Cosmic string signals: computing CMB maps

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Outline

Nambu–Goto strings dynamics
A small matter era run (movie)
Small angles and flat sky limit
Real space signatures
Filling the transparent universe with strings
Massively parallel ray tracing method
After a million of cpu-hours
Comparison between flat and full sky
Can we do GW cosmic string maps?

Planck 2013 results XXV: `arXiv:1303.5085`
CR, F. R. Bouchet: `arXiv:1204.5041`
CR: `arXiv:1005.4842`

Nambu–Goto strings dynamics

- Two-dimensional worldsheet surface located at $x^\mu = X^\mu(\xi^a)$.
- Lorentz invariance along the string $(\tau \equiv \xi^0 \; \sigma \equiv \xi^1)$

$$S = -U \int \mathrm{d}\tau \mathrm{d}\sigma \sqrt{-\gamma}, \quad \gamma_{ab} = g_{\mu\nu} X^\mu_{,a} X^\nu_{,b} \text{ (induced metric)}$$

- String motion in FLRW (TT gauge: $X^0 = \tau$, $\dot{\boldsymbol{X}} \cdot \acute{\boldsymbol{X}} = 0$)

$$\ddot{\boldsymbol{X}} + 2\mathcal{H}\left(1 - \dot{\boldsymbol{X}}^2\right) - \frac{1}{\varepsilon}\left(\frac{\acute{\boldsymbol{X}}}{\varepsilon}\right)' = 0, \quad \dot{\varepsilon} + 2\mathcal{H}\varepsilon\dot{\boldsymbol{X}}^2 = 0, \quad \varepsilon = \sqrt{\frac{\acute{\boldsymbol{X}}^2}{1 - \dot{\boldsymbol{X}}^2}}$$

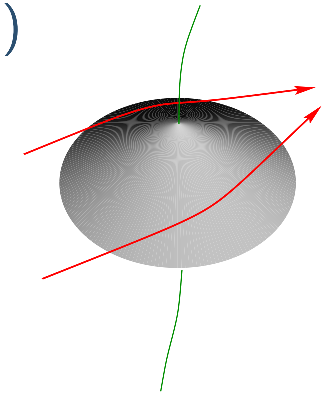
- Hubble damped propagation of left- right-moving waves

$$\mathcal{H} = 0 \quad \Rightarrow \quad \acute{\boldsymbol{X}}(\tau, \sigma) = \frac{1}{2}\left[\vec{p}(\sigma + \tau) + \vec{q}(\sigma - \tau)\right]$$

Induced CMB distortions

- Vacuum tubes $\Rightarrow$ no static gravitational effects ($T = U$)

- Do have General Relativity effects on light and thus on CMB! (Gott-Kaiser-Stebbins)



- Nambu-Goto stress tensor

$$\sqrt{-g}\,T^{\mu\nu} = U \int \mathrm{d}\tau \mathrm{d}\sigma \left(-\frac{1}{\epsilon} \acute{X}^{\mu} \acute{X}^{\nu} + \epsilon \dot{X}^{\mu} \dot{X}^{\nu} \right) \delta^{4}(x - X)$$

