eman ta zabal zazu

UPV EHU

Field theory simulations of cosmic defects

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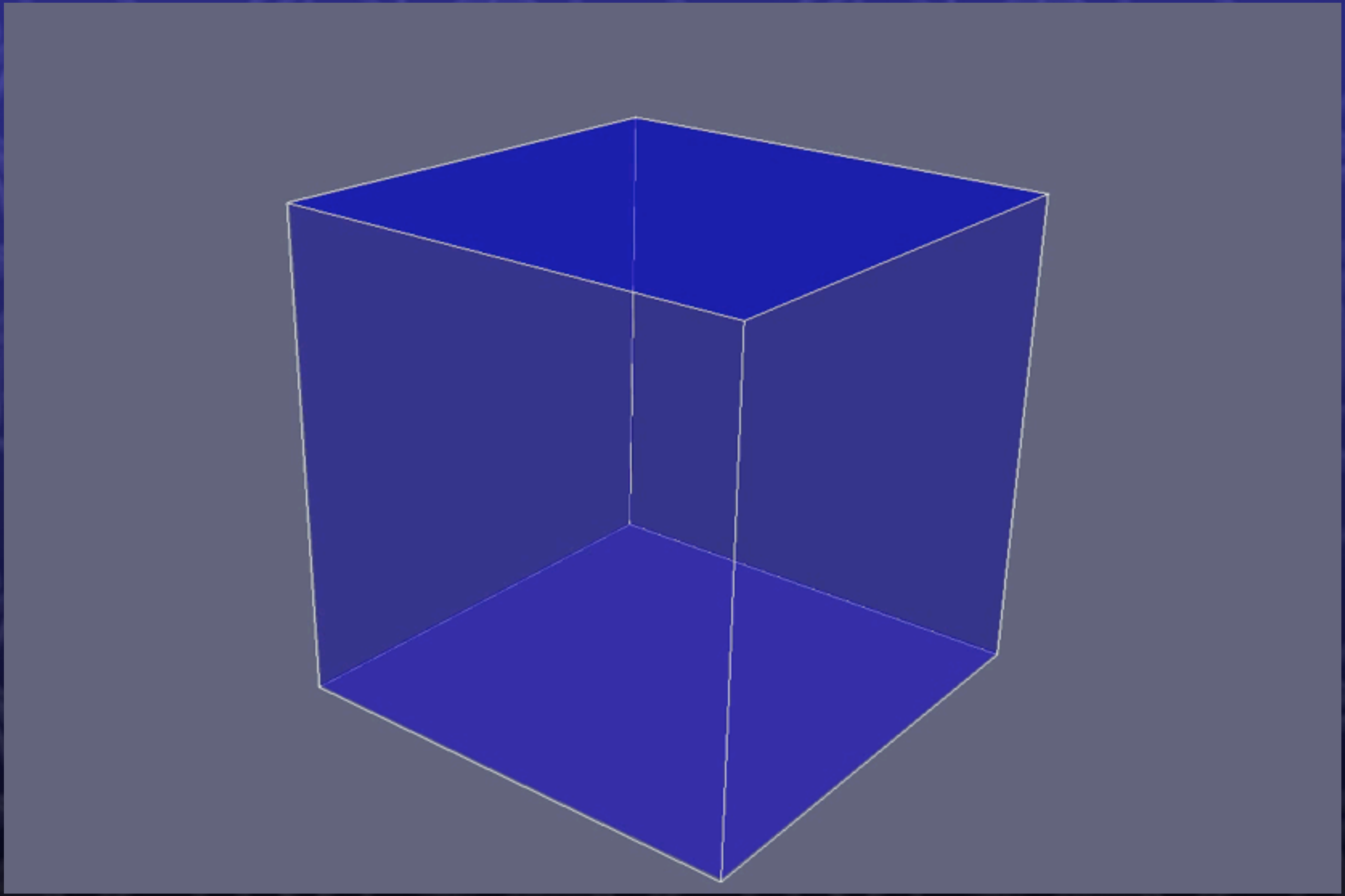
eLISA Cosmology Working Group
Workshop, 15-04-15

Defects: Abelian Higgs strings

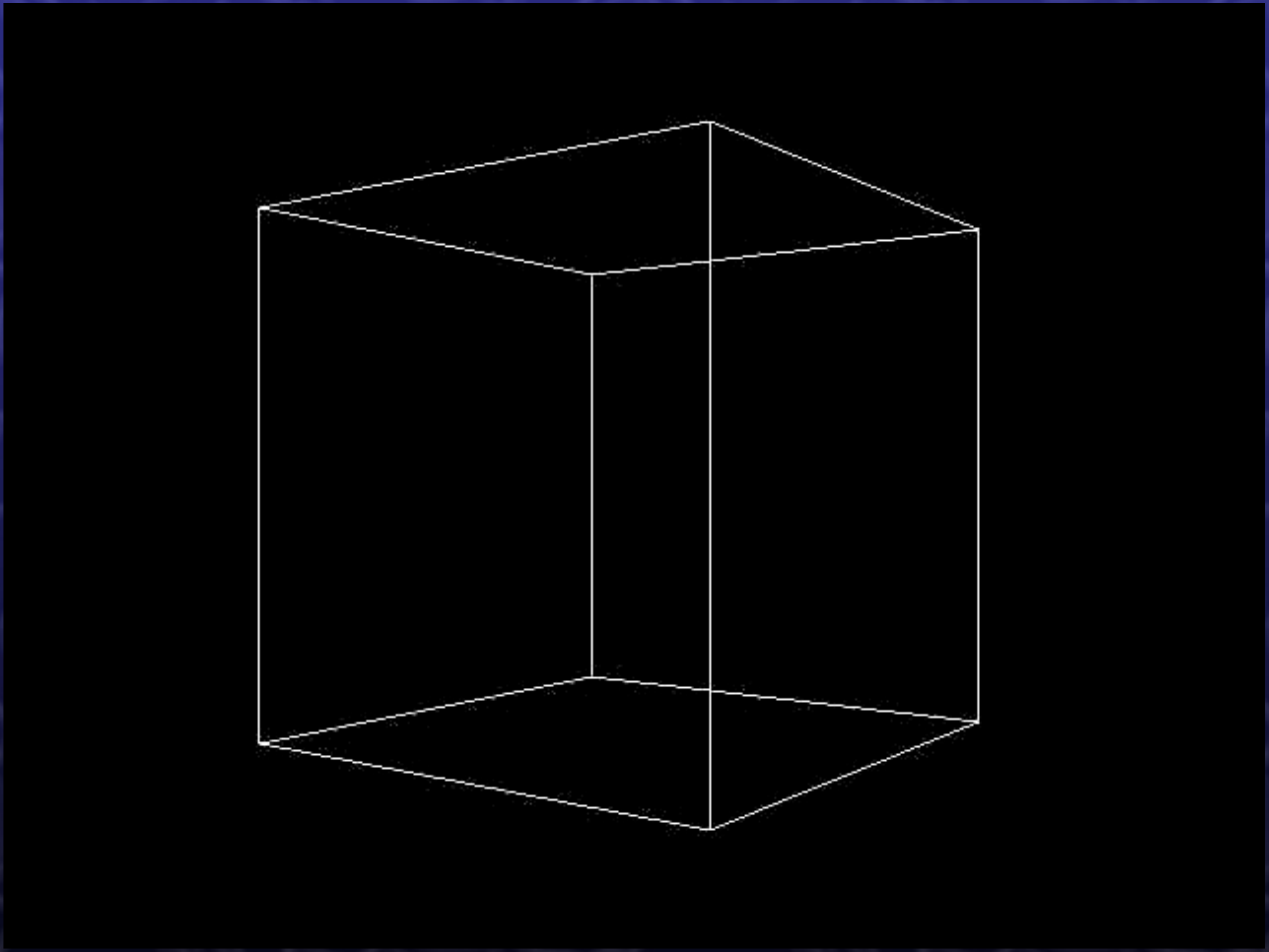
Classical solutions localised into tubes



Animation by David Daverio

Defects: Semilocal strings

Classical solutions localised into tubes (can have ends)



Achucarro, Borrill, Liddle; http://crd.lbl.gov/~borrill

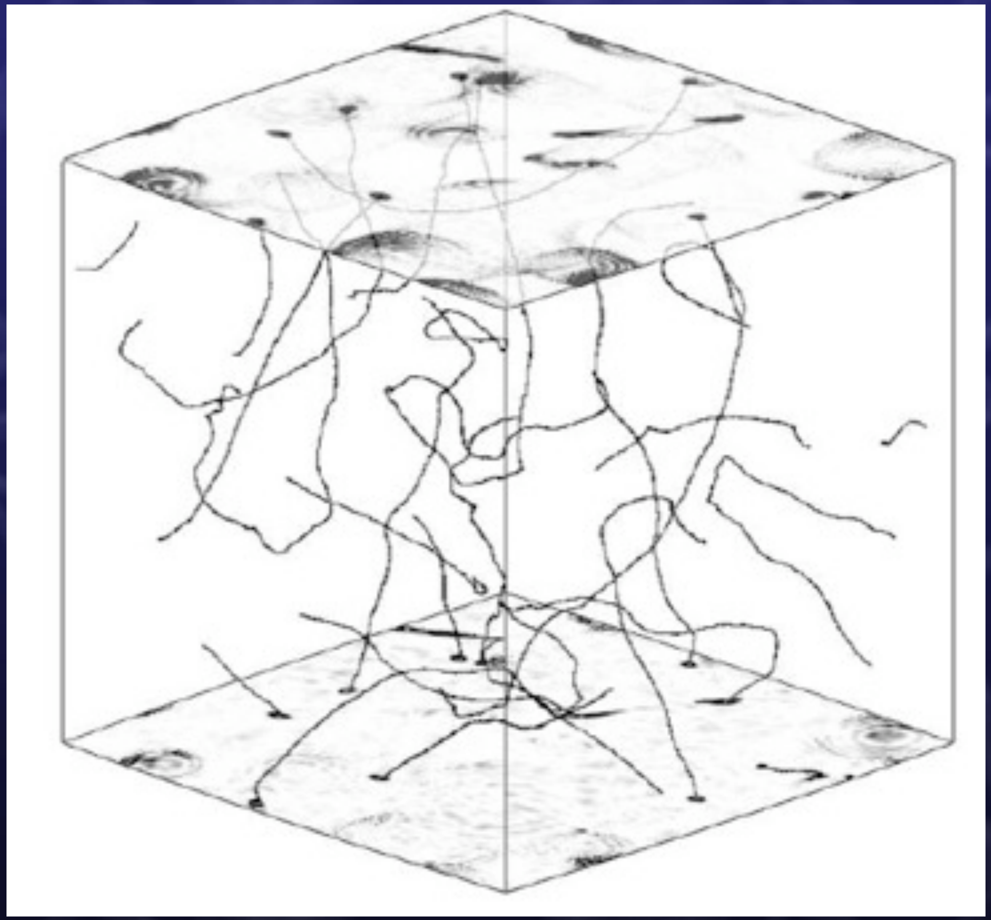
Defects: Abelian Higgs strings

$$\mathcal{L} = |D_\mu \phi|^2 - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{4} \left(|\phi|^2 - \eta^2 \right)^2$$

1 complex scalar field ϕ

1 vector field A_μ

Covariant derivative $D_\mu = \partial_\mu - i A_\mu$



“Strings” much better studied:

Nambu-Goto, unconnected segments...

String tension = μ

Adimensional number => $G\mu/c^2$

Defects: semilocal strings

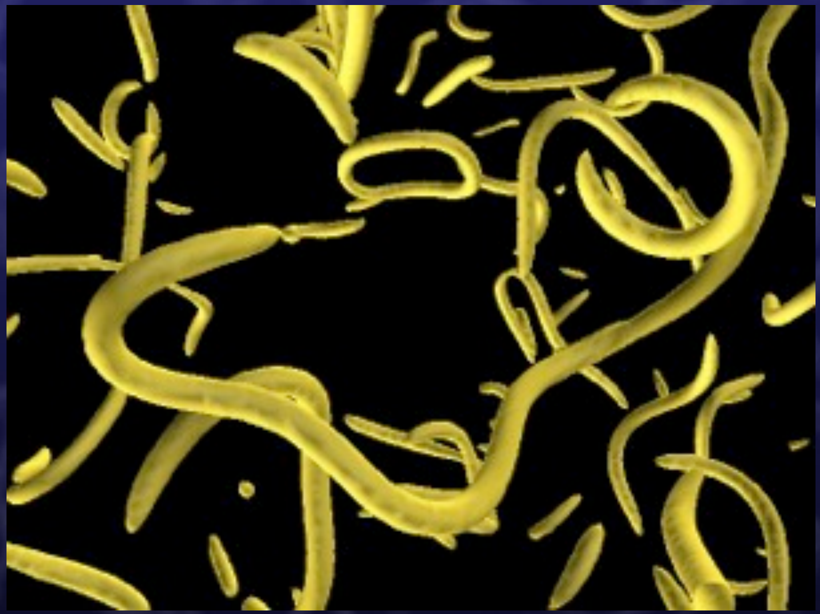
$$\mathcal{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{4}\left(|\phi_1|^2 + |\phi_2|^2 - \eta^2\right)^2$$

2 complex scalar field $\phi_1, \ \phi_2$

1 vector field A_μ

Covariant derivative $D_\mu = \partial_\mu - iA_\mu$

Only field theory simuations



Appear in D-term inflation [b] , P-term inflation [c], D branes [d]...

