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Nambu-Goto Cosmic Strings

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What is a cosmic string?

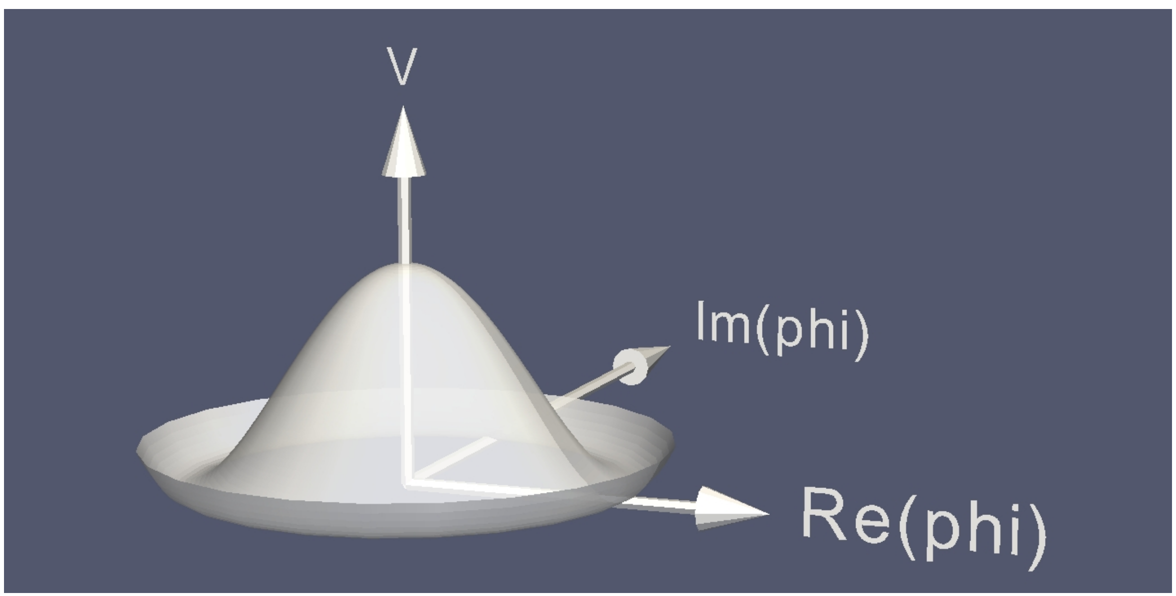
- Topological defects in field theory.
    - Relativistic version of Abrikosov vortices.
    - They exist in many extensions of the standard model.
- Cosmic Superstrings.
    - Fundamental Strings can be stretched to cosmological sizes and behave basically as classical objects.

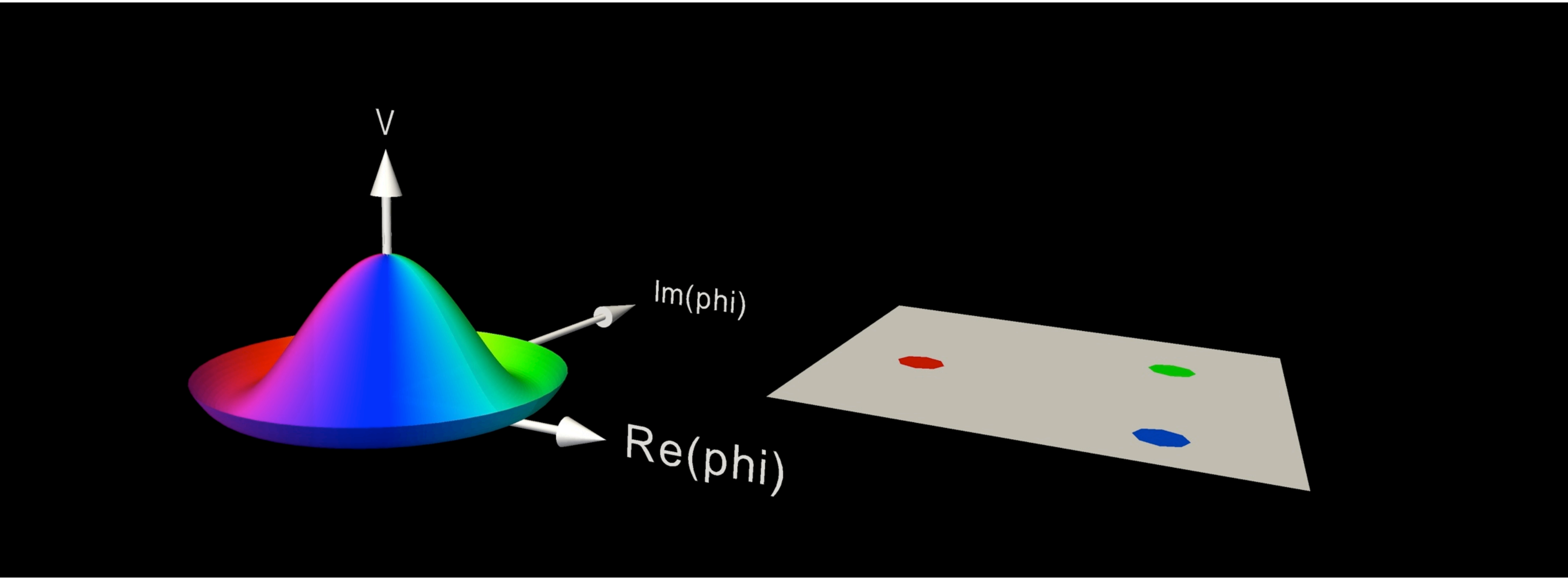
What is a cosmic string?

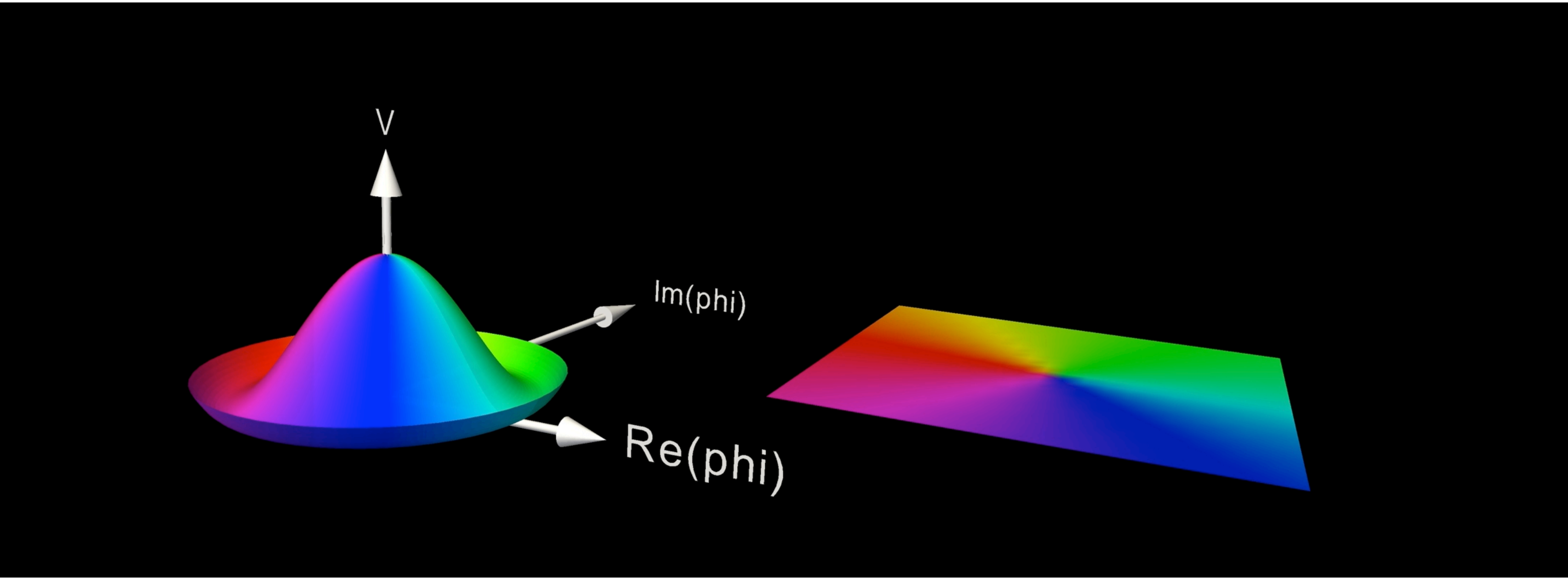
- Simplest model:

$$S_{U(1)} = \int d^4x \left[\partial_\mu \phi \partial^\mu \phi^* - V(|\phi|^2)\right]$$

