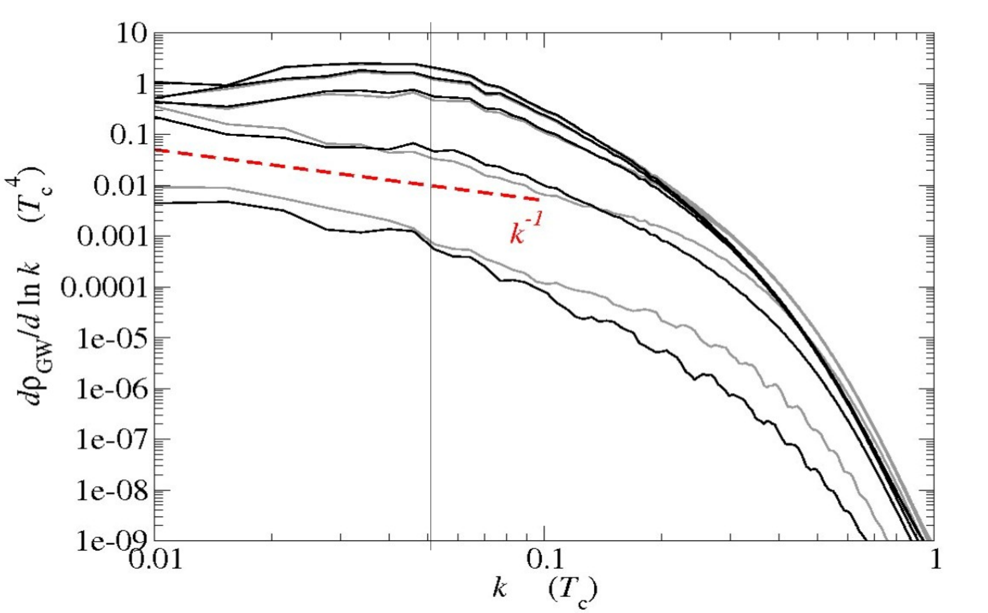
The peak frequency depends on the bubble size that is a few times smaller than the **Hubble horizon**

(B) Hydrodynamics

[Hindmarsh et.al. '13]

For small wall velocities, the system can be descibed using hydrodynamics. $v_{bubble} \ll c$





The overall features are quite similar to the envelope approximation.

For large wall velocities turbulence is expected [turbulence: see talk by R. Durrer]

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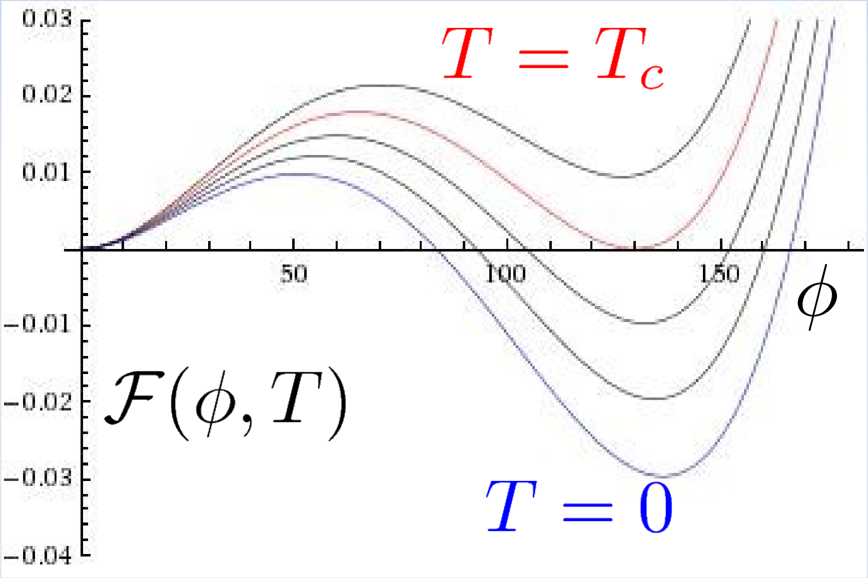
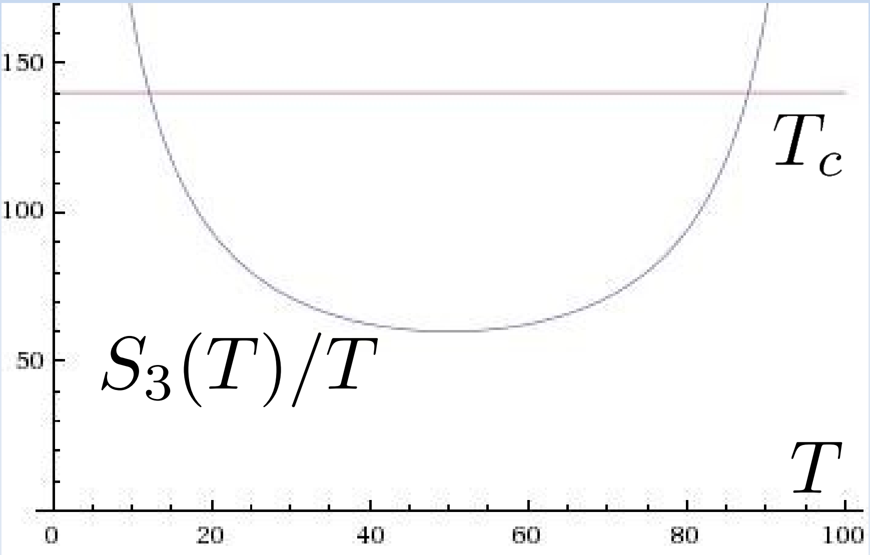
Tunnel probability