[a] Vachaspati, Achucarro (1991)
[b] JU, Achucarro, Davis (2004)
[c] Burrage, Davis (2007)
[d] Dasgupta, Hsu, Kallosh, Linde, Zagermann (2004)

Defects: global strings

$$\mathcal{L} = \left(\partial_\mu \psi_1\right)^2 + \left(\partial_\mu \psi_2\right)^2 - \frac{\lambda}{4}\left(\psi_1^2 + \psi_2^2 - \eta^2\right)^2$$

Real Scalar fields ψ_i

Defects: global defects

$$\mathcal{L} = \sum_{i=1}^{N} (\partial_\mu \psi_i)^2 - \frac{\lambda}{4} \left(\sum_{i=1}^{N} \psi_i^2 - \eta^2 \right)^2$$

Real Scalar fields ψ_i

$i = 2$	strings
$i = 3$	monopoles
$i = 4$	textures
$i > 4$	non-topological textures
$i \to \infty$	self-ordering scalar fields

Calculation difficulties: Approximations

String/M-theory

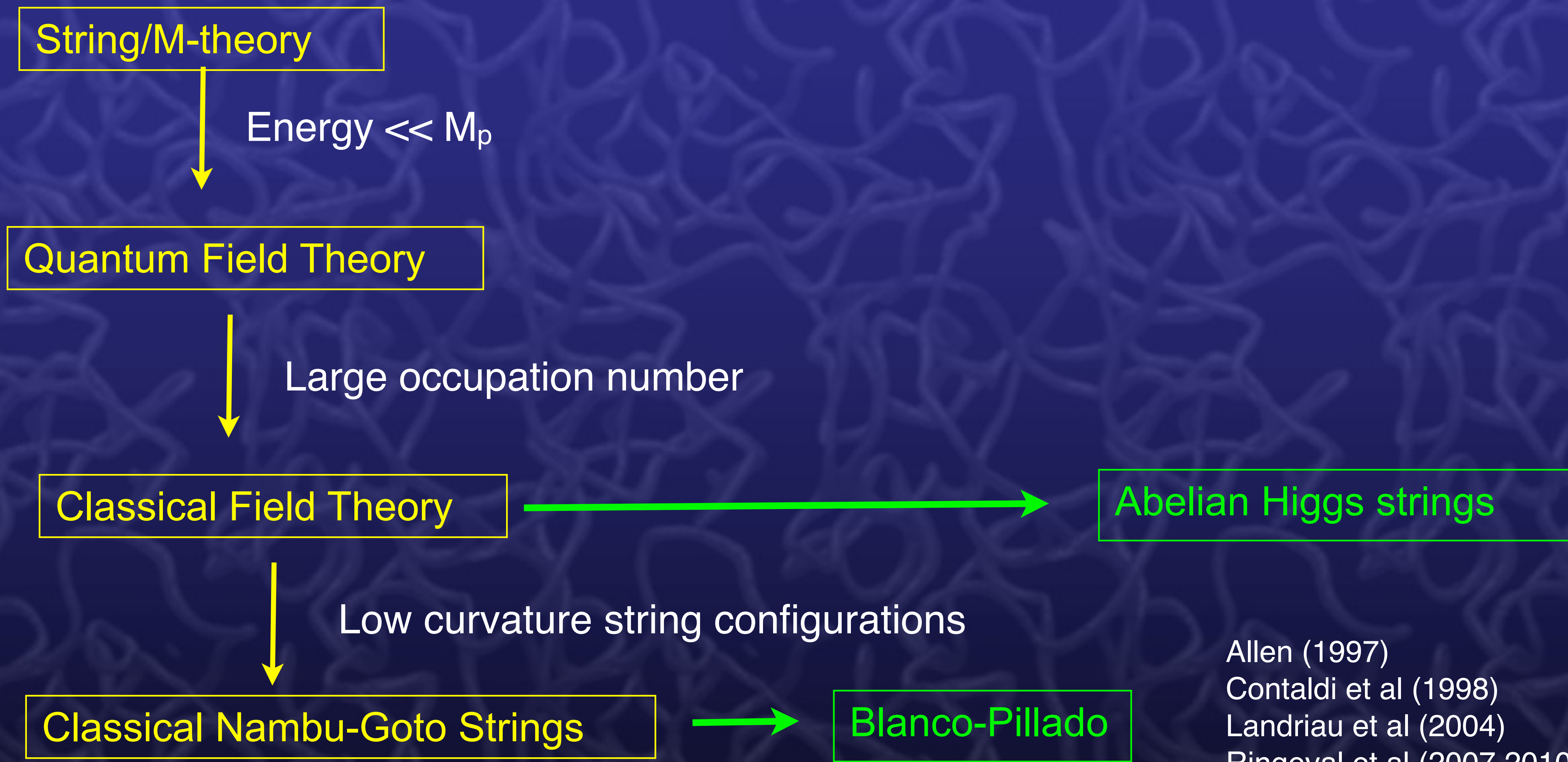
↓ Energy << M_p

Quantum Field Theory

↓ Large occupation number

Classical Field Theory → Abelian Higgs strings

Calculation difficulties: Approximations



Allen (1997)
Contaldi et al (1998)
Landriau et al (2004)
Ringeval et al (2007,2010)
Blanco-Pillado et al (2011,2014)
Lozanu et al (2014)

Calculation difficulties: Approximations

String/M-theory

↓ Energy << M_p

Quantum Field Theory

↓ Large occupation number

Classical Field Theory → Abelian Higgs strings

↓ Low curvature string configurations

Classical Nambu-Goto Strings → Blanco-Pillado

↓ Phenomenological

Unconnected segment model

Allen (1997)
Contaldi et al (1998)
Landriau et al (2004)
Ringeval et al (2007,2010)
Blanco-Pillado et al (2011,2014)
Lozanu et al (2014)

Perivolaropoulos (1995)
Albrecht et al (1997)
Wyman et al (2005,2006)

Calculation difficulties: Approximations

String/M-theory

↓ Energy << M_p

Quantum Field Theory

↓ Large occupation number

Classical Field Theory → Semilocal strings

Calculation difficulties: Approximations

String/M-theory

↓ Energy << M_p

Quantum Field Theory

↓ Large occupation number

Classical Field Theory → Semilocal strings

↓

????

Calculation difficulties: Approximations

String/M-theory

↓ Energy << M_p

Quantum Field Theory

↓ Large occupation number

Classical Field Theory →

Global strings
Global monopoles
Global textures
...
Self-ordering scalar fields O(N)

Calculation difficulties: Approximations

String/M-theory

↓ Energy << M_p

Quantum Field Theory

↓ Large occupation number

Classical Field Theory

→ Global strings
Global monopoles
Global textures
...
Self-ordering scalar fields O(N)

↓

non-linear σ model (textures)

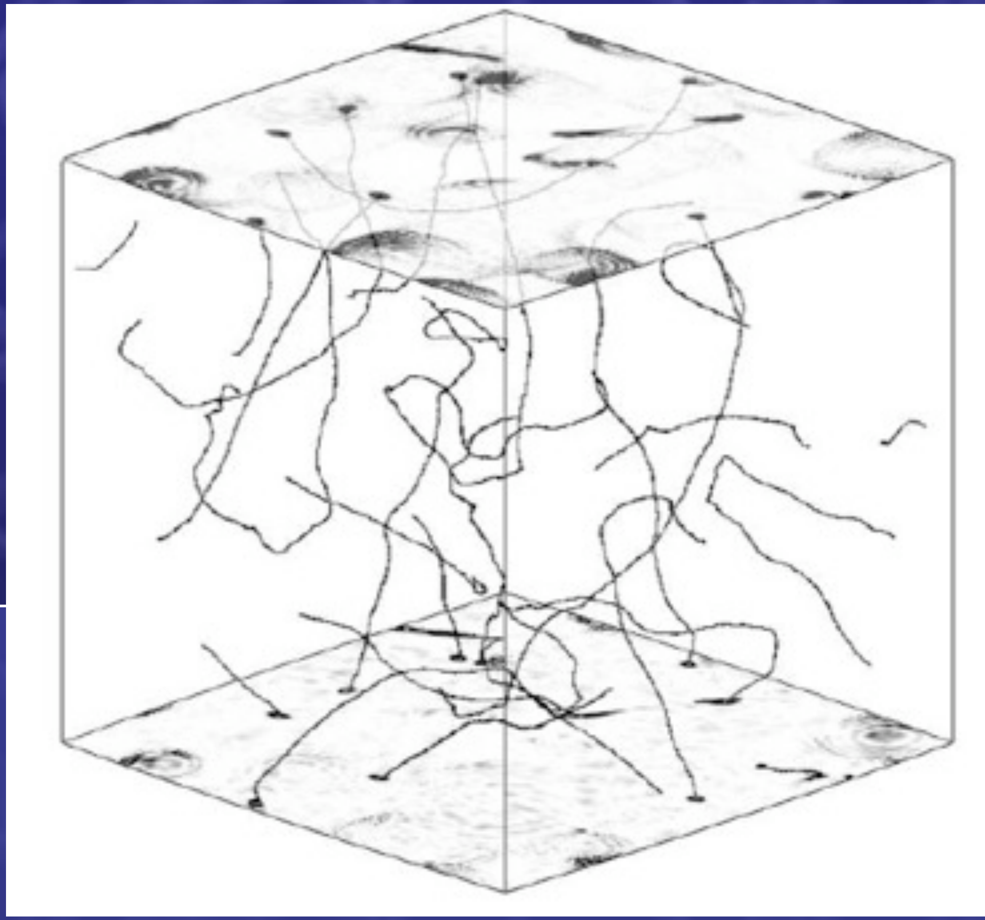
Pen, Seljak, Turok (1996)
Durrer, Kunz, Melchiorri (1997)

↘

analytic approximation large N

Durrer, Kunz, Melchiorri (1997)
Fenu, Figueroa, Durrer, Garcia-Bellido, Kunz (2014)

Defects in the early universe: CMB



- Form at $t \sim 10^{-36}$ sec
- Observe at $t_0 \sim 10^{17}$ sec.
- Scaling hypothesis: dimensional analysis based on physical scales

  checked numerically

Abelian Higgs Model: CMB

$$S = -\int d^4x \sqrt{-g} \left(g^{\mu\nu} D_\mu \phi^* D_\nu \phi + V(\phi) + \frac{1}{4e^2} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \right)$$

Complex scalar field $\phi(\mathbf{x}, t)$, vector field $A_\mu(\mathbf{x}, t)$

Covariant derivative $D_\mu = \partial_\mu - iA_\mu$.

Potential $V(\phi) = \frac{1}{2}\lambda(|\phi|^2 - v^2)^2$.

Metric $g_{\mu\nu} = a^2(\tau)\eta_{\mu\nu}$

Abelian Higgs Model: Shrinking String

comoving string shrinks as a^{-1} → strings slip through lattice points

$$
\begin{aligned}
\ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} - D^2\phi + \lambda a^2(|\phi|^2 - v^2)\phi &= 0, \\
\partial^\mu \left(\frac{1}{e^2}F_{\mu\nu}\right) - i a^2(\phi^* D_\nu \phi - D_\nu \phi^* \phi) &= 0,
\end{aligned}
$$

$(A_0 = 0)$

Abelian Higgs Model: Shrinking String

comoving string shrinks as a^{-1} → strings slip through lattice points

$$\ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} - D^2\phi + \lambda a^{2s}(|\phi|^2 - v^2)\phi = 0,$$

$$\partial^\mu\left(\frac{a^{2(1-s)}}{e^2}F_{\mu\nu}\right) - ia^2(\phi^* D_\nu\phi - D_\nu\phi^*\phi) = 0,$$

$(A_0 = 0)$

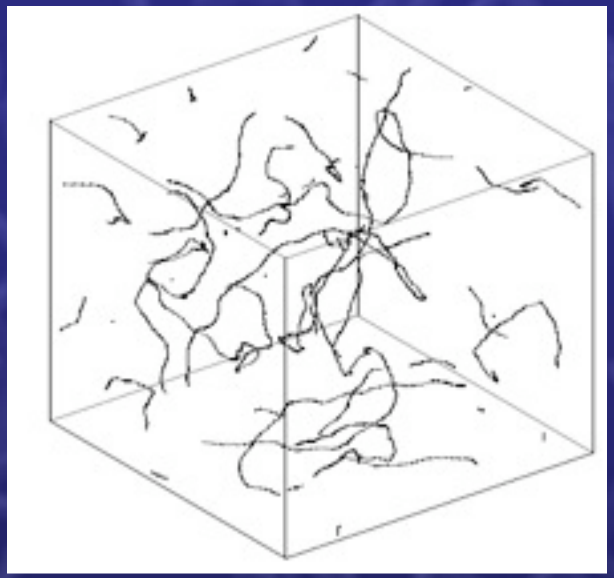
"Real value" $\longrightarrow$ s=1

For s<1 $\longrightarrow$ string "fattens"

