Acoustic production of gravitational waves at phase transitions

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with

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Gravitational waves in the early universe

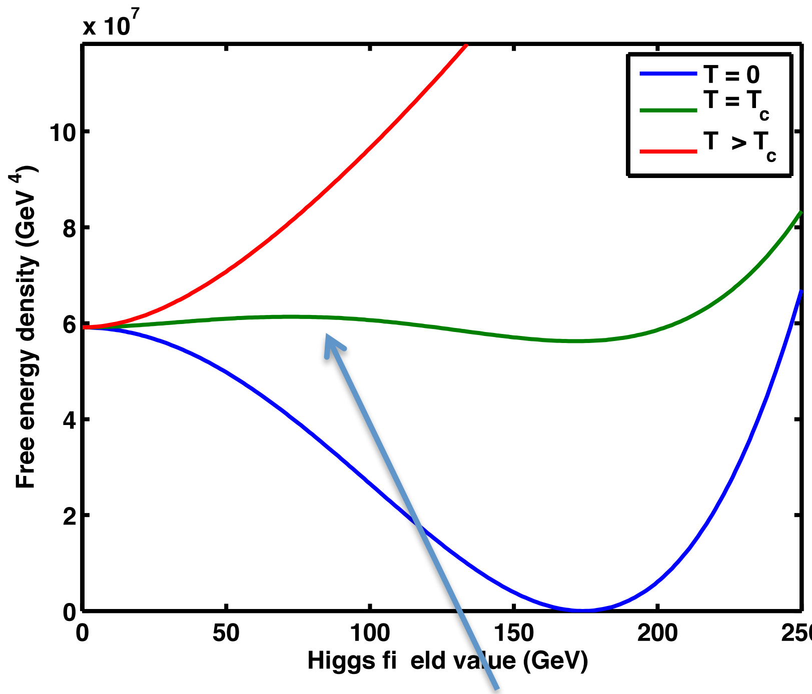
- Sources
  - Inflation
  - Preheating after inflation
  - Topological defects
  - First-order phase transitions
- Early Universe is transparent to GWs
  - Unique probe of physics at high energies

Direct detection of gravity waves in Brighton

- Here they are!
- No detection of gravitational waves yet

Electroweak phase transition

- Free energy density of plasma depends on
  - Temperature T
  - Particle masses $m_i(\phi)$
- High T: reduce free energy by forcing Higgs ϕ to zero
- Phase transition in weakly coupled gauge theories: Kirzhnits 1972
- Electroweak phase transition: $T_c \approx v_{EW} \approx 100$ GeV

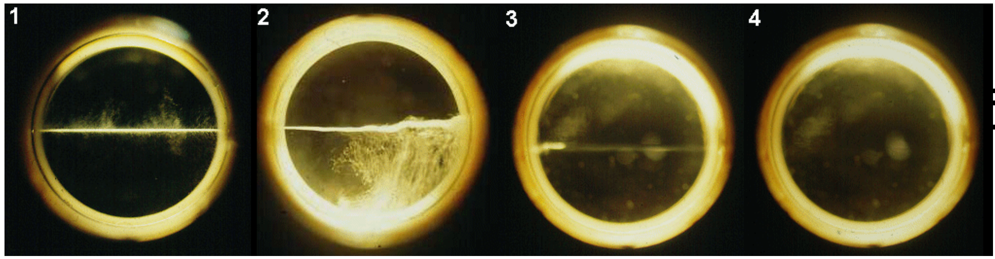


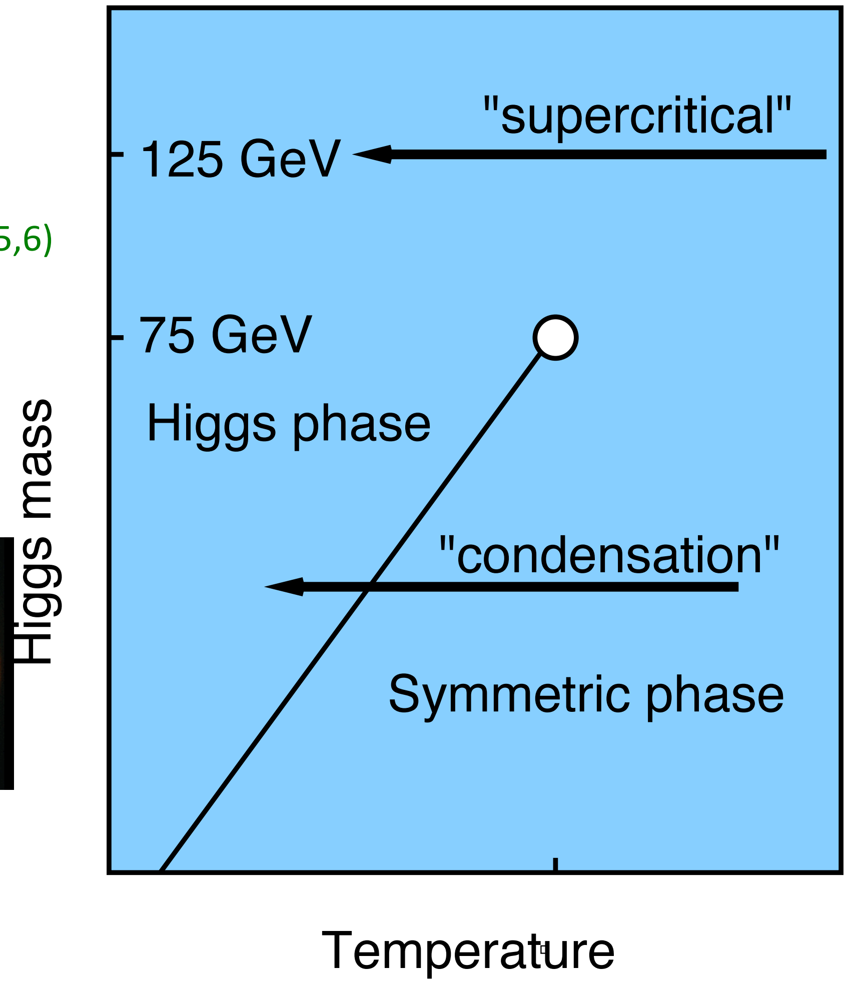
Potential barrier from cubic term in perturbative high-T expansion. First order transition?

- High T (>> $m_i(\phi)$): $V(\phi, T) \simeq \frac{1}{2}\alpha(T^2 - T_0^2)\phi^2 - \frac{1}{3}\gamma T\phi^3 + \frac{1}{4}\lambda\phi^4$

Standard Model electroweak phases

- SM is not weakly coupled at high T
- Non-perturbative techniques:
  - Dimensional reduction + effective field theory + 3D lattice
    Kajantie, Laine, Rummukainen, Shaposhnikov (1995,6)
  - 4D lattice
    Czikor, Fodor, Heitger (1998)
- SM transition at $m_h \approx 125$ GeV
  like a supercritical fluid

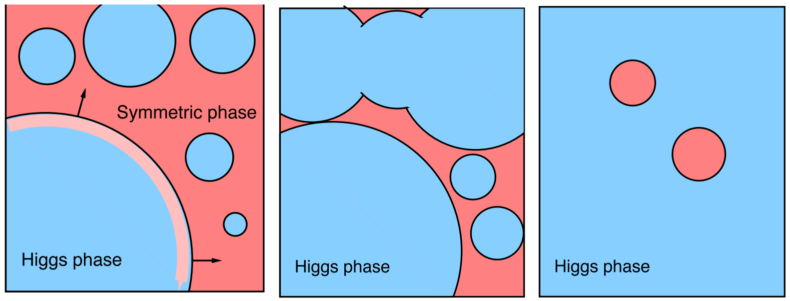


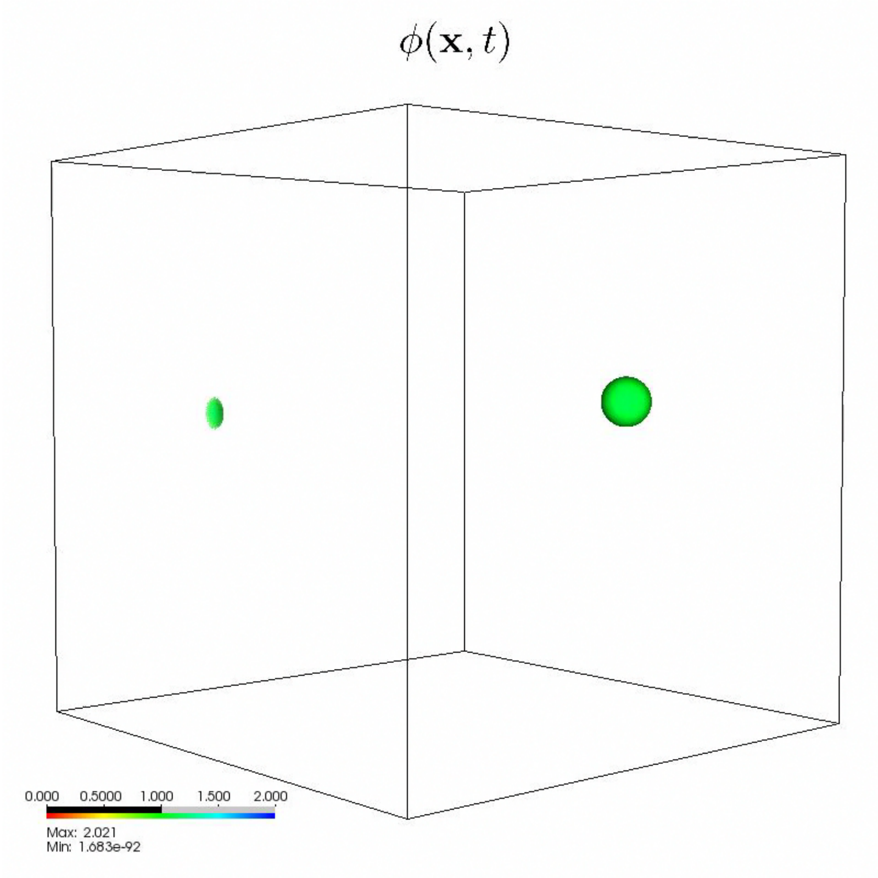


- 1st order transition = beyond the Standard Model physics
  (see talk by S Huber)

Little bangs in the Big Bang

- 1st order transition proceeds by nucleation of bubbles of Higgs phase
- Expanding bubbles generate pressure waves in hot fluid
- Detectable gravitational waves?





