Gravitational waves from phase transitions: Bubble collisions and turbulence

Ruth Durrer

Department of Theoretical Physics and Center for Astroparticle Physics
Geneva University
Switzerland

eLISA workshop, April 14, 2015
Outline

1. Introduction

2. Sources of gravitational waves

3. The spectrum
   - Causality
   - Peak position

4. The electroweak phase transition
   - Bubble collisions
   - Turbulent MHD

5. Conclusions
The Universe has expanded and cooled down from a very hot initial state to (presently) 2.7°K. It seems likely that it underwent several phase transitions during its evolution of adiabatic expansion.
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In this talk I present a semi-analytical evaluation of the GW signal from a first order phase transition in terms of the parameters characterizing the transition.
If a phase transition at $T = T_*$ generates a (relativistic) source of gravitational waves with energy density $\rho_X$, the GW spectrum has the following properties:

- **The peak frequency** (correlation scale) is larger than the Hubble rate, $k_* \gtrsim H_*$. 

### Ruth Durrer (Université de Genève)

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- Its amplitude is of the order of 

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- The behaviour of the spectrum on small scales, $k \gg k_*$ depends on the details of the source.
Sources of gravitational waves

Gravitational waves are sourced by fluctuations in the energy momentum tensor which have a non-vanishing spin-2 contribution.

\[ ds^2 = a^2 \left( d\eta^2 + (\gamma_{ij} + 2h_{ij})dx^i dx^j \right) \]

where \( h_{ij} \) is transverse and traceless. In Fourier space \( k^i h_{ij} = h^i = 0 \).
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\[ \left( \partial^2_\eta + 2\mathcal{H} \partial_\eta + k^2 \right) h_{ij} = 8\pi G a^2 \Pi_{ij} \]

Here \( \Pi_{ij}(k) \) is the tensors type (spin-2) anisotropic stress and \( \mathcal{H} = a' a \).

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Duration: \( \Delta \eta_* \leq \mathcal{H}_*^{-1} \), bubble size: \( R = v_b \Delta \eta_* \), \( v_b = \) bubble velocity.
\( k_* = (\Delta \eta_*)^{-1} \) or \( R^{-1} \).
Because of causality, the correlator \( \langle \Pi_{ij}(\eta_1, x)\Pi_{lm}(\eta_2, y) \rangle = M_{ijlm}(\eta_1, \eta_2, x - y) \) is a function of compact support. For distances \( |x - y| > \max(\eta_1, \eta_2) \), \( M \equiv 0 \).
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We decompose \( \Pi_{ij} \) into two helicity modes which we assume to be uncorrelated (parity),

\[
\Pi_{ij}(\eta, k) = e_{ij}^+ \Pi_+ (\eta, k) + e_{ij}^- \Pi_- (\eta, k)
\]

\[
\langle \Pi_+ (\eta, k)\Pi^*_+ (\eta', k') \rangle = \langle \Pi_- (\eta, k)\Pi^*_- (\eta', k') \rangle = (2\pi)^3 \delta^3 (k - k') \rho^2_X P(\eta, \eta', k)
\]

\[
\langle \Pi_+ (\eta, k)\Pi^*_- (\eta', k') \rangle = 0.
\]

Here \( \rho_X \) is the energy density of the component \( X \) with anisotropic stress \( \Pi \) which has been factorized in order to keep \( k^3 P(\eta, \eta', k) \) dimensionless.
Typical frequencies and the spectrum

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  Here \( \rho_X \) is the energy density of the component \( X \) with anisotropic stress \( \Pi \) which has been factorized in order to keep \( k^3 P(\eta, \eta', k) \) dimensionless.
- Causality implies that the function \( P(\eta, \eta', k) \) is analytic in \( k \). We therefore expect it to start out as white noise and to decay beyond a certain correlation scale \( k_*(\eta, \eta') > \min(1/\eta, 1/\eta') \).
The anisotropic stress power spectrum from Kolmogorov turbulence. On small scales $P \propto k^{-11/3}$. 
If the gravitational wave source is active only for a short duration $\Delta \eta^*$ (less than one Hubble time), we can neglect the damping term $2\mathcal{H}^*$ in the equation of motion for $h$. 

The gravitational wave energy density is given by

$$\rho_{gw}(\eta, x) = \frac{1}{8\pi} G a^2 \langle \partial_\eta h_{ij}(\eta, x) \partial_\eta h_{ij}^*(\eta, x) \rangle$$

If the Universe is radiation dominated during the phase when the gravitational waves are generated, this gives on large scales, $k < k^*$

$$d \Omega_{gw} d \log(k)(\eta_0) = \frac{3}{4} \Omega_{\text{rad}}(\eta_0) 4\pi^2 \left(\Omega_X(\eta^*) \Omega_{\text{rad}}(\eta^*)\right)^2 H^2 k^3 \Re\left[\hat{P}(k, k, k)\right].$$
If the gravitational wave source is active only for a short duration $\Delta \eta_*$ (less than one Hubble time), we can neglect the damping term $2H_*$ in the equation of motion for $h$.