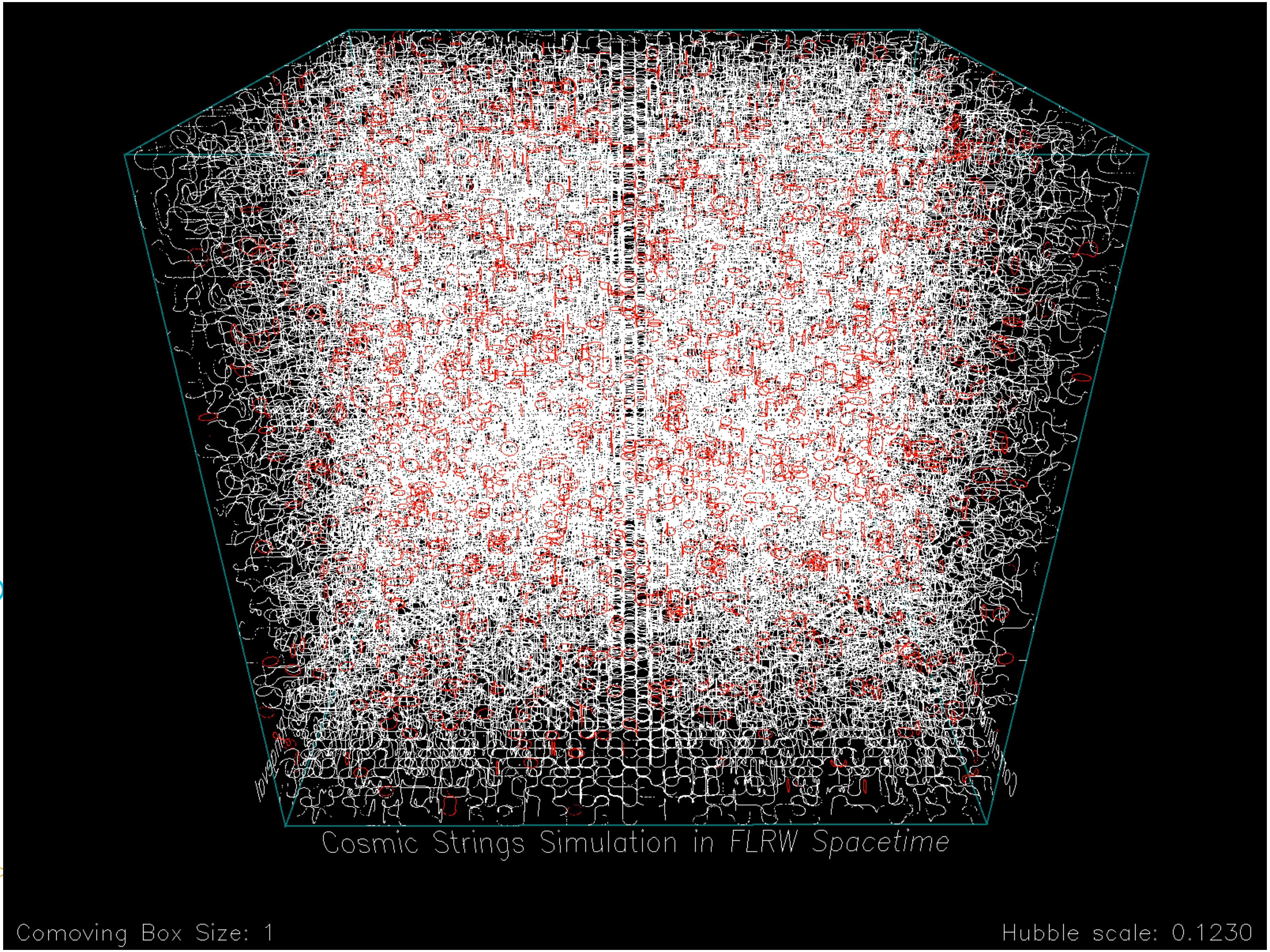
- ISW from Nambu–Goto stress tensor + linearized Einstein equations:

[Hindmarsh 94, Stebbins 95]

$$\Theta(\hat{\boldsymbol{n}}) \equiv \frac{\delta T}{T_{\mathrm{CMB}}} = -4GU \int_{\boldsymbol{X} \cap \boldsymbol{x}_{\gamma}} \left[\boldsymbol{u}(\hat{\boldsymbol{n}}) \cdot \frac{X\hat{\boldsymbol{n}} - \boldsymbol{X}}{(X\hat{\boldsymbol{n}} - \boldsymbol{X})^{2}} \right] \epsilon \mathrm{d}\sigma$$

$$\boldsymbol{u} = \dot{\boldsymbol{X}} - \frac{(\hat{\boldsymbol{n}} \cdot \boldsymbol{X}') \cdot \boldsymbol{X}'}{1 + \hat{\boldsymbol{n}} \cdot \dot{\boldsymbol{X}}}$$

A small matter era run (movie)