First-order phase transitions proceed by tunneling
$\rightarrow$ bubble nucleation





The tunneling probability in a Hubble volume is given by

$$P \sim \frac{T^4}{H^4} e^{-S_3/T}$$

And the PT happens at

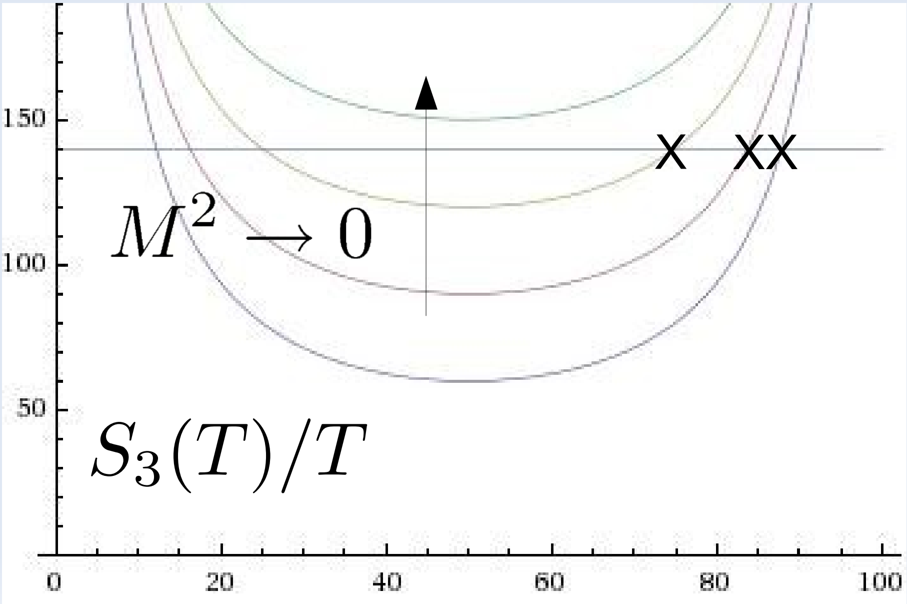
$$S_3/T \sim \text{Log}\, T^4/H^4 \sim \text{Log}\, M_{Pl}^4/T^4 \sim 140$$

Effective model

In the SM, the electroweak breaking happens by a cross-over and a first-order transition signals beyond SM physics.

A simple toy model for a strong electroweak phase transition is given by the Standard Model equipped with a higher-dimensional operator in the Higgs potential