The solution with vanishing initial conditions is then

$$h(k, \eta) = \frac{4i\pi G\alpha_\ast^3}{ak} \left[ e^{-ik\eta} \int_{\eta_*}^{\eta_* + \Delta \eta_*} d\eta' e^{ik\eta'} \Pi(\eta', k) + e^{ik\eta} \int_{\eta_*}^{\eta_* + \Delta \eta_*} d\eta' e^{-ik\eta'} \Pi(\eta', k) \right]$$

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$$ \frac{d\Omega_{gw}}{d \log(k)}(\eta_0) = \frac{3\Omega_{rad}(\eta_0)}{4\pi^2} \left( \frac{\Omega_X(\eta_*)}{\Omega_{rad}(\eta_*)} \right)^2 H_\ast^2 k^3 \text{Re}[\hat{\mathcal{P}}(k, k, k)]. $$
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\hat{P}(\omega, \omega', k) \equiv \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta' P(\eta, \eta', k) e^{i(\omega \eta - \omega' \eta')}. 
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On large scales, \( k < k_* > \mathcal{H}_* \) the GW energy density from a 'causal' source always scales like \( k^3 \). This remains valid also for long duration sources. \( 1/k_* \) is the correlation scale which is smaller than the co-moving Hubble scale \( 1/\mathcal{H}_* = \eta_* \).

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- The behavior or the spectrum close to the peak and its decay rate on smaller scales depends on the source characteristics, on its temporal behavior (continuity, differentiability) and its power spectrum.
For a totally incoherent source, \( P(\eta, \eta', k) = \delta(\eta - \eta') \Delta \eta \star P(\eta, \eta, k) \) the peak position of the GW spectrum is determined by the peak of the spatial Fourier transform of the source.
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For a coherent source, \( P(\eta, \eta', k) = \sqrt{P(\eta', \eta', k) P(\eta, \eta, k)} \), when \( P(\eta, \eta, k) \) is continuous in time but not differentiable (bubble collisions) the peak position of the GW spectrum \( \propto k^3 P(k, k, k) \) is determined by the peak of the temporal Fourier transform of the source.

(see Caprini, RD and Servant, 2009, arXiv:0909.0622 for details)
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For a source with finite coherence time, 
\[ P(\eta, \eta', k) = \sqrt{P(\eta', \eta', k)P(\eta, \eta, k)}\Theta(x_c - |\eta - \eta'|k), \quad x_c \sim 1 \] the GW spectrum is again determined by the peak of the spatial Fourier transform of the source.

(see Caprini, RD and Servant, 2009, arXiv:0909.0622 for details)
The GW energy density spectrum in the incoherent (red, dashed), tophat (black, dotted) and coherent (blue solid). The parameters are: $T_* = 100$ GeV, $\Delta \eta_* H_* = 0.01$, $\Omega_X/\Omega_{\text{rad}} = 2/9$ ($\langle v^2 \rangle = 1/3$). Caprini, RD and Servant, 2009, arXiv:0909.0622
According to the standard model, the electroweak transition is not even second order, but only a cross-over. If this is true, the ew transition does not lead to the formation of gravitational waves.

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The peak amplitude of the resulting GW spectrum depends on the strength of the phase transition, $\alpha = \rho V / \rho_{\text{rad}}$ and the velocity of the bubble wall, $v_b$. The spectrum goes like $d^2 \Omega_{GW} / d \ln (k) \propto k^3$, $k < k^\ast \approx \pi / \Delta \eta$ and $d^2 \Omega_{GW} / d \ln (k) \propto k^{-1}$, $k > k^\ast$. The peak sensitivity of eLISA is supposed to be about $h_2^2 d^2 \Omega_{GW} / d \ln (k) \bigg|_{k = k_p} \approx 10^{-12}$, $k_p \sim 10^{-3}$ Hz.


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Caprini, RD, Konstandin, Servant, 2009 arXiv:0901.1661 ($\beta = (\Delta \eta_* )^{-1}$)
The electroweak phase transition: GW’s from bubble collisions

Huber & Konstandin 2008

\( \Omega_{GW} \) from colliding bubbles, numerical results, \( \Omega_X/\Omega_{\text{rad}} = 0.03 \).
The electroweak phase transition: GW’s from turbulence and magnetic fields

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Furthermore, in the broken phase the electromagnetic field does generically not vanish. The high conductivity rapidly damps the electric fields so that we are left with a magnetic field in a turbulent plasma, MHD turbulence.

Because both, the vorticity and the magnetic field are divergence free, causality requires that both,

$$P_v(k) \propto k^2 \quad \text{for small } k,$$

and the functions

$$\langle v_i(k)v_j(k') \rangle = \left(\frac{2\pi}{2\pi}\right)^3 \delta^3(k-k')\hat{k}_j\hat{k}_i - \delta_{ij}P_v(k),$$

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must be analytic because of causality.