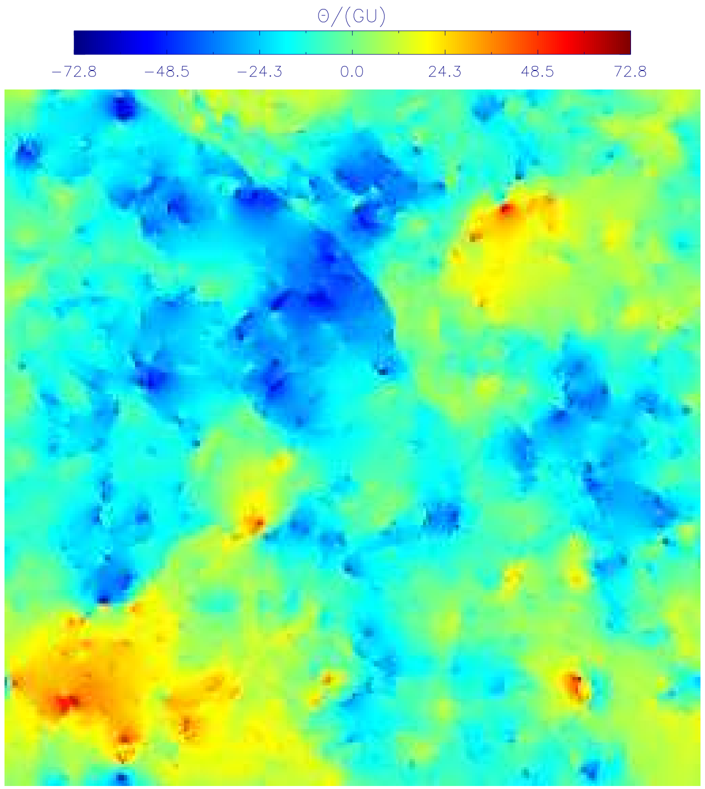


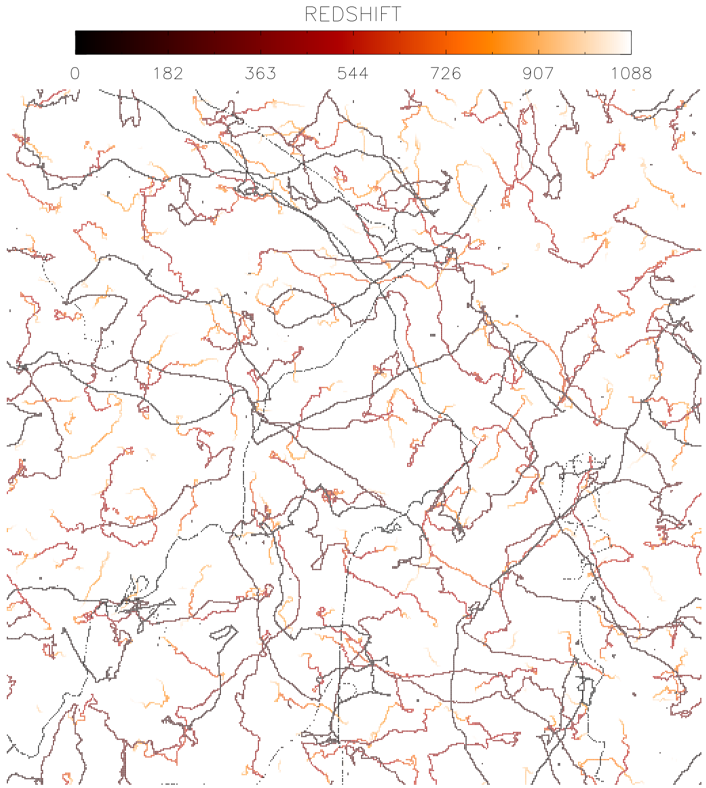
Small angles and flat sky limit

- At small angular scales, in 2D transverse Fourier space ($\boldsymbol{k}\cdot\hat{\boldsymbol{n}} \simeq 0$):

$$\Theta \simeq \frac{8\pi i\, GU}{\boldsymbol{k}^2} \int_{\boldsymbol{X}\,\cap\,\boldsymbol{x}_\gamma} (\boldsymbol{u}\cdot\boldsymbol{k})\, e^{-i\,\boldsymbol{k}\cdot\boldsymbol{X}}\, \mathrm{d}\sigma$$

- Flat sky simulation over 7.2° [Fraisse, CR, Spergel, Bouchet 07]