Scalar field

Steinhardt (1982); Gyulassy et al (1984); Witten (1984); Enqvist et al (1992);

MH, Huber, Rummukainen, Weir (2013)
Scalar only: Child, Giblin (2012)

First order phase transitions

- Bubble nucleation rate/volume Linde (1983)

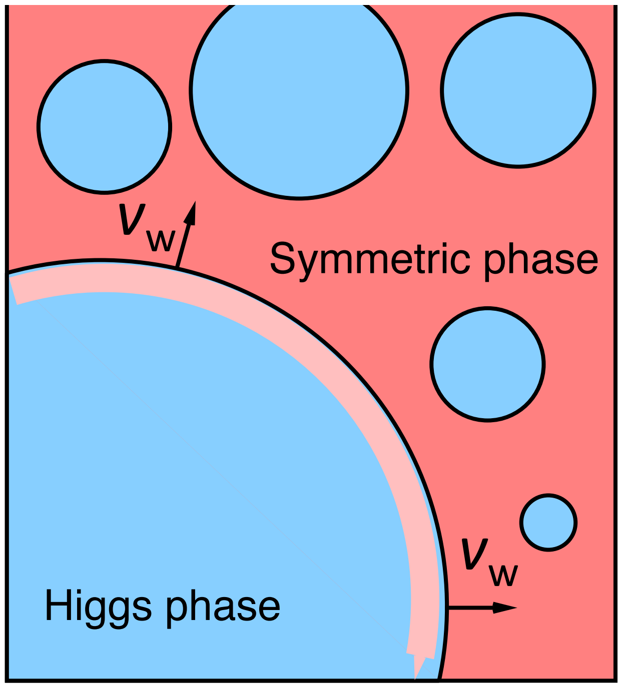
$$p(t) = \Gamma(T)e^{-S(T)} \simeq \Gamma(T_N)e^{-S(t_\mathrm{N})+\beta(t-t_\mathrm{N})},$$

- Thermal EW transition:
  - $S(T_\mathrm{N}) \sim 10^2$, $\beta \sim 10^4 H(T_\mathrm{N})$

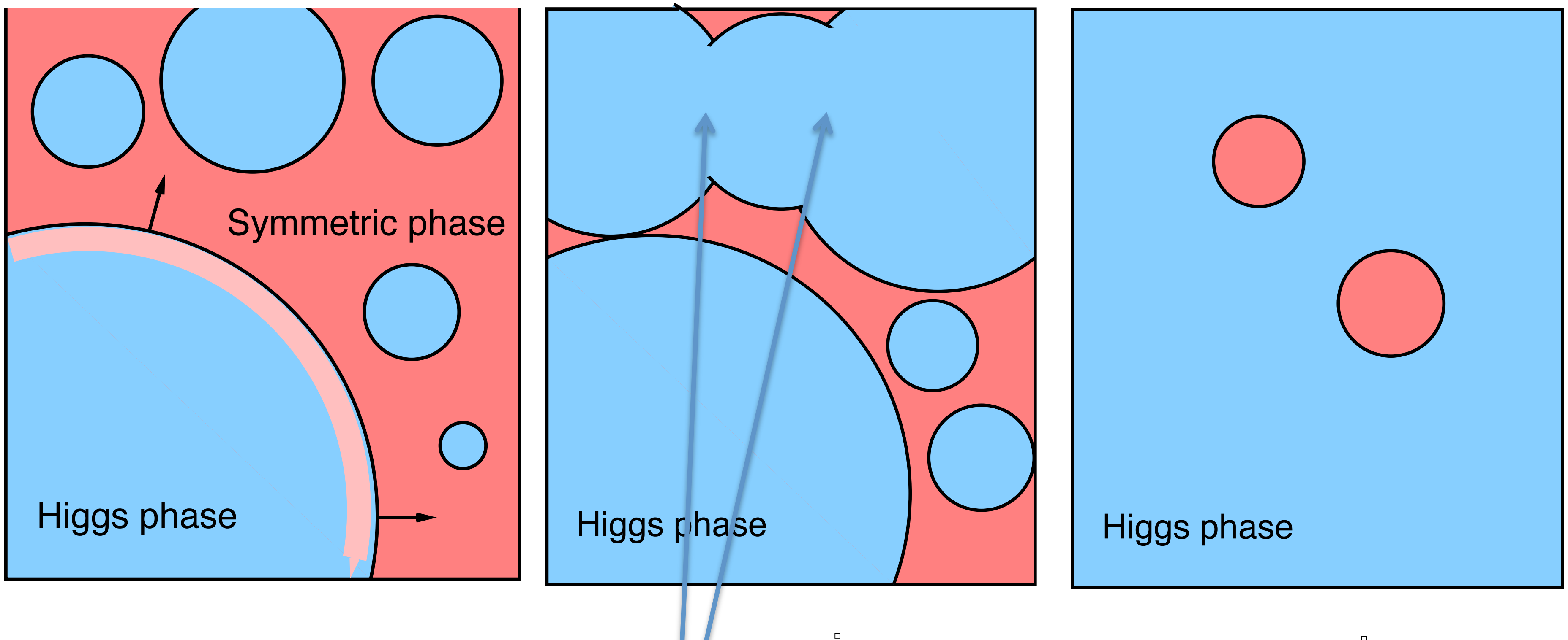
    Hogan (1983)
    Moore & Rummukainen (2000)
- Duration of transition: β^{-1}
- Average bubble separation

  $R_* \sim v_\mathrm{w}\beta^{-1}$
- Wall speed v_w $< c_s$ deflagration, $> c_s$ detonation

  Steinhardt (1982)



GWs from first order transitions: envelope approximation



Spherical matter distributions do not radiate

Assume: Gravitational waves generated by colliding bubbles
(Kosowski, Turner, Watkins 1992
Kamionkowski, Kosowsky, Turner 1994)

GWs from first order transition: envelope approximation

Kosowsky, Turner, Watkins 1992; Kamionkowski, Kamionkowsky, Turner 1994

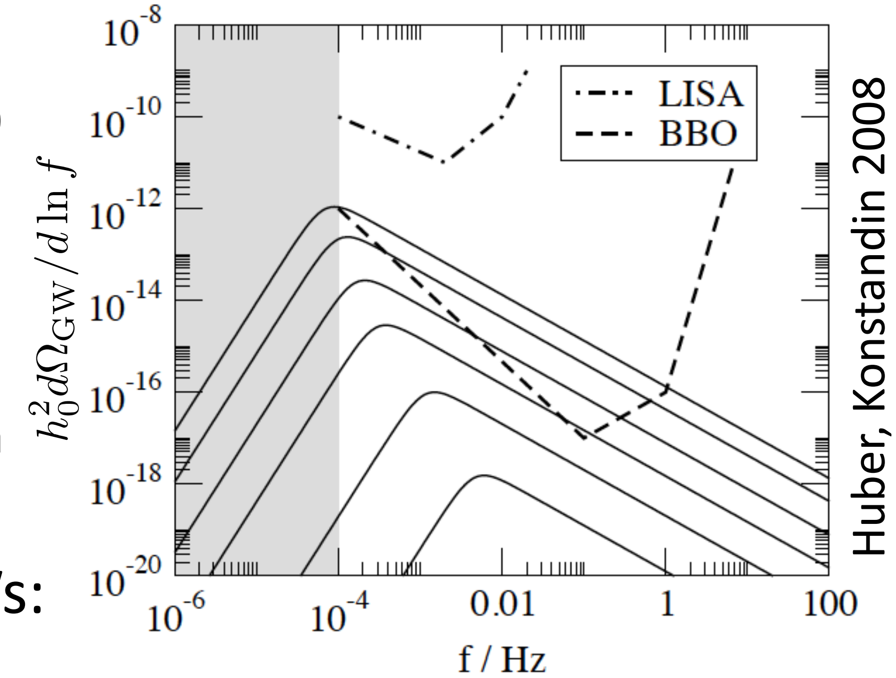
- Parametrise transition:
  - α = (Vacuum energy)/(Total energy)
  - v_w = Bubble wall speed
  - H_* = Hubble rate at transition
  - β = bubble nucleation rate
  - κ = conversion efficiency of vacuum energy to fluid kinetic energy

    Espinosa et al 2008
- Fraction of energy density in GWs:

$$\Omega_{\rm GW} = \frac{\rho_{\rm GW}}{\rho_{\rm Tot}}$$

$$\Omega_{\rm GW}^{\rm ea} \simeq \frac{0.11 v_{\rm w}^3}{0.42 + v_{\rm w}^2} \left(\frac{H_*}{\beta}\right)^2 \frac{\kappa^2\alpha^2}{(\alpha+1)^2},$$

Huber, Konstandin 2008



GW power spectrum:
α = 0.2 … 0.03
H_*/β = 0.1 … 0.003

Direct numerical simulation of an early universe phase transition

- Ingredients:

  Ignatius et al (1994), Kurki-Suonio, Laine (1996)

  - Higgs field $$-\ddot{\phi} + \nabla^2\phi - \frac{\partial V}{\partial \phi} = \eta W(\dot{\phi} + V^i\partial_i\phi)$$
    - η coupling to fluid (models energy transfer)
  - Relativistic fluid

$$\dot{E} + \partial_i(EV^i) + P[\dot{W} + \partial_i(WV^i)] - \frac{\partial V}{\partial \phi}W(\dot{\phi} + V^i\partial_i\phi) = \eta W^2(\dot{\phi} + V^i\partial_i\phi)^2.$$

$$\dot{Z}_i + \partial_j(Z_iV^j) + \partial_iP + \frac{\partial V}{\partial \phi}\partial_i\phi = -\eta W(\dot{\phi} + V^j\partial_j\phi)\partial_i\phi.$$

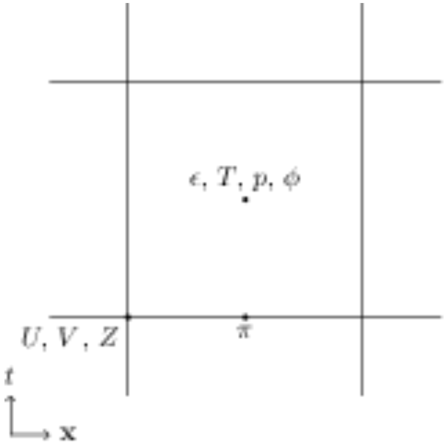
    - E energy density, Z_i momentum density, V_i velocity, W γ-factor
  - Metric perturbation $$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\mathrm{TT}}$$
  - Metric projected from u_{ij} $$\ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi G T_{ij}^{\mathrm{Traceless}}$$