$$V(\phi) = \frac{\lambda}{4}(|\phi|^2 - \eta^2)^2$$




V
Im(phi)
Re(phi)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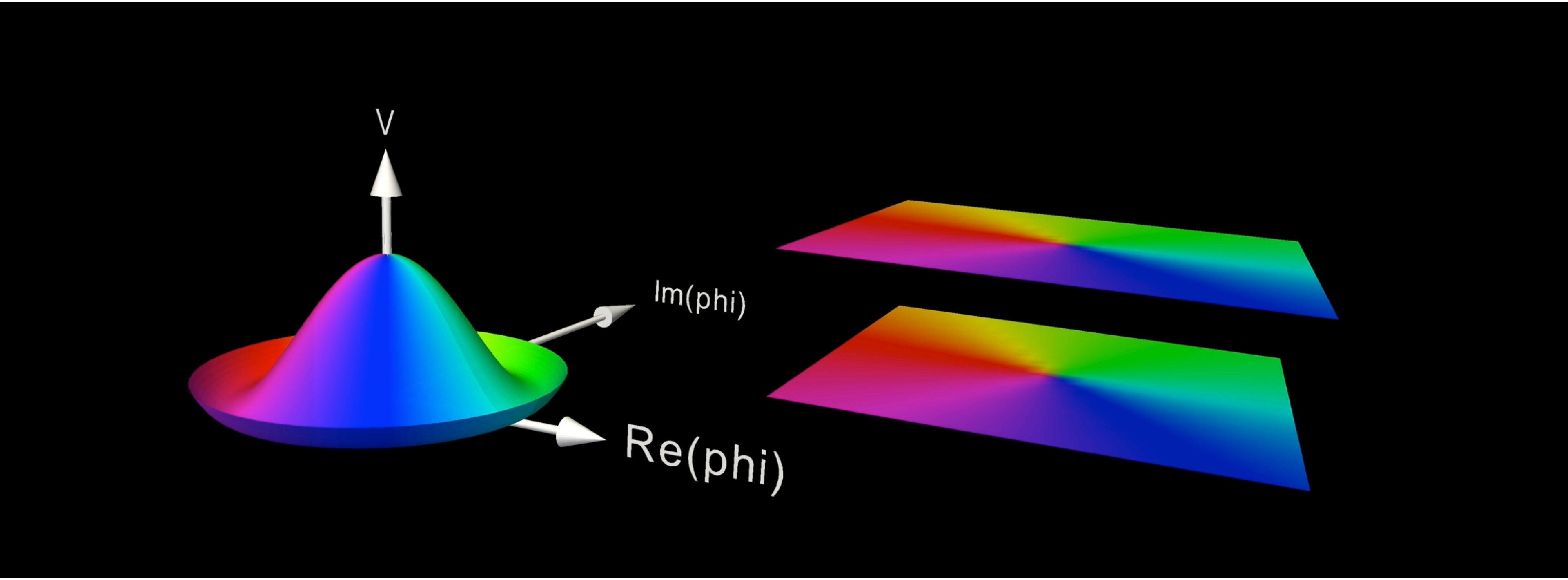
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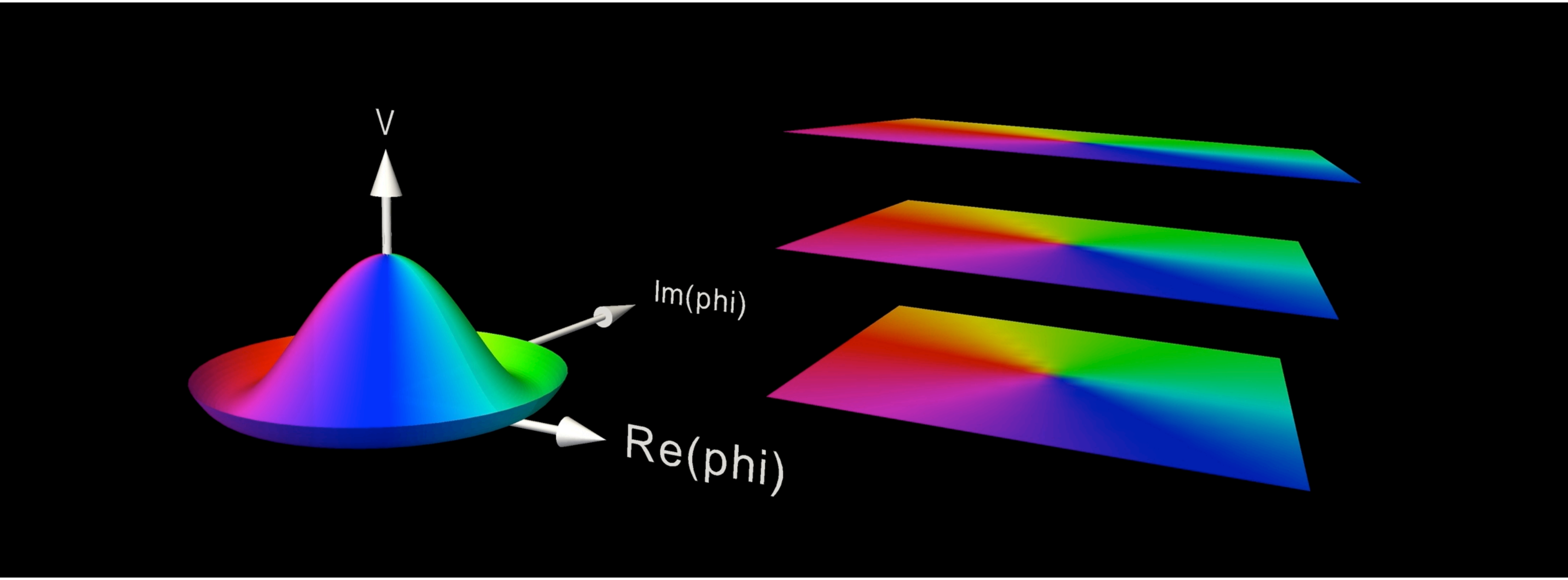
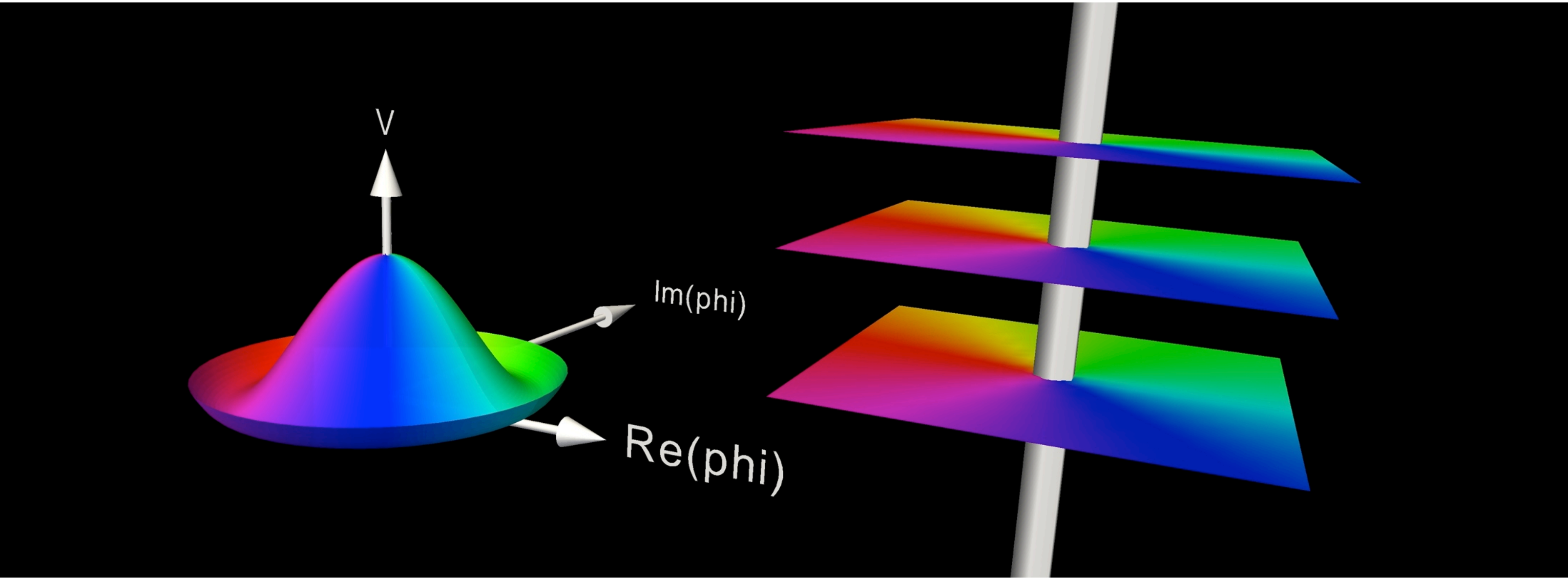

V
Im(phi)
Re(phi)


V
Im(phi)
Re(phi)

What is a cosmic string?

- Physical properties of the strings:
  - They are topological stable objects, they have no ends.
  - They are Lorentz invariant.

    Tension = Energy density per unit length

  - They are not coupled to any massless mode, except gravity.

(This is the simplest version of strings that we will consider here)

The String Scale

- Thickness, energy density and tension of the string are controlled by the symmetry breaking scale. η
- For a Grand Unified Theory scale: $\eta \approx 10^{16}\mathrm{GeV}$
- Thickness: $\delta = 10^{-30}\mathrm{cm}$
- Linear mass density: $\mu = 10^{22}\mathrm{gr/cm}$
- Tension : $T = 10^{37}N$
- Gravitational effects depend on: $G\mu = \left(\frac{\eta}{M_{Pl}}\right)^2 \sim 10^{-6}$