$$V(\phi) = m^2\phi^2 - \lambda\phi^4 + \frac{1}{M^2}\phi^6$$

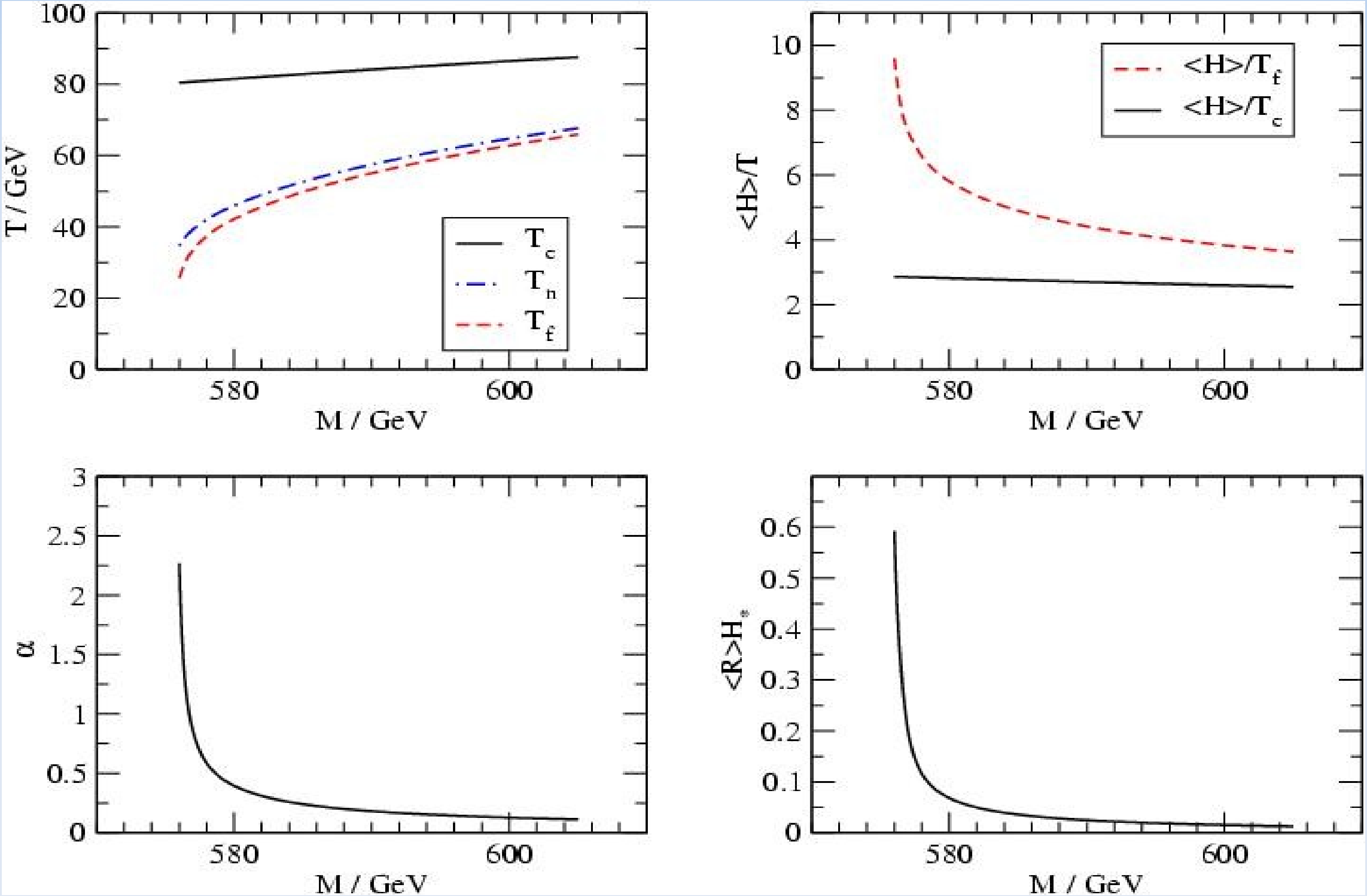


For smaller M

T_{PT}	decreases
$\alpha \propto \frac{\rho_{vac}}{T^4}$	increases
$\beta/H = T\frac{d}{dT}\frac{S_3}{T}$	decreases

Parameters

[Huber&TK '08]

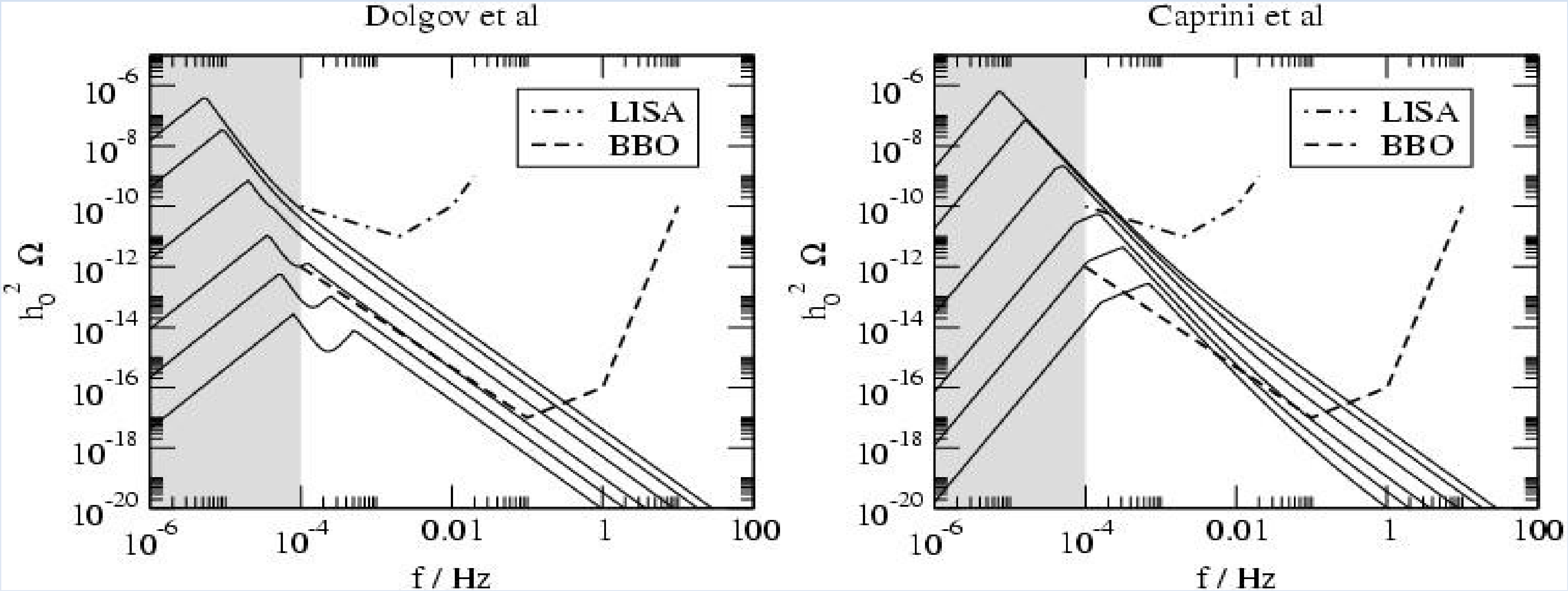


Reasonably strong PTs, $\alpha \gtrsim 0.1$, need tuning on the percent level.

Spectrum

[Huber&TK '08]

On first sight, the redshifted **peak frequency** of the GWs matches the best sensitivity of LISA nicely.