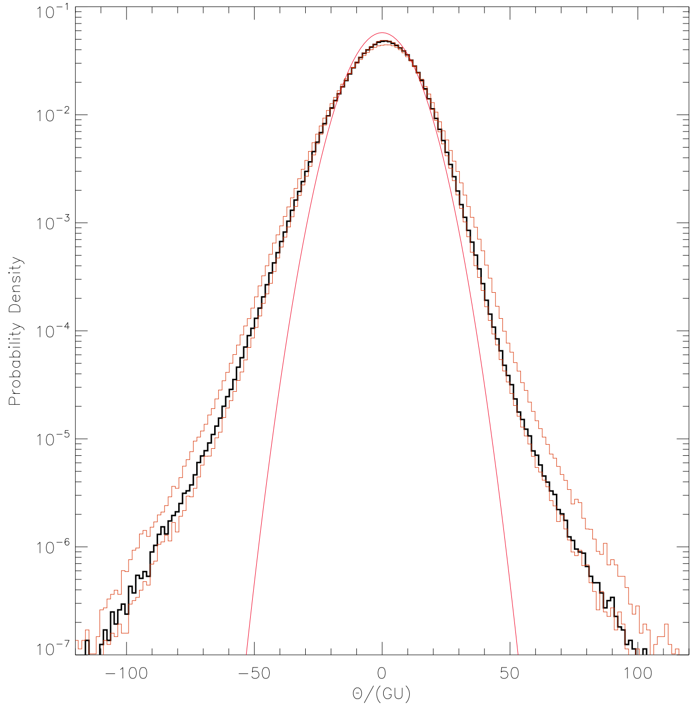


Real space signatures

- One-point functions

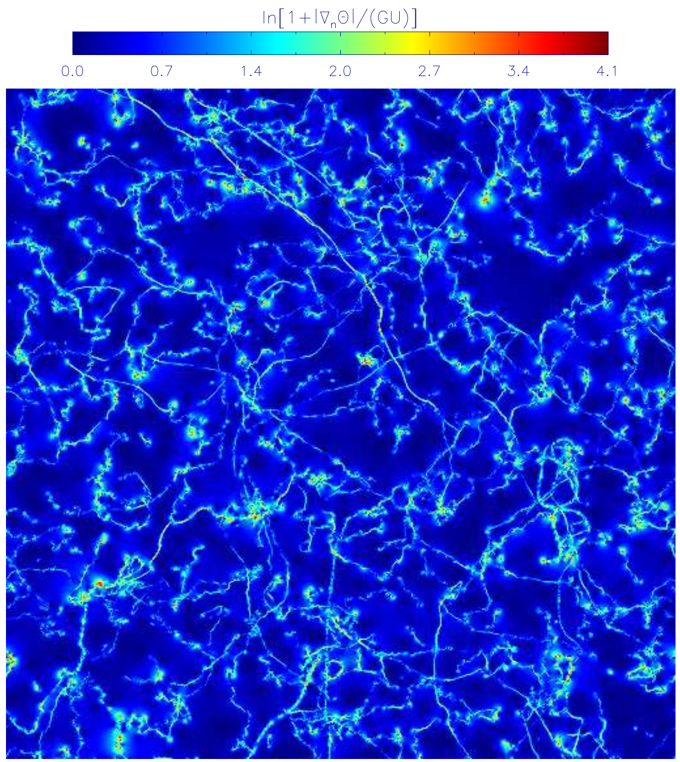
$$g_1 \equiv \left\langle \frac{\overline{(\Theta - \bar{\Theta})^3}}{\sigma^3} \right\rangle \simeq -0.22 \pm 0.12$$

$$g_2 \equiv \left\langle \frac{\overline{(\Theta - \bar{\Theta})^4}}{\sigma^4} \right\rangle - 3 \simeq 0.69 \pm 0.29.$$