Preserves Gauss's Law,
but violates EM conservation

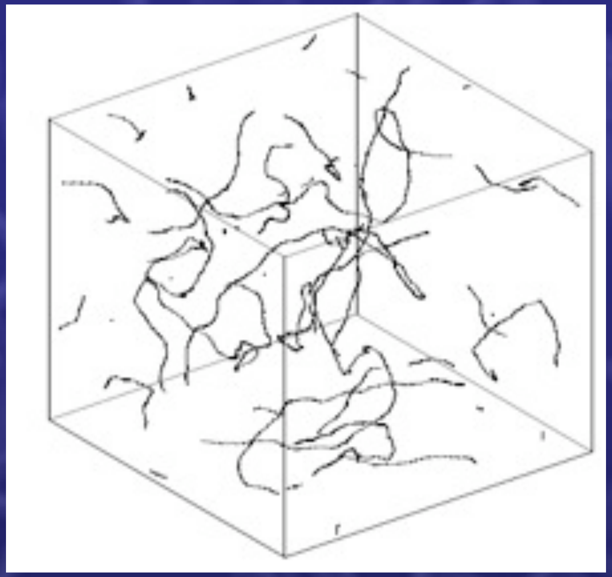
Press, Ryden, Spergel (1989); Moore, Shellard, Martins (2001); Bevis, Hindmarsh, Kunz, JU (2007-2010)

Abelian Higgs Model



Numerical simulations
$N=1024^3$ (2010)
$s<0.3$

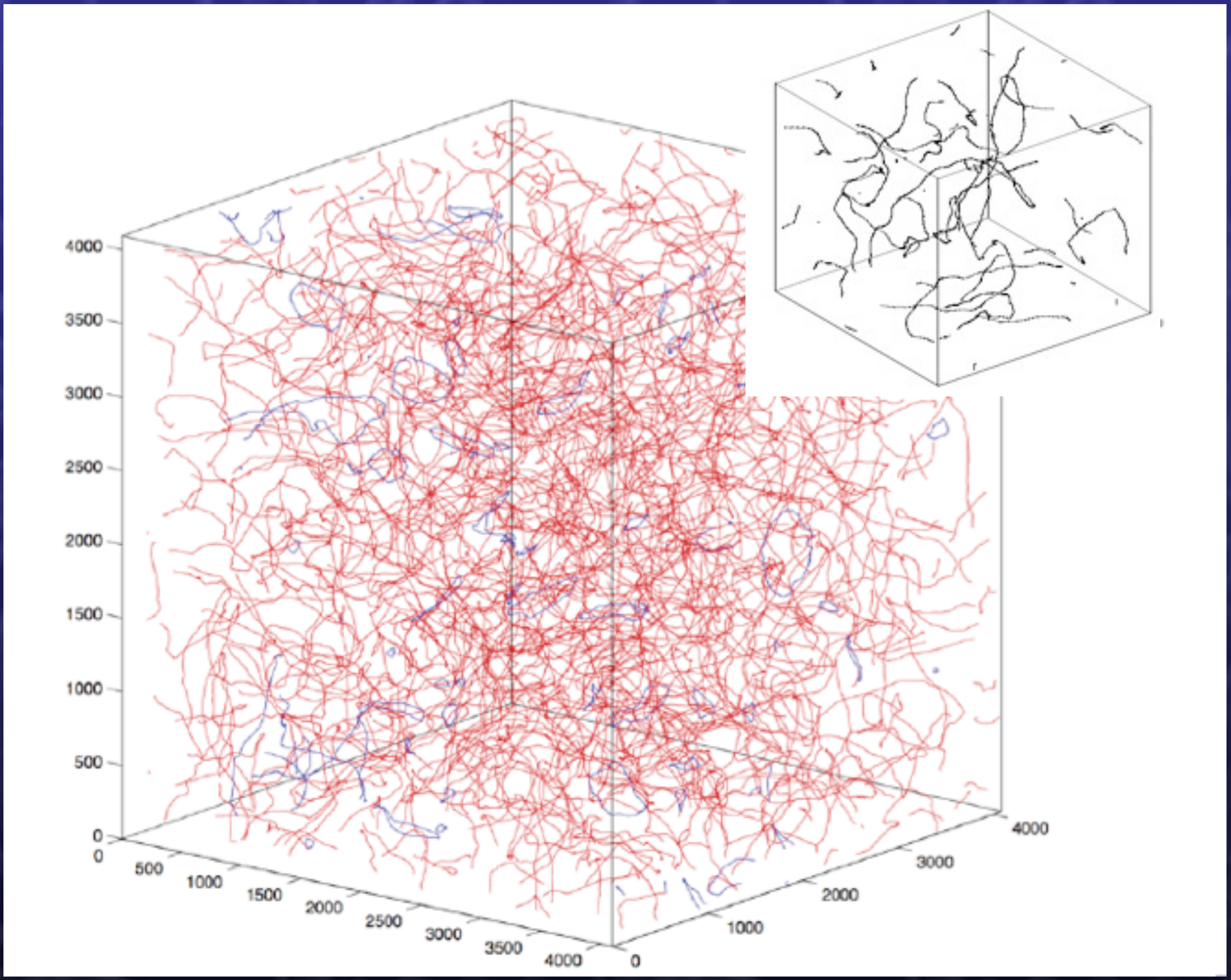
Abelian Higgs Model



Numerical simulations
$N=1024^3$ (2010)
$s<0.3$

Numerical simulations*
$N=4096^3$ (2014/5)
$s=1$

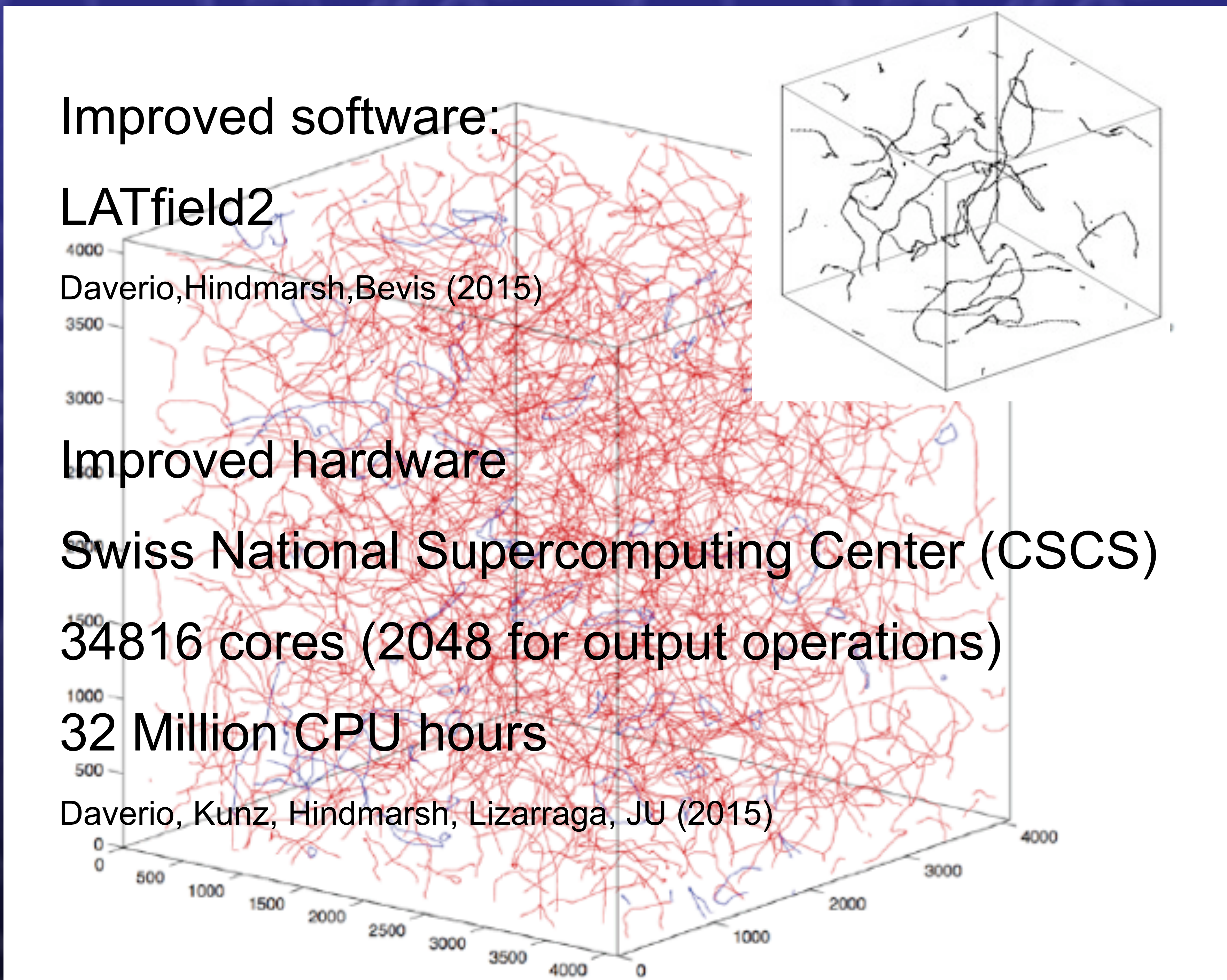
Abelian Higgs Model



Numerical simulations
N=1024^3 (2010)
s<0.3

Numerical simulations*
N=4096^3 (2014/5)
s=1

Abelian Higgs Model



Improved software:

LATfield2

Daverio,Hindmarsh,Bevis (2015)

Improved hardware

Swiss National Supercomputing Center (CSCS)

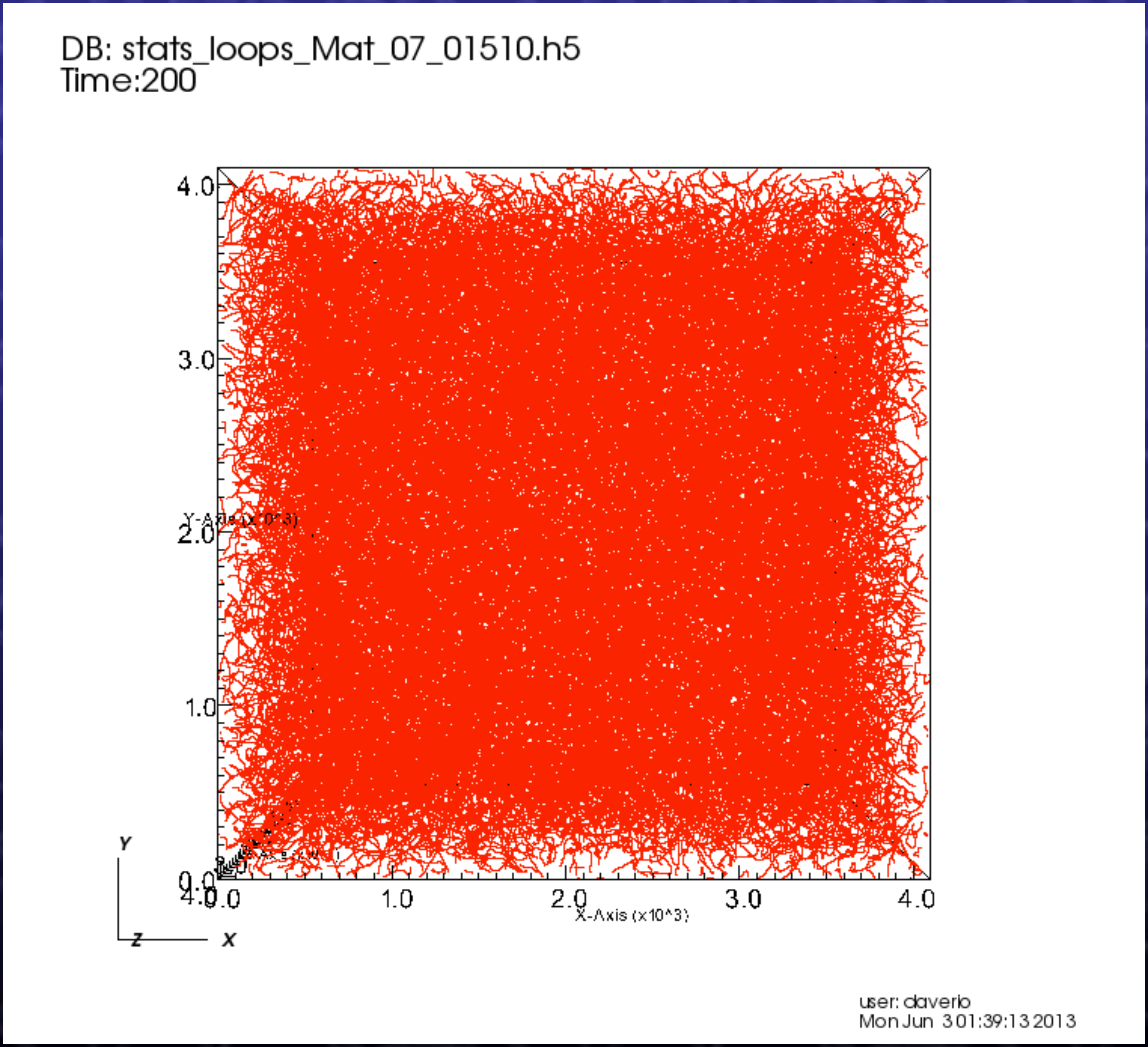
34816 cores (2048 for output operations)

32 Million CPU hours

Daverio, Kunz, Hindmarsh, Lizarraga, JU (2015)

Numerical simulations
$N=1024^3$ (2010)
s<0.3

Numerical simulations*
$N=4096^3$ (2014/5)
s=1


DB: stats_loops_Mat_07_01510.h5
Time:200
Y-Axis (x10^3)
X-Axis (x10^3)
0.0
1.0
2.0
3.0
4.0
Y
Z
X
user: daverio
Mon Jun 3 01:39:13 2013