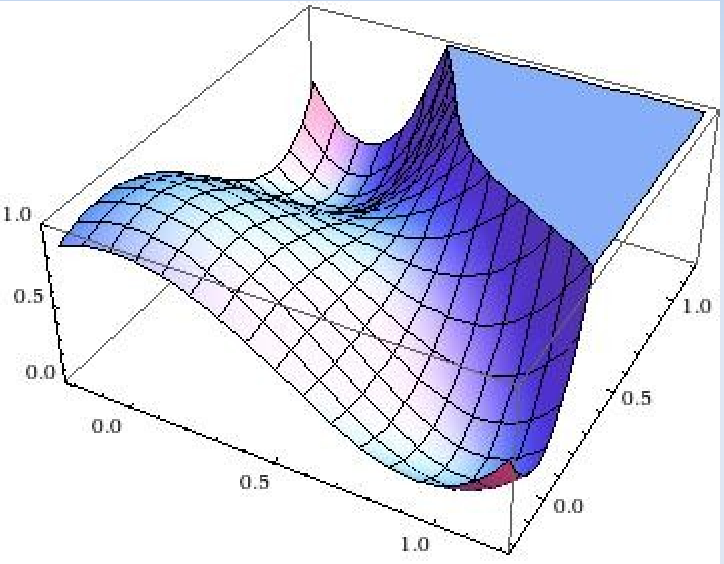
For very strong PTs, the peak of the spectrum is shifted out of the sensitivity range of LISA and BBO. [other sources? Larger T_{PT}?]

Singlet extension with 2-stage PT

[Espinosa&TK&Riva '13]

The Standard Model only features a electroweak crossover.

A potential barrier and hence first-order phase transitions are quite common in extended scalar sectors:



$$V(h,s) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2 + m_s^2 s^2 + \lambda_s s^4 + \lambda_m s^2 h^2$$

The singlet field has an additional $\mathbb{Z}_2$ symmetry and is a viable DM candidate.

The phase transition proceeds via

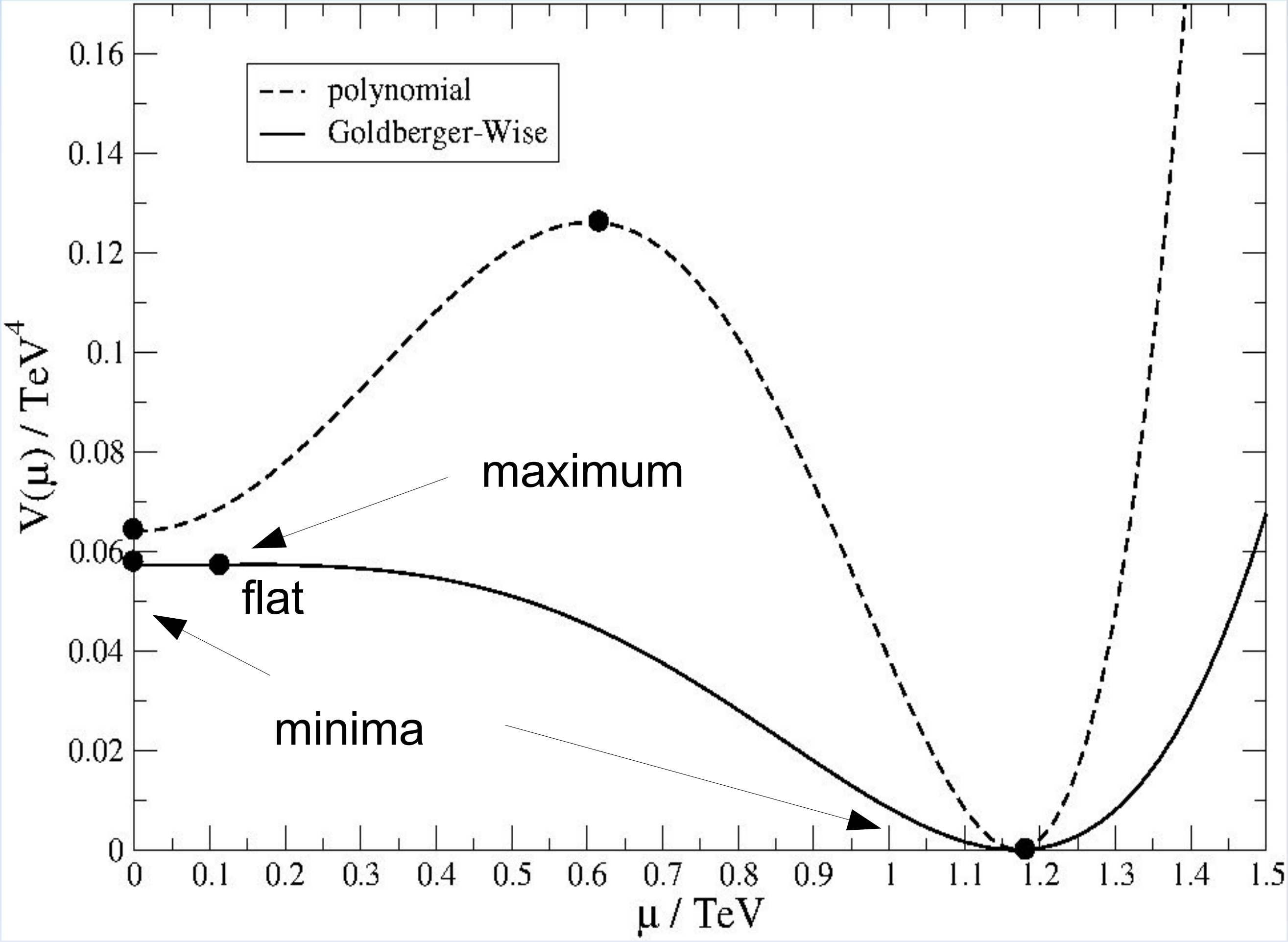
$$(h,s) = (0,w) \to (h,s) = (v,0)$$

Radion potential in warped extra-dimensions

The radion potential produces the electroweak scale by dimensional transmutation

$$V(\mu) \sim \mu^4 \times \text{polynomial}(\log \mu/M_{Pl})$$

In particular, there is a hierarchy between the maximum and the EW minimum.



