Gravitational waves from phase transitions: Bubble collisions and turbulence

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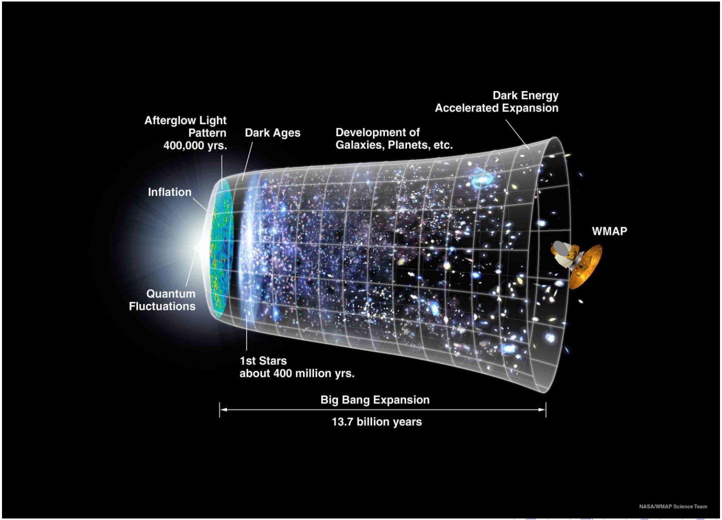
Outline

Introduction

The Universe has expanded and cooled down from a very hot initial state to (presently) 2.7^oK. It seems likely that it underwent several phase transitions during its evolution of adiabatic expansion.



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- The electroweak transition $T_{ew} \simeq 10^2$GeV, $\quad t_{ew} \simeq 10^{-10}$sec, ,
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In this talk I present a semi-analytical evaluation of the GW signal from a first order phase transition in terms of the parameters characterizing the transition.

Generic results

If a phase transition at $T = T_*$ generates a (relativistic) source of gravitational waves with energy density ρ_X, the GW spectrum has the following properties

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- The behaviour of the spectrum on small scales, $k \gg k_*$ depends on the details of the source.

Sources of gravitational waves

Gravitational waves are sourced by fluctuations in the energy momentum tensor which have a non-vanishing spin-2 contribution.

$$ds^2 = a^2\left(d\eta^2 + (\gamma_{ij} + 2h_{ij})dx^i dx^j\right)$$

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Einstein's eqn. to first order in h_{ij} give

$$\left(\partial_\eta^2 + 2\mathcal{H}\partial_\eta + k^2\right)h_{ij} = 8\pi G a^2 \Pi_{ij}$$

Here $\Pi_{ij}(\mathbf{k})$ is the tensors type (spin-2) anisotropic stress and $\mathcal{H} = \frac{a'}{a}$.

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Duration: $\Delta\eta_* < \mathcal{H}_*^{-1}$, bubble size: $R = v_b\Delta\eta_*$, v_b = bubble velocity.
$k_* = (\Delta\eta_*)^{-1}$ or R^{-1}.

Typical frequencies and the spectrum

- Because of causality, the correlator $\langle \Pi_{ij}(\eta_1, \mathbf{x})\Pi_{lm}(\eta_2, \mathbf{y})\rangle = \mathcal{M}_{ijlm}(\eta_1, \eta_2, \mathbf{x} - \mathbf{y})$ is a function of compact support. For distances $|\mathbf{x} - \mathbf{y}| > \max(\eta_1, \eta_2)$, $\mathcal{M} \equiv 0$.

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- We decompose Π_{ij} into two helicity modes which we assume to be uncorrelated (parity),

$$\Pi_{ij}(\eta, \mathbf{k}) = e^+_{ij}\Pi_+(\eta, k) + e^-_{ij}\Pi_-(\eta, k)$$

$$\langle \Pi_+(\eta, k)\Pi^*_+(\eta', k')\rangle = \langle \Pi_-(\eta, k)\Pi^*_-(\eta', k')\rangle = (2\pi)^3\delta^3(\mathbf{k} - \mathbf{k}')\rho_X^2 P(\eta, \eta', k)$$