UETC method for CMB anisotropies power spectrum

Time dependent diff operator

Source (Energy momentum)

$$\mathcal{D}_{\alpha\beta}(\tau,k)h_\beta(\tau,k) = S_\alpha(\tau.k)$$

Linear perturbations

Power spectrum [a]

$$\langle|h_\alpha(\tau_0,k)|^2\rangle = \int\int \mathcal{D}^{-1}\mathcal{D}^{-1}\langle S_\alpha(\tau,k)S^*_\alpha(\tau',k)\rangle$$

Need unequal-time correlators (UETCs) of energy-momentum tensor

$$C_{\mu\nu\rho\lambda}(k,\tau,\tau') = \langle T_{\mu\nu}(k,\tau)T^*_{\rho\lambda}(k,\tau')\rangle$$

[a] Pen, Seljak, Turok (1997); Durrer, Kunz, Melchiorri (1998, 2002)

Calculate UETCs

$$\langle S_\alpha(\tau,k)S^*_\alpha(\tau',k)\rangle \sim C_{\mu\nu\rho\sigma} \sim \langle T_{\mu\nu}(\tau,k)T^*_{\rho\sigma}(\tau',k)\rangle$$

Isotropy + EM conservation + parity: Only 3 scalars, 1 vector, 1 tensor

For example:

$$S^S_\Phi = T_{00} - 3\frac{\dot{a}}{a}\frac{i\hat{k}_m}{k}T_{0m}$$

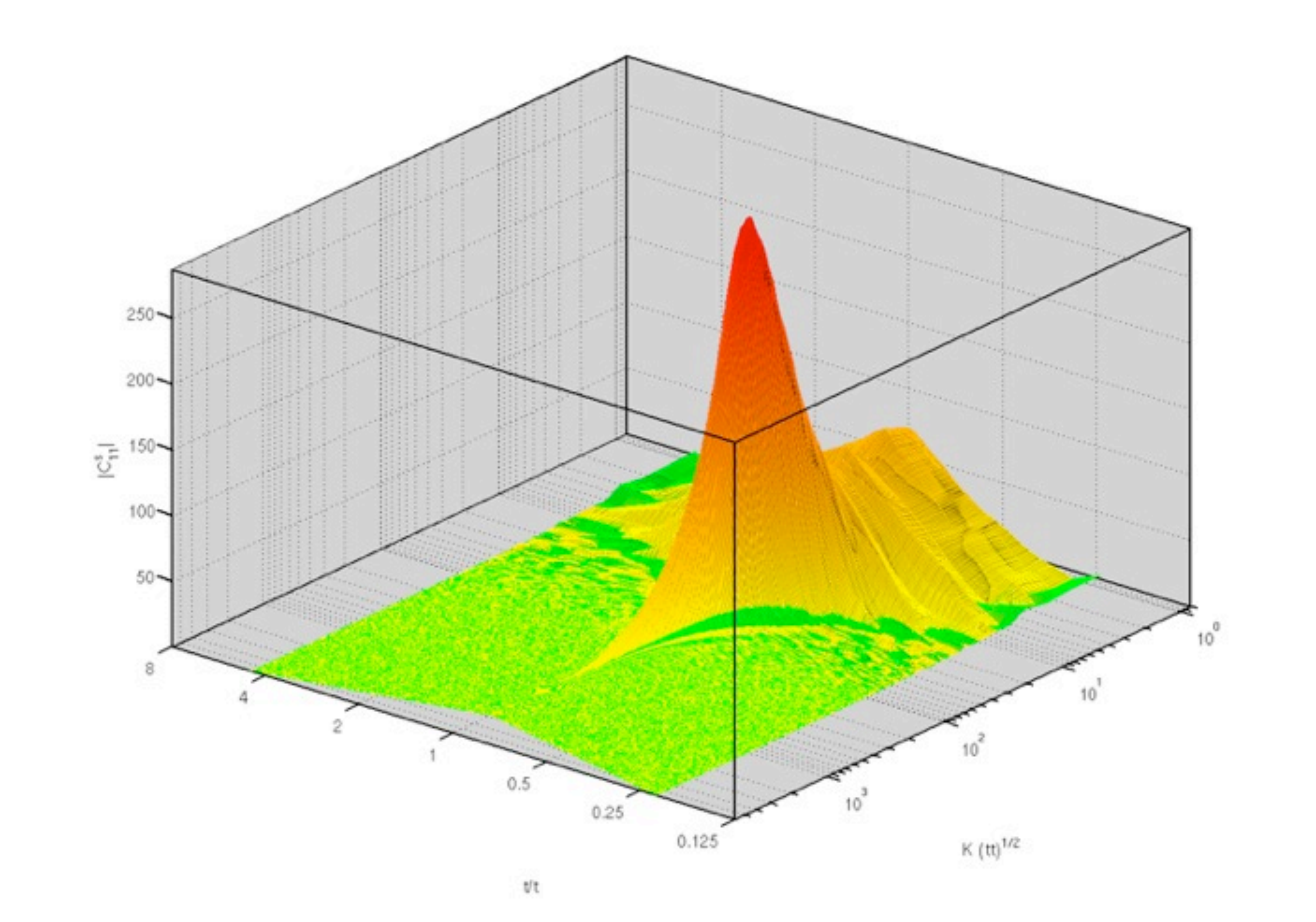
$$\langle S^S_\Phi(\mathbf{k},\tau)S^{S*}_\Phi(\mathbf{k},\tau')\rangle \sim C^S_{11}\left(k\sqrt{\tau\tau'},\tau/\tau'\right)$$

Calculate UETCs

$$\langle S_\alpha(\tau,k) S^*_\alpha(\tau',k)\rangle \sim C_{\mu\nu\rho\sigma} \sim \langle T_{\mu\nu}(\tau,k) T^*_{\rho\sigma}(\tau',k)\rangle$$