Tunnel probability

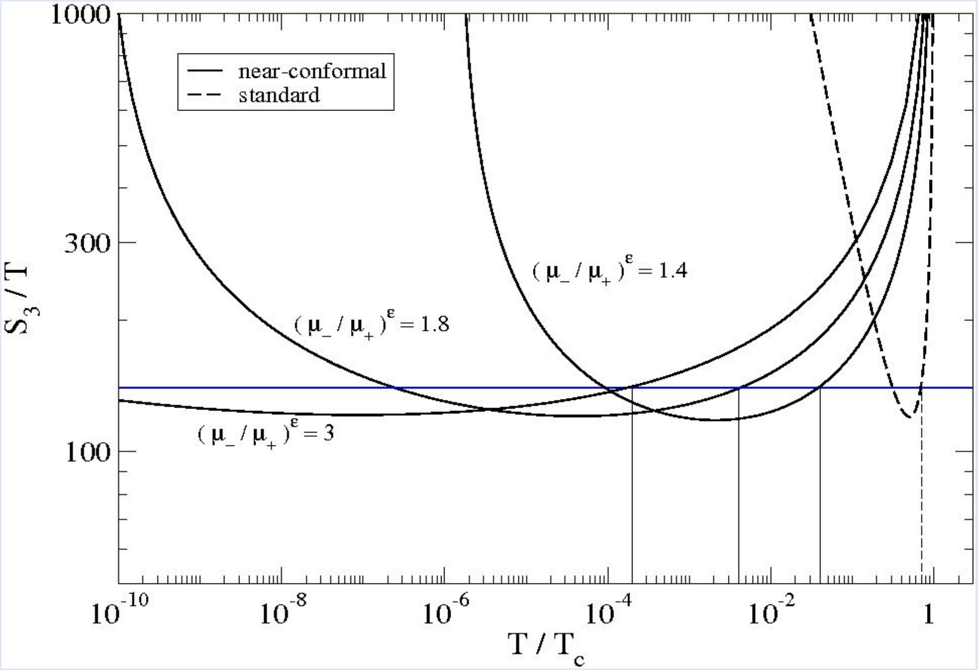
The semi-classical tunnel probability is given by the Euclidean action

$$P \sim \frac{T^4}{H^4} e^{-S_3/T}$$

The tunnel action inherits the nearly conformal behavior of the scalar potential

$$S_3/T \sim 4 \log T/H \sim 140$$

$$S_3/T \sim \mathrm{polynomial}(\log(T/T_c))$$



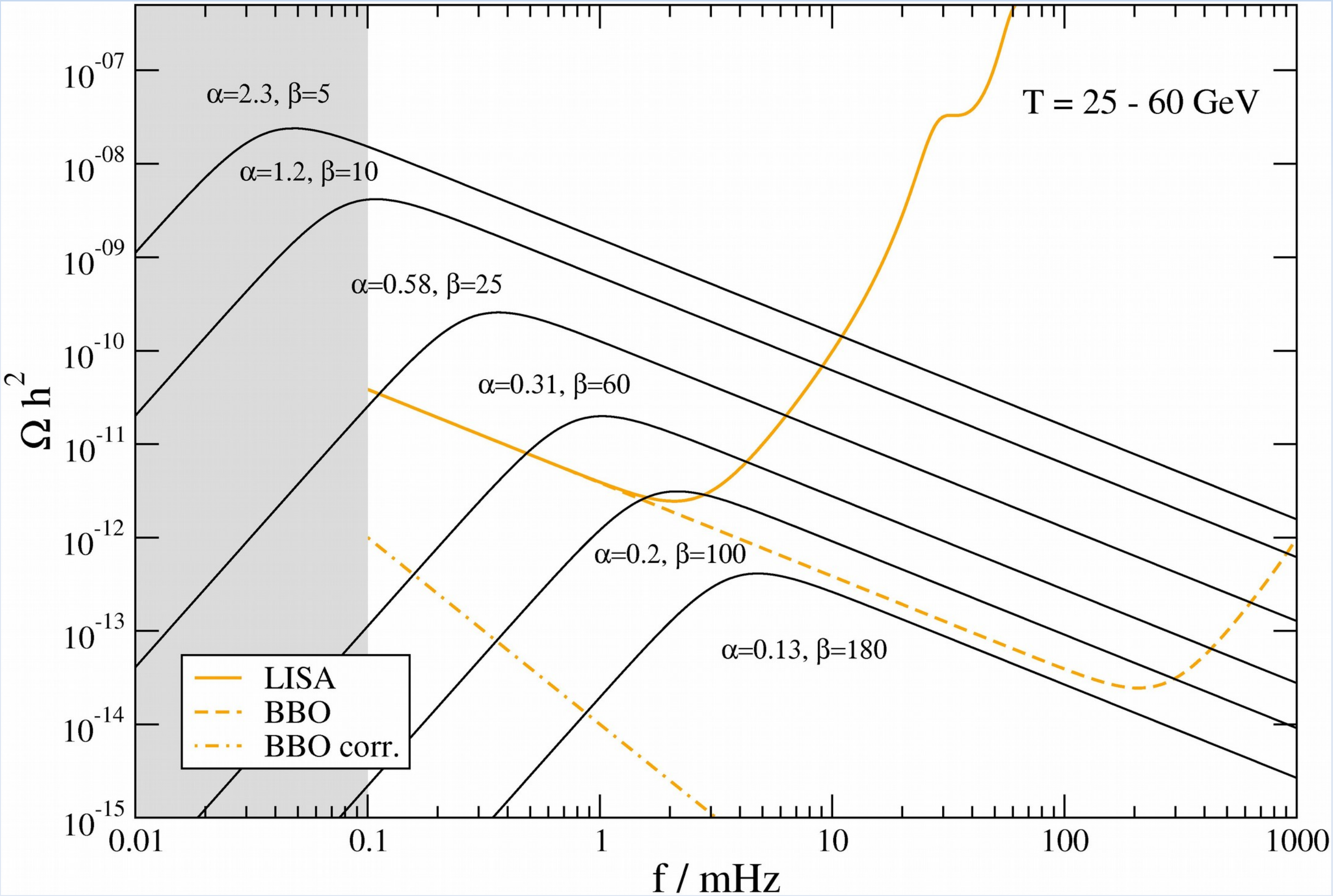
→ large supercooling and extremely strong phase transitions generic