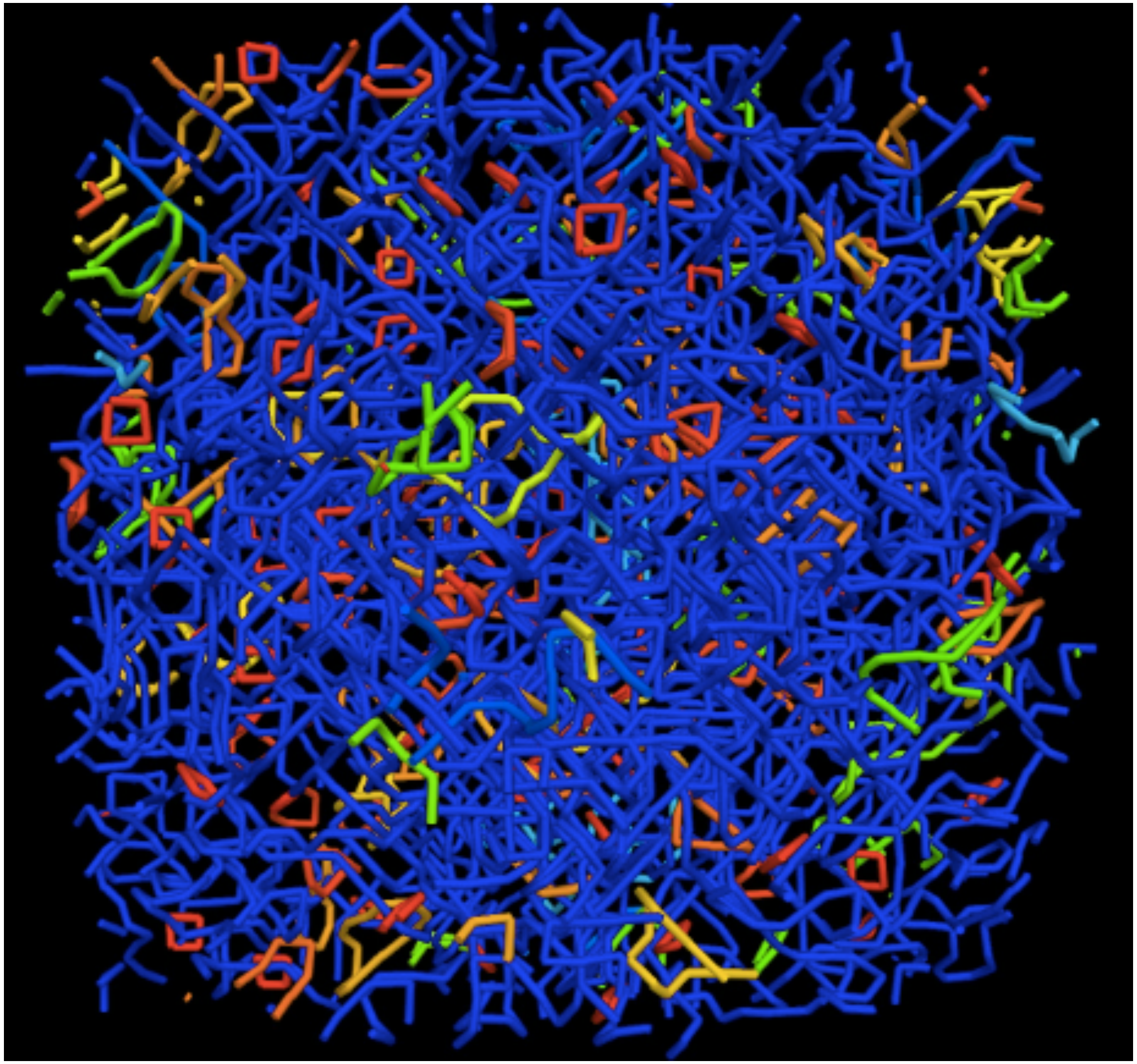
Cosmological Formation

(Kibble ‘76).

- Strings get formed at a cosmological phase transition.
- In order for strings to survive this should happen at the end of inflation.
- They could be formed in models of hybrid inflation or during reheating.
- In String Theory they are produced at the end of brane inflation.

Initial Conditions

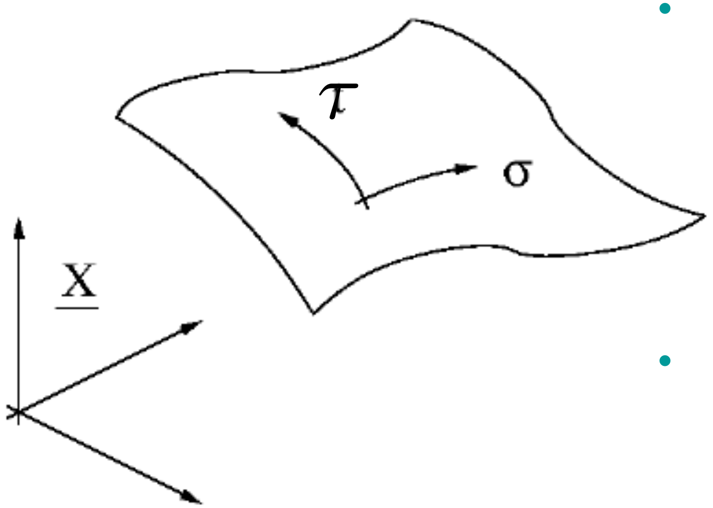
(Vachaspati & Vilenkin ‘84).



(B-P., Olum and Shlaer ‘12).

Cosmic String Dynamics

(Nambu, '71; Goto '70).



- A relativistic string dynamics has an action of the form,

$$S_{NG} = -\mu \int \sqrt{-\gamma}\, d^2\xi$$

- The string is described by: $\mathbf{X}(\sigma, \tau)$

- We use the gauge conditions:

$$\mathbf{X}'^{\mathbf{2}} + \dot{\mathbf{X}}^{\mathbf{2}} = \mathbf{1}$$

$$\mathbf{X}' \cdot \dot{\mathbf{X}} = \mathbf{0}$$

- The e.o.m. become:

$$\mathbf{X}'' = \ddot{\mathbf{X}}$$

$$\mathbf{X}(\sigma, \tau) = \frac{1}{2}\left[\mathbf{a}(\sigma - \tau) + \mathbf{b}(\sigma + \tau)\right]$$

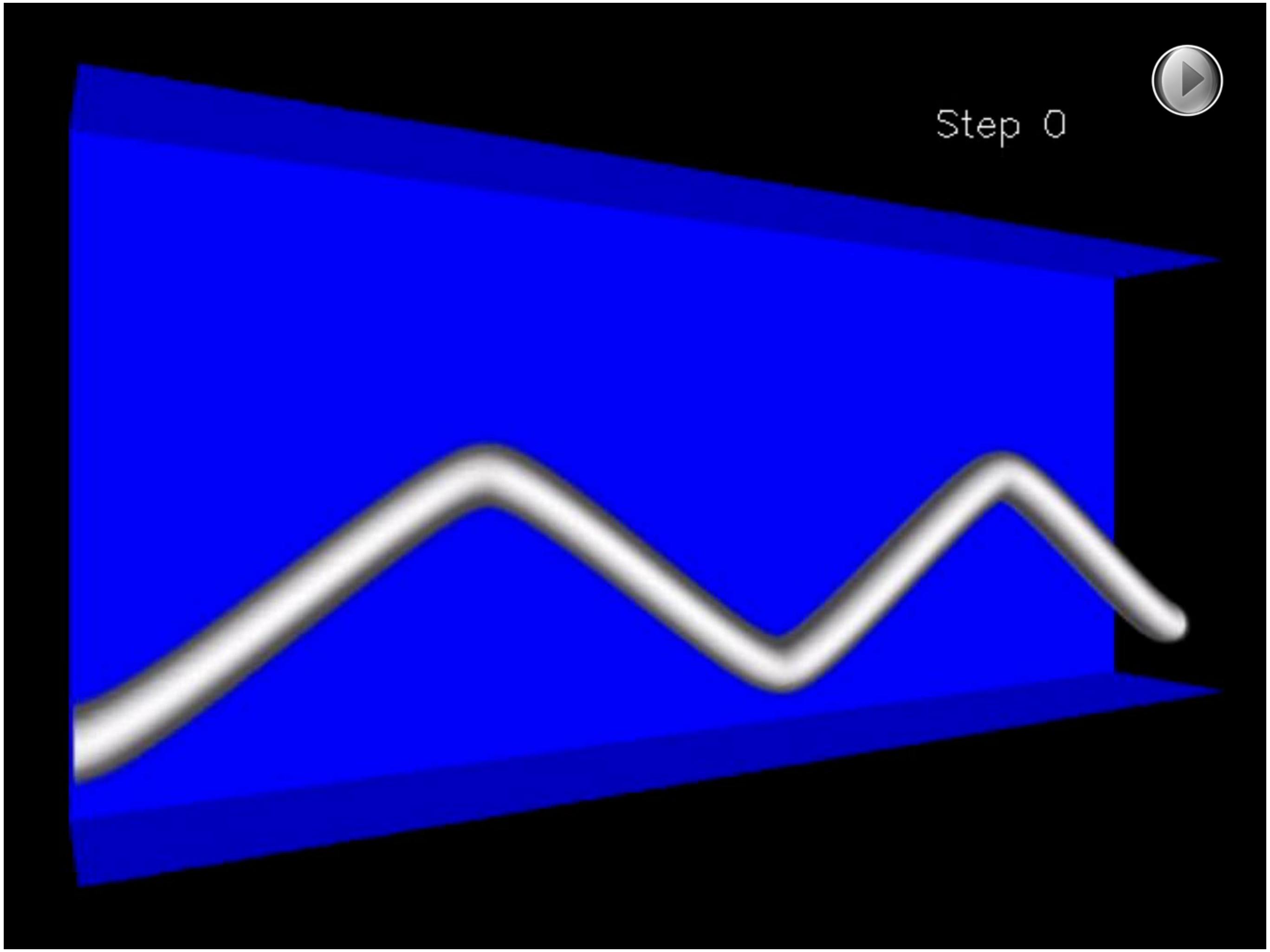
$$|\mathbf{a}'| = |\mathbf{b}'| = \mathbf{1}$$

Cosmic String Dynamics

$$\mathbf{X}(\sigma, \tau) = \frac{1}{2}\left[\mathbf{a}(\sigma - \tau) + \mathbf{b}(\sigma + \tau)\right]$$

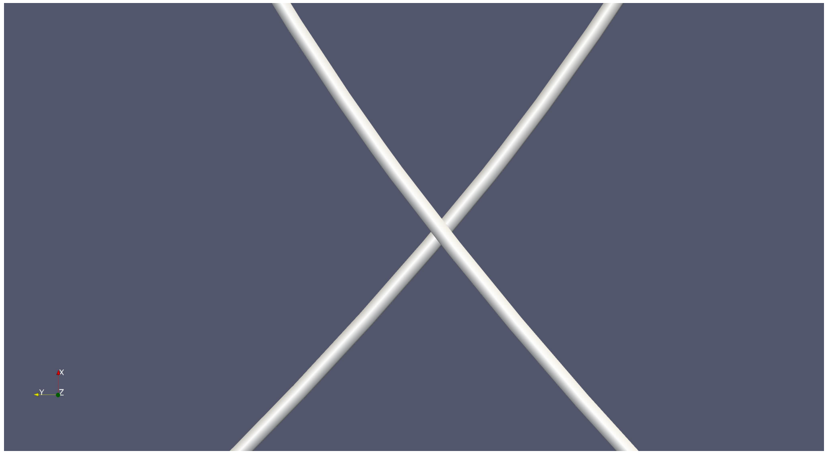
The effect of finite thickness: Field Theory

(B-P. & Olum '99).



What about interactions?

- Strings interact by exchanging partners, creating loops.



- This mechanism produces kinks on strings.
- This builds up small scale structure on strings.