Isc ... or

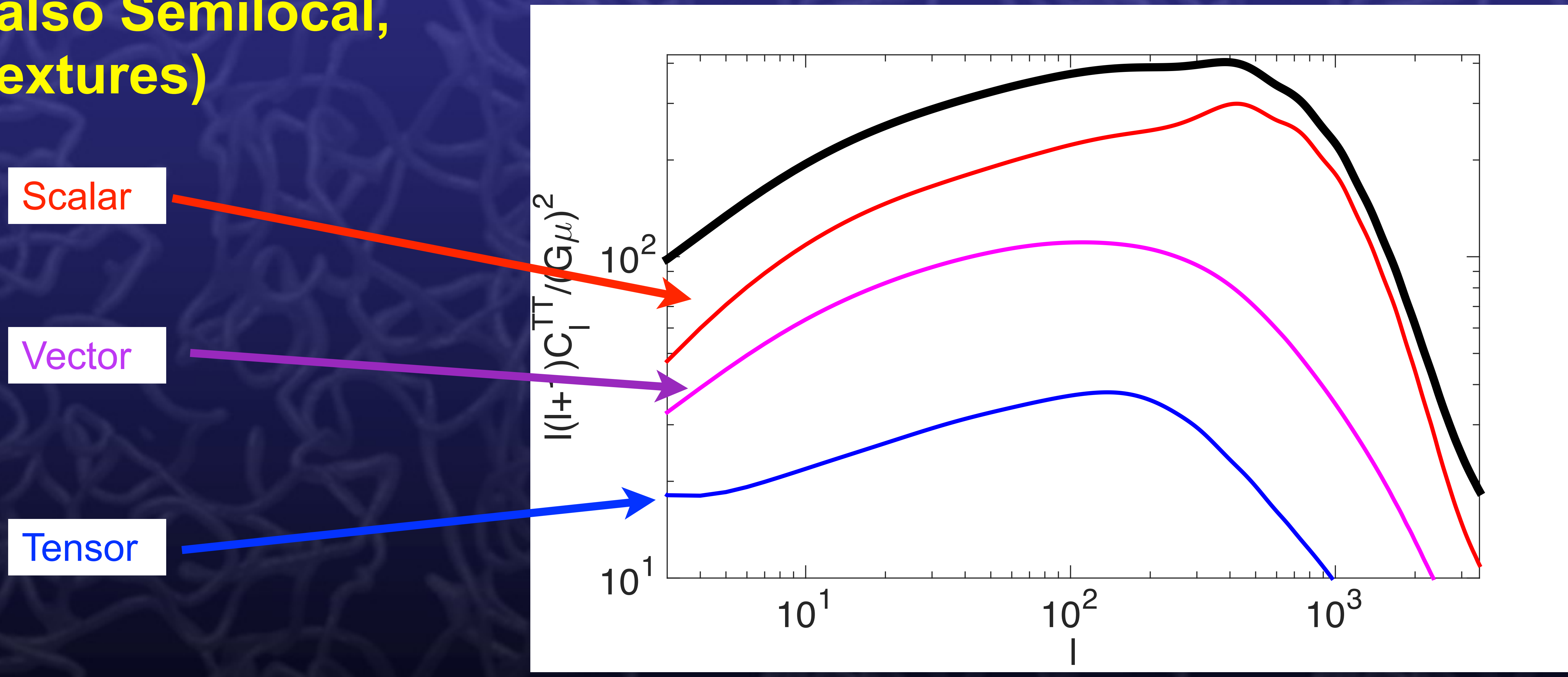
For exam



Temperature power spectrum

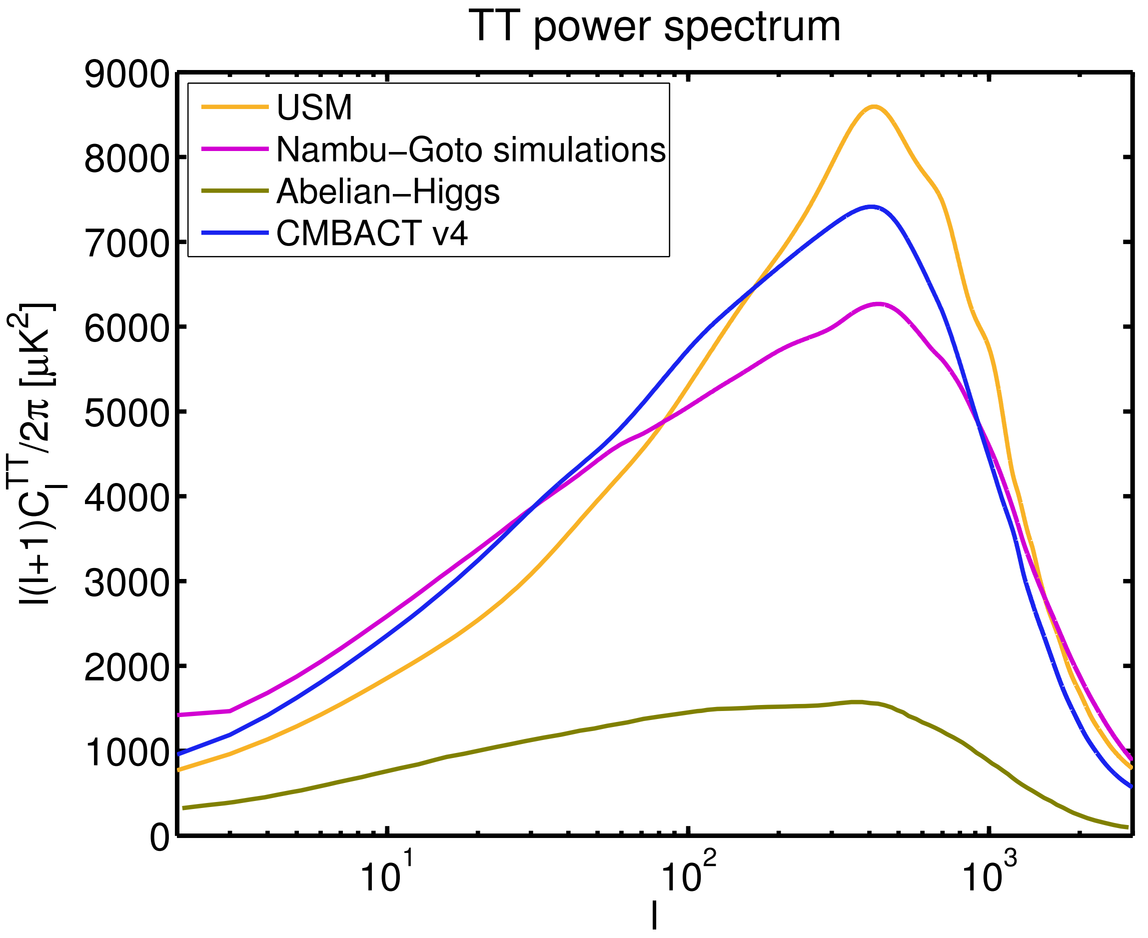
scalar-vector-tensor

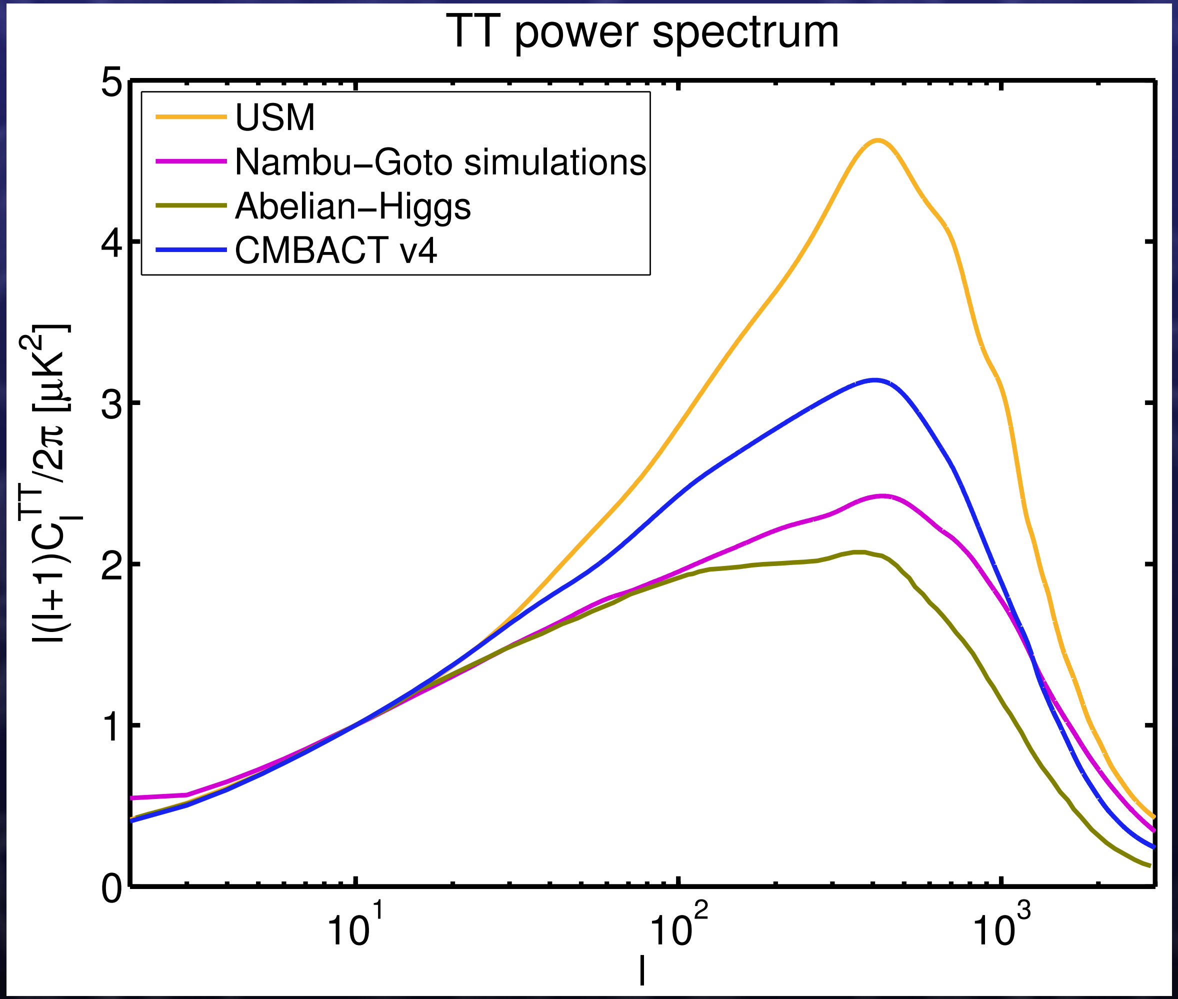
ABELIAN HIGGS (also Semilocal, textures)



Temperature power spectrum

AH, NG, USM

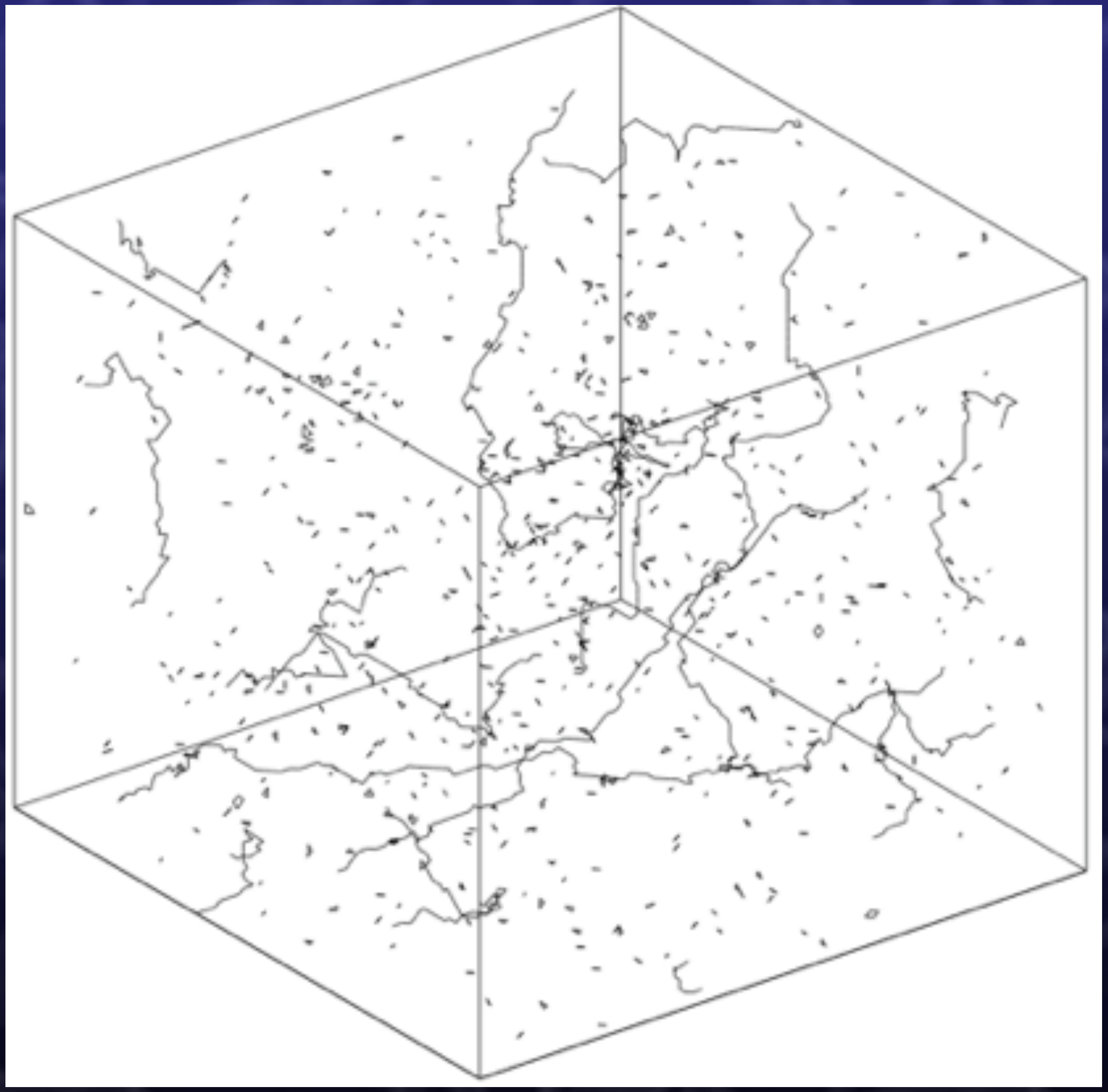


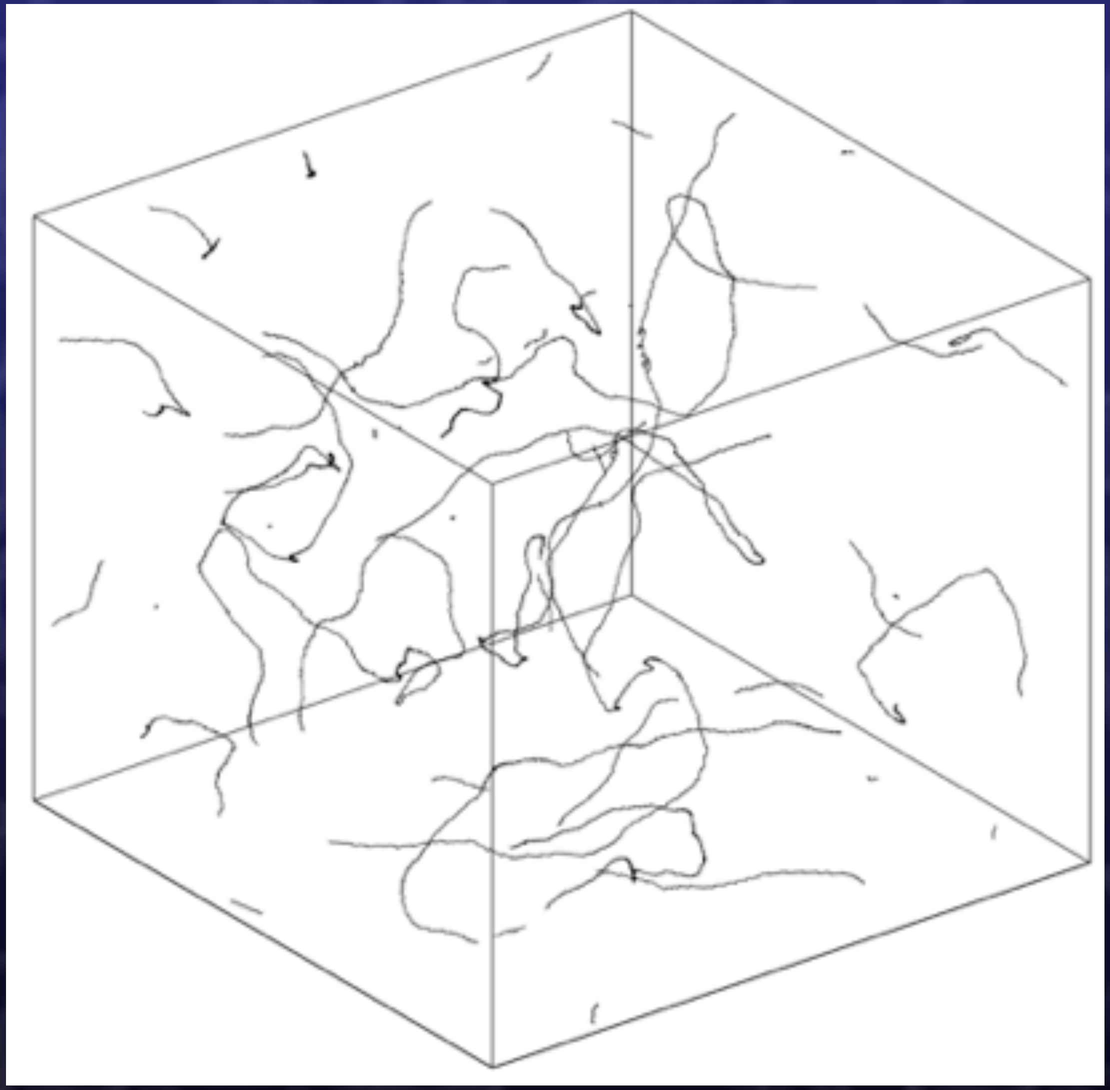


Battye, Moss (2010);
Bevis, Kunz, Hindmarsh, JU (2010);
Lazanu, Shellard (2015)