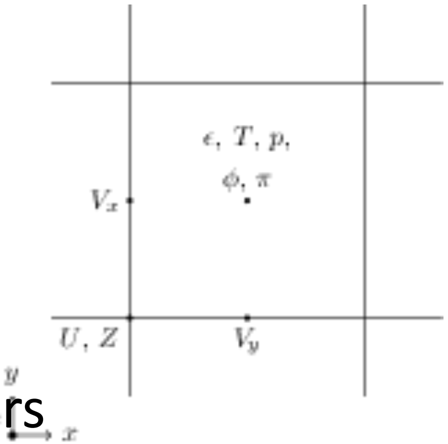
Garcia-Bellido, Figueroa, Sastre (2008)

Numerical simulation details 1

- Discretisation:
  - Fluid: Wilson & Matthews (2003)

    Different approach: Giblin, Mertens (2013)
  - Scalar field and metric:
    $O(\delta x^2)$ finite difference, leapfrog
- Lattice spacing: $\delta x = 2/T_c$
- Time step: $\delta t = 0.2/T_c$
- Lattice size: 2400^3
- Machine: Cray XC40, CSC Finland
  - CSC Special Grand Challenge 750k CPU-hours
- Code: D Weir





Numerical simulation details 2

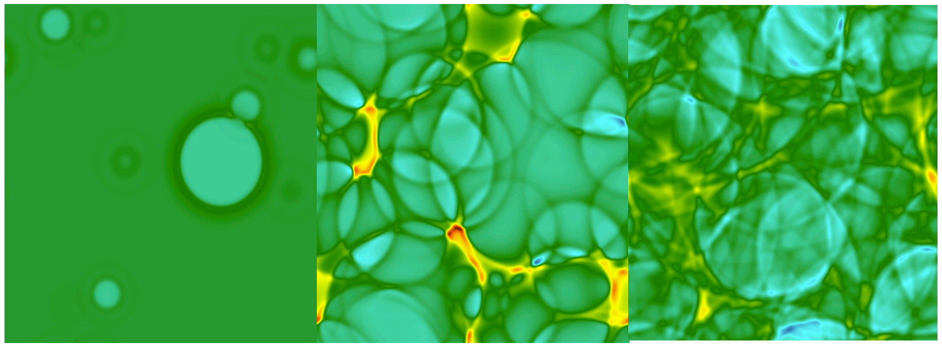
- Potential $V(\phi, T) = \frac{1}{2}\gamma(T^2 - T_0^2)\phi^2 - \frac{1}{3}AT\phi^3 + \frac{1}{4}\lambda\phi^4.$
- Domain wall surface tension σ, width ℓ
- Nucleation temperature T_N, latent heat $\mathcal{L}$
- Transition strength parameter $\alpha_{TN} = \mathcal{L}/\varepsilon$
- Critical bubble radius R_c

	Weak	Weak (scaled)	Intermediate
T_0	$T_c/\sqrt{2}$	$T_c/\sqrt{2}$	$T_c/\sqrt{2}$
γ	$1/18$	$4/18$	$2/18$
A	$\sqrt{10}/72$	$\sqrt{10}/9$	$\sqrt{10}/72$
λ	$10/648$	$160/648$	$5/648$
σ/T_c^3	$1/10$	$1/20$	$4\sqrt{2}/10$
ℓT_c	6	3	$6/\sqrt{2}$
T_N/T_c	0.86	0.86	0.8
α_{T_N}	0.010	0.010	0.084
$R_c T_c$	16	8.1	8.6

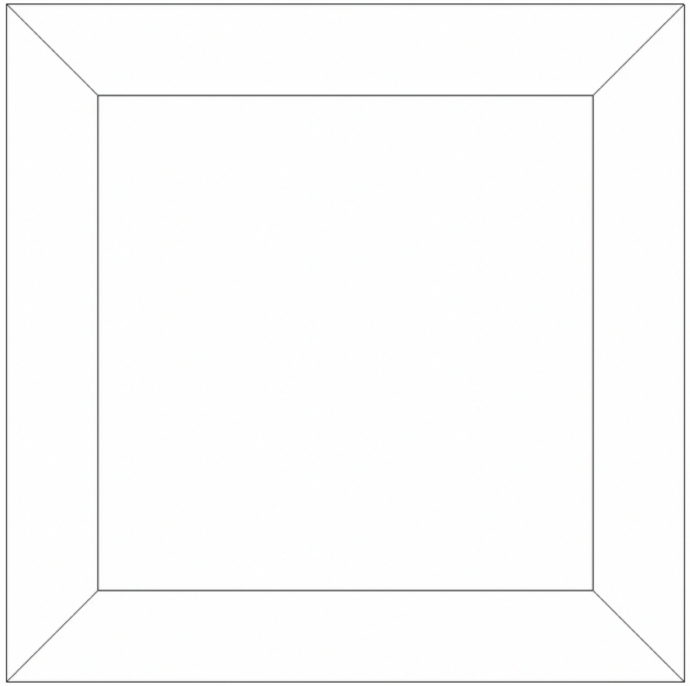
Hydrodynamics of phase transitions

- Latent heat of transition goes into fluid compression waves – **sound** Hogan (1986)
- Sound waves remain long after transition is complete

MH, Huber, Rummukainen, Weir (2014)



Fluid energy density as transition proceeds



Fluid energy density

MH, Huber, Rummukainen, Weir (2013,2015)
See also Giblin, Mertens (2013)

Sources of gravitational waves

- GW source contained in:

$$\tau_{ij}^{\phi} = \partial_i\phi\partial_j\phi$$

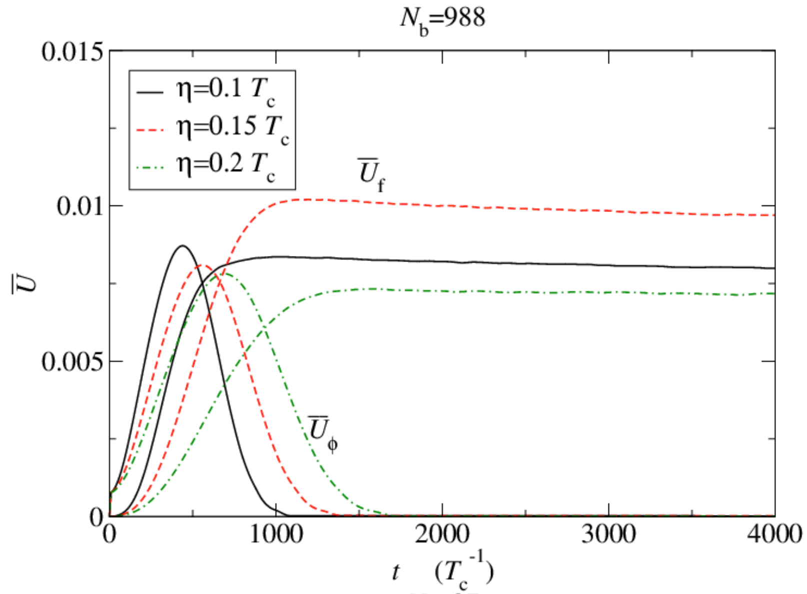
$$\tau_{ij}^{\mathrm{f}} = W^2(\epsilon + p)V_iV_j$$

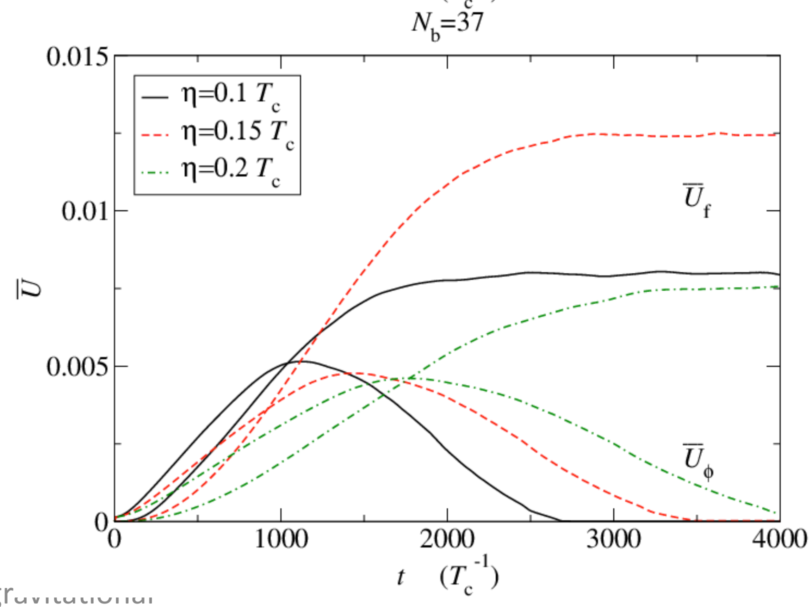
- Look at
  - rms fluid velocity

$$(\bar{\epsilon} + \bar{p})\bar{U}_{\mathrm{f}}^2 = \frac{1}{V}\int d^3x\tau_{ii}^{\mathrm{f}},$$

  - equivalent field quantity

$$(\bar{\epsilon} + \bar{p})\bar{U}_{\phi}^2 = \frac{1}{V}\int d^3x\tau_{ii}^{\phi},$$





Lifetime of sound waves

- Sound damped by viscosity and Hubble damping

$$\left(\frac{4}{3}\eta_{\rm s} + \zeta\right)\nabla^2 V_\parallel^i$$

- Shear viscosity $\eta_{\rm s} \sim e^4 T^3/\ln(1/e)$,
- Lifetime of scale *R*: $\tau_\eta(R) \sim R^2\epsilon/\zeta \sim \ln(1/e) R^2 T/e^4$.
- Longer than Hubble time for scales