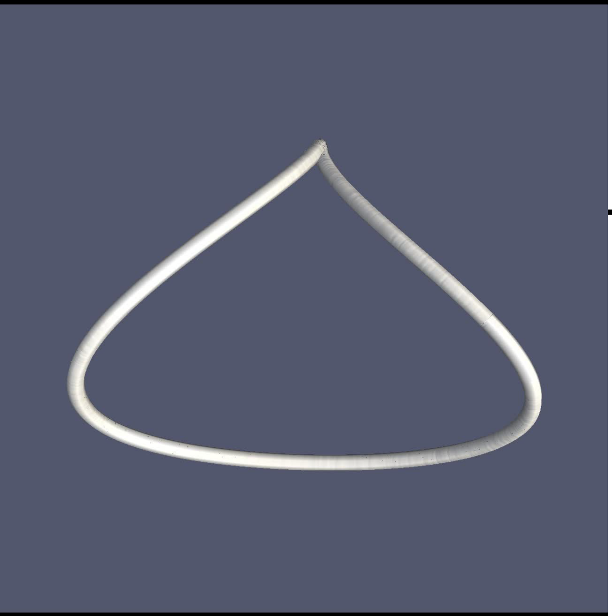
Cosmic String Dynamics (Loops)

$$\mathbf{X}(\sigma, \tau) = \frac{1}{2}\left[\mathbf{a}(\sigma - \tau) + \mathbf{b}(\sigma + \tau)\right]$$

- The solutions for closed loops are periodic.
- The loops oscillate under their tension.
- The strings move typically relativistically.
- During its evolution a loop may have points where the string reaches the speed of light: A cusp

$$|\dot{\mathbf{X}}| = \frac{\mathbf{1}}{\mathbf{2}}|\mathbf{b}' - \mathbf{a}'| \qquad \mathbf{b}' = -\mathbf{a}'$$

Cosmic String Cusps

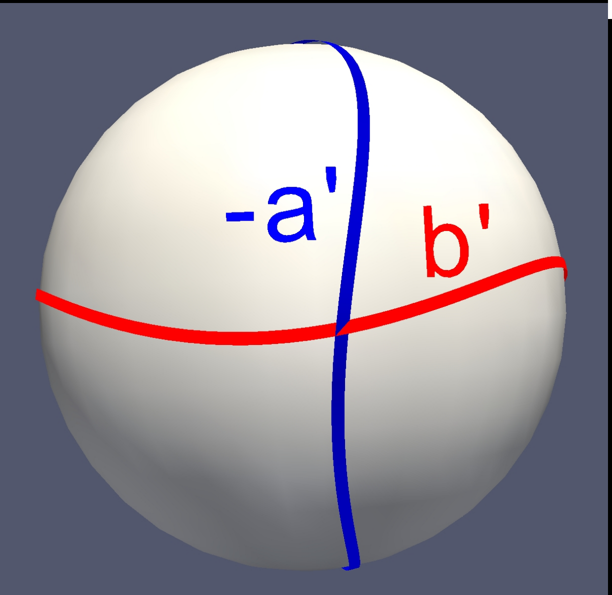


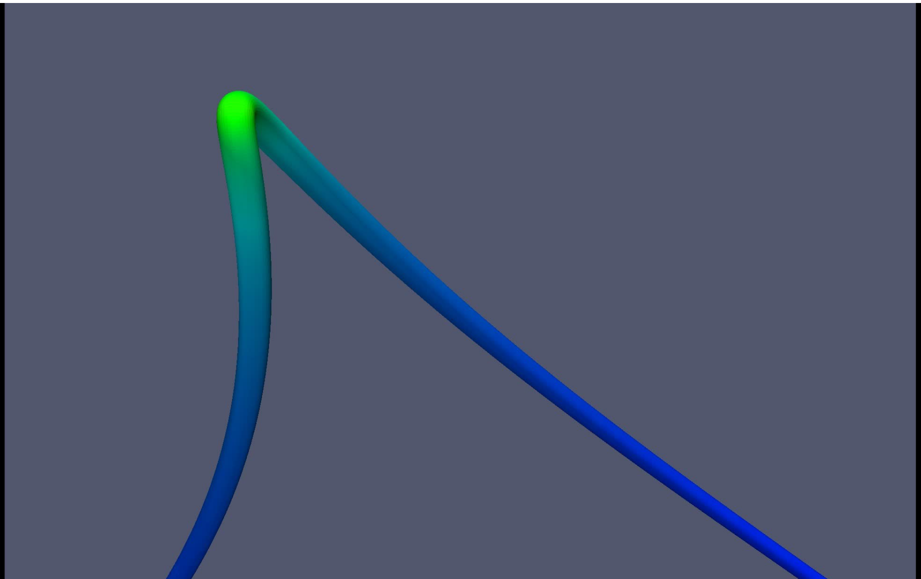
(Turok '84).

- Loops will typically have a cusp in each oscillation.

- The string doubles back on itself.

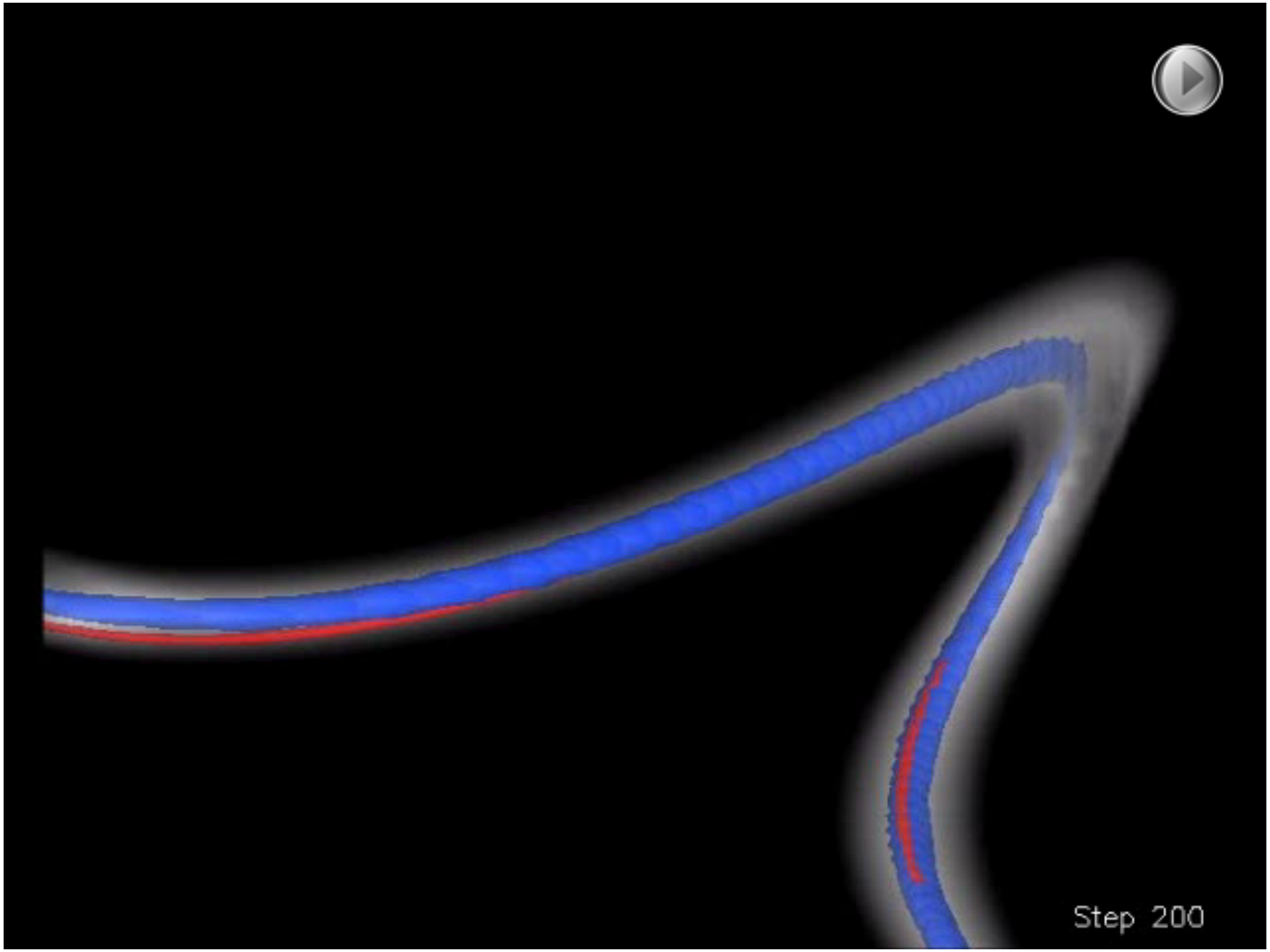
$$\mathbf{X}'^{2} + \dot{\mathbf{X}}^{2} = 1$$





Field Theory Cusps

(B-P. & Olum '99).



The importance of Loops

- Without any mechanism for energy loss strings would dominate the energy density of the universe.
- Loops oscillate under their tension and lose energy by gravitational radiation.
- This mechanism allows strings to be subdominant part of the energy budget.
- No “monopole problem” for strings.

Observational Signatures

- Many different ideas:
  - Gravitational Waves.
  - Cosmic Rays.
  - Structure Formation.
  - Effects on the CMB.
  - Lensing
  - Several other ideas…

Gravitational Radiation by Loops

- The power of gravitational waves will affect the size of the loops:

$$\dot{M} \sim G(\dddot{Q})^2 \sim GM^2L^4w^6 \sim \Gamma G\mu^2$$

- The total power has been calculated with several sets of loops:

$$P \sim \Gamma G\mu^2 \qquad\qquad \Gamma \sim 50 - 100$$

- Loops will therefore shink is size or the rest mass of the loop will be:

$$m(t) \sim m(t') - \Gamma G\mu^2(t - t')$$

Gravitational Radiation by Loops

- Loops are periodic sources so they emit at specific frequencies

$$P = \sum_n P_n$$

- The power of radiation does not depend on the size of the loop, only on its shape.

- We have to determine: P_n

- Kinks and cusps have different spectrum:

$$P_n^{cusps} \sim G\mu^2 n^{-4/3}$$
$$P_n^{kinks} \sim G\mu^2 n^{-5/3}$$

Gravitational Waves from the Network

- There are 2 different contributions to gravitational waves from a network of strings:

  - Stochastic background generated by all the modes in the loop.

    (Vilenkin ‘81, Hogan and Rees ’84, Caldwell et al. ‘92, Siemens et al.; Battye et al., Sanidas et al., Binetruy et al.; and many more…).

  - Burst signals from individual cusps.

    (Damour & Vilenkin ‘01).

Stochastic background of Gravitational Waves

- The whole network of strings contributes to the stochastic background of GW.