The behavior of the spectrum on scales smaller than the correlation scale $k > k_c$ is expected to be a Kolmogorov spectrum for the vorticity field, $P_v \propto k^{-11/3}$ and an Iroshnikov–Kraichnan spectrum, $P_B \propto k^{-7/2}$ or a Kolmogorov spectrum, $P_B \propto k^{-11/3}$ for the magnetic field.

For the induced GW spectrum this yields

$$\frac{d\Omega_{GW}}{d\ln(k)} \simeq \Omega_{rad}(\eta_0) \left(\frac{\Omega_{\text{rad}}(\eta_0)}{\Omega_{\text{rad}}(\eta_0)}\right)^2 \times \left\{\begin{array}{ll}
\left(k/k^*_\eta_0\right)^3 & \text{for } k < k^*_\eta_0 \\
(k/k^*_\eta_0)^{-\alpha} & \text{for } k > k^*_\eta_0
\end{array}\right.$$

For $\bullet = v$ we have $\alpha = 11/3 - 1 = 8/3$ and for $\bullet = B$ we have $\alpha = 7/2 - 1 = 5/2$.

(See Caprini & RD, 2006, astro-ph/0603476)
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$\omega_{GW}$ from magnetic fields (red) and turbulence (blue), total (black). Modelling the time-decorrelation of the source (Kraichnan decorrelation) by a ’top-hat’ in Fourier space. Sensitivity curves from A. Buonanno 2003.

Caprini, RD, Servant, arXiv:0909.0622
We also consider a phase transition at $T = 5 \times 10^6$ GeV with $\Delta \eta_\ast = 0.02/H_\ast$. 

Caprini, RD, Servant, arXiv:0909.0622
It is difficult to estimate $\Omega_B(\eta_*)$ or $\Omega_V(\eta_*)$ accurately, but since causality requires the spectra to be so blue, $\frac{d\Omega_B(k,\eta_*)}{d\ln(k)} \propto k^5$, the limit on gravitational waves (which comes from small scales $k \simeq k_*$) yields very strong limits on primordial magnetic fields on large scales. Already from the simple nucleosynthesis constraint, $\Omega_{GW} \lesssim 0.1 \Omega_{\text{rad}}$. 
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This is purely a consequence of the redness of the magnetic field spectrum. The simple requirement that $\rho_B(t_*) < \rho_c(t_*)$ yields

$$k_1^{3/2}B(k_1) < 10^{-29}\text{Gauss} \left( \frac{k_1 \cdot 0.1\text{Mpc}}{k_* \cdot 10^3\text{sec}} \right)^{5/2}.$$

$(10^3\text{sec} = 10^{-11}\text{Mpc})$
There is a possible way out of these stringent constraints: During the electroweak phase transition parity is broken. Actually, the Chern-Simon winding number of the gauge field, $N_{CS} \propto \int F \wedge A$, which is related to the baryon number, has an electromagnetic part to it which is nothing else than the helicity of the magnetic field, $H = V^{-1} \int_V A \cdot B d^3x$ (Vachaspati, 2001).
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Such helical magnetic fields and or turbulence lead to $T$-$B$ and $E$-$B$ correlations in the CMB, and they also generate gravitational waves with non-vanishing helicity (Caprini, Kahniashvili, RD, 2004; Kahniashvili, Gogoberize & Ratra, 2005).
The electroweak phase transition: helical magnetic fields and parity violation

Helicity conservation for a helical field leads to an inverse cascade in the evolution of the magnetic field:

This can move power from small to larger scales. However, this is not quite sufficient to lead to sufficient large scale magnetic fields for the electroweak phase transition, but it can work for the QCD phase transition (Caprini, RD, Fenu 2009).
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In this case, the GW background is not parity symmetric. There are more GW’s of one helicity than of the other.
First order phase transitions stir the relativistic cosmic plasma sufficiently to induce the generation of a (possibly observable) stochastic gravitational wave background.
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Generically, the density parameter of the GW background is of the order of

\[ \Omega_{GW}(t_0) \simeq \Omega_{rad}(t_0) \left( \frac{\Omega_X(t_*)}{\Omega_{rad}(t_*)} \right)^2 \left( \mathcal{H}_* \Delta \eta_* \right)^2 \]
Conclusions

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- The spectrum grows like \( \frac{d\Omega_{GW}(k,t_0)}{d \ln(k)} \propto k^3 \) on large scales and decays on scales smaller than the correlations scale \( k_* \gtrsim 1/\eta_* \). The decay law depends of the physics of the source.
Conclusions

If the SM holds, the electroweak phase transition is not of first order and does (probably) not generate an appreciable gravitational wave background. However, simple deviations from the SM can make it first order (like adding a Higgs singlet (Ashoorioon & Konstandin 2009)).
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- In this case we also expect a parity violating gravitational wave background,
  $|h_+(k)|^2 \neq |h_-(k)|^2$. 