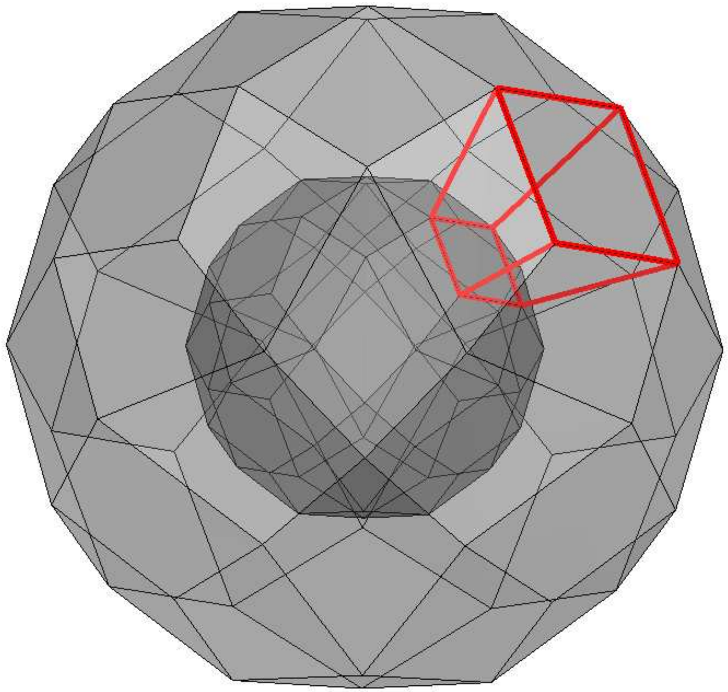
- Gradient magnitude

$$|\nabla\Theta| \equiv \sqrt{\left(\frac{\mathrm{d}\Theta}{\mathrm{d}\alpha}\right)^2 + \left(\frac{\mathrm{d}\Theta}{\mathrm{d}\beta}\right)^2}$$



Filling the transparent universe with strings

- Dedicated to string NG with Planck ⇒ all sky
  - ✦ Each simulation is a box of initial resolution 2000^3 (movie box)
  - ✦ Have to be stacked to fill 13 billion light years (HEALpix)



- This can be done with 3072 CS runs
- In which we propagate the CMB. . .

Massively parallel ray tracing method

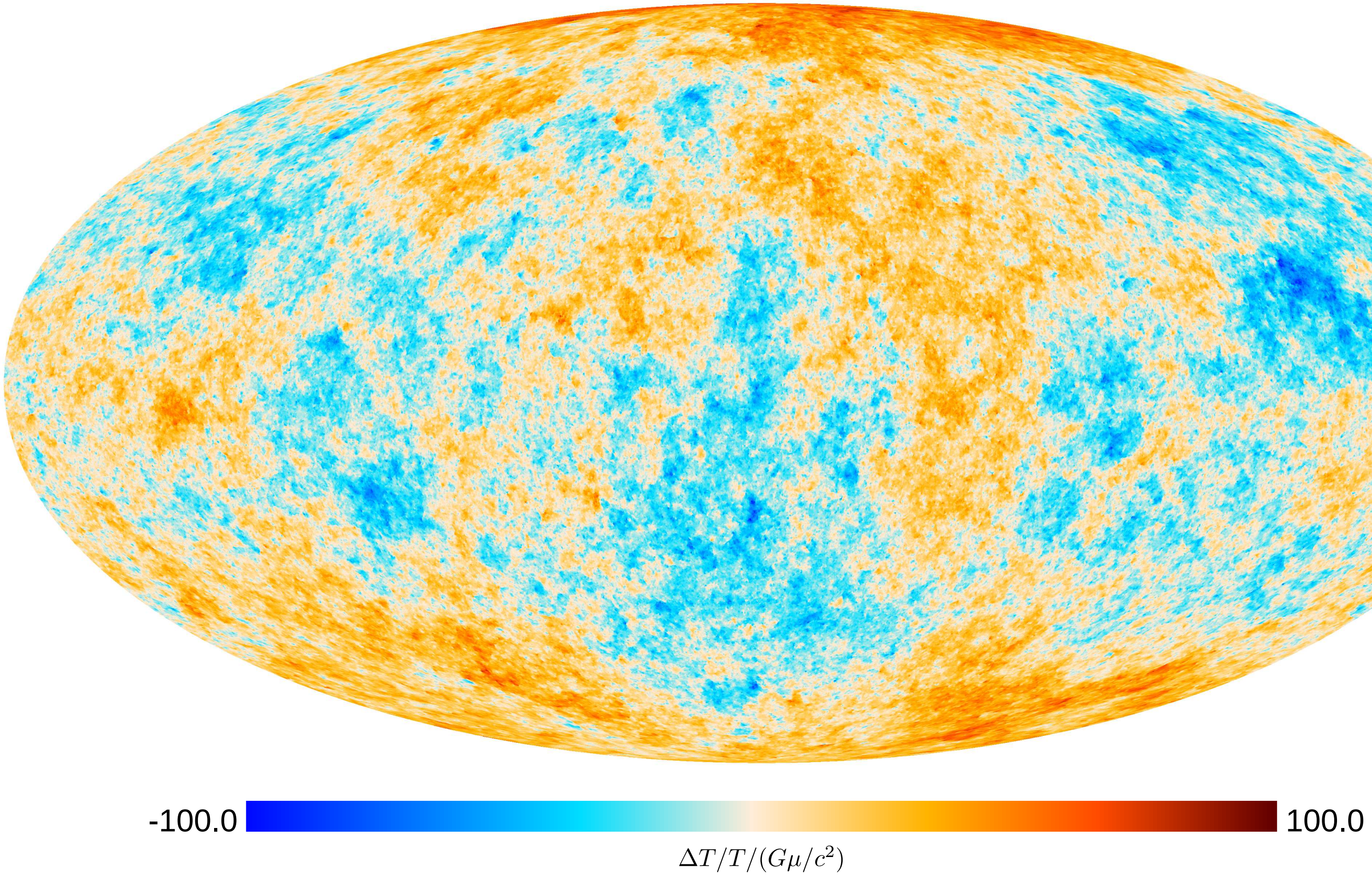
- Sky pixelized with $200\,000\,000$ lines of sight
  - Each direction receives cumulative contributions from all CS
  - Account for roughly 10^{17} iterations
- Parallelization implementation
  - `MPI` over the 3072 boxes + reduction
  - `OpenMP` over the $200\,000\,000$ pixels
  - `Vectorization` of the most inner loop (string segments)
- Code development performed on the `CP3-cosmo` cluster (100 cores)
- Reasonable computing time demands a 100 TeraFlops computer :-/
  - The Planck collaboration has a few... (thanks to J. Borrill)