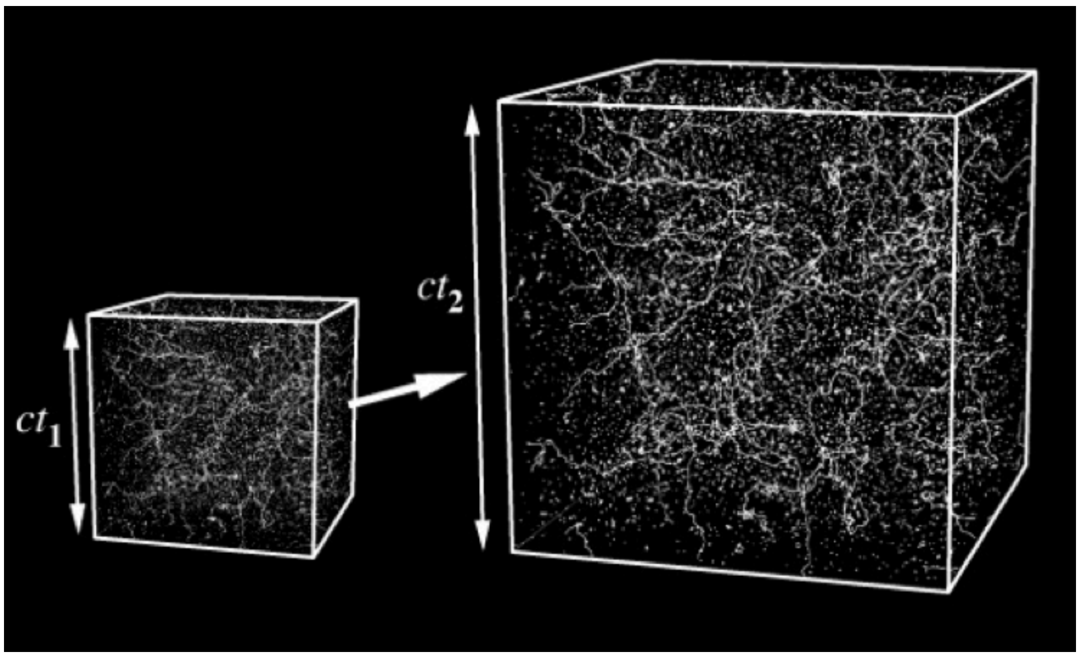
$$\Omega_{gw}(\ln f) = \frac{8\pi G}{3H_o^2} f \int_0^{t_0} dt \left(\frac{a(t)}{a(t_0)} \right)^3 \int_0^{m_{max}} dm \; n(t,m) \left(\frac{dP}{df} \right)$$

$n(t,m)$ ⟸ It depends directly on the number of loops.

$\left(\frac{dP}{df} \right)$ ⟸ It also depends on the spectrum of gw emission by the surviving loops.

Cosmic String Networks

- As the string network evolves it reaches a scaling solution where the energy density of strings is a constant fraction of the energy density in the universe.

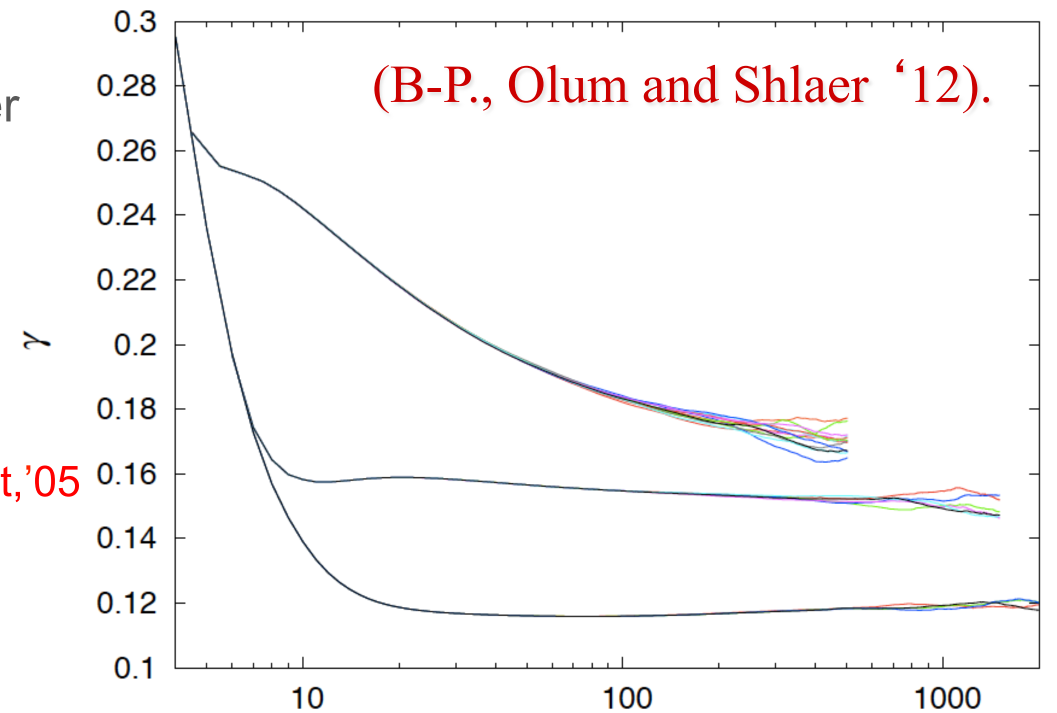


$$\frac{\rho_\infty}{\rho} = \text{constant}$$

- All statistical properties scale with the horizon distance.

The Scaling of Cosmic String Networks

- Long strings seem to scale relatively fast.
- This was also found earlier by smaller simulations:
  - Albrecht & Turok, '89
  - Bennet and Bouchet, '89
  - Allen & Shellard, '90
- And more recently:
  - Ringeval, Sakellariadou and Bouchet,'05
  - Vanchurin, Olum and Vilenkin, '05.
  - Martins and Shellard, '06



- Also from field theory simulations.
  - Bevis, Hindmarsh , Kunz and Urrestilla, '10.

$$\gamma = \frac{1}{d_h}\sqrt{\frac{\mu}{\rho_\infty}}$$

- On the other hand, the properties of loops and small scale structure on the strings approach scaling much slower.

A long standing question:
What is the size of the loops?

- Most loops are created with a size comparable to the horizon.

$$l \sim d_h$$

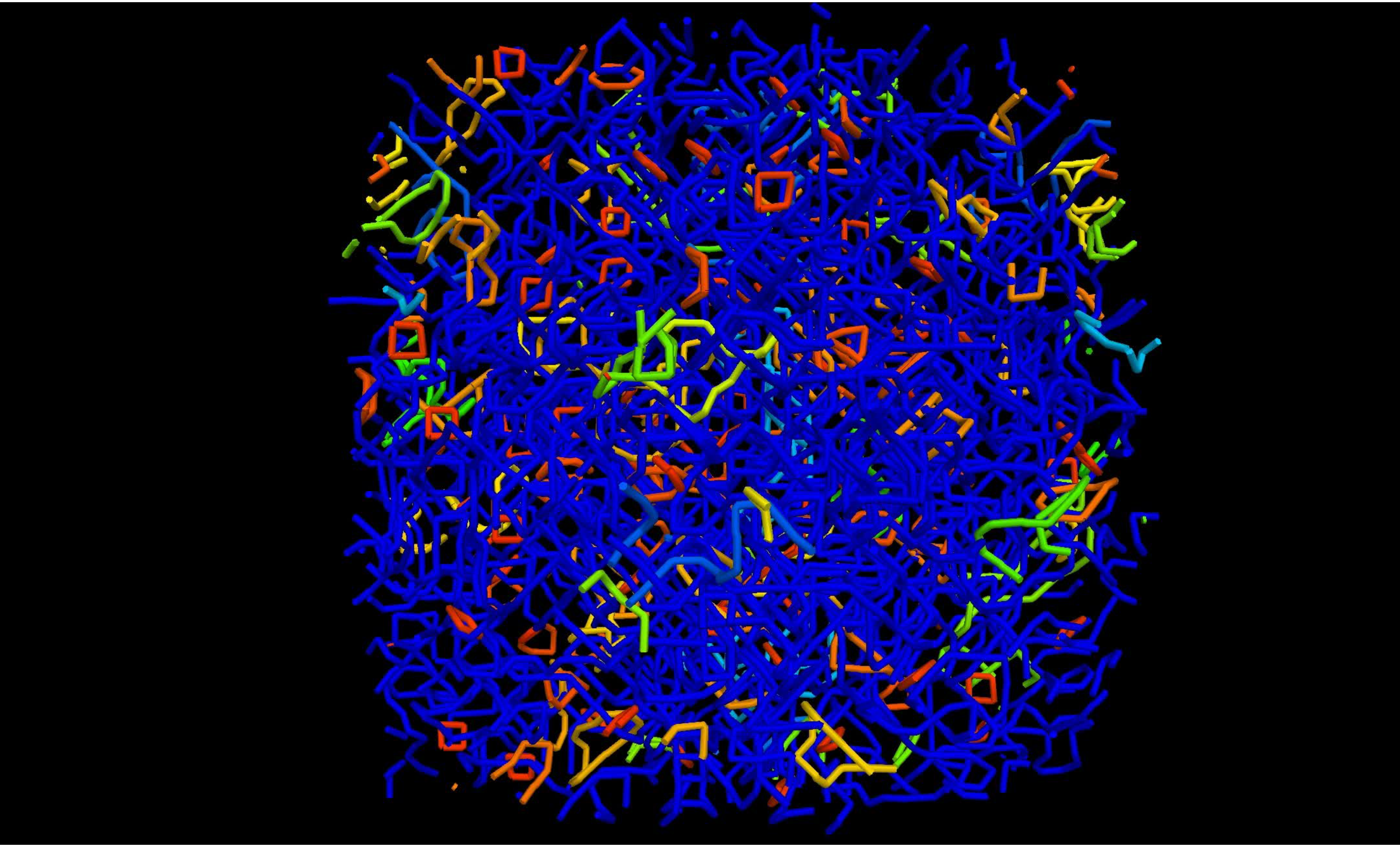
- Most loops are created at the smallest possible scales. The scale of the string thickness.

$$l \sim \delta$$

- Each of these cases will lead to different observational constraints.

Nambu-Goto Cosmic String Networks

(B-P., Olum and Shlaer '12).