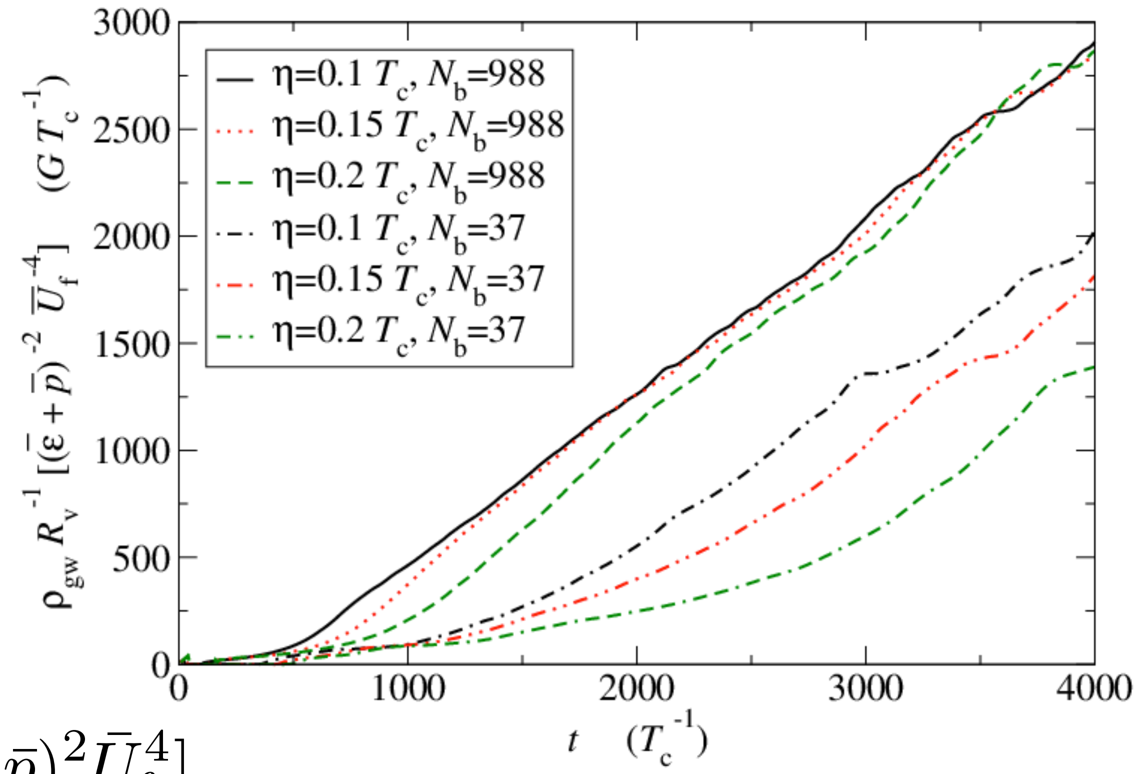
$$R \gg \frac{v_{\rm w}}{H_*}\left(\frac{T_{\rm c}}{m_{\rm Pl} e^4}\right) \sim 10^{-11}\frac{v_{\rm w}}{H_*}\left(\frac{T_{\rm c}}{100\ {\rm GeV}}\right),$$

- Hence sound wave lifetime is Hubble time

$$\tau_{\rm v} \sim H_*^{-1} \ll \tau_\eta(R_*).$$

GW energy density

- Fluid sources GWs continuously
- Source goes as (enthalpy density)^2
- GW energy density grows with time



$$\rho_{\rm GW} \propto t[GL_{\rm f}(\bar\epsilon + \bar p)^2 \bar U_{\rm f}^4]$$

- Fluid length scale L_f

Acoustic GW production formula

- Spectral density of gravitational radiation:

$$P_{\dot{h}}(k,t) = (16\pi G)^2 \int_0^t dt_1 dt_2 \frac{\cos[k(t_1 - t_2)]}{2} \Pi^2(k, t_1, t_2).$$

  - Π^2 – shear stress UETC

Caprini, Durrer, Konstandin, Servant (2009)

$$P_{\dot{h}}(k,t) = \left[16\pi G(\bar{\epsilon} + \bar{p})\bar{U}_{\mathrm{f}}^2\right]^2 t k^{-1} L_{\mathrm{f}}^3 \int dz \frac{\cos(z)}{2} \tilde{\Pi}^2(kL_{\mathrm{f}}, z).$$

  - $\tilde{\Pi}^2$ – dimensionless shear stress UETC

- Hence: $\rho_{\mathrm{GW}} = t[GL_{\mathrm{f}}(\bar{\epsilon} + \bar{p})^2 \bar{U}_{\mathrm{f}}^4 (8\pi\tilde{\Omega}_{\mathrm{GW}})]$

\- where $\tilde{\Omega}_{\mathrm{GW}} = \int \frac{dk}{k} \frac{(kL_{\mathrm{f}})^2}{2\pi^2} \int dz \frac{\cos(z)}{2} \tilde{\Pi}^2(kL_{\mathrm{f}}, z).$

GW energy density

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- Source goes as (energy density)^2
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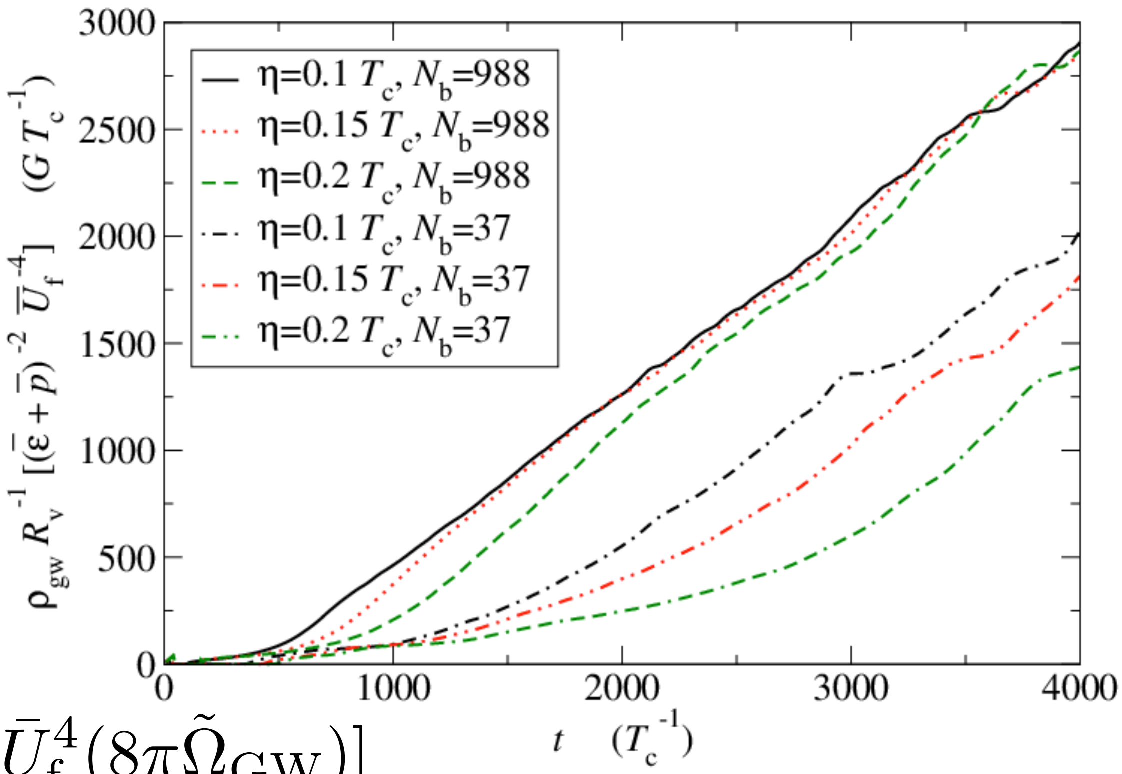
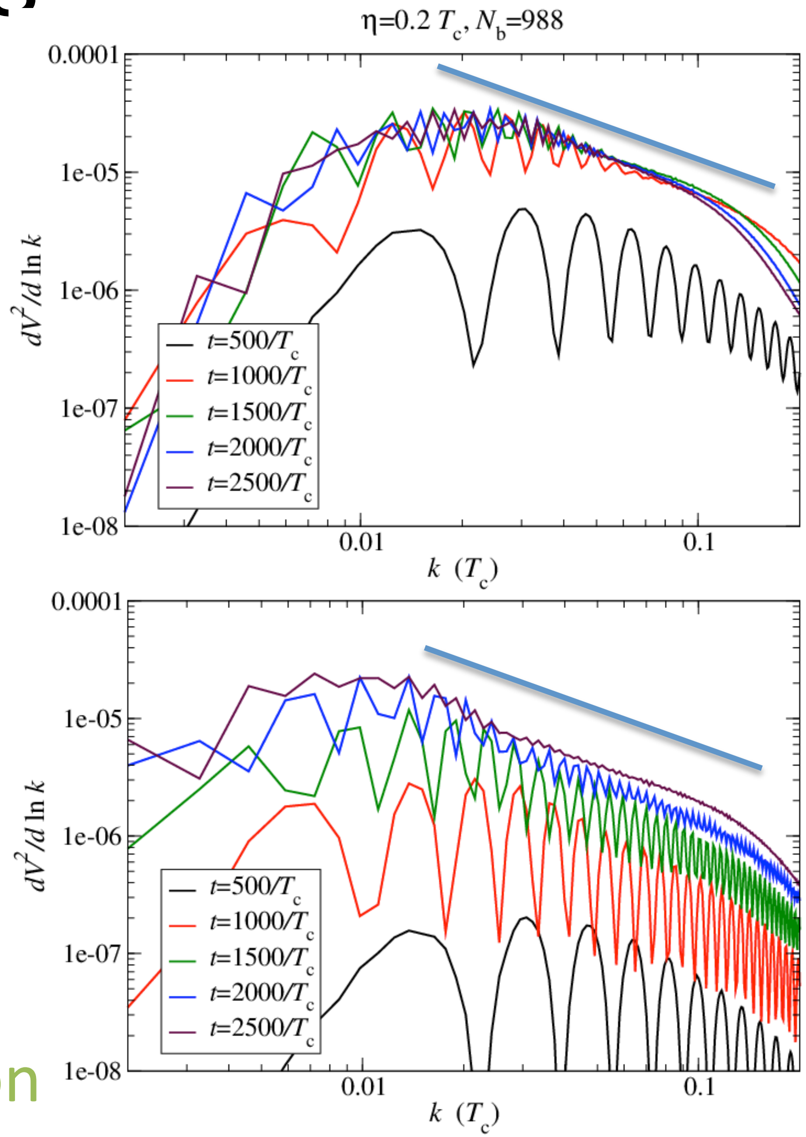


$$\rho_{\rm GW} = t[GL_{\rm f}(\bar\epsilon + \bar p)^2 \bar U_{\rm f}^4 (8\pi\tilde\Omega_{\rm GW})]$$

- Fluid length scale L_f
  - Integral scale $\xi_{\rm f} = \frac{1}{\langle V^2\rangle}\int \frac{d^3k}{(2\pi)^3}|k|^{-1}P_V(k),$
- $8\pi\tilde\Omega_{\rm GW} = 0.8 \pm 0.1$