512 nodes / 12K cores runs at NERSC

- National Energy Research Scientific Computing Center (Berkeley U.S.)
- The "Hopper" Cray XE6 machine (world rank 8 in Nov 2011)
  - More than 6000 nodes with Dual processor 24 cores
  - 3D Cray Gemini: Maximum injection bandwidth per node 20 GB/s

After a million of cpu-hours

- Full sky synthetic string map of 2×10^8 pixels [Ringeval:2012tk, Ade:2013xla]
- Temperature anisotropies



- $\times 4$ for tests and string challenges

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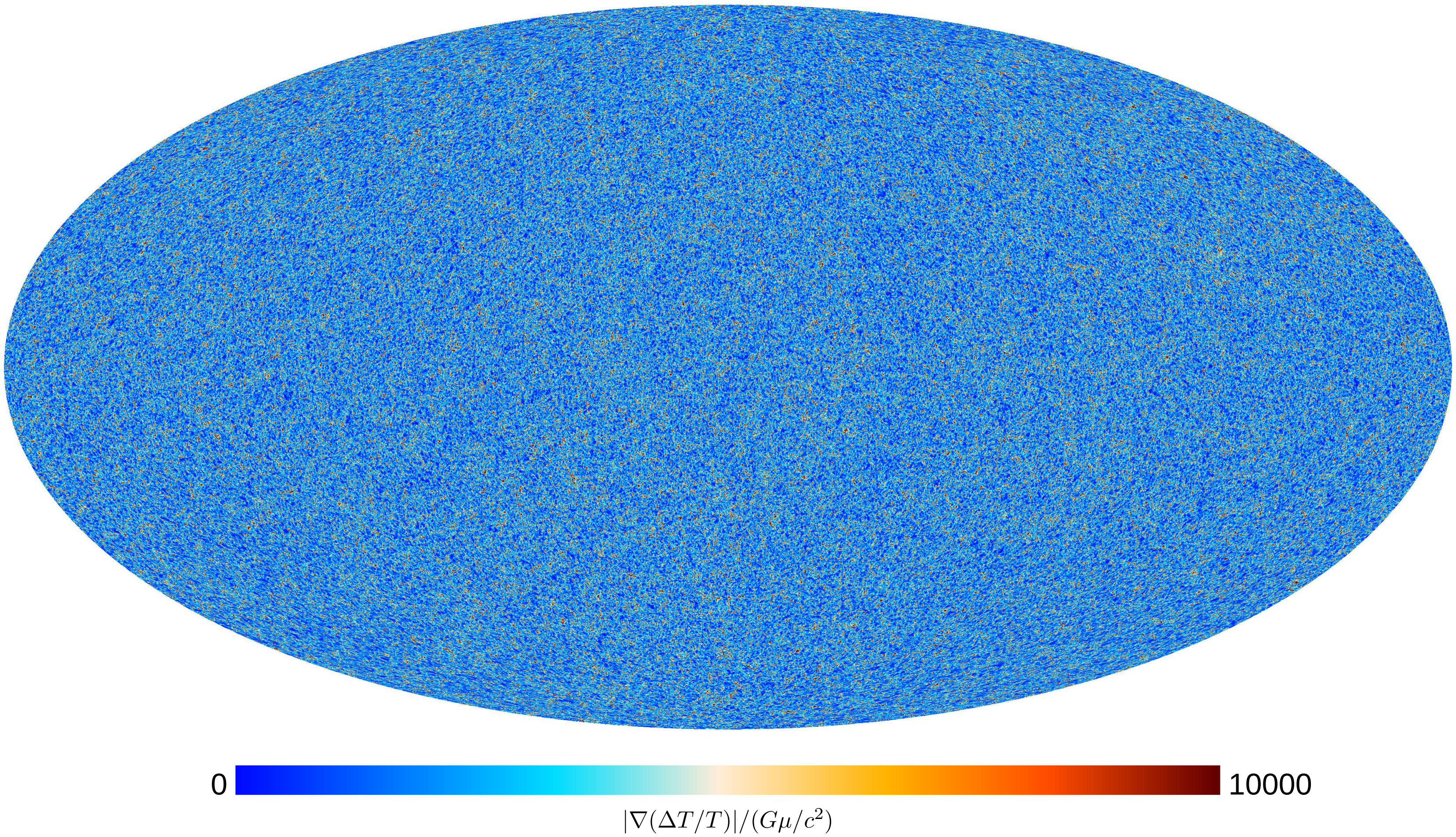
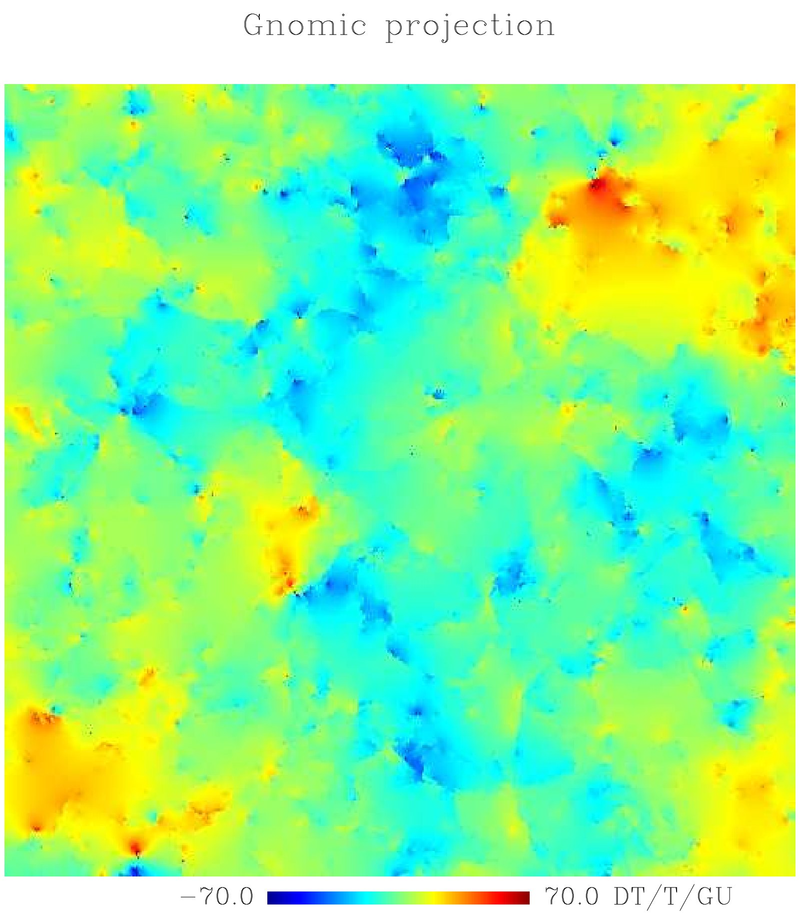
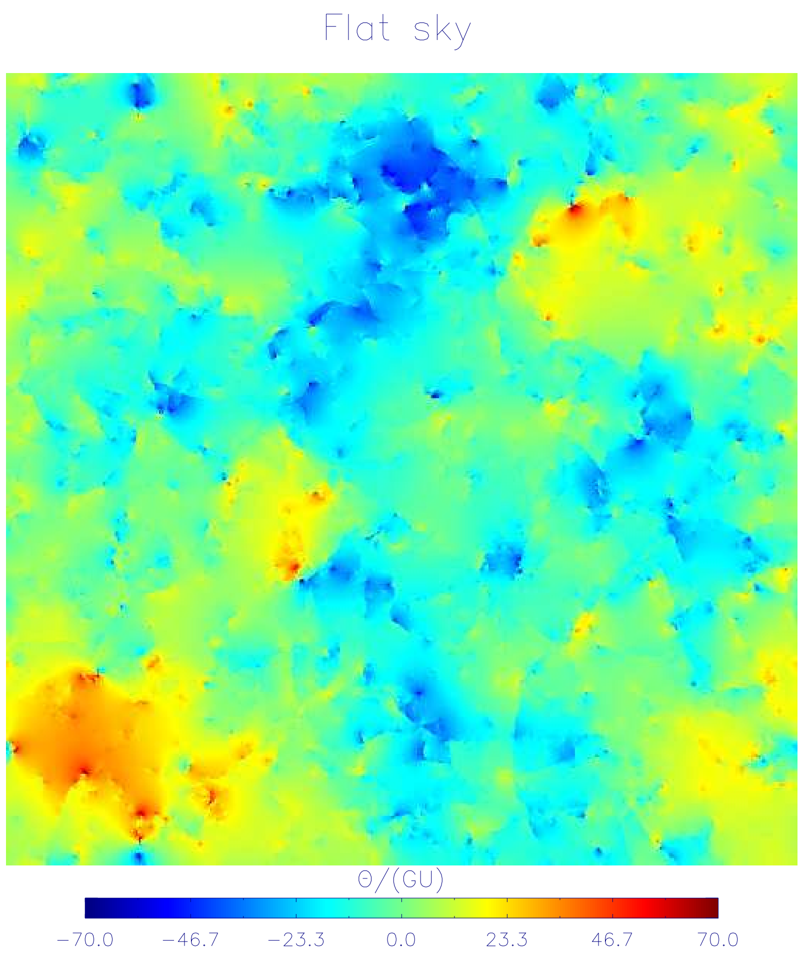