The number of cosmic string loops

(B-P., Olum and Shlaer ‘13).

- There is a scaling population of loops being produced at large sizes.

$$l \approx 0.1 d_h$$

- It takes a long time to find this scaling regime for loops.

- They become the dominant contribution to the number density of loops.

- We can extract the normalization of the number of loops from simulations.

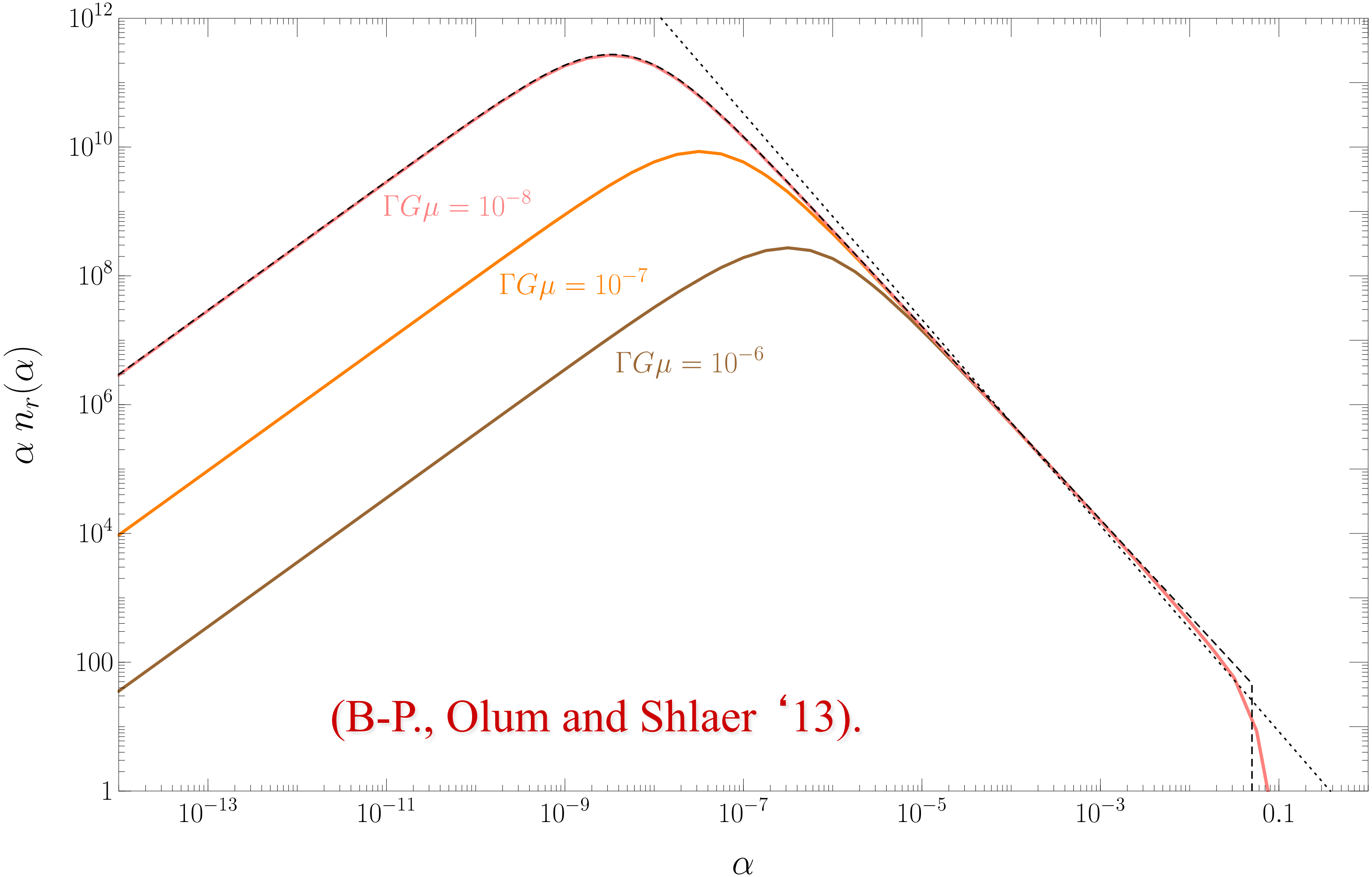
- This is in agreement with other Nambu-Goto simulations.

(Ringeval, Sakellariadou and Bouchet ‘05).

The number of cosmic string loops

- In the radiation era:

(Dotted line is from: Ringeval et al.).



(B-P., Olum and Shlaer '13).

Stochastic background of Gravitational Waves

- The whole network of strings contributes to the stochastic background of GW.

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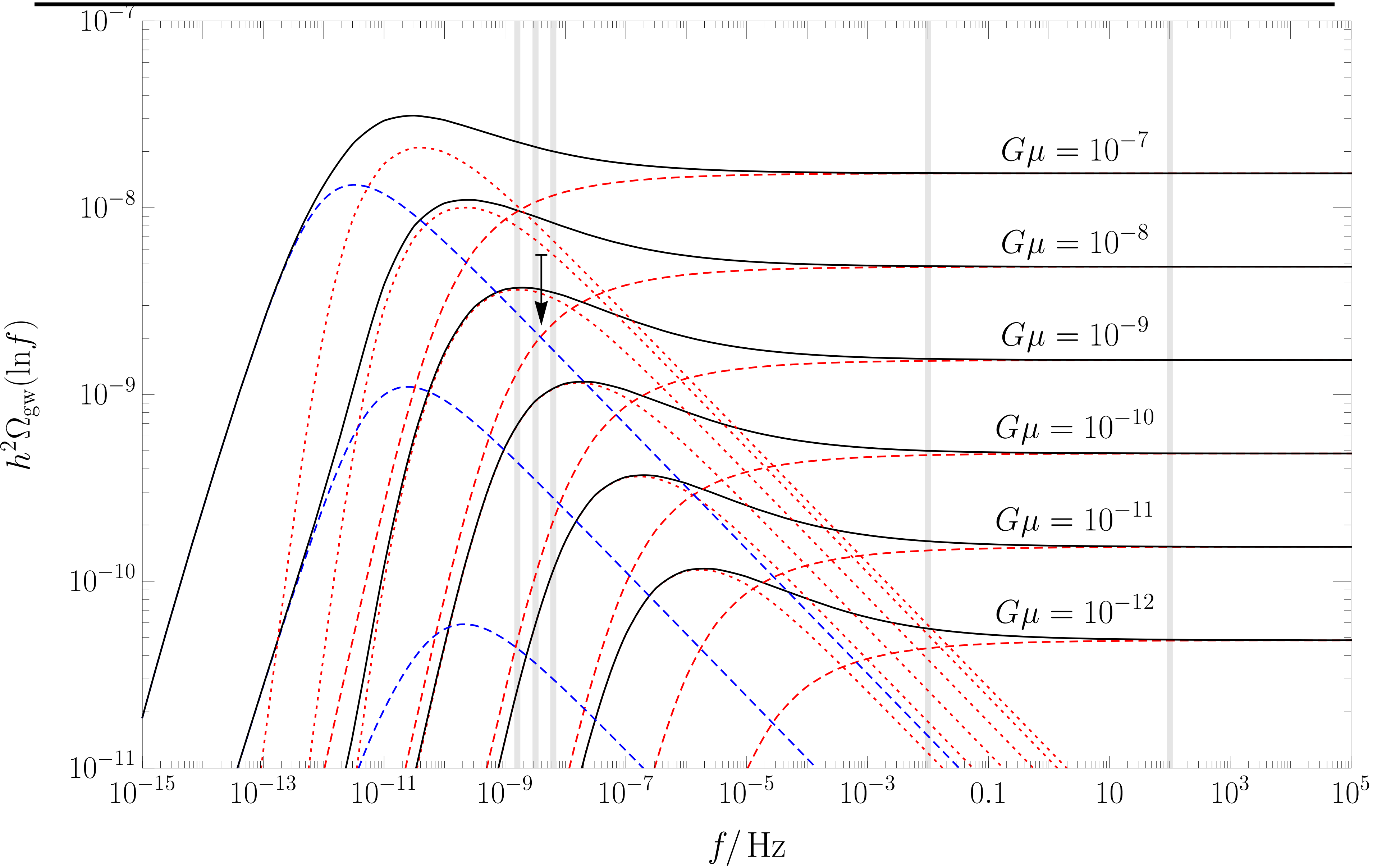
$n(t,m)$ ⟸ It depends directly on the number of loops.

$\left(\frac{dP}{df}\right)$ ⟸ It also depends on the spectrum of gw emission by the surviving loops.

$$P_n^{cusps} \sim G\mu^2 n^{-4/3}$$

Estimate Stochastic background of Gravitational Waves

(B-P., Olum and Shlaer '13).



