The bubble wall velocity

Cosmological phase transition front propagation

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$V(\phi) = -m^2\phi^2 + \lambda\phi^4$ (tree-level)

$$+\sum_i \frac{\pm g_i}{64\pi^2}\left[m_i^4(\phi)\left(\log\left(\frac{m_i^2(\phi)}{m_i^2(v)}\right) - \frac{3}{2}\right) + 2m_i^2(\phi)m_i^2(v)\right]$$

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$$\Delta V(\phi, T) = \sum_i (\pm g_i)\int \frac{d^3p}{(2\pi)^3}\log\left(1 \mp e^{-\sqrt{p^2+m_i(\phi)^2}/T}\right)$$

finite-temperature corrections

symmetry restoration at high T

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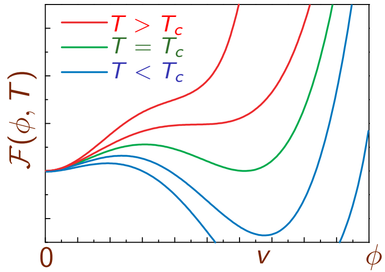

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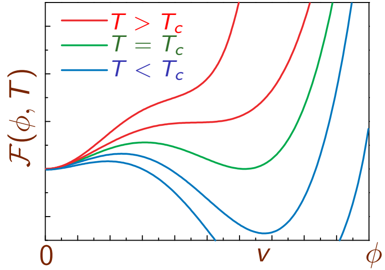

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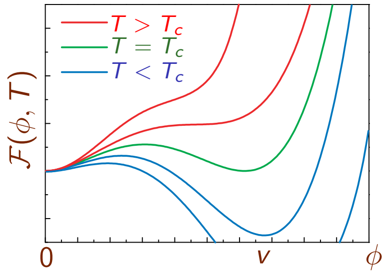

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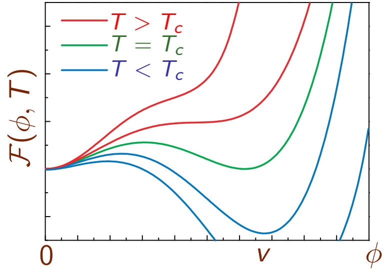

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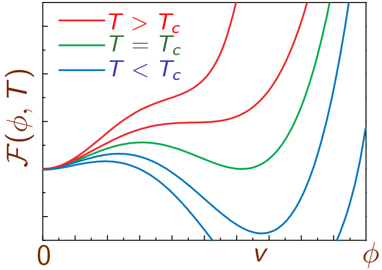

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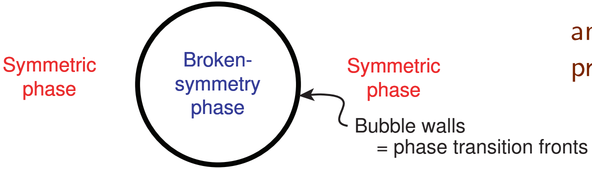
- The two phases are characterized by two different EOS
  $\mathcal{F}_u(T) = \mathcal{F}(0, T)$ and $\mathcal{F}_b(T) = \mathcal{F}(\phi_b(T), T)$
  unbroken-symmetry phase broken-symmetry phase

Bubble nucleation and expansion

At a temperature $T_n < T_c$ (supercooling) bubbles nucleate



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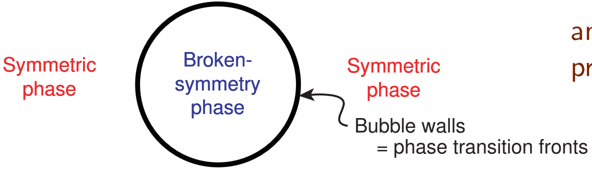

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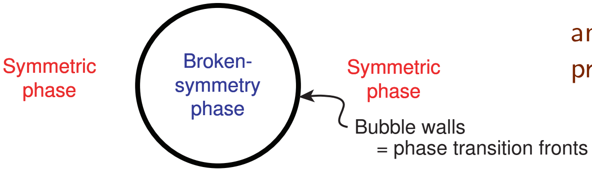