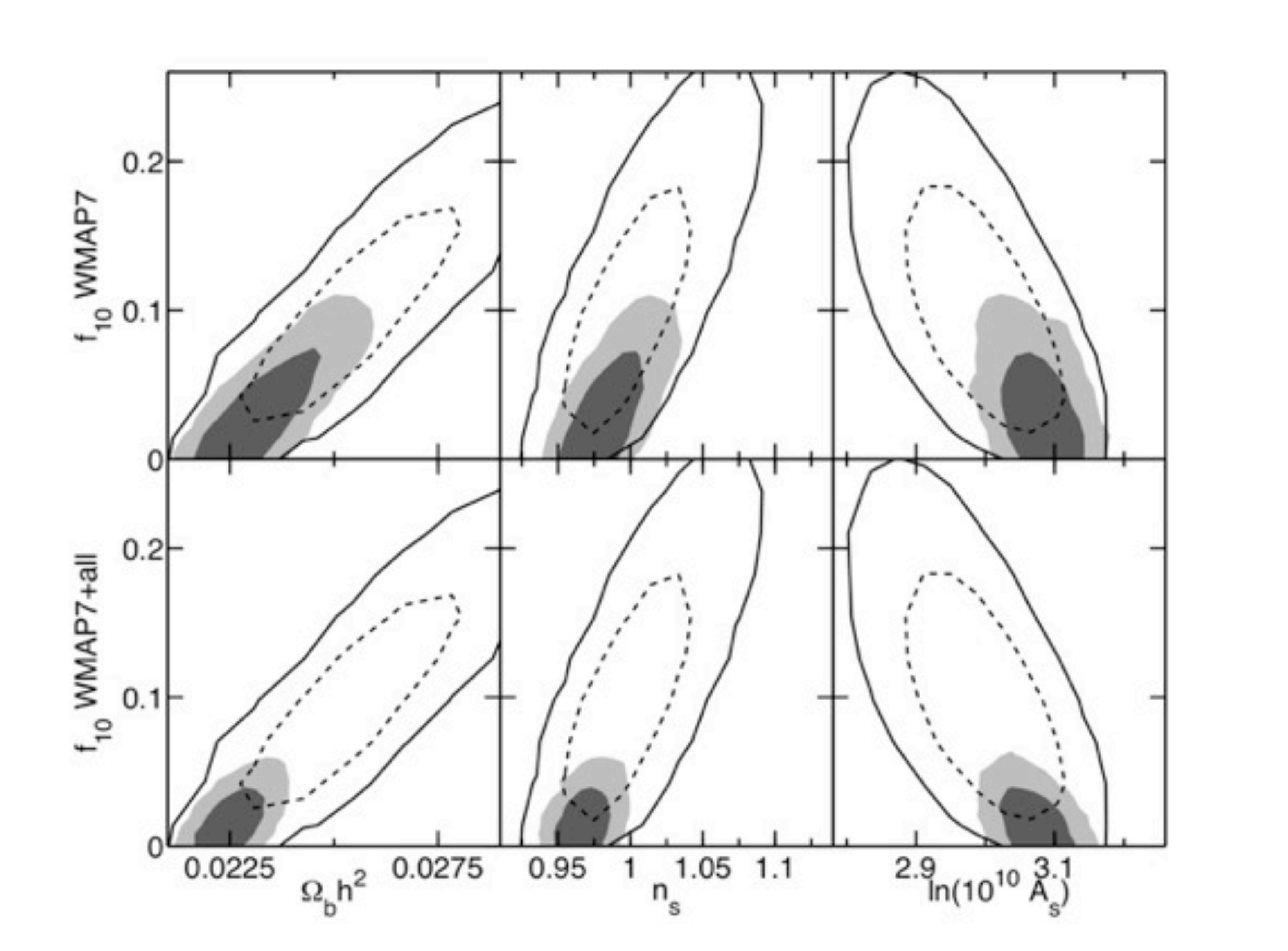
Nambu-Goto simulation

Abelian Higgs simulation





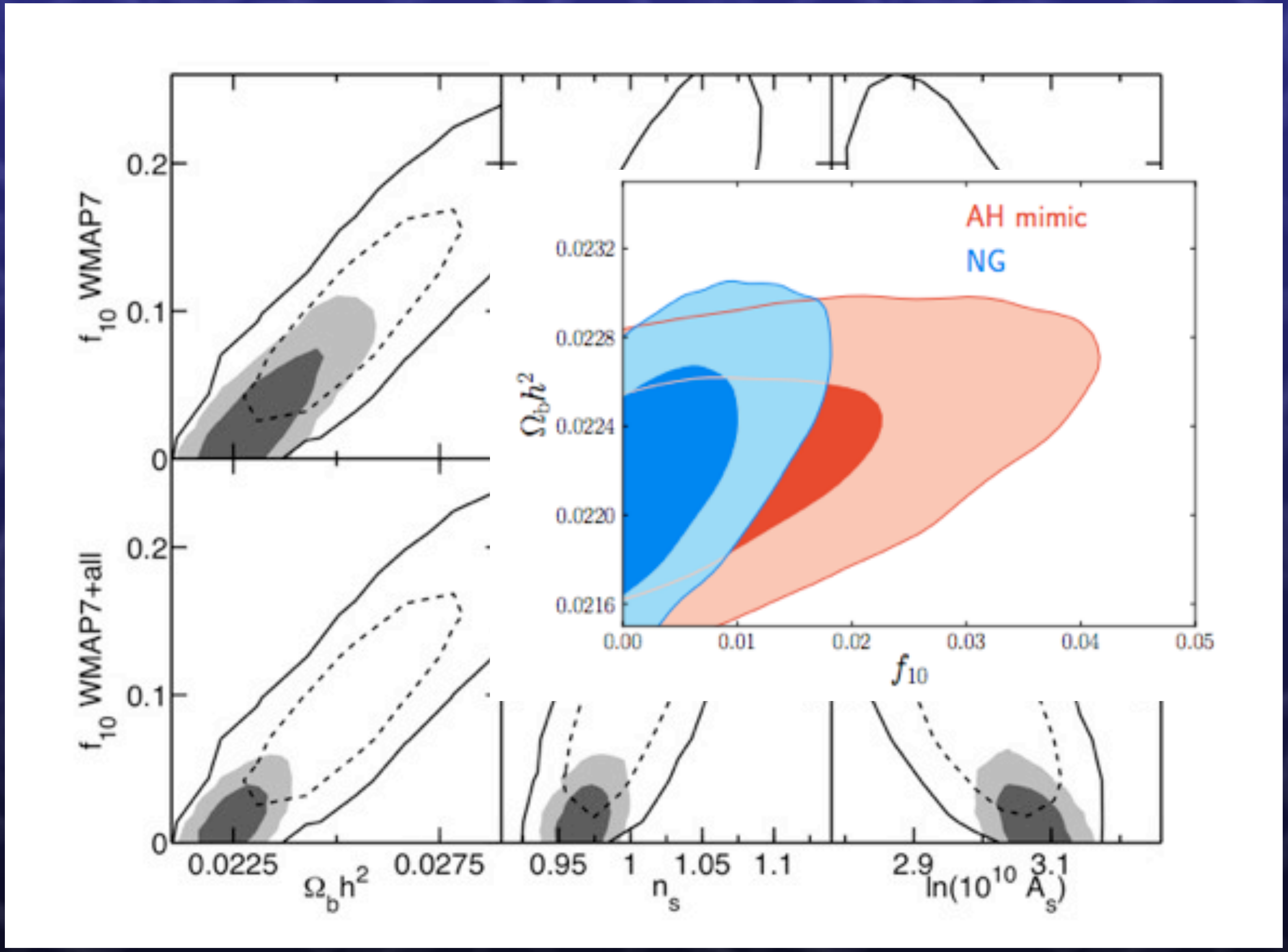
Fitting CMB with inflation + defects



Unshaded (2008) WMAP3
Shaded (2011) WMAP7

Bevis, Kunz, Hindmarsh, JU (2008)
JU, Bevis, Kunz, Hindmrash (2011)

Fitting CMB with inflation + defects



Unshaded (2008) WMAP3
Shaded (2011) WMAP7
Planck (2013/4)

Bevis, Kunz, Hindmarsh, JU (2008)

JU, Bevis, Kunz, Hindmrash (2011)

Fitting CMB with inflation + defects

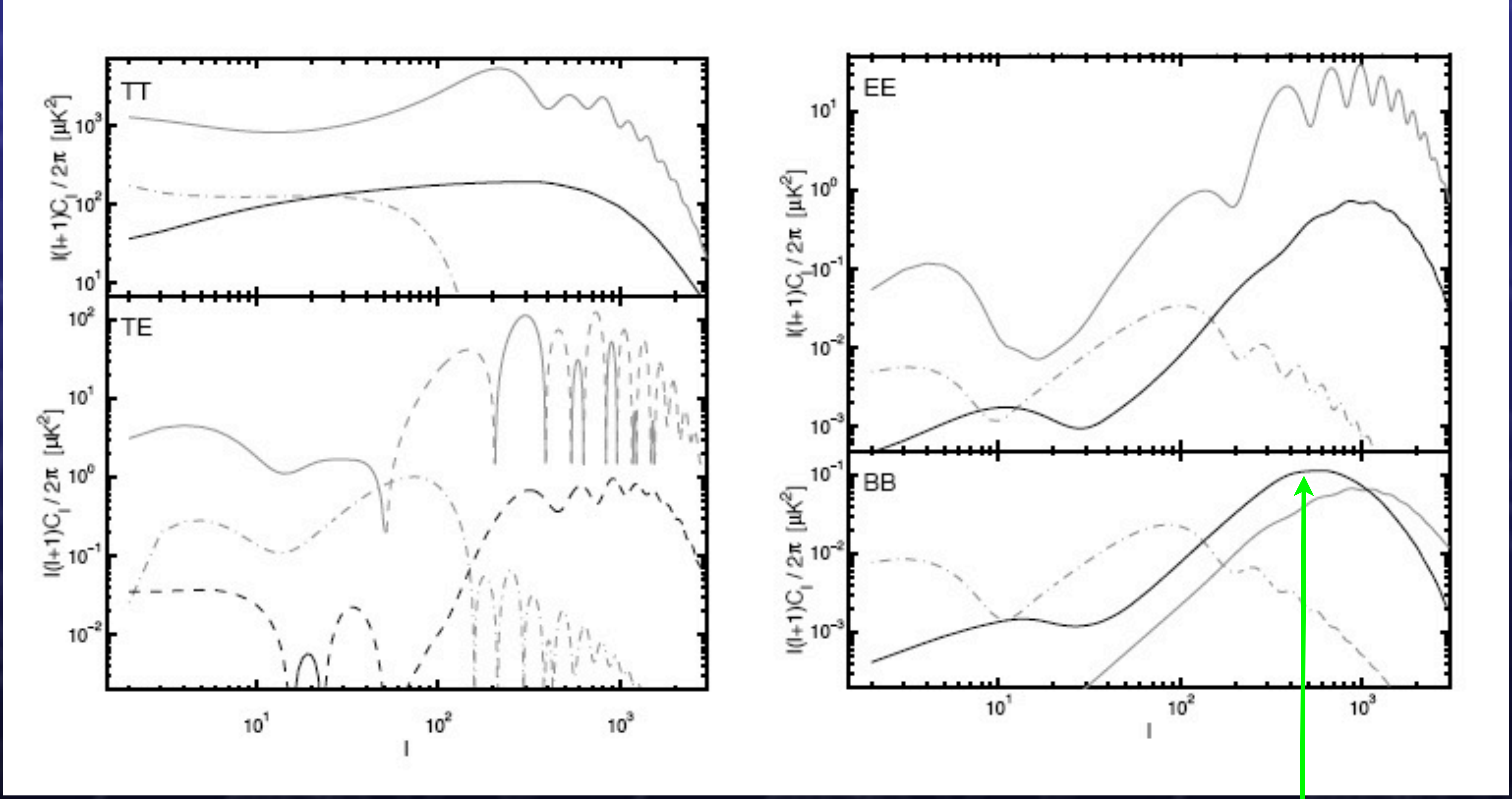
Some numerical results for strings:

Model	Data set	$10^6\, G\mu$ (95%)
AH[a]	WMAP3+BOOMERANG+CBI+ACBAR+VSA	0.7
USM-AH[b]	WMAP5	0.68
USM-NG[b]	WMAP5	0.28
AH[c]	WMAP7	0.57
USM-NG[d]	WMAP7+ACT	0.16
AH[e]	Planck+WP	0.32
USM-AH[e]	Planck+WP	0.36
USM-NG[e]	Planck+WP	0.15
NG[f]	Planck+WP	0.15
AH[g]	BICEP+Planck+WP	0.27

$G\mu \sim 10^{-7} \sim 10^{15}$ GeV

a) Bevis, Hindmarsh, Kunz, JU (2008)
b) Battye, Moss (2010)
c) Dunkley et al (ACT) (2010)
d) JU, Bevis, Hindmarsh, Kunz (2011)
e) Planck (2013/4)
f) Lazanu,Shellard (2015)
g) Lizarraga et al (2014)

Temperature and Polarization CMB Power Spectra

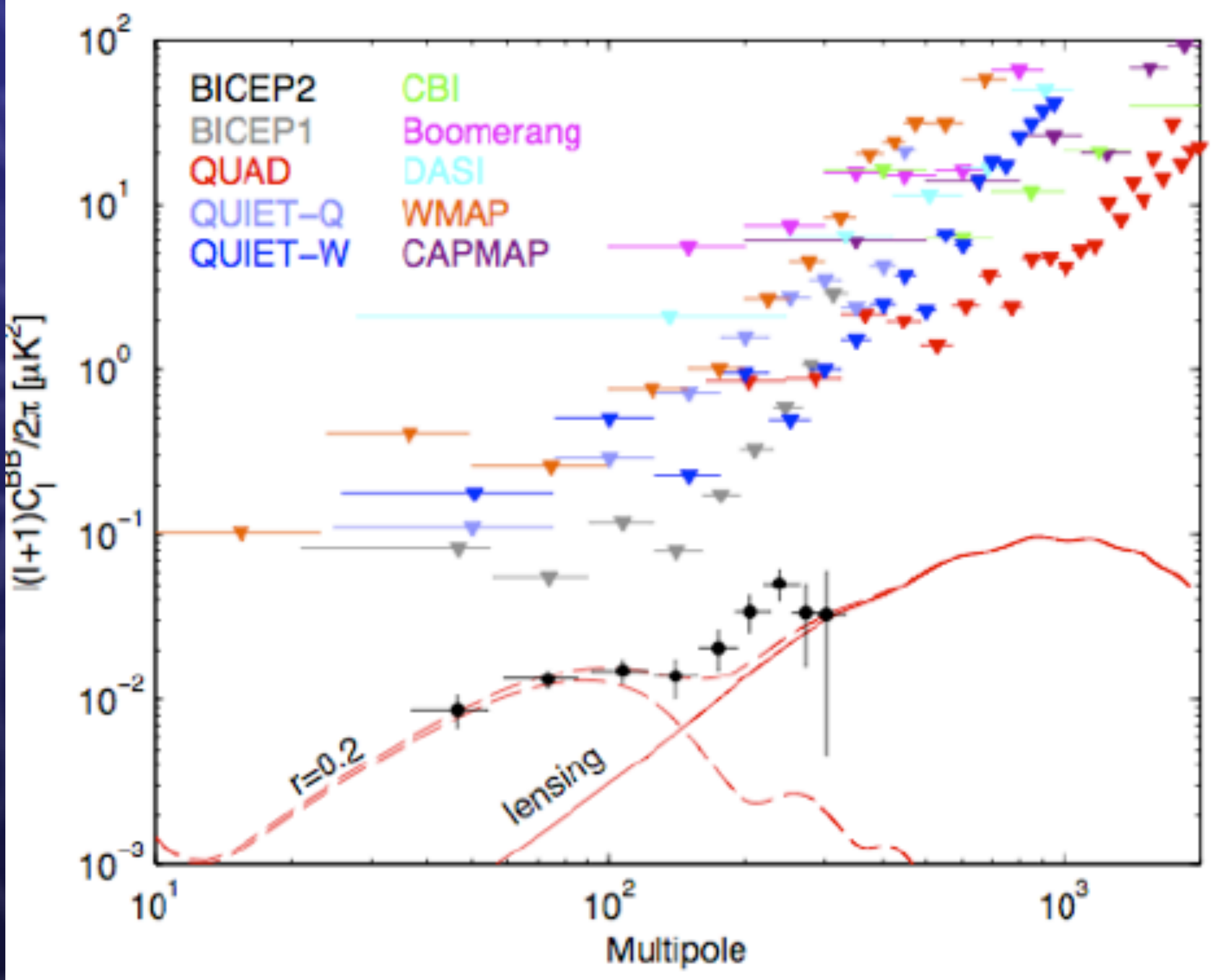


Inflation r=0.4 and strings f_{10}=0.1

STRINGS!