The nearly conformal potential evades the graceful exit problem of old inflation, but only ~15 efolds of inflation possible.

[Nardini, Wulzer, Quiros '07] [TK, Nardini, Quiros '10] [TK, Servant '11]

GW spectra examples

$$\alpha = \rho_{vac}/\rho_{rad}, \quad \beta \sim \tau^{-1}, \quad v_b, \quad T$$



Conclusions

A strong electroweak first-order phase transition can produce gravitational waves that are observable by eLISA.

This requires a Higgs potential that is close to meta-stability with large super-cooling

$$\alpha \gtrsim 1, \quad \beta \lesssim 100\, H, \quad v_b \simeq 1$$

Even better are prospects for phase transitions that are slightly above the electroweak scale e.g. from new physics that solves the hierarchy problem.

Comment on Holography

In AdS/CFT this phase transition is identified with the confining phase transition of the strongly coupled (alsmost CFT) gauge theory

[Arkani-Hamed, Porrati, Randall '00]
[Rattazzi, Zaffaroni, '01]

However, a large number of degrees of freedom imply a large tunnel action

$$\Delta g \propto N^2$$

$$S_3/T \propto (M_5 l)^3 = N^2/16\pi^2$$

that leads to a bound from meta-stability

[Creminelli, Nicolis, Rattazzi '01] $N < 3$ thermal / thin wall
[Randall, Servant, '06] quantum / thick wall
[Hassanain, March-Russell, Schvellinger '07] throats
[Nardini, Quiros, Wulzer '09] $N < 13$ numerical
[TK, Nardini, Quiros '10] backreactions