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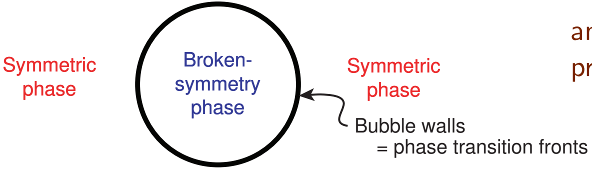

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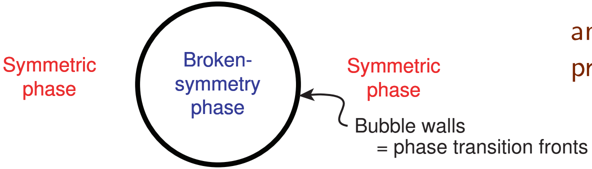

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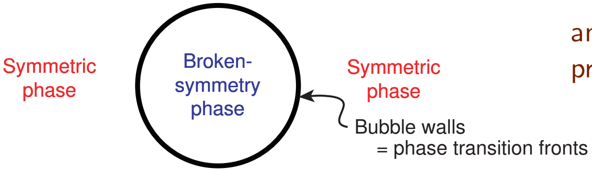

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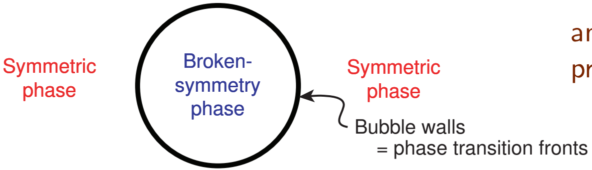

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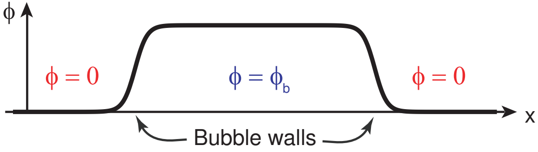

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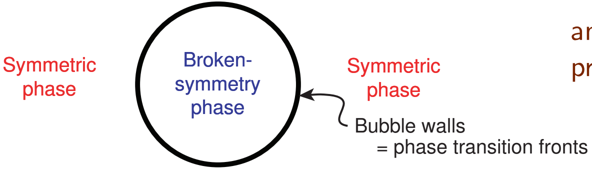

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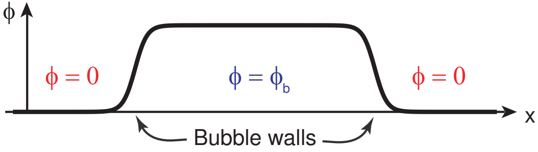

- Bubbles expand (bubble walls move): $\phi = \phi(\mathbf{x}, t)$

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- So we must also consider the fluid equations

$$\partial_\mu \left(T^{\mu\nu}_{\text{plasma}} + T^{\mu\nu}_{\phi} \right) = 0$$

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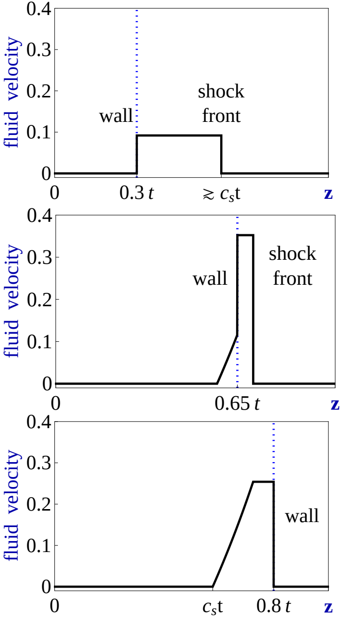
- The fluid equations depend on $c_s^2 = dp/de$
  (c_s = speed of sound; for radiation, $c_s = \sqrt{1/3}$)

Steady state solutions (planar walls)



Weak deflagration:

- subsonic: $v_w < c_s$
- preceded by a shock front
- $T_u > T_n$

Jouguet deflagration
[Kurki-Suonio & Laine, 1995]