B Polarization CMB Power Spectra

BICEP2 : Primordial (inflationary) GW?

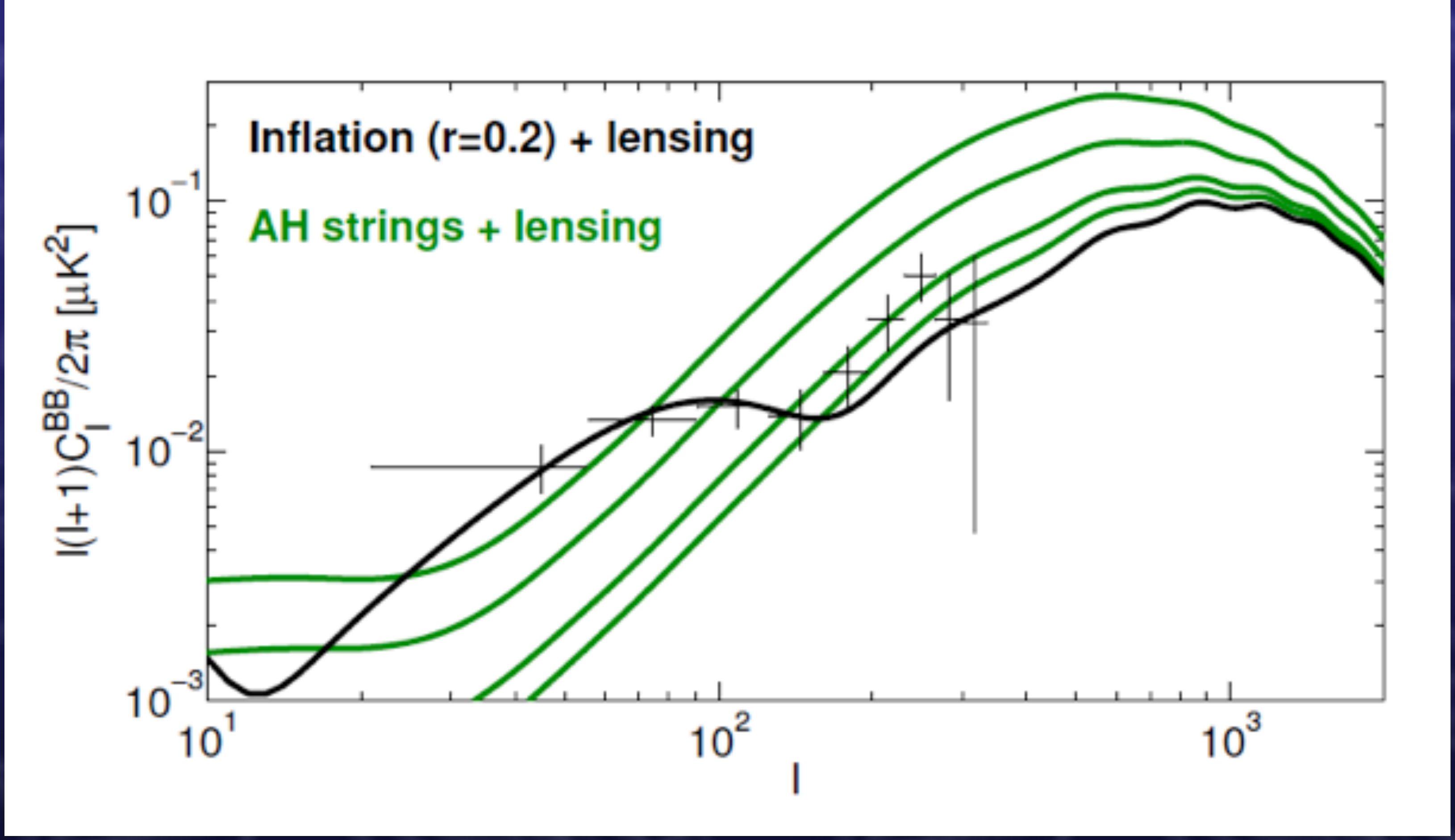


$$r = 0.2^{+0.07}_{-0.05}$$

Ade et al, BICEP colaboration (2014)

B Polarization CMB Power Spectra

BICEP2 : Why not cosmic defects?
(cosmics string, semilocals,textures, self-ordering scalar fields?



Lizarraga, JU, Daverio, Kunz, Hindmarsh, Liddle (2014)
Moss, Pogosian (2014)
Durrer, Figueroa, Kunz (2014)

B Polarization CMB Power Spectra

BICEP2 : Why not cosmic defects?
(cosmics string, semilocals,textures, self-ordering scalar fields?

Dataset	Planck + WP + BICEP2		
Param	r	strings	textures
n_s	$0.962^{+0.007}_{-0.007}$	$0.955^{+0.007}_{-0.008}$	$0.962^{+0.007}_{-0.007}$
r	$0.15^{+0.03}_{-0.04}$	-	-
$10^{12}(G\mu)^2$	-	$0.084^{+0.026}_{-0.025}$	$0.73^{+0.14}_{-0.15}$
$-\ln \mathcal{L}_{\max}$	5265.8	5280.1	5266.8

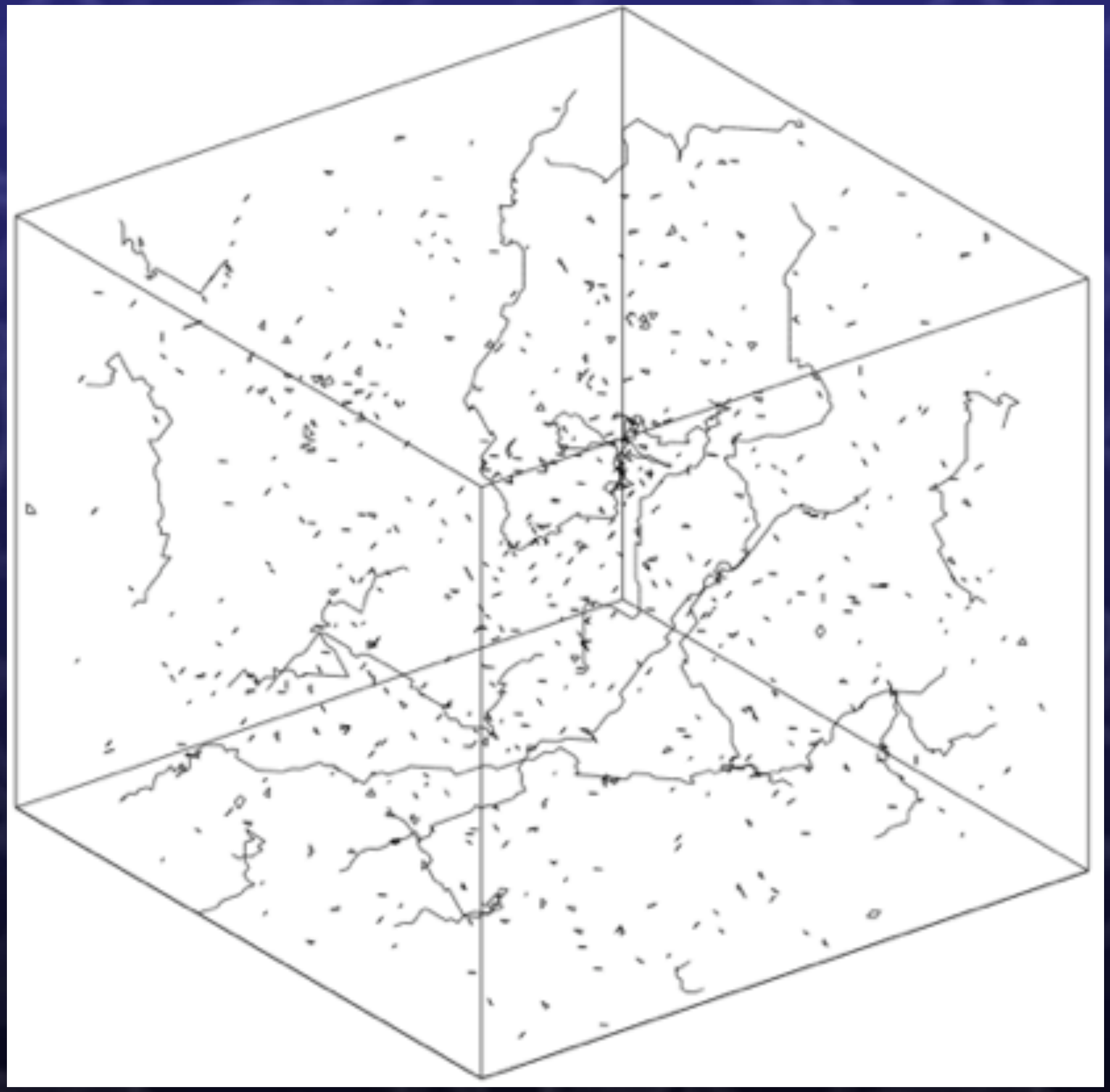
Lizarraga, JU, Daverio, Kunz, Hindmarsh, Liddle (2014)
Moss, Pogosian (2014)
Durrer, Figueroa, Kunz (2014)

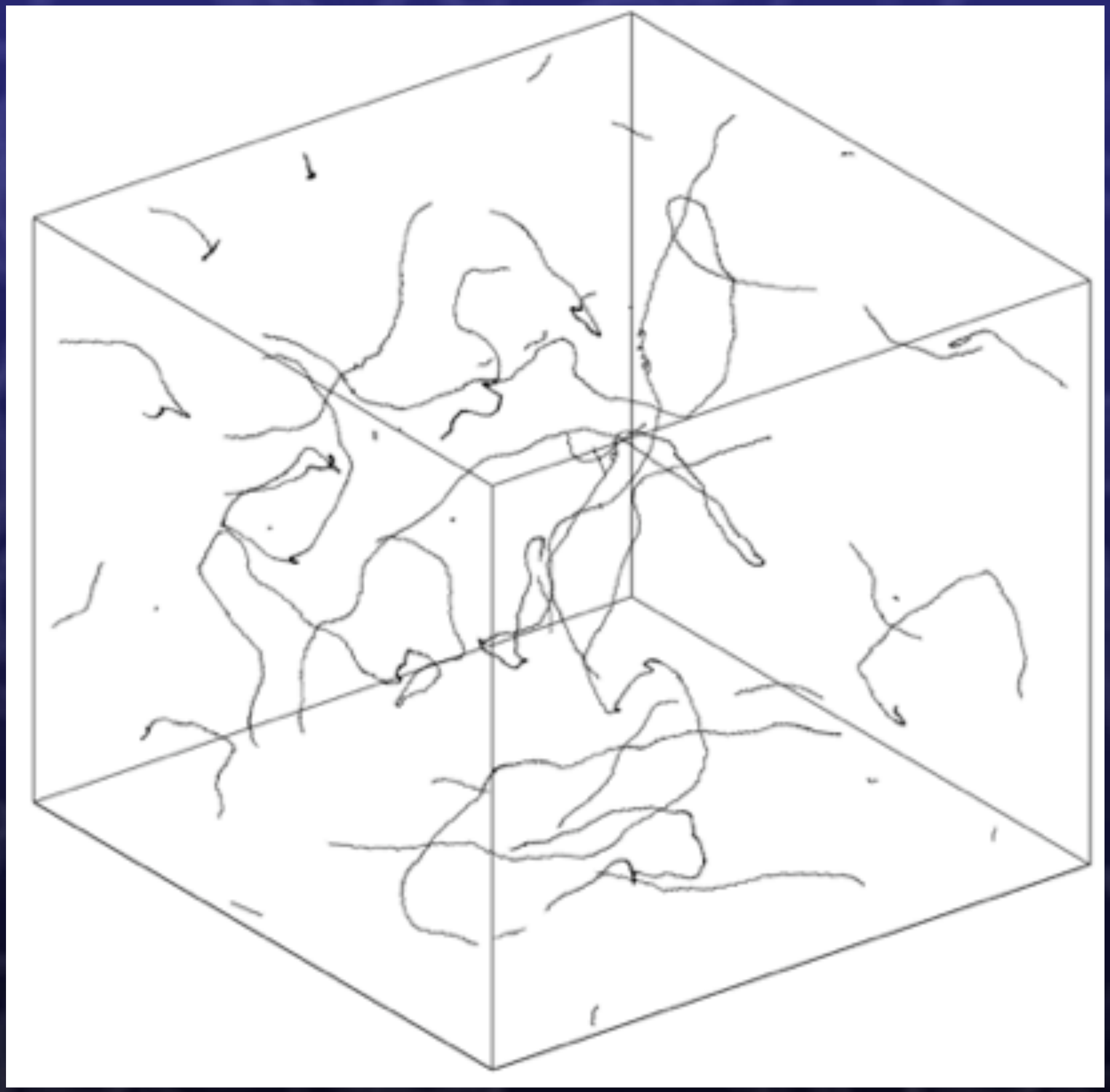
New window: gravitational waves

New window: gravitational waves

Nambu-Goto simulation

Abelian Higgs simulation





New window: gravitational waves

Nambu-Goto strings

Network loses energy by loops
Loops lose energy into GW
Lots of loops!
High Gravitational Wave background!
Loop size?
Need backreaction!