Stochastic background of Gravitational Waves

- Using the limit from Pulsar Timing Array:

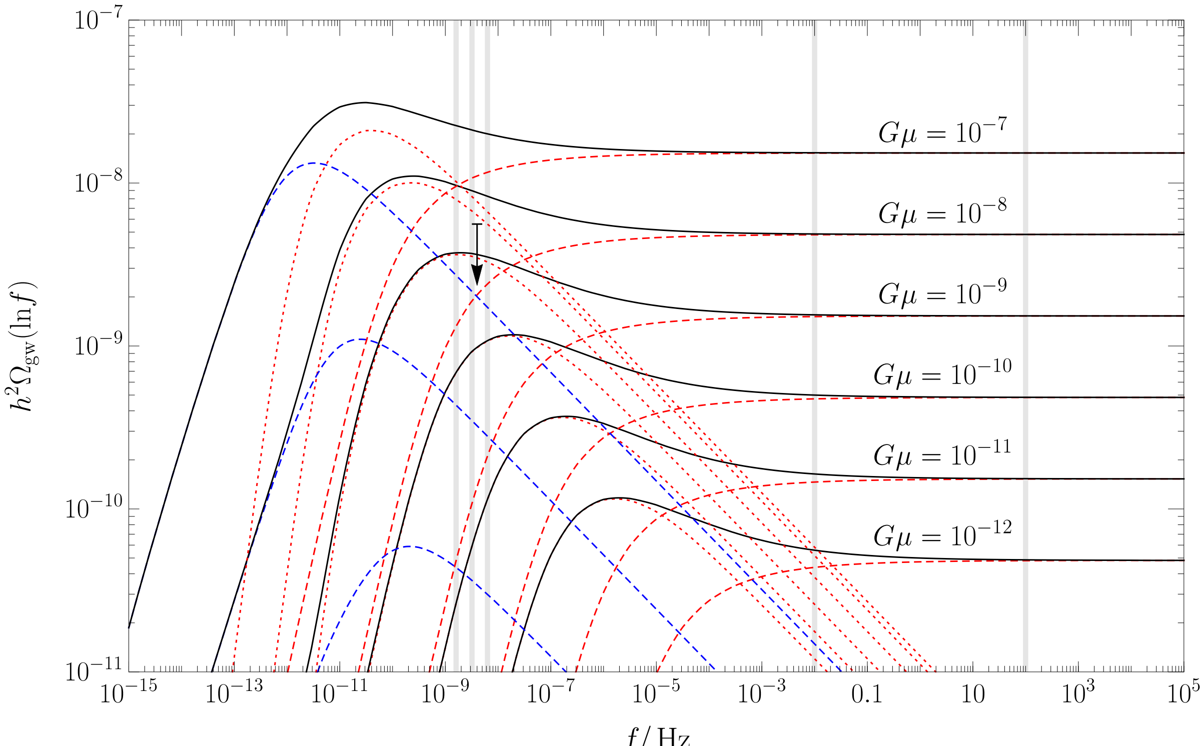
(R. van Haasteren et al., ‘11).

$$h^2\Omega_{gw}(f = 4.0 \times 10^{-9}\text{Hz}) \leq 5.6 \times 10^{-9}$$

- We arrive to a new bound on the cosmic string tension:

$$G\mu \leq 2.8 \times 10^{-9}$$

(B-P., Olum and Shlaer ‘13.).



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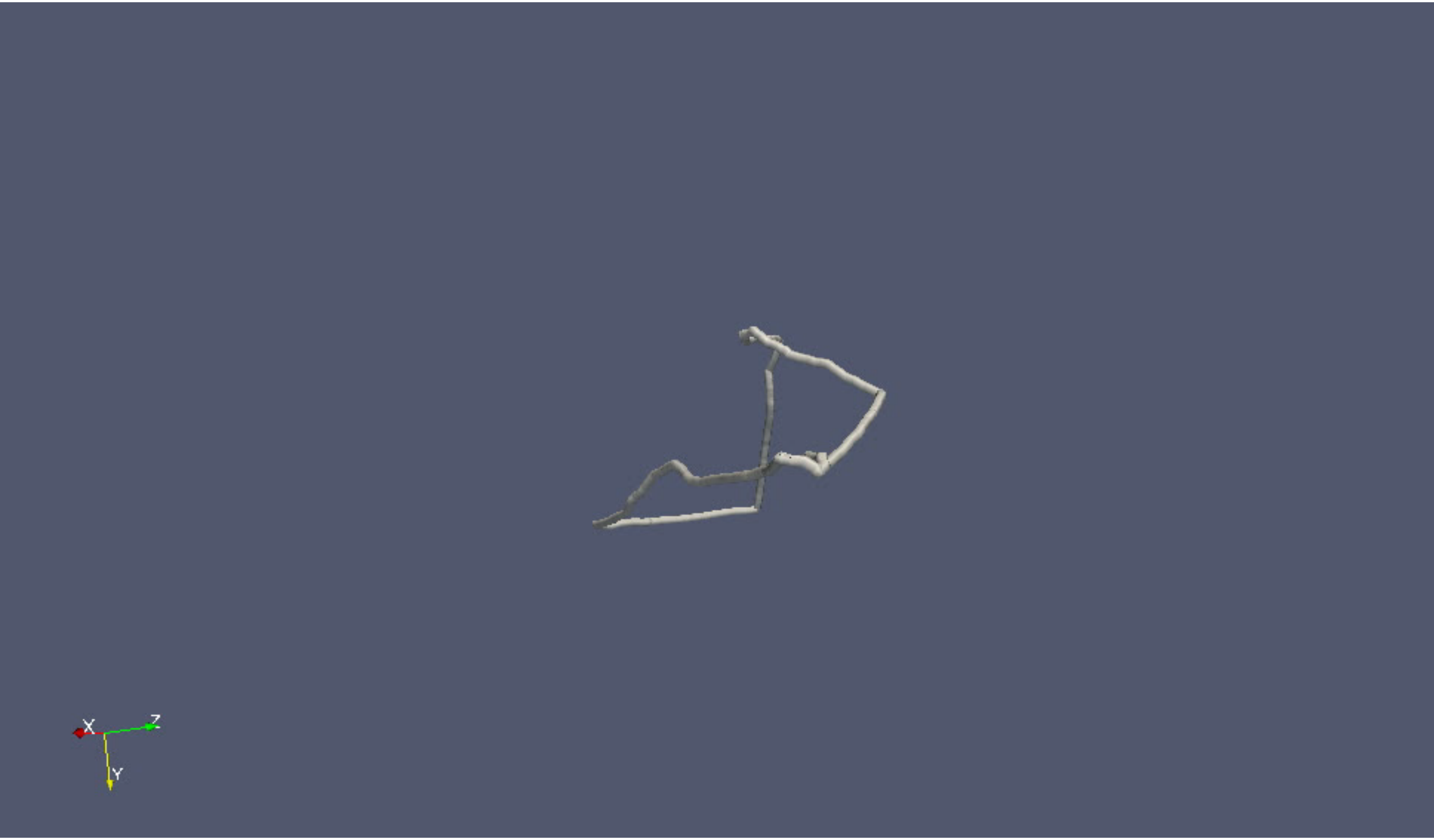
Loops from the Simulation

(B-P., Olum and Shlaer, work in progress).

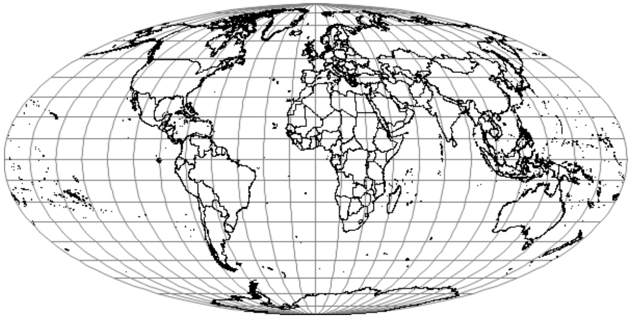


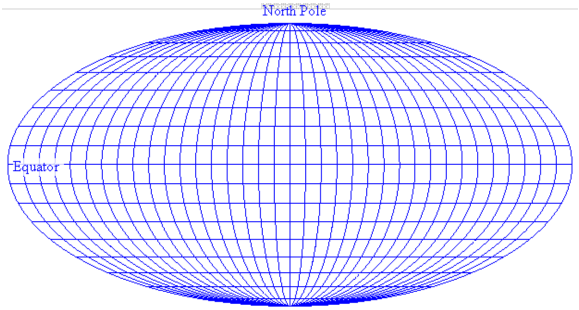
Describing the loops from the simulation

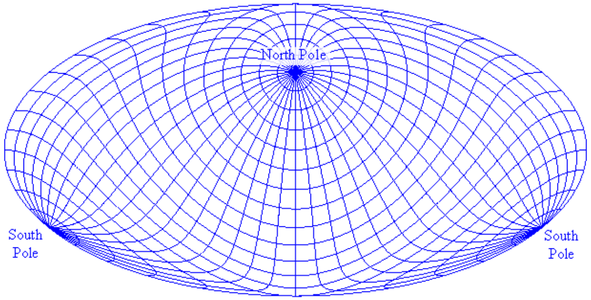
(B-P., Olum and Shlaer, work in progress).

- One way to describe the loop is to look at the a' and b' trajectories on the Kibble-Turok sphere.

- We will use the Mollweide projection.



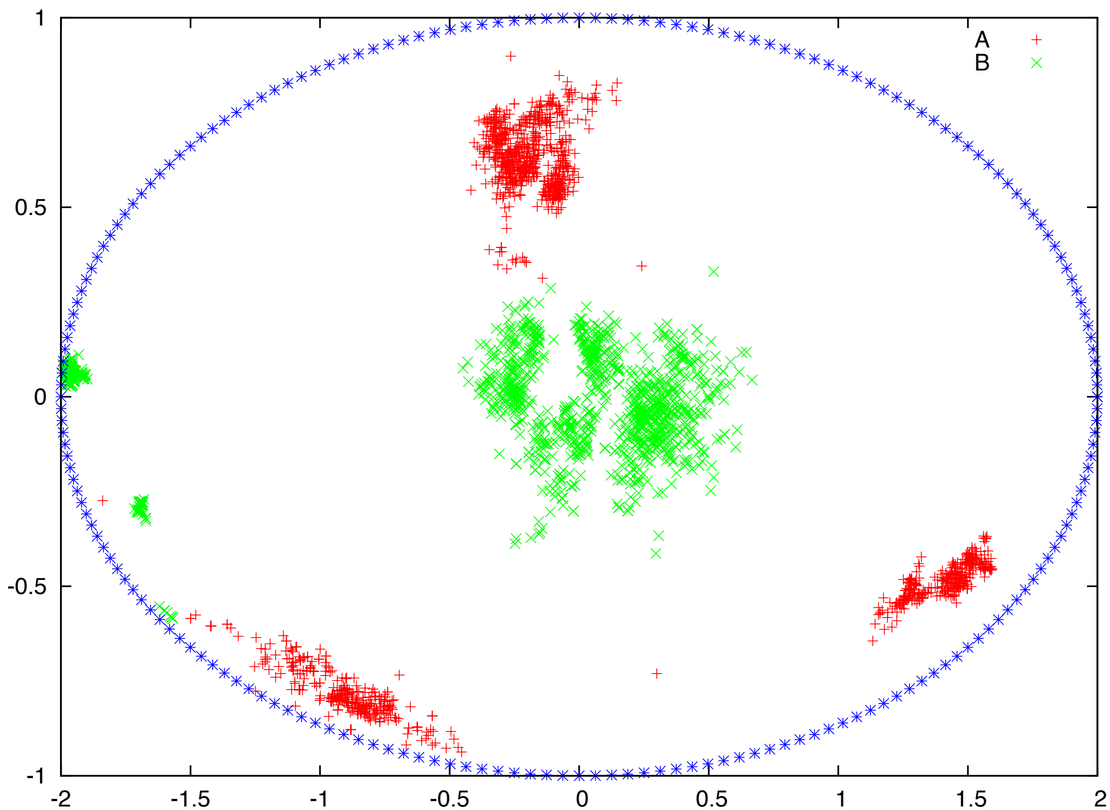


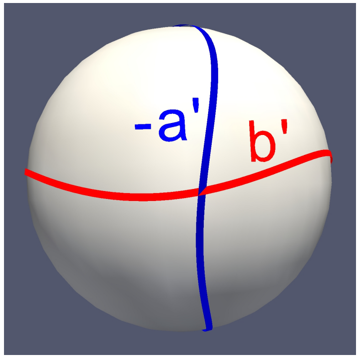


Describing the loops from the simulation

(B-P., Olum and Shlaer, work in progress).