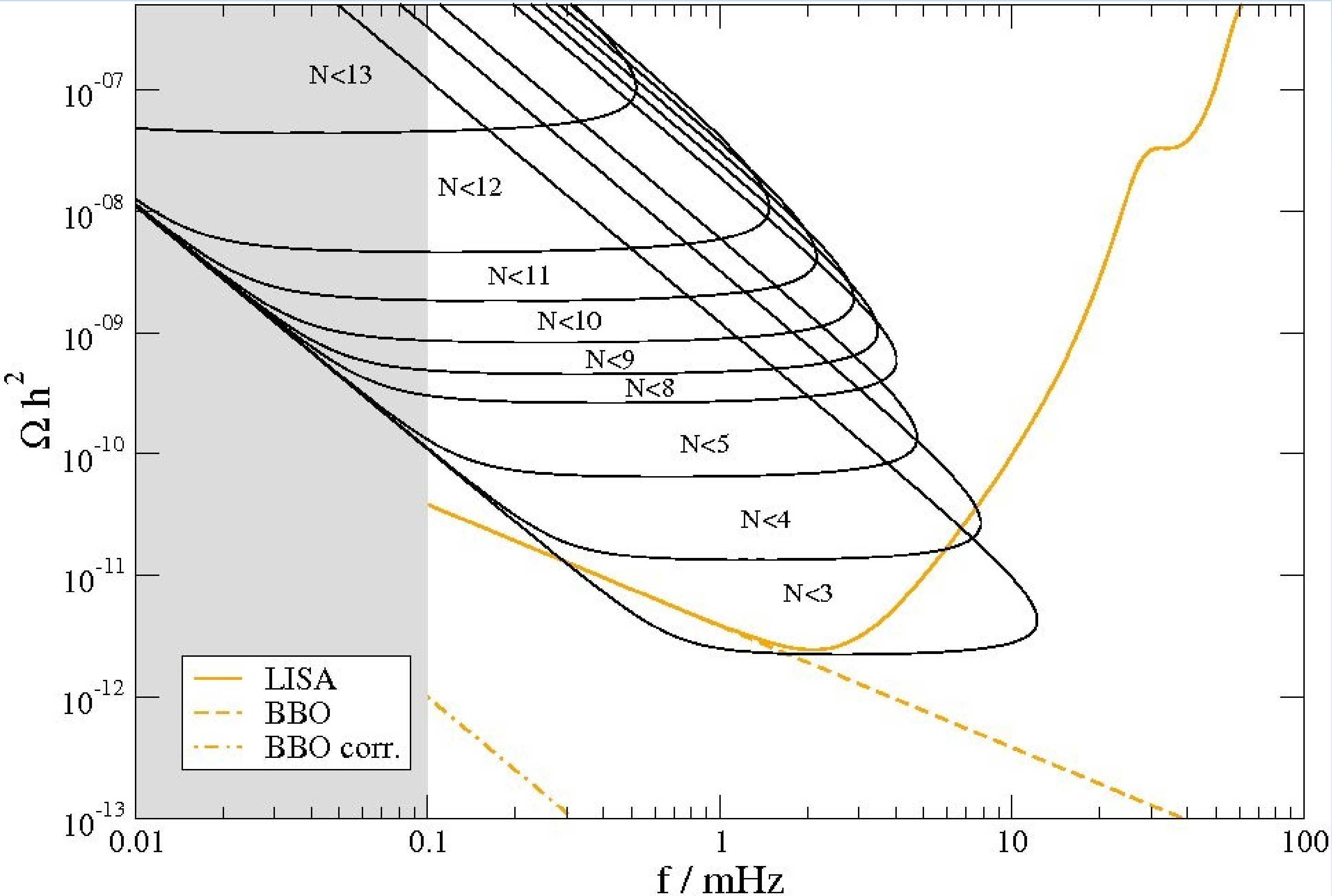
Gravitational waves

Sizable supercooling is the optimal condition for large gravitational wave production. *[TK, Nardini, Quiros '10]*

$$\alpha = \frac{\rho_{vac}}{\rho_{rad}} \gg 1$$

$$\tau^{-1} \gtrsim \mathrm{a\,few}\, H$$

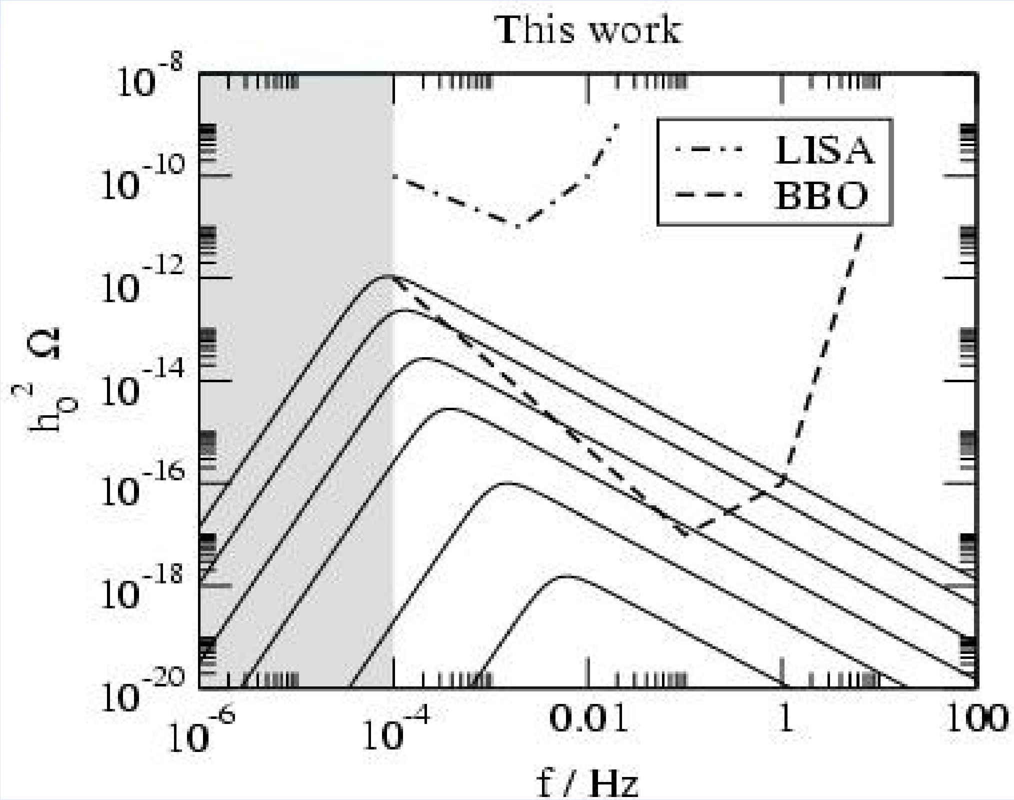
Even very large KK scales (~100 TeV) accessible



More realisitc models: nMSSM

[Huber&TK '08]

The nMSSM is an extension of the MSSM that solves the μ - problem. Typically, there is no barrier at zero T.