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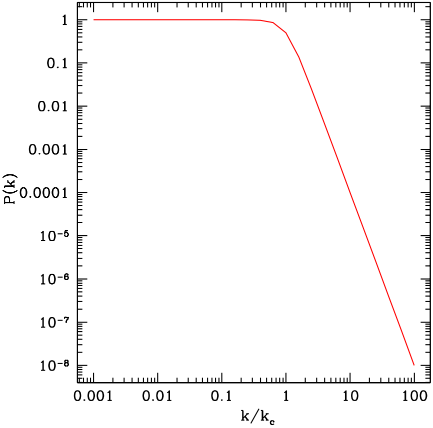
  Here ρ_X is the energy density of the component X with anisotropic stress Π which has been factorized in order to keep $k^3 P(\eta, \eta', k)$ dimensionless.
- Causality implies that the function $P(\eta, \eta', k)$ is analytic in k. We therefore expect it to start out as white noise and to decay beyond a certain correlation scale $k_*(\eta, \eta') > \min(1/\eta, 1/\eta')$.

The spectrum



The anisotropic stress power spectrum from Kolmogorov turbulence. On small scales $P \propto k^{-11/3}$.

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h(\mathbf{k},\eta) &= \frac{4i\pi G a_*^3}{ak}\left[e^{-ik\eta}\int_{\eta_*}^{\eta_*+\Delta\eta_*} d\eta' e^{ik\eta'}\Pi(\eta',\mathbf{k}) + \right. \\
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- The gravitational wave energy density is given by

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- If the Universe is radiation dominated during the phase when the gravitational waves are generated, this gives on large scales, $k < k_*$

$$\frac{d\Omega_{gw}}{d\log(k)}(\eta_0) = \frac{3\Omega_{\rm rad}(\eta_0)}{4\pi^2}\left(\frac{\Omega_X(\eta_*)}{\Omega_{\rm rad}(\eta_*)}\right)^2 \mathcal{H}_*^2 k^3 \mathrm{Re}[\hat{P}(k,k,k)]\,.$$

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- The behavior or the spectrum close to the peak and its decay rate on smaller scales depends on the source characteristics, on its temporal behavior (continuity, differentiability) and its power spectrum.

Peak position

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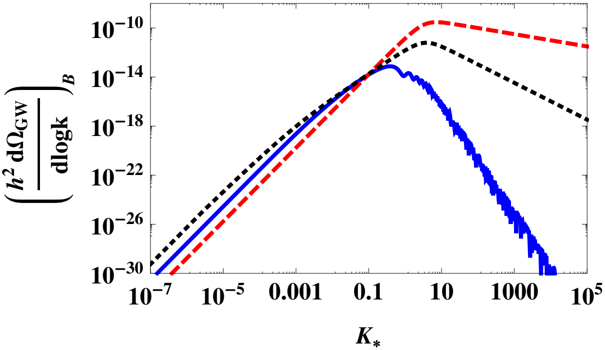
- For a source with finite coherence time, $P(\eta, \eta', k) = \sqrt{P(\eta', \eta', k)P(\eta, \eta, k)}\Theta(x_c - |\eta - \eta'|k)$, $\quad x_c \sim 1$ the GW spectrum is again determined by the peak of the spatial Fourier transform of the source.

(see Caprini, RD and Servant, 2009, arXiv:0909.0622 for details)



The GW energy density spectrum in the incoherent (red, dashed), tophat (black, dotted) and coherent (blue solid). The parameters are: $T_* = 100$ GeV, $\Delta\eta_*\mathcal{H}_* = 0.01$, $\Omega_X/\Omega_{\rm rad} = 2/9 \quad (\langle v^2\rangle = 1/3)$

. Caprini, RD and Servant, 2009, arXiv:0909.0622

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  The spectrum is supposed to peak at the correlation scale $k_* = 1/\Delta\eta_* \simeq 100/\eta_* \sim 10^{-3}$Hz, which is close to the frequency of the peak sensitivity for the space born gravitational wave antenna eLISA.