- The distribution of a’ and b’ on the sphere looks like:





Compare to a smooth loop

Backreaction (a toy model)

(B-P., Olum and Shlaer, work in progress).

- We would like to simulate the effect of backreaction by smoothing the a’ ad b’ functions on the sphere.

- What is the effect of this process on ?

    - Fragmentation.

    - Number of Cusps.

    - Cusp parameters.

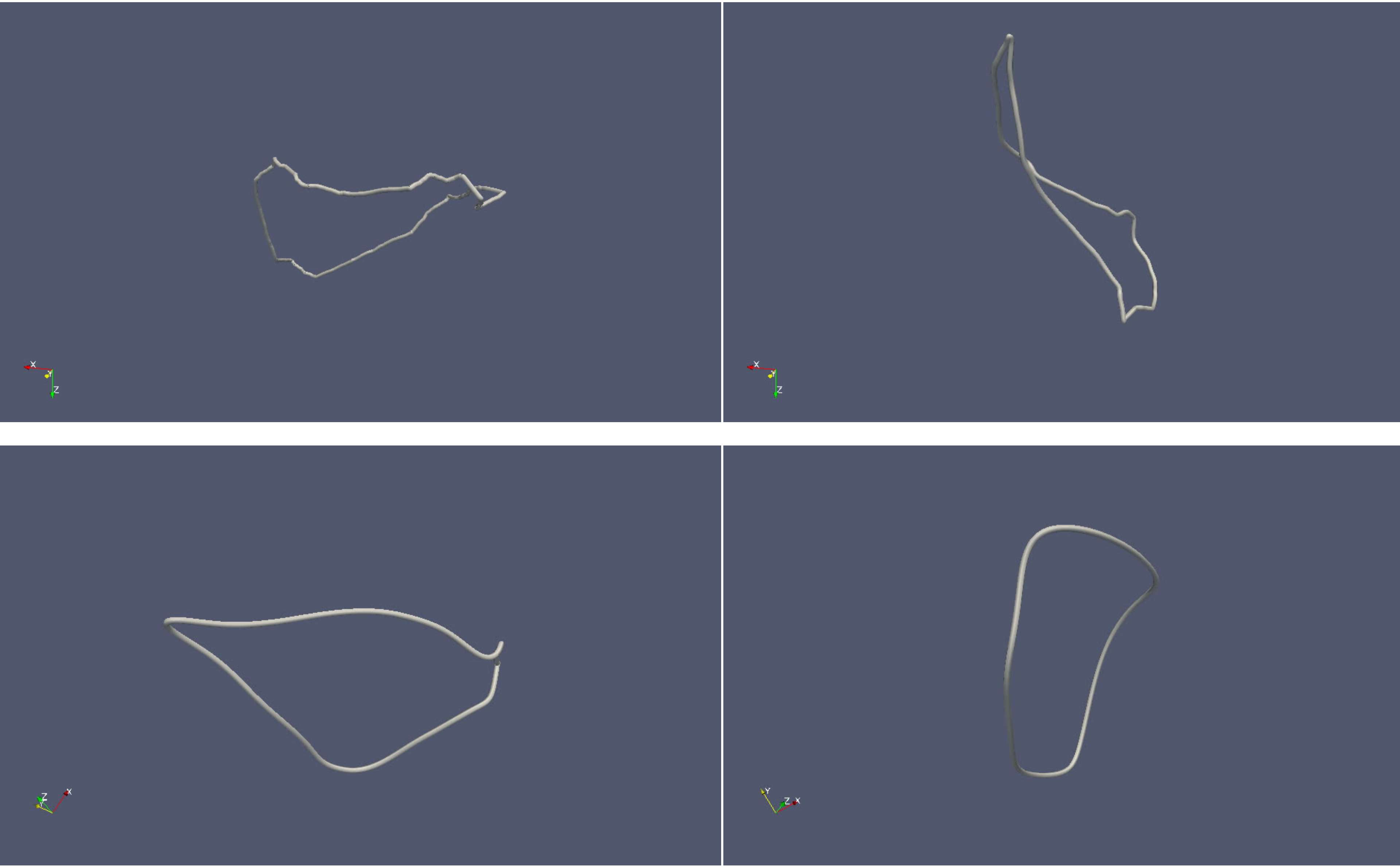
Smoothing the loops

(B-P., Olum and Shlaer, work in progress).

- What do we do in practice?
  - We first smooth the a’ and b’ functions with a particular scale.
  - We evolve the new loop until it falls into a non-self-intersecting trajectory.
  - We smooth again with a larger smoothing scale.
  - We evolve again…
  - We repeat these for many loops to get statistical results.

Smoothing the loops

(B-P., Olum and Shlaer, work in progress).



Results

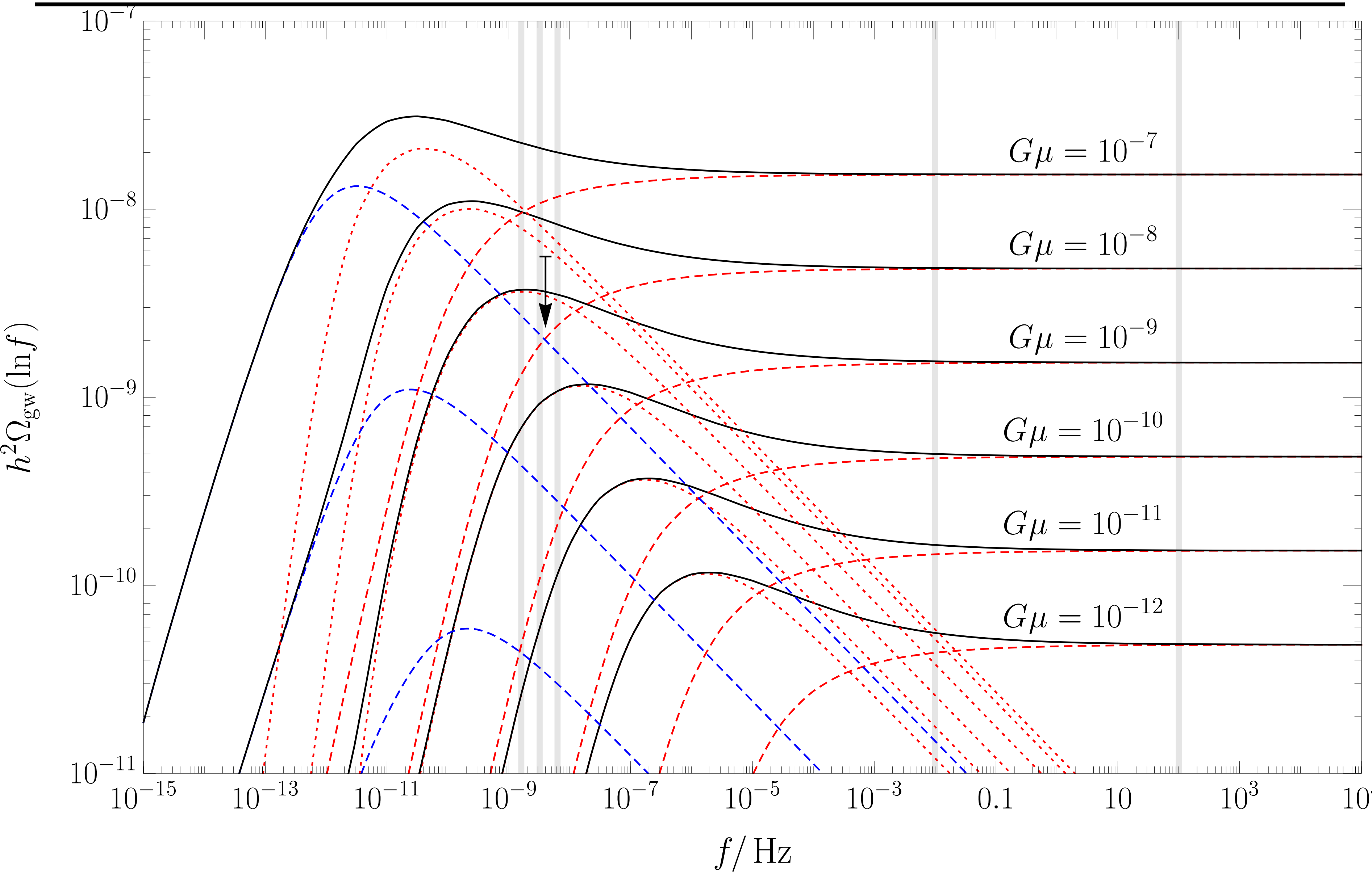
(B-P., Olum and Shlaer, work in progress).

- There is no significant self-intersections at any stage after smoothing.
- The percentage of loop length lost to this process is very small.
- The typical loop after the half of the energy is emitted is smooth and has of the order of 2 (loop scale) cusps.

The conclusions for gravitational waves are not drastically modified, most of its contribution is coming from large loops that have evolved for some time.

Estimate Stochastic background of Gravitational Waves

(B-P., Olum and Shlaer '13).



Conclusions

- We are entering an era of precision cosmology in cosmic string simulations.
- We have reached a consensus on the number and size of the important loops consistent with other NG simulations.
- We can impose important constraints on the scale of the string from PTA.
- We should study the evolution of the spectrum of radiation of loops including backreaction.
- Preliminary work in this direction indicates this may not change the picture too much.
- We should also look for rare bursts from realistic loops.