- supersonic: $c_s < v_w < v_J$
- shock front and rarefaction wave

Weak detonation:

- supersonic: $v_w > v_J(T_u) > c_s$
- followed by a rarefaction wave
- $T_u = T_n$ $(T_b > T_u)$

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- The equation for the wall is derived from the field equation

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- Go to the reference frame of the wall: $\phi = \phi(z)$,
  $\partial_\mu \partial^\mu \phi = \phi''(z)$
- Multiply $\times \phi'(z)$, integrate $\int dz$ across the wall
  $\Rightarrow$ the first term vanishes

The wall equation

We obtain

$$\int_b^u dz \frac{d\phi}{dz} \frac{\partial \mathcal{F}(\phi, T)}{\partial \phi} + \sum_i \int_b^u dz \frac{dm_i^2}{dz} \int \frac{d^3p}{(2\pi)^3 2E_i} \delta f_i = 0$$

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- F_{dr} does not depend on the wall velocity

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- An inhomogeneous temperature, so $F_{\rm dr} = p_b(T_b) - p_u(T_u) + \int_b^u s(\phi, T) dT$

Friction force: the non-relativistic regime

$$F_{\mathrm{fr}} = \sum_i \int_b^u dz \frac{dm_i^2}{dz} \int \frac{d^3p}{(2\pi)^3 2E_i} \delta f_i(z,p)$$

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For the SM, this infrared boson contribution gives a larger friction than the thermal particles contribution

Friction force: the non-relativistic regime

Parametric dependence (analytic approximations)

To lowest order in v_w we have $F_{\rm fr} = -\eta_{NR} v_w$
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- thermal particles: $\eta_{\rm th} \sim \sum_i (g_i h_i^4/\bar{\Gamma})(\phi_b^2 \sigma)$
  - h_i = coupling of particles with Higgs
  - $\bar{\Gamma}$ = effective interaction rate $\sim 10^{-2} T$
- infrared bosons: $\eta_{\rm ir} \sim \sum_i (g_i m_D^2/L_w) \log(m_i L_w)$

[See, e.g., A.M. & A.Sánchez, NPB 825, 151 (2010)]

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$$\partial_\mu \partial^\mu \phi + \frac{\partial \mathcal{F}(\phi, T)}{\partial \phi} + \tilde{\eta}(\phi)\, u^\mu\, \partial_\mu \phi = 0$$

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- To convert the field equation into a wall equation, assume $\phi = \phi(z)$ in the rest frame of the wall, multiply $\times \partial_z \phi$, integrate $\int dz$. We obtain

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Here, γv comes from u^z ($v \equiv v_z \sim -v_w$)

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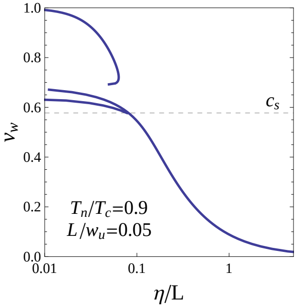
- With these approximations, v_w depends only on:
  - the friction coefficient η,
  - the latent heat L,
  - the amount of supercooling T_n/T_c

The model $\tilde{\eta}\, u^\mu \partial_\mu \phi$

Wall velocity as a function of the friction

Solutions

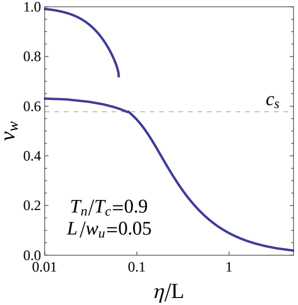


Agree with numerical computations
Kurki-Suonio & Laine, 1995-1996, *static case*

The model $\tilde{\eta}\, u^{\mu}\partial_{\mu}\phi$

Wall velocity as a function of the friction

Stable solutions (A.M., A. Membiela, 2014)