Velocity power spectra: weak deflagration

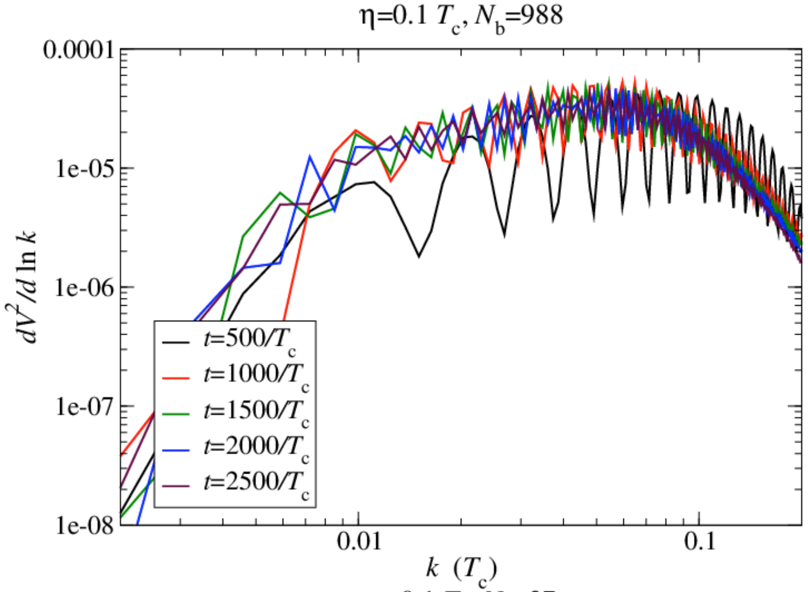
- Weak transition:
  $\alpha_T = 0.01$
- Subsonic wall:
  $v_w = 0.44$
- Approx k^{-1} spectrum

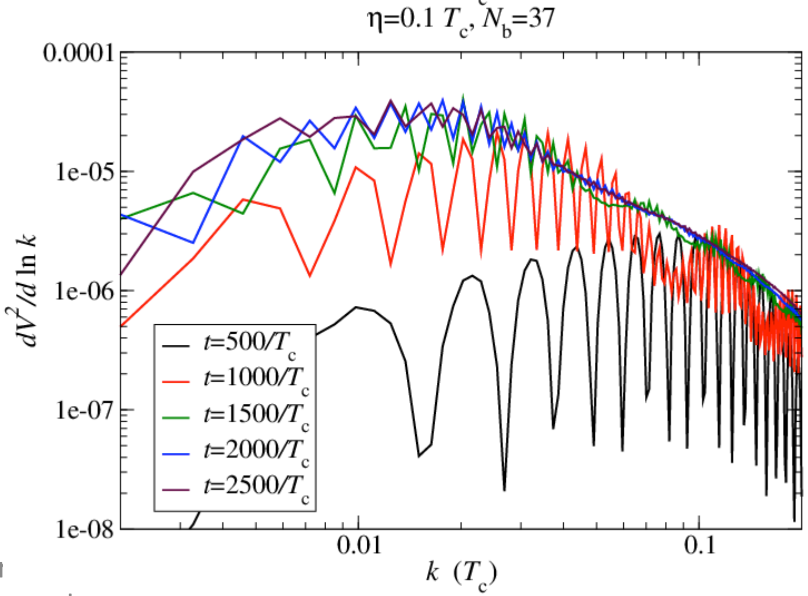


- NB "ringing" due to simultaneous bubble nucleation

Velocity power spectra: weak detonation

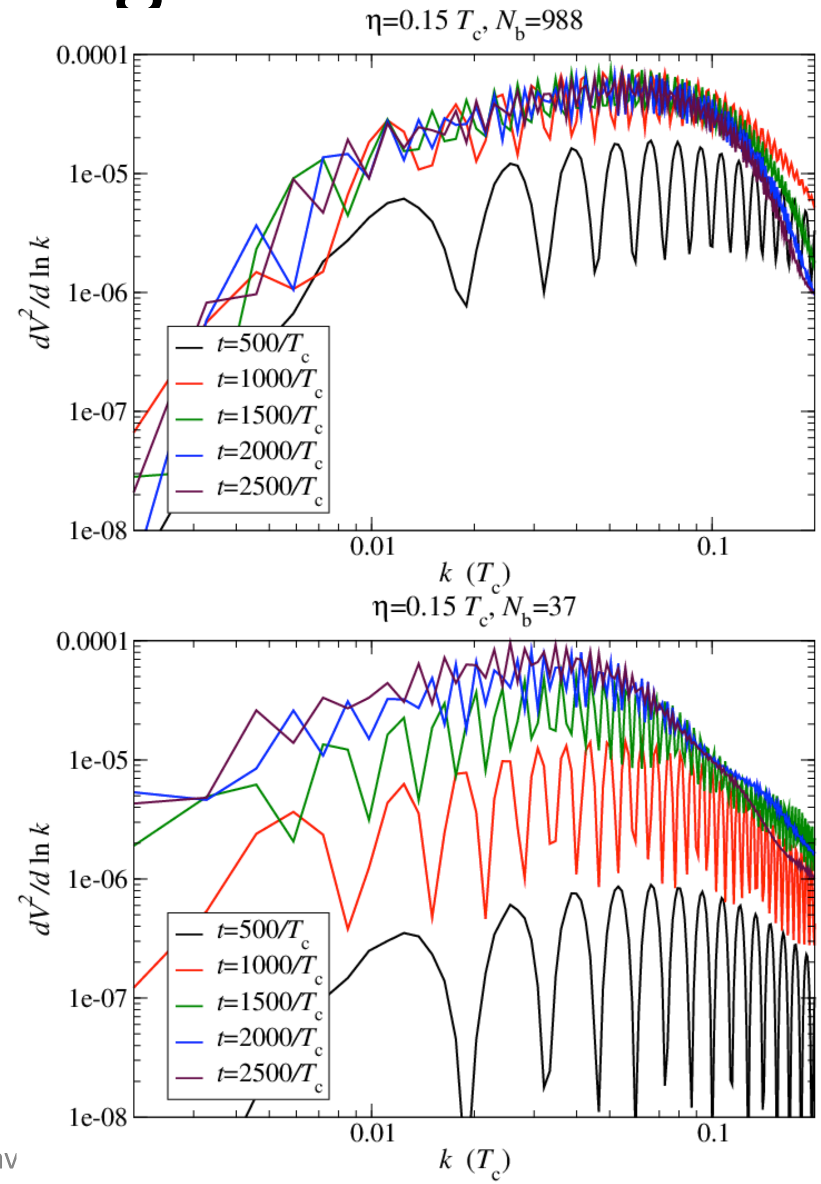
- Weak transition: $\alpha_T = 0.01$
- Supersonic wall: $v_w = 0.68$
- Power spectrum still evolving with R_*





Velocity power spectra: weak near-Jouguet

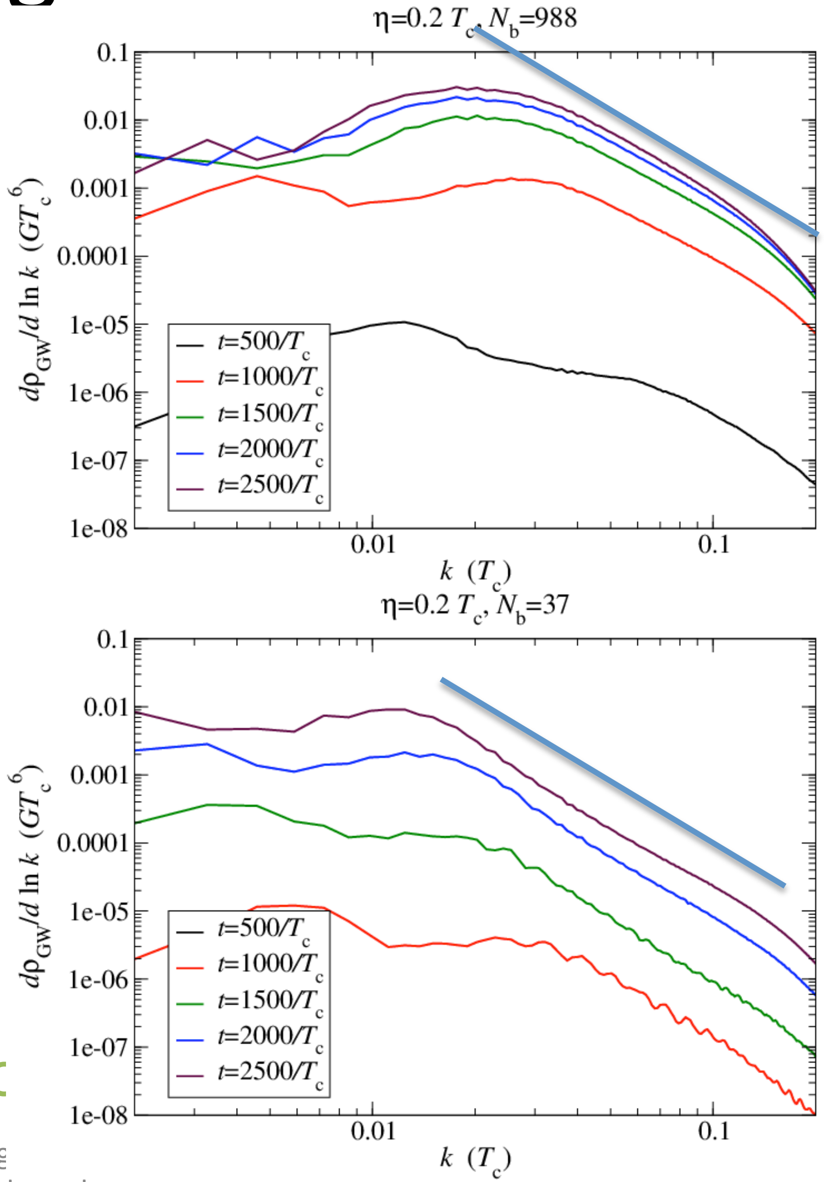
- Weak transition:
  $\alpha_T = 0.01$
- Just subsonic wall:
  $v_w = 0.54$
- Power spectrum still evolving with R_*



GW power spectra: weak deflagration

- Weak transition: $\alpha_T = 0.01$
- Subsonic wall: $v_w = 0.44$
- Approx k^{-3} spectrum