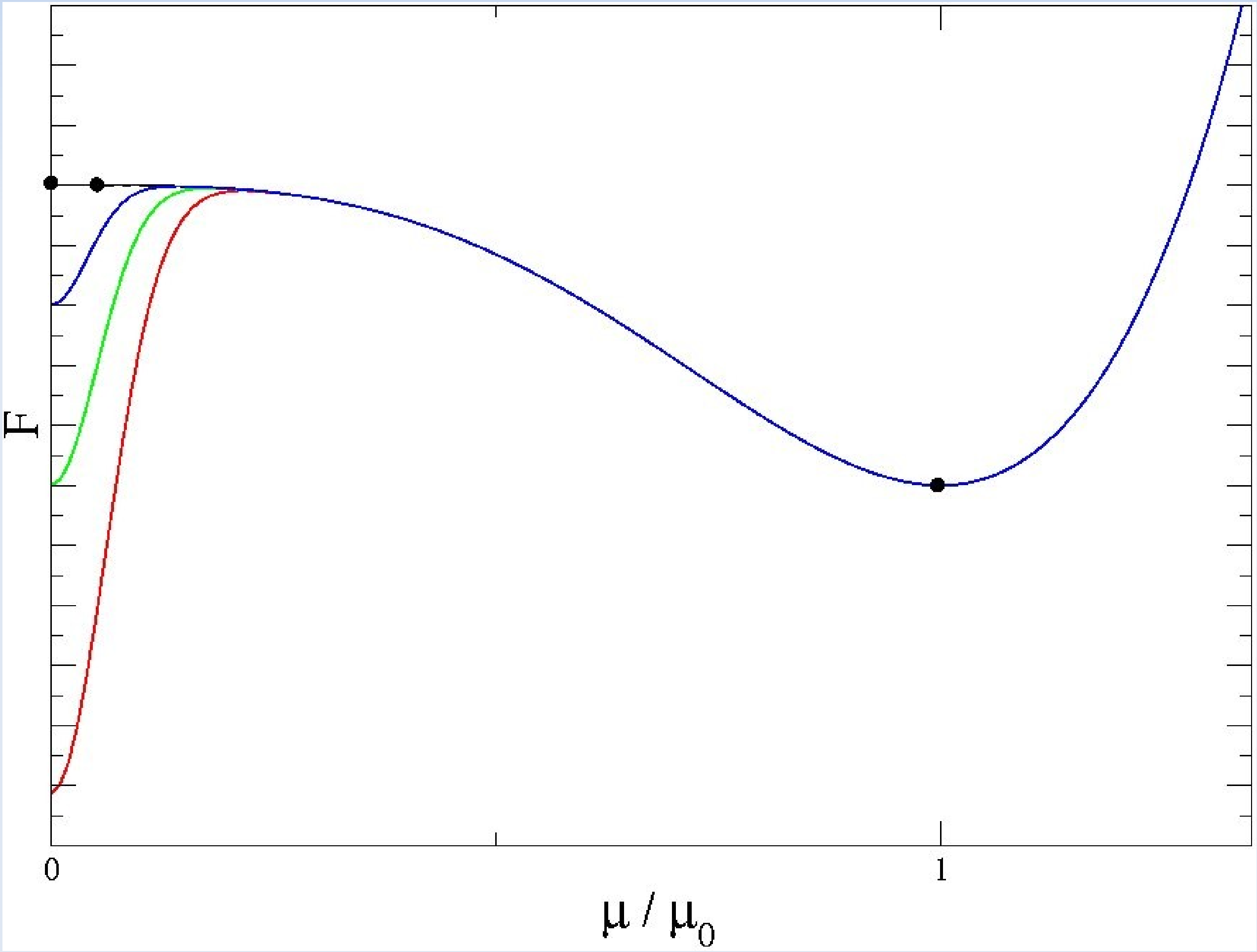
The potential of observing GWs from the EWPT with eLISA are rather small. For BBO a tuning ~ few % is required.

Free energy

In a thermal system a phase transition will connect the two stable phases of the system.

KK particles that are massive in the broken phase induce a difference in free energy between the two phases

$$\Delta F = \frac{\pi^2}{90}\,\Delta g\,T^4$$



Warped extra dimensions

Warped extra dimensions provide one possible solution to the hierarchy problem

$$\mathcal{L} \ni -M_5^3 \int d^5x \sqrt{g} \left[R + \frac{12}{l^2} \right] + \int d^5x \sqrt{g} \frac{1}{2} \left[(\partial\phi)^2 + m^2\phi^2 \right]$$

$$\epsilon = (ml)^2/4 \sim 1/30$$

5D RS

$$ds^2 = e^{-2r/l} dx^2 - dr^2$$

UV

$$r_0 \sim 35\, l$$

IR

$$\phi \sim A e^{-\epsilon r/l} + B e^{(4+\epsilon) r/l}$$

boundary conditions $\longrightarrow$ stabilization

[Randall&Sundrum '99], [Goldberger&Wise '99]

Goldberger-Wise mechanism

Using the canonical normalization of the radion

$$\mu \propto l^{-1} e^{-r/l}$$

and appropriate boundary conditions for the bulk scalar leads to a nearly conformal radion potential

$$V(\mu) \sim \mu^4 \lambda((\mu l)^\epsilon)$$

This solves the hierarchy problem as long as

$$dV/d\mu = 0 \leftrightarrow \lambda((\mu l)^\epsilon) \simeq 0$$

$$\leftrightarrow \mu_0 \simeq l^{-1}\; O(1)^{1/\epsilon} \simeq l^{-1} 10^{-16} \simeq \mathrm{TeV}$$