Also in agreement with Kurki-Suonio & Laine, 1995-1996, *dynamic calculation*

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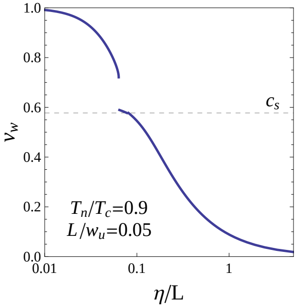
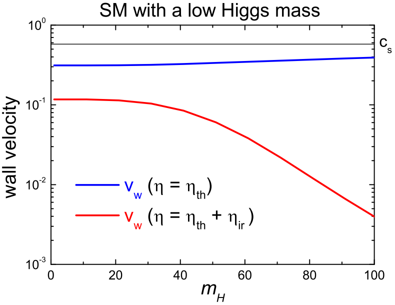
Wall velocity as a function of the friction

Realized in the phase transition



(according to the dynamic calculation of Kurki-Suonio & Laine, 1995-1996)

Application to physical models (electroweak PT)



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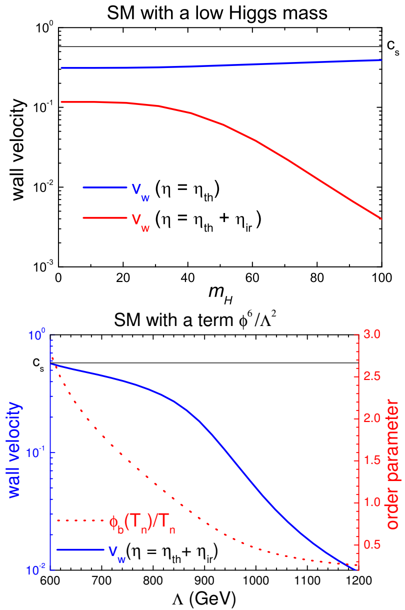

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- No stationary state: if $F_{\rm net}(T_n) > 0$ we have an accelerated wall (runaway)

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- The term $\tilde{\eta}(\phi)\, u^\mu \partial_\mu \phi$ is too simplistic, since $F_{\rm fr} \sim v\gamma$ does not saturate in the UR limit

A friction which saturates

Espinosa, Konstandin, No, Servant, 2010

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A friction which saturates

Espinosa, Konstandin, No, Servant, 2010

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Notice that the vector $\partial_\mu \phi$ is $(0, 0, 0, \partial_z \phi)$ in the wall frame

A friction which interpolates between the NR and UR limits

- Consider instead the term [A.M., 2013]

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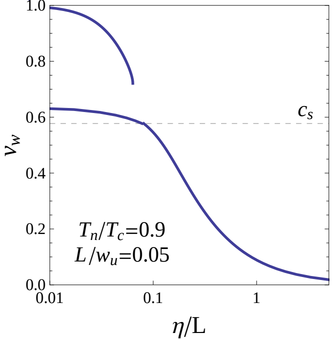
- To lowest order in $m(\phi)/T$, we have

$$F_{\rm fr} = \sum_{\rm bos}(g_i T/12\pi)\left[m_i^3(\phi_b) - m_i^3(\phi_u)\right] \sim \boxed{\sum g_i h_i^3 \phi_b^3 T}$$

A friction which interpolates between the NR and UR limits

A.M., 2013

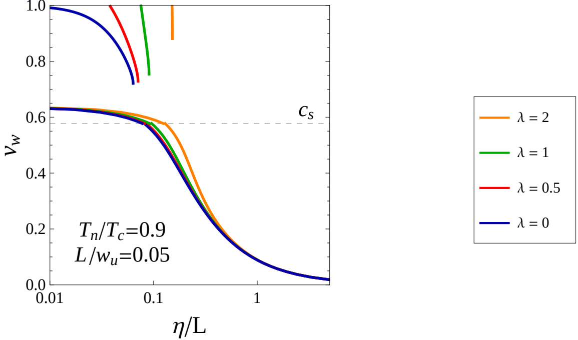


Old phenomenological model:

$\lambda = 0$

A friction which interpolates between the NR and UR limits

A.M., 2013



A friction which interpolates between the NR and UR limits

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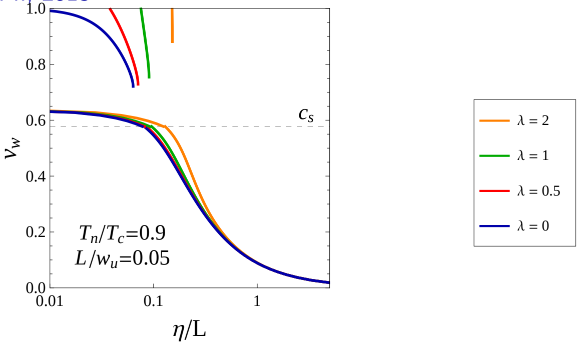