- NB "ringing" due to simultaneous bubble nucleatic



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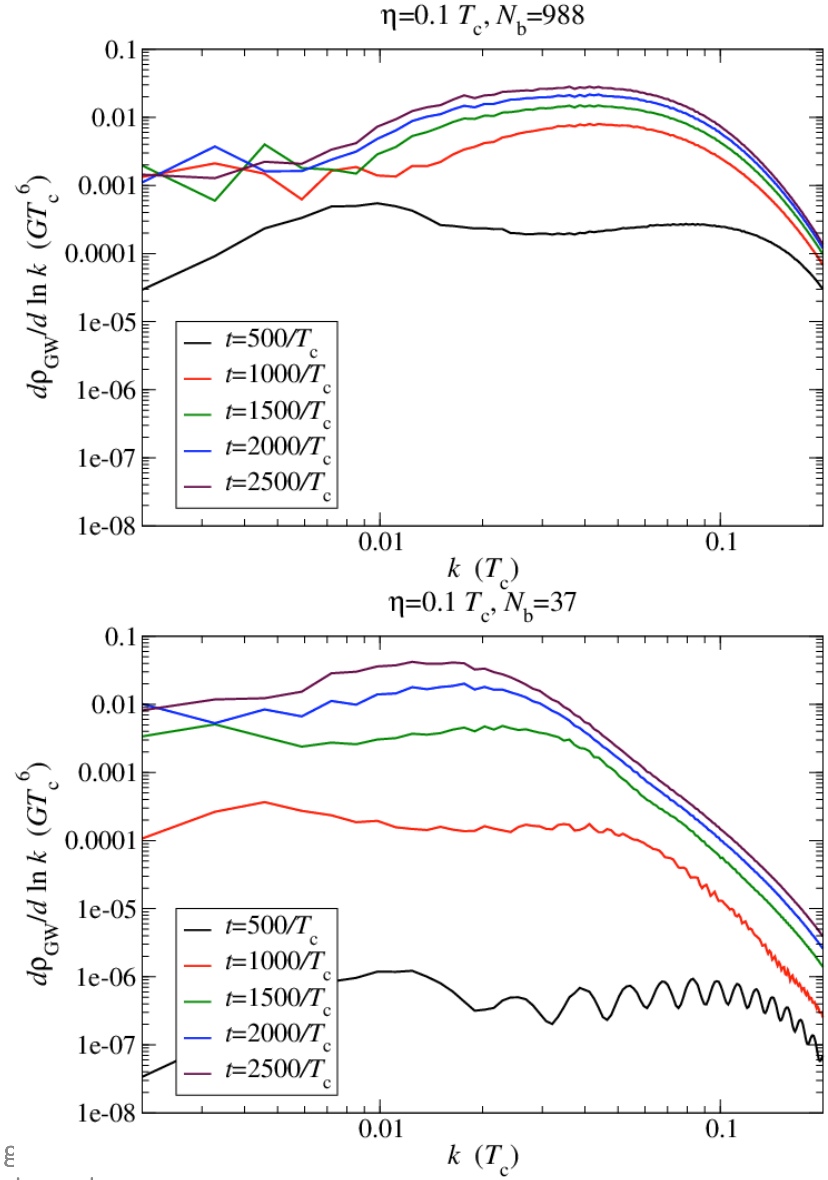

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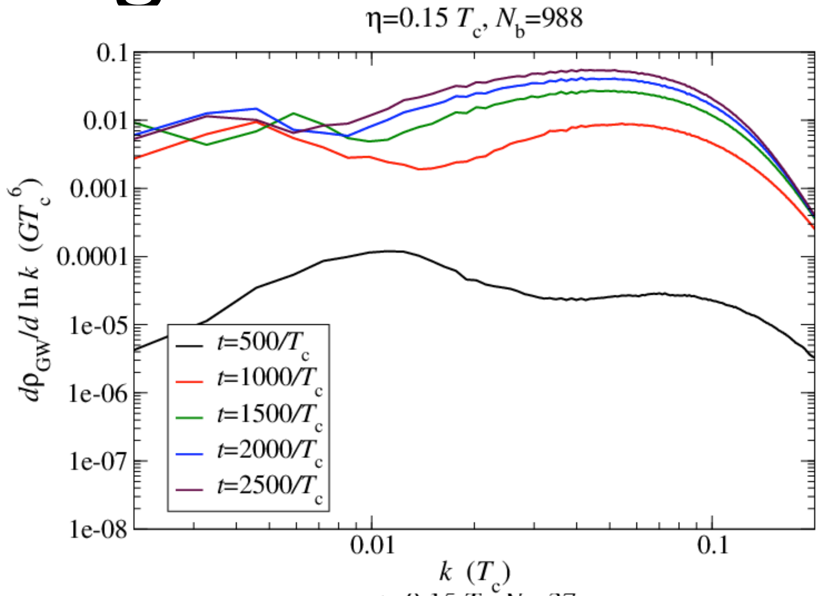
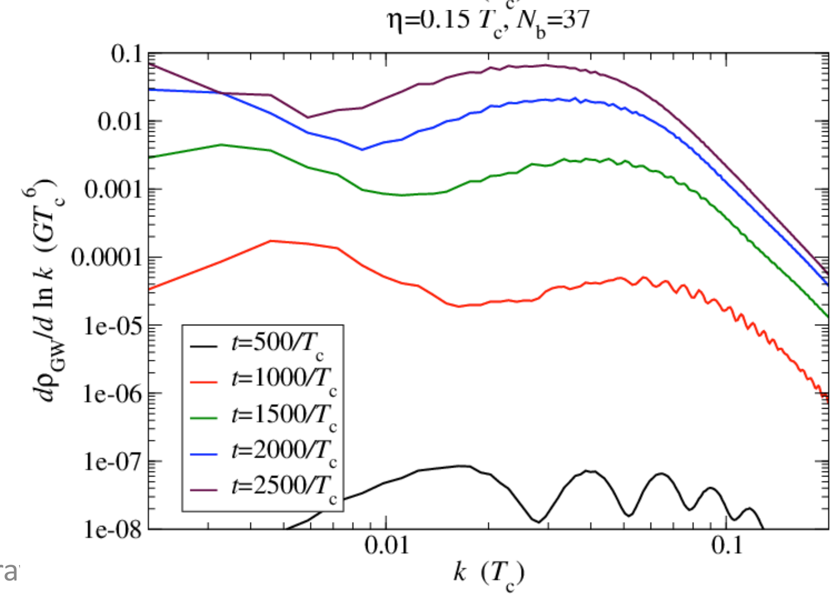




A new model for GW production

- Acoustic production:

$$\Omega_{\rm GW} = 3(1+w)^2 \bar{U}_{\rm f}^4 (H_* \tau_{\rm v})(H_* L_{\rm f}) \tilde{\Omega}_{\rm GW}$$

  - Fluid source "on" for time $\tau_{\rm v}$ (~ Hubble time)
  - Fluid source length scale $L_{\rm f}$ (~ bubble separation R_*)
  - Fluid kinetic energy density fraction $(1+w)\bar{U}_{\rm f}^2$
  - Dimensionless constant $8\pi\tilde{\Omega}_{\rm GW} = 0.8 \pm 0.1$

- Compare with envelope approximation:

$$\Omega_{\rm GW}^{\rm ea} \simeq \frac{0.11 v_{\rm w}^3}{0.42 + v_{\rm w}^2} \left(\frac{H_*}{\beta}\right)^2 \frac{\kappa^2 \alpha^2}{(\alpha+1)^2},$$

Note $(1+w)\bar{U}_{\rm f}^2 = \dfrac{\kappa\alpha}{\alpha+1}$

Acoustic GW signal boost

- Ratio of acoustic production to envelope approximation
  NB $R_* = (8\pi)^{1/3}\, v_w/\beta$

$$\frac{\Omega_{\rm GW}}{\Omega^{\rm ea}_{\rm GW}} \simeq \frac{3(8\pi)^{\frac{1}{3}}\tilde\Omega_{\rm GW}}{0.11 v_{\rm w}^2(0.42+v_{\rm w}^2)}\,(\beta\tau_{\rm v})\,.$$

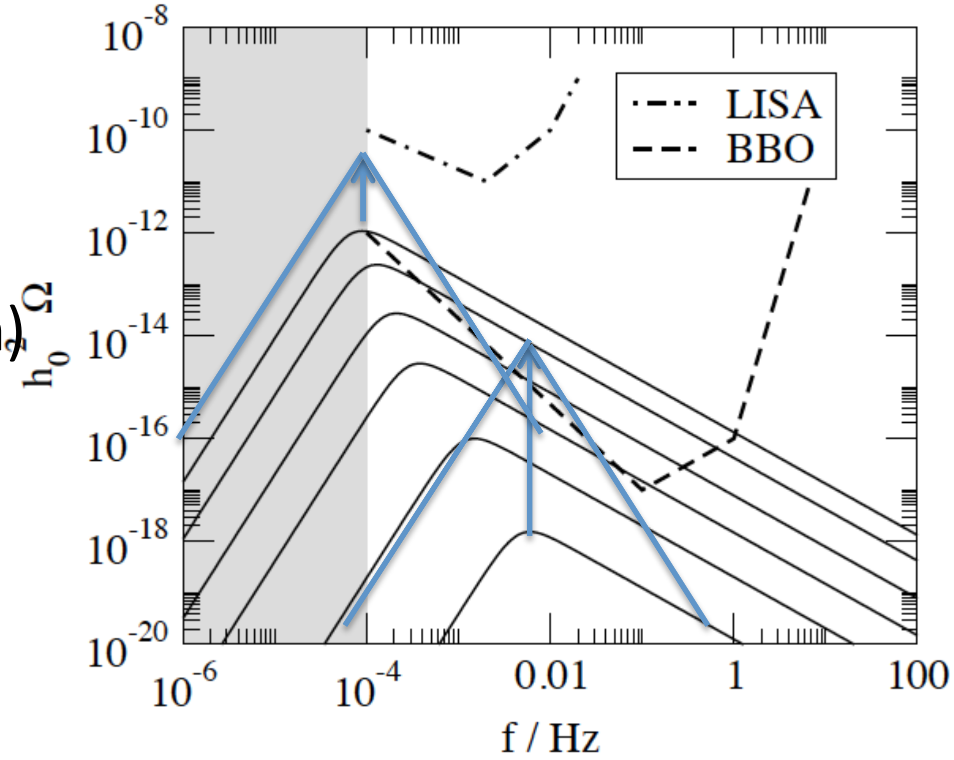
- Using
  - sound lifetime τ_v ~ Hubble time
  - wall speed $v_w < 1$

$$\frac{\Omega_{\rm GW}}{\Omega^{\rm ea}_{\rm GW}} \gtrsim 60\tilde\Omega_{\rm GW}\frac{\beta}{H_*}.$$