- $\times 4$ for tests and string challenges

Comparison between flat and full sky

- Small spherical distorsions on the edges and smoother temperature contrasts





Can we do GW cosmic string maps?

- Source term is more complex
  - ✦ Transverse and traceless stress tensor in Fourier space

$$k^4\tau_{ij}(t,\boldsymbol{k}) = U\int \epsilon \mathrm{d}\sigma \left\{ \frac{1}{2}\left(k_i k_j + k^2\delta_{ij}\right)\left[\left(\boldsymbol{k}\cdot\dot{\boldsymbol{X}}\right)^2 - \left(\frac{\boldsymbol{k}\cdot\acute{\boldsymbol{X}}}{\epsilon}\right)^2\right] + k^4\left(\dot{X}^i\dot{X}^j - \frac{\acute{X}^i\acute{X}^j}{\epsilon^2}\right) + k^2\left(k_ik_j - k^2\delta_{ij}\right)\left(\dot{\boldsymbol{X}}^2 - \frac{1}{2}\right) + k^2\frac{\boldsymbol{k}\cdot\dot{\boldsymbol{X}}}{\epsilon^2}k_{(i}\acute{X}_{j)} - k^2\left(\boldsymbol{k}\cdot\dot{\boldsymbol{X}}\right)k_{(i}\dot{X}_{j)}\right\} e^{-i\boldsymbol{k}\cdot\boldsymbol{X}}$$

- Main difficulty: time sampling
  - ✦ One needs $h_{ij}(f,\hat{\boldsymbol{n}}) = G_{\mathrm{ret}} * \tau_{ij}$ today
  - ✦ A given f today requires a zf all sky time sampling at redshift z
  - ✦ However GW maps do not need CMB angular resolution...