The electroweak phase transition: GW's from bubble collisions

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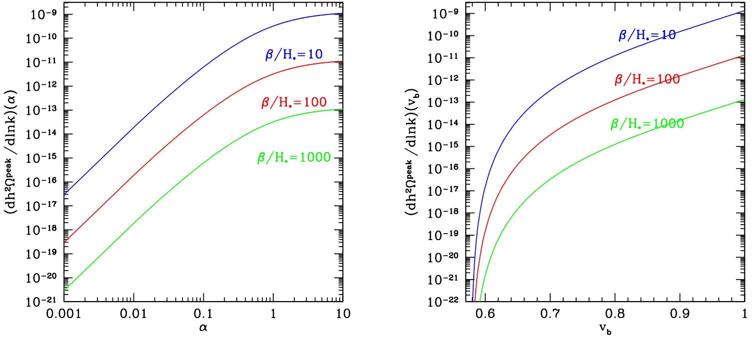
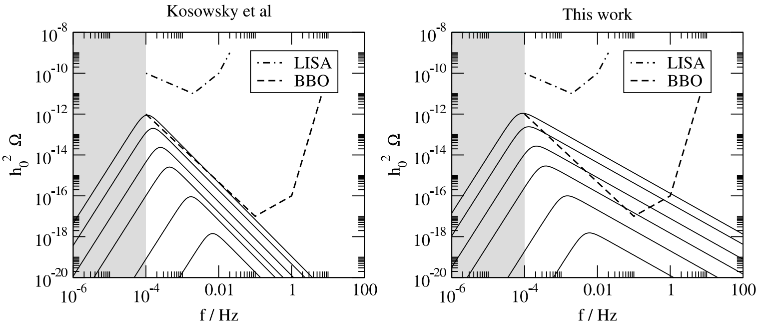


Caprini, RD, Konstandin, Servant, 2009 arXiv:0901.1661 $(\beta = (\Delta\eta_*)^{-1})$



Huber & Konstandin 2008

Ω_{GW} from colliding bubbles, numerical results, $\Omega_X/\Omega_{\rm rad} = 0.03$.

The electroweak phase transition: GW's from turbulence and magnetic fields

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- Because both, the vorticity and the magnetic field are divergence free, causality requires that both, $P_v(k)$ and $P_B(k) \propto k^2$ for small k.
  $\langle v_i(\mathbf{k})v_j(\mathbf{k}')\rangle = (2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')(\hat{k}_j\hat{k}_i - \delta_{ij})P_v(k)$,
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- The behavior of the spectrum on scaler smaller than the correlations scale $k > k_c$ is expected to be a Kolmogorov spectrum for the vorticity field, $P_v \propto k^{-11/3}$ and an Iroshnikov–Kraichnan spectrum, $P_B \propto k^{-7/2}$ or a Kolmogorov spectrum, $P_B \propto k^{-11/3}$ for the magnetic field.

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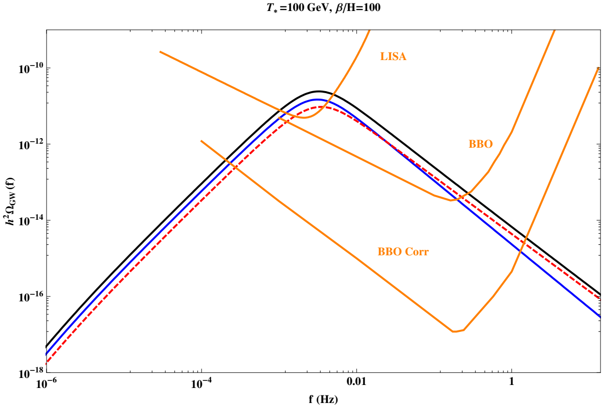
- For the induced GW spectrum this yields

$$\frac{d\Omega_{GW\bullet}(k,\eta_0)}{d\ln(k)} \simeq \Omega_{\rm rad}(\eta_0)\left(\frac{\Omega_\bullet(\eta_*)}{\Omega_{\rm rad}(\eta_*)}\right)^2 \times \begin{cases} (k/k_*)^3 & \text{for } k < k_* \\ (k/k_*)^{-\alpha} & \text{for } k > k_* \end{cases}$$

For $\bullet = v$ we have $\alpha = 11/3 - 1 = 8/3$ and for $\bullet = B$ we have $\alpha = 7/2 - 1 = 5/2$. (See Caprini & RD, 2006, astro-ph/0603476)

The electroweak phase transition: GW's from turbulence and magnetic fields



Caprini, RD, Servant, arXiv:0909.0622

Ω_{GW} from magnetic fields (red) and turbulence (blue), total (black). Modelling the time-decorrelation of the source (Kraichnan decorrelation) by a 'top-hat' in Fourier space. Sensitivity curves from A. Buonanno 2003.