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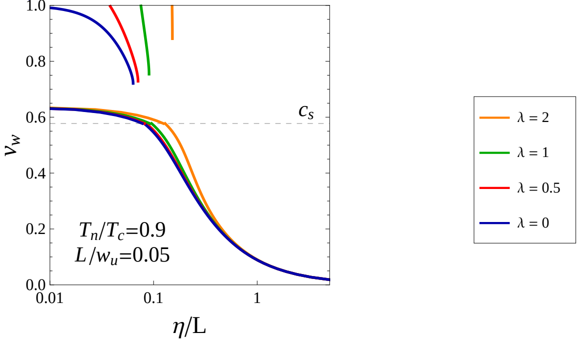

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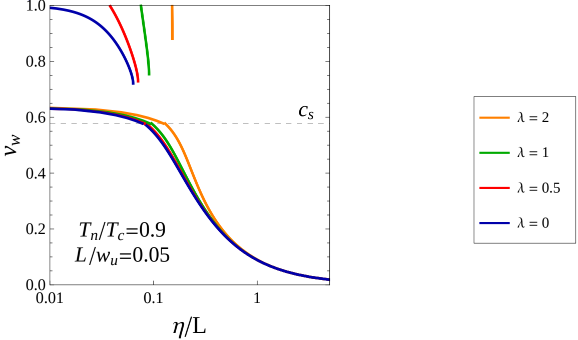

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- This happens for $F_{\rm dr}(T_u, T_b) = \eta/\lambda$ (i.e., for $v_w = 1$) with $F_{\rm dr} \simeq p_b(T_b) - p_u(T_u) - \langle s \rangle (T_b - T_u)$

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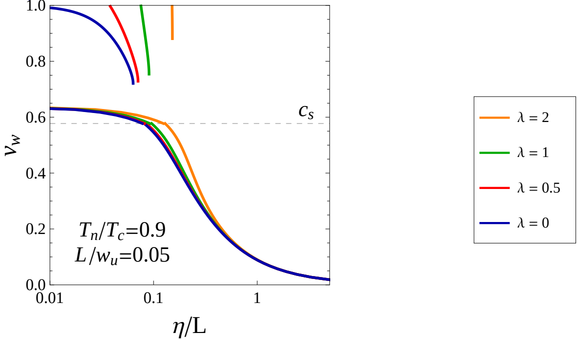

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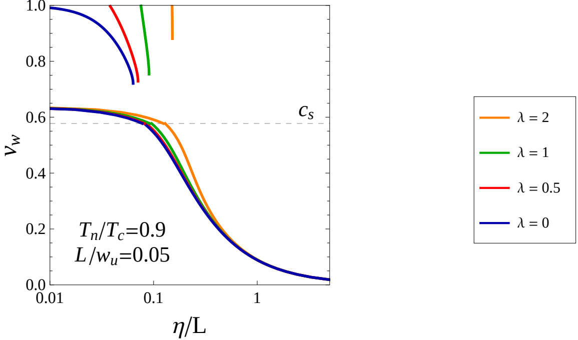

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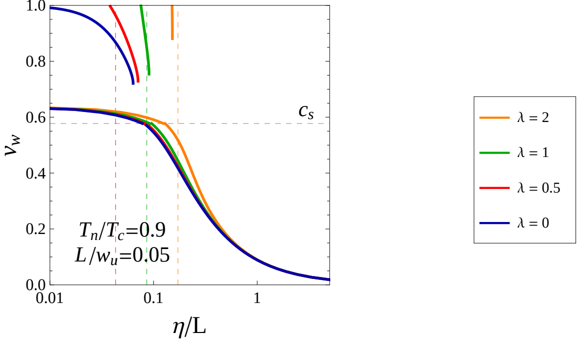