Abelian Higgs strings

Loses energy through particle emission
No loops (new BIG simulation?)
Low Gravitational Waves
Need understanding small scale
Will eventually behave like NG? Not yet!

Global defects (strings included)

Lose energy through massless radiation
Low Gravitational Waves

Temperature power spectrum

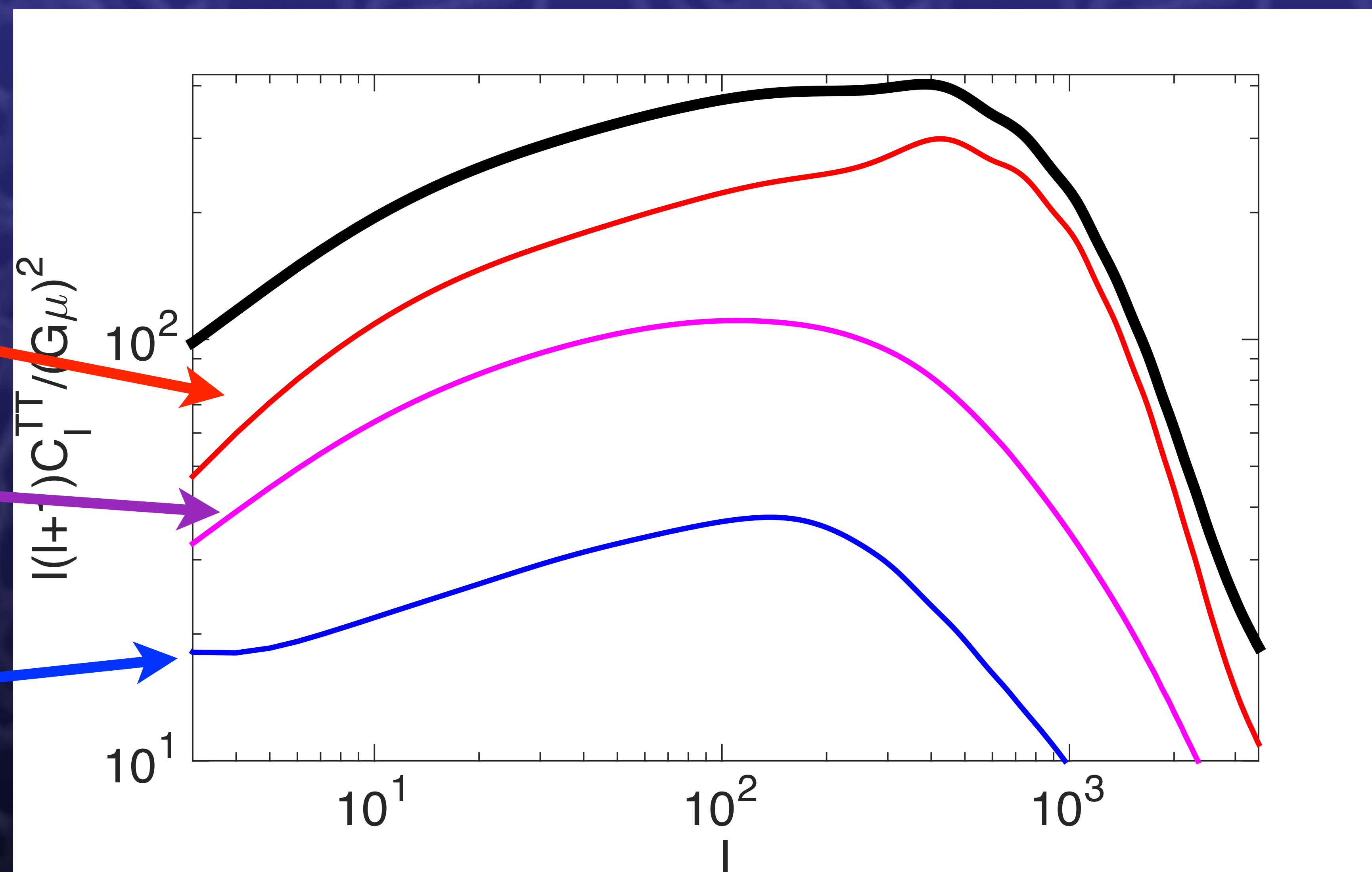
scalar-vector-tensor

ABELIAN HIGGS
(also Semilocal,
textures)

Scalar

Vector

Tensor



Temperature power spectrum

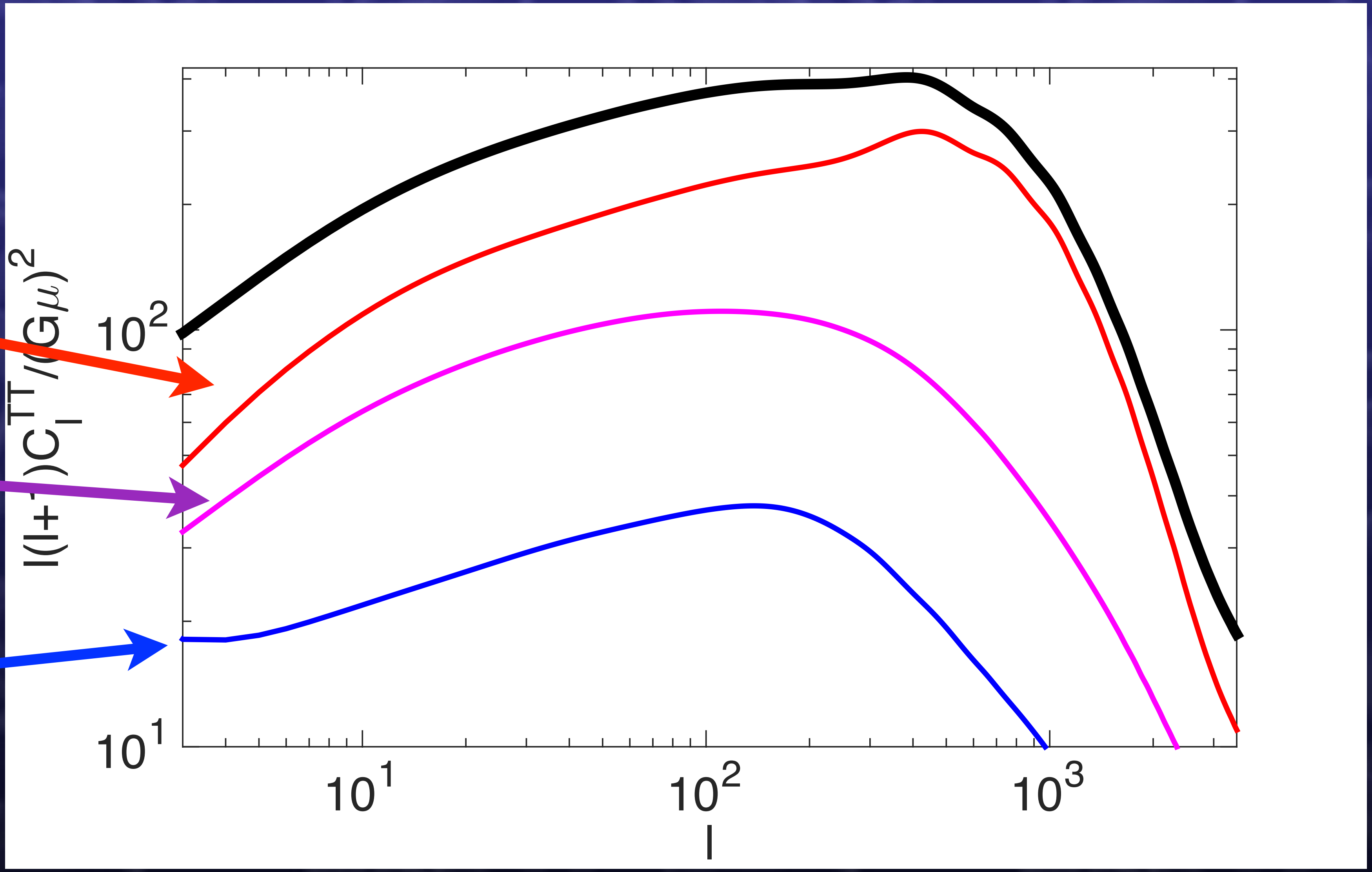
scalar-vector-tensor

ABELIAN HIGGS
(also Semilocal, textures)

Scalar

Vector

Transverse, traceless, energy-momentum tensor



Gravitational wave background from scaling defects

Scalar Large O(N) self-ordering:
Flat GW spectrum

$$\Omega_{\rm GW}(f) \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f}(f) \simeq \frac{650}{N}\,\Omega_{\rm rad}\left(\frac{v}{M_P}\right)^4$$

Krauss (1992)
Jones-Smith, Krauss, Mathur (2008)
Fenu, Figueroa, Durrer, García-Bellido (2009)

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Krauss (1992)
Jones-Smith, Krauss, Mathur (2008)
Fenu, Figueroa, Durrer, García-Bellido (2009)

Any scaling cosmological source during radiation gives flat GW spectrum

Figueroa, Hindmarsh, JU (2013)

$$\Omega_{\rm GW}(k) \equiv \frac{1}{\rho_c}\left(\frac{d\rho_{\rm GW}}{d\log k}\right) = \frac{32}{3}\,\Omega_{\rm rad}\left(\frac{v}{M_P}\right)^4 F_\infty^T$$

Depends on model

Gravitational wave background from scaling defects

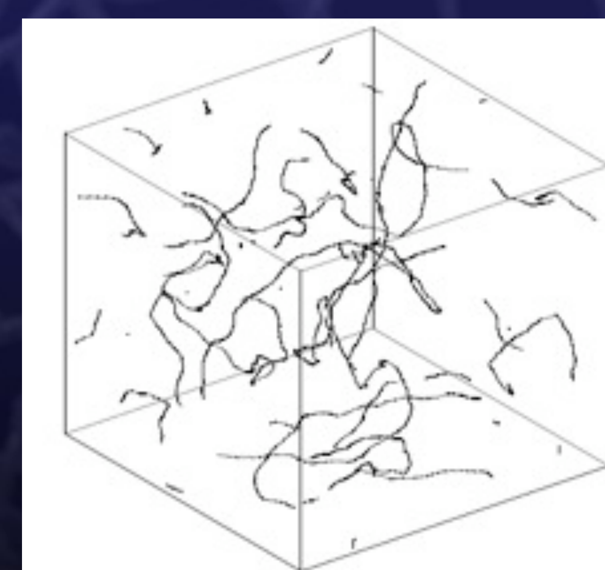
$F^{T}_{\infty} \leftarrow T^{\mathrm{TT}}_{\mathrm{ij}}$

Field theory simulations:
global fields N=2,3,4,8,20

$$\mathcal{L}(\Phi) = \partial_\mu \Phi^{\mathrm{T}} \partial_\mu \Phi - \lambda \left(\Phi^{\mathrm{T}} \Phi - v^2/2 \right)^2$$

$$T_{\mu\nu}(\mathbf{x}, t) = 2\partial_\mu \Phi^{\mathrm{T}} \partial_\nu \Phi - g_{\mu\nu} \mathcal{L}(\Phi)$$

1024^3 Cube
$\Delta x=0.5$; $\Delta t=0.2\Delta x$
Checked scaling
Obtain Tensor Correlators



Figueroa, Hindmarsh, JU (2013)

Gravitational wave background from scaling defects

$F_\infty^T \leftarrow T_{\rm ij}^{\rm TT}$

Large O(N) theory:

$$\Omega_{\rm GW}^{\rm th} \sim \frac{650}{N}\Omega_{\rm rad}\left(\frac{\nu}{M_p}\right)^4$$

Simulations:

For N>4

$$\frac{\Omega_{\rm GW}^{\rm num}}{\Omega_{\rm GW}^{\rm th}} \sim 1.1 + \frac{45}{N^2}$$

$$\Omega_{\rm GW}^{\rm texture} \sim 600\Omega_{\rm rad}\left(\frac{\nu}{M_p}\right)^4$$

For strings (N=2)

$$\frac{\Omega_{\rm GW}^{\rm num}}{\Omega_{\rm GW}^{\rm th}} \sim 130$$

$$\Omega_{\rm GW}^{\rm string} \sim 4\times 10^4\Omega_{\rm rad}\left(\frac{\nu}{M_p}\right)^4$$

$$\Omega_{\rm GW}^{\rm string} \sim (G\mu)^2$$

Figueroa, Hindmarsh, JU (2013)

Summary

Field theory evolution of many models
(some good points, some shortcomings...)

CMB results quite robust

Scaling sources (defects) give a flat GW background.
Amplitude depends on defect type
(calculated for global defects)

Irreducible minimum for defects,
important for global defects, Abelian Higgs;
but subdominant for Nambu-Goto

eman ta zabal zazu

UPV EHU

Field theory simulations of cosmic defects

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Bilbao, Spain

eLISA Cosmology Working Group
Workshop, 15-04-15