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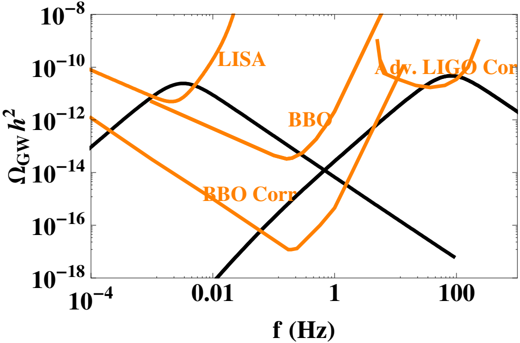

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We also consider a phase transition at $T = 5 \times 10^6$ GeV with $\Delta\eta_* = 0.02/\mathcal{H}_*$.

It is difficult to estimate $\Omega_B(\eta_*)$ or $\Omega_v(\eta_*)$ accurately, but since causality requires the spectra to be so blue, $\frac{d\Omega_B(k,\eta_*)}{d\ln(k)} \propto k^5$, the limit on gravitational waves (which comes from small scales $k \simeq k_*$) yields very strong limits on primordial magnetic fields on large scales.

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This is purely a consequence of the redness of the magnetic field spectrum. The simple requirement that $\rho_B(t_*) < \rho_c(t_*)$ yields

$$k_1^{3/2}B(k_1) < 10^{-29}\text{Gauss}\left(\frac{k_1 \cdot 0.1\text{Mpc}}{k_* \cdot 10^3\text{sec}}\right)^{5/2} .$$

($10^3\text{sec} = 10^{-11}\text{Mpc}$)

The electroweak phase transition: helical magnetic fields and parity violation

- There is a possible way out of these stringent constraints:
  During the electroweak phase transition parity is broken. Actually, the Chern-Simon winding number of the gauge field, $N_{CS} \propto \int F \wedge A$, which is related to the baryon number, has an electromagnetic part to it which is nothing else than the helicity of the magnetic field, $H = V^{-1} \int_V A \cdot B d^3x$ (Vachaspati, 2001).

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- This relates the baryon number the the magnetic helicity.
- Such helical magnetic fields and or turbulence lead to T-B and E-B correlations in the CMB, and they also generate gravitational waves with non-vanishing helicity (Caprini, Kahniashvili, RD, 2004; Kahniashvili, Gogoberize & Ratra, 2005).

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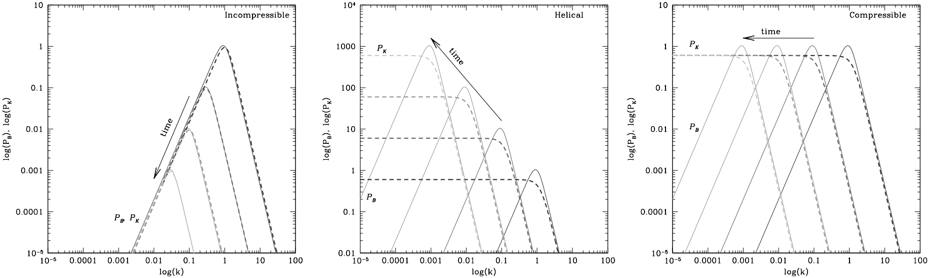
Helicity conservation for a helical field leads to an inverse cascade in the evolution of the magnetic field:



RD & A Neronony, 2012

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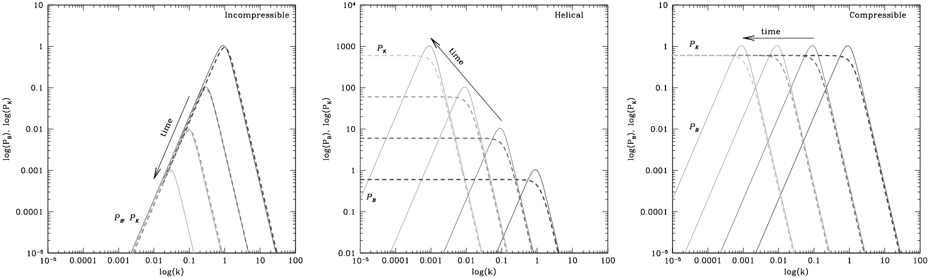

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In this case, the GW background is not parity symmetric. There are more GW's of one helicity than of the other.

Conclusions

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- The spectrum grows like $\frac{d\Omega_{GW}(k,t_0)}{d\ln(k)} \propto k^3$ on large scales and decays on scales smaller than the correlations scale $k_* \gtrsim 1/\eta_*$. The decay law depends of the physics of the source.

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- In this case we also expect a parity violating gravitational wave background, $|h_+(k)|^2 \neq |h_-(k)|^2$.