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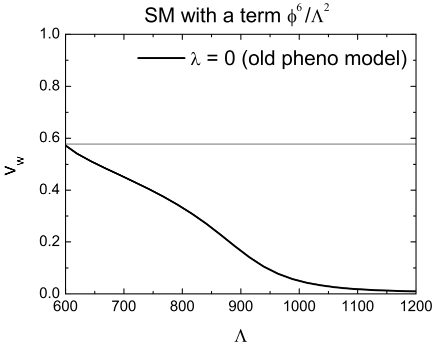
- The runaway wall solution is possible if $F_{\rm dr}(T_n, T_n) = \eta/\lambda$ i.e., $F_{\rm dr} = p_b(T_n) - p_u(T_n)$
- As a consequence, stationary and runaway solutions coexist

Application to physical models (preliminary results)



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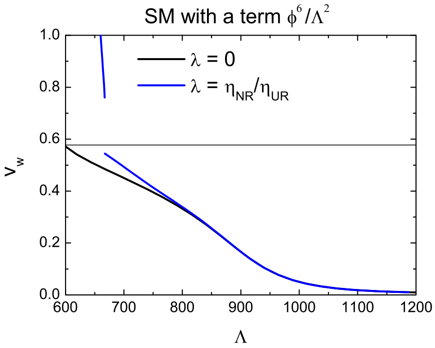

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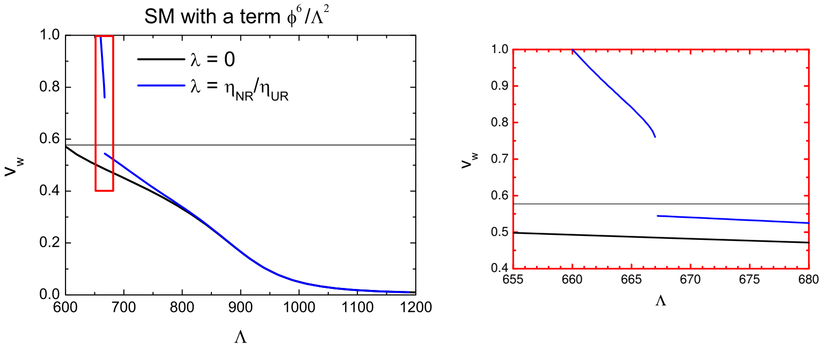

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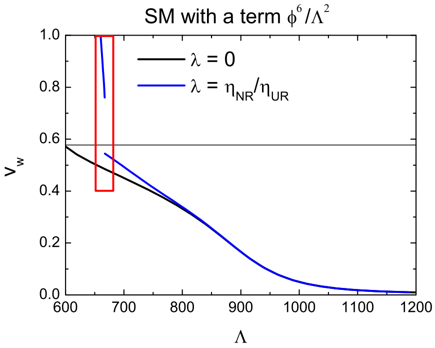

Outlook

Apply to GW generation in the electroweak phase transition

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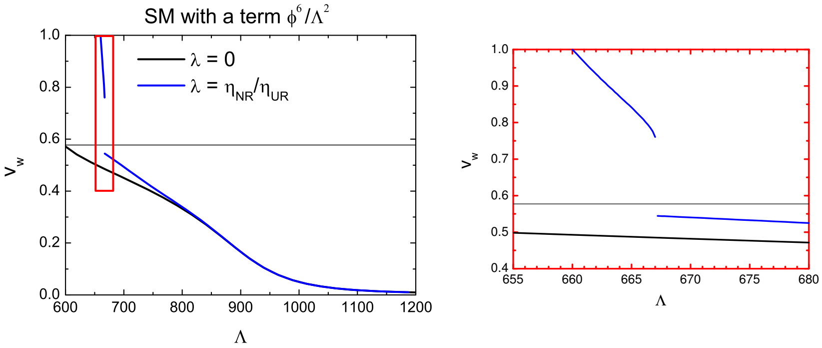

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Extra slides

Weak deflagrations are unstable for $v_w < v_{\rm crit}$ (good for GWs)