- GWs are parametrically larger by a factor which is $O(10^2)$ for a thermal electroweak-scale transition

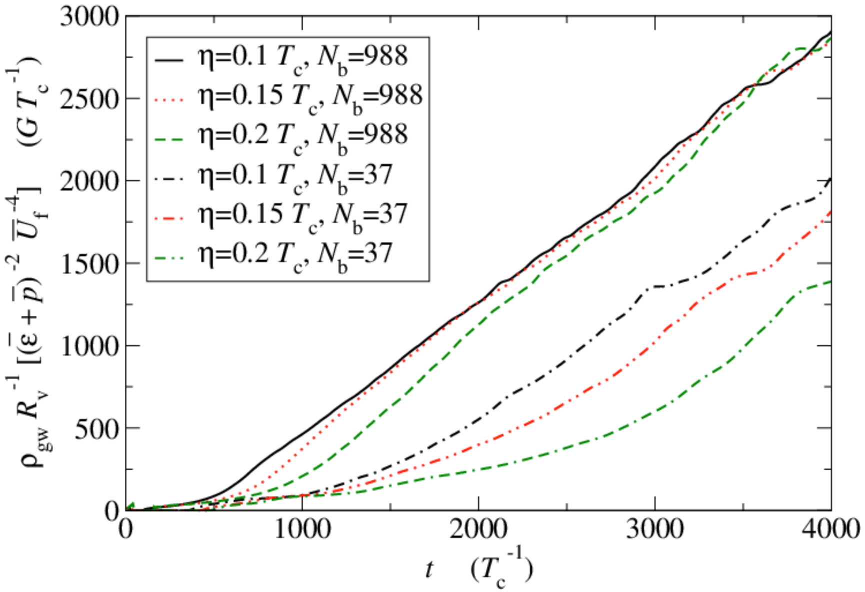
Implications for GW detection

- Preliminary sketch:
  - Peak amplitude increases
  - High frequency GW power spectrum ~ f^{-3} (deflagration)



Summary: GWs from phase transitions

- Source: **sound waves** from the nucleating droplets of the low temperature phase
- New O(1) "constant" $8\pi\Omega_{\rm GW}$
  - slope of $\rho_{\rm GW}/\left[(\bar\epsilon+\bar p)^2\bar U_{\rm f}^4 L_{\rm f}\right]$ vs. *t*
- GWs O(10^2) larger than old estimates (weak transitions)
- Approx k^{-3} spectrum (weak deflagrations)
- c.f. envelope approx k^{-1} (OK for vacuum transition?)

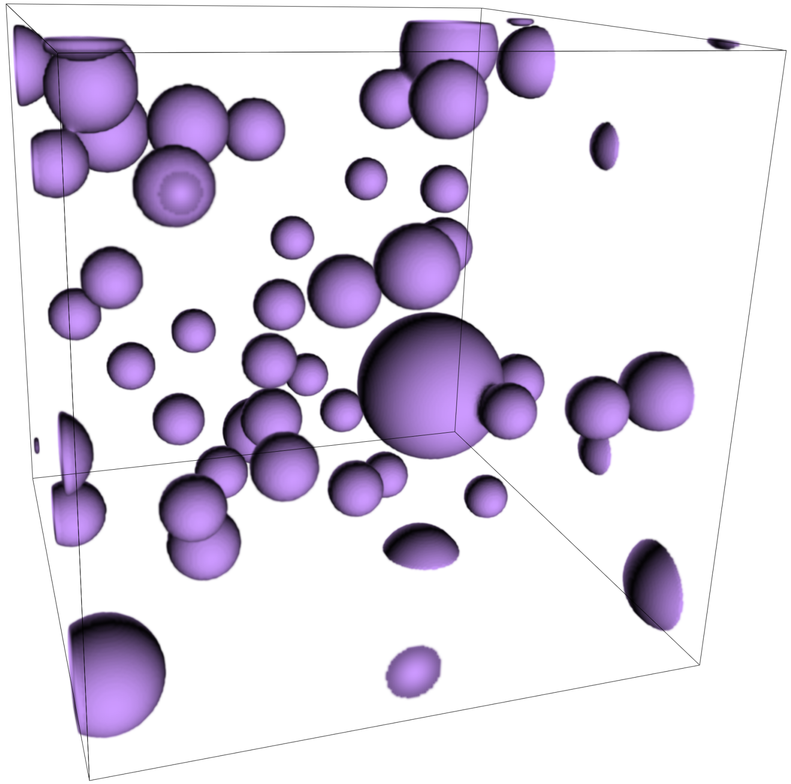


Outlook: GWs from phase transitions

- Larger simulations needed
  - 18M CPU-hours PRACE Tier 0
  - Detonation power spectra
  - Long wavelengths
  - Wall instabilities?

    Megevand, Membiela, Sanchex (2014)
  - Strong transitions
  - Turbulence? (Next talk: R Durrer)
  - Inverse acoustic cascade?

    Kalaydzhyan, Shuryak (2014)
  - Vacuum, runaway transitions
- Implications for eLISA 2034, DECIGO, BBO

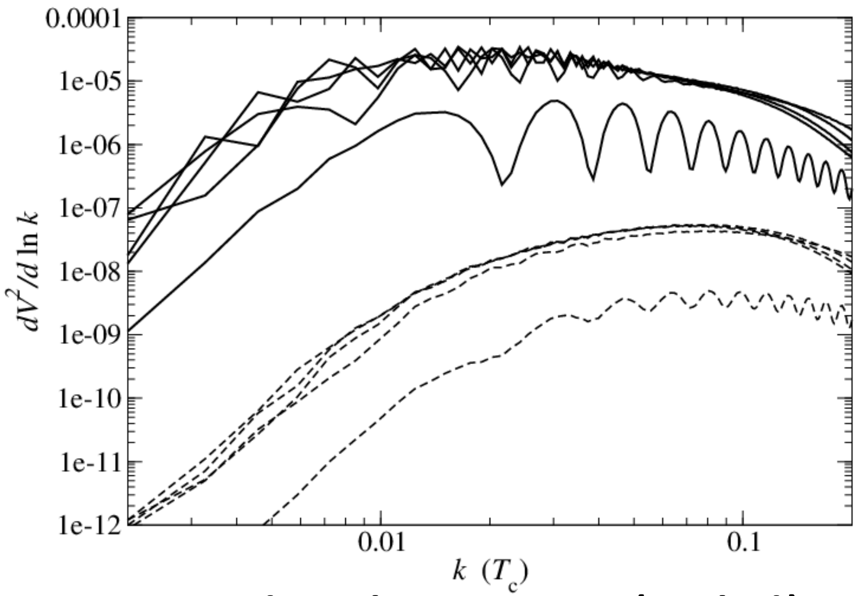


Back-up slides

Sound waves dominate rotational motion of fluid

- Longitudinal motion = sound waves

Type	η/T_c	v_w	N_b	$\bar{U}_{\mathrm{f,max}}$	$\bar{U}^{\perp}_{\mathrm{f,max}}$
Weak	0.06	0.83	988	0.0052	0.00037
			125	0.0052	0.00028
	0.1	0.68	988	0.0084	0.00036
			125	0.0082	0.00026
			37	0.0080	0.00021
	0.121	0.59	988	0.0116	0.00052
	0.15	0.54	988	0.0102	0.00037
			37	0.0120	0.00025
	0.2	0.44	32558	0.0059	0.00047
			988	0.0073	0.00031
			125	0.0075	0.00023
			37	0.0078	0.00019
	0.4	0.24	988	0.0036	0.00049
Wk. (Sc.)	0.4	0.44	988	0.0075	0.00029
Interm.	0.4	0.44	988	0.0595	0.00328



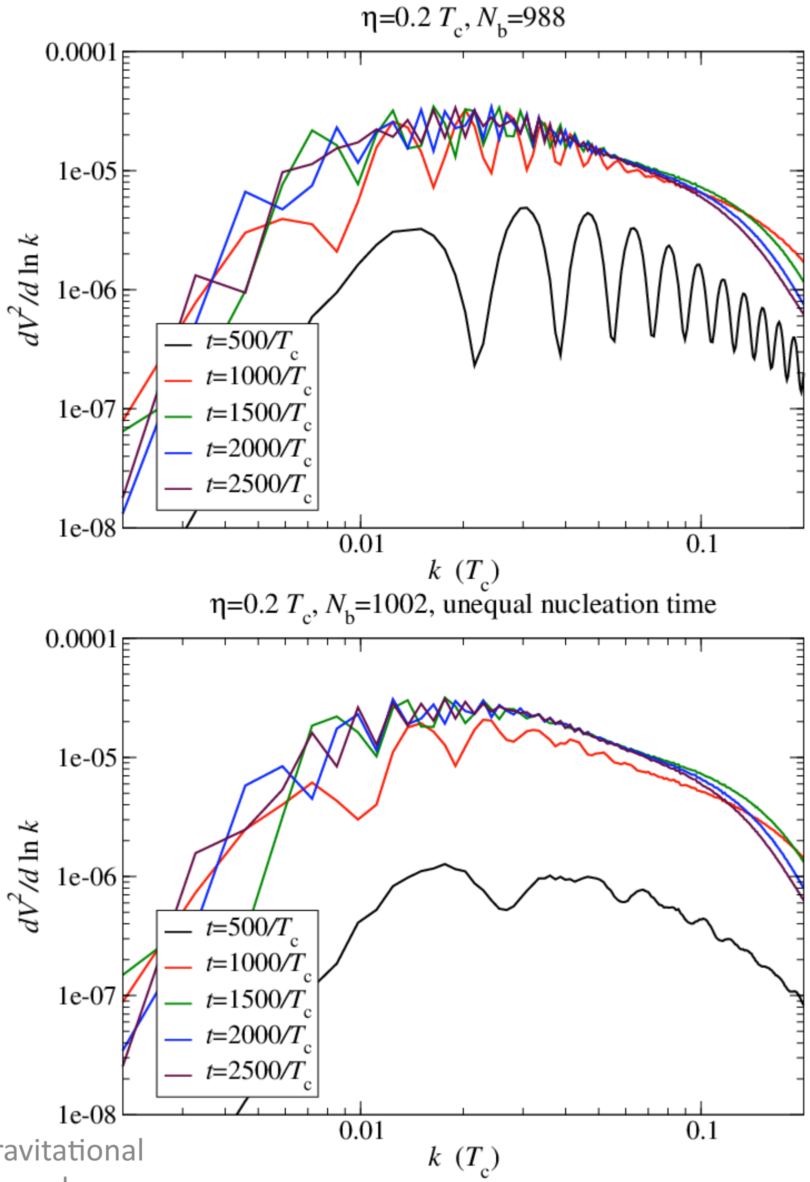
Longitudinal motion (solid)
Rotational motion (dashed)
[weak deflagration]

Ringing in velocity power spectrum

- Simultaneous bubble nucleation makes bubbles closer in size
- Causes "ringing" in velocity power spectrum
- Check by nucleating with probability density

$$p(t) = p_0 e^{\beta t}$$

- ($\beta = 200\ T_c$)



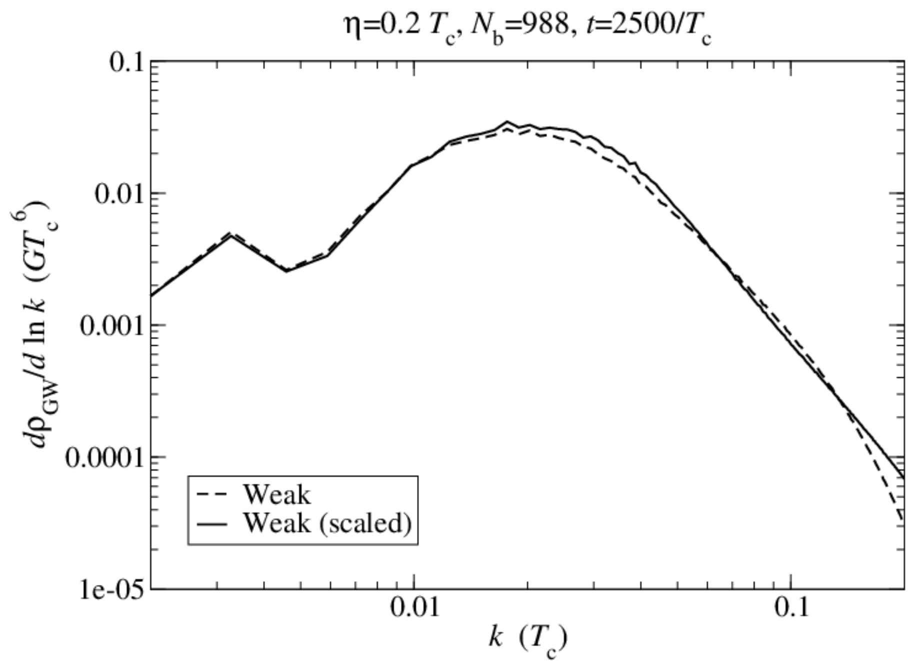
Scaling to physical parameter values

$$\gamma \to r^2\gamma, \quad A \to r^3 A, \quad \lambda \to r^4\lambda, \quad \eta \to r\eta,$$
$$\phi(x) \to r^{-1}\phi(rx), \quad V^i(x) \to V^i(rx),$$
$$E(x) \to E(rx), \qquad Z_i(x) \to Z_i(rx),$$

- Leaves latent heat alone
- Surface tension σ
  wall width ℓ

$$\sigma \to r^{-1}\sigma, \quad \ell \to r^{-1}\ell$$

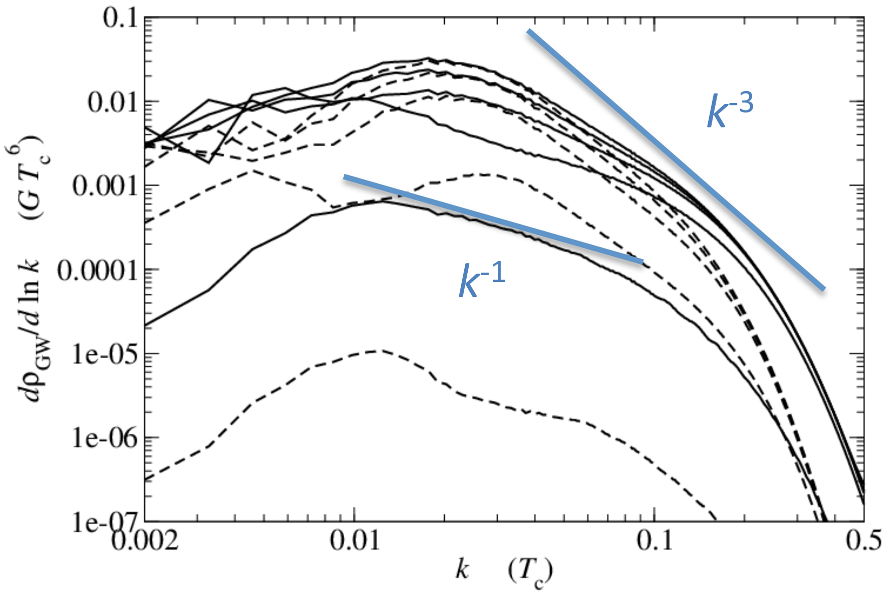
- Physical limit: r large
  (constant bubble sep. R_*)



Scaling with r = 2

GWs from field + fluid vs. fluid only

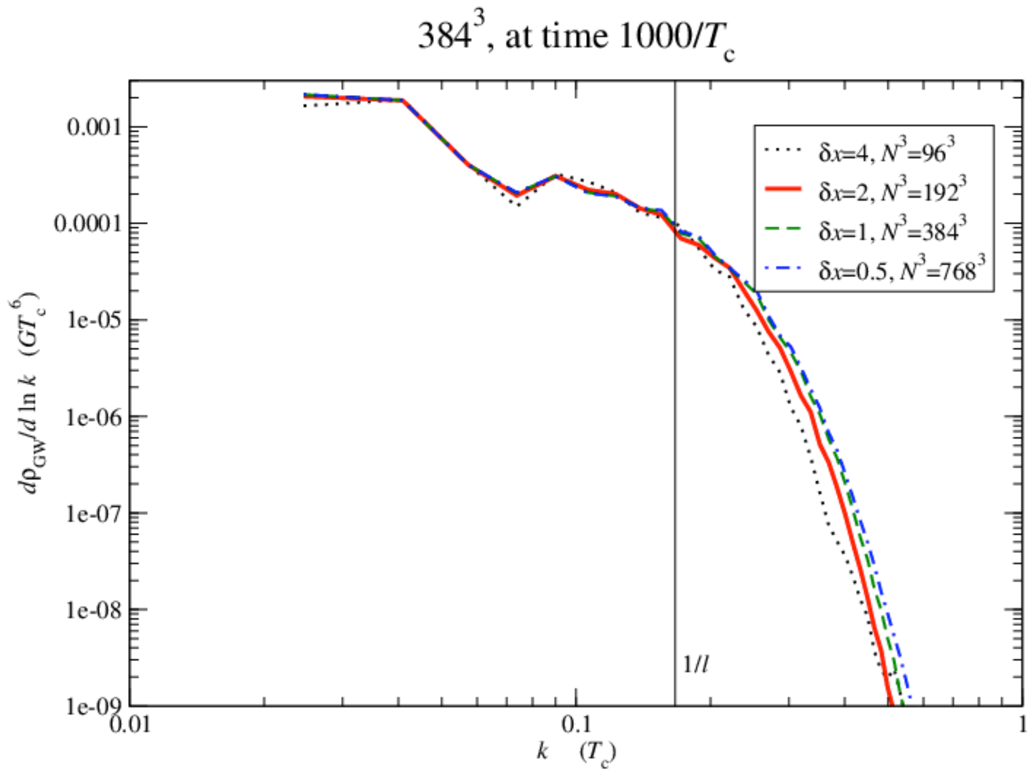
- Early stages dominated by field (GW pow spec ~ k^{-1})
- Num sims exaggerate relative importance of GWs from field
- Surface tension σ
  wall width ℓ
  latent heat $\mathcal{L}$
  bubble radius R
- Energy in field ~ $R^2\sigma$
- Energy in fluid ~ $R^3\mathcal{L}$
- Ratio ~ ℓ/R_*



Fluid only (dashed)
Field + fluid (solid)

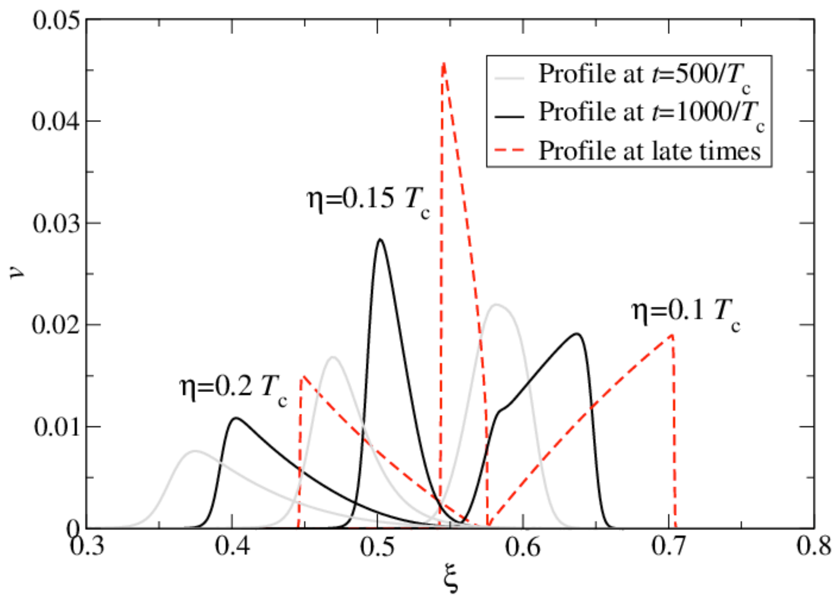
Lattice spacing checks

- Calculate GWs from single bubble self-collision
- Lattice spacings up to $\delta x = 4/T_c$
- Wall width $\ell = 6/T_c$
- Run with $\delta x = 2/T_c$



Radial velocity profiles

- Bubble surrounded by a radial velocity field $V_r(r,t)$
- Tends towards self-similar *asymptotic profile*
- Function of $\xi = r/t$
- Higher speed bubbles have shorter time to reach asymptotic profile before collision



Fluid length scale

- Fluid length scale L_f extracted from velocity spectral density $P_V(k)$
- Use integral scale

$$\xi_{\rm f} = \frac{1}{\langle V^2 \rangle} \int \frac{d^3k}{(2\pi)^3} |k|^{-1} P_V(k),$$

- $\xi_f \sim R_*$ = (average bubble separation)