# GW propagation in the inhomogeneous universe

**Camille Bonvin**

CERN

eLISA Cosmology Working Group Workshop

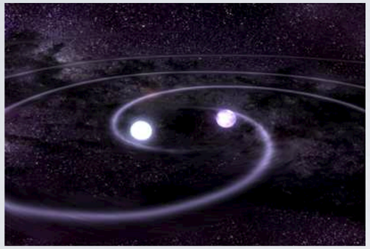
April 2015

# Outline

- Inspiral **binary** system
- Large-scale **structure** affect the wave-form and the amplitude of the wave
- Two main effects
  - Change in the **redshift** → frequency and mass
  - Change in the luminosity **distance** → amplitude

# Inspiral binary system

euclidian universe expanding perturbed universe



x ⟶ x observer

**First stage**:

- We solve the trajectory of the two masses assuming a quasi-circular orbit. The system looses energy through GW emission, leading to an **inspiral**.

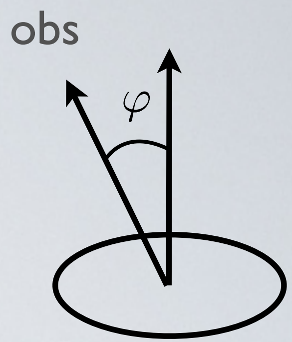
- Assuming that the change in radius is slow, we calculate the energy momentum tensor and find the two **polarisations**.

see e.g. Maggiore, Gravitational Waves, 2008

# Polarisations

$$h_+(t) = \frac{2}{r} M_c^{5/3} \Big(\pi f(t_{\rm ret})\Big)^{2/3} (1+\cos^2\varphi)\cdot\cos\Phi(t_{\rm ret})$$

$$h_\times(t) = \frac{4}{r} M_c^{5/3} \Big(\pi f(t_{\rm ret})\Big)^{2/3} \cos\varphi\cdot\sin\Phi(t_{\rm ret})$$




Chirp mass $\quad M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$ $\qquad t_{\rm ret} = t - r/c$

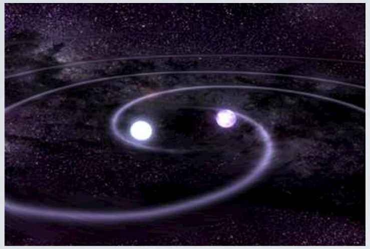
The frequency **evolves** $\quad \dot f = \frac{96}{5}\pi^{8/3}\left(\frac{GM_c}{c^3}\right)^{5/3} f^{11/3}$

$$f(\tau) = \frac{1}{\pi}\left(\frac{5}{256\,\tau}\right)^{3/8}\left(\frac{GM_c}{c^3}\right)^{-5/8} \qquad \tau = t - t_{\rm coal}$$

$$\Phi(\tau) = \int^\tau d\tau' f(\tau') = -2\left(\frac{5GM_c}{c^3}\right)^{-5/8}\tau^{5/8} + \Phi_0$$

# Second stage: propagation

euclidian universe expanding perturbed universe



x ⟶ x observer

GWs propagate on **null geodesics**, that depend on the **metric.**

**Three effects:** $h_+(\tau) = \frac{2}{r} M_c^{5/3} \Big(\pi f(\tau)\Big)^{2/3} (1+\cos^2\varphi)\cdot\cos\Phi(\tau)$

- frequency $f_S \to f_O$
- phase $\Phi_S \to \Phi_O$
- distance $1/r$ governs the dilution of energy, which is different in a non-euclidian universe.

# Frequency and phase

- Frequency

$$1+z \equiv \frac{E_S}{E_O} = \frac{f_S}{f_O} \quad \rightarrow \quad f_O = \frac{f_S}{1+z}$$

- Phase

$$\Phi_S(\tau_S) = \int^{\tau_S} d\tau'_S \, f_S(\tau'_S) = \int^{\frac{\tau_O}{1+z}} \frac{d\tau'_O}{1+z}(1+z) f_O(\tau'_O) = \Phi_O(\tau_O)$$

The phase is **constant** on null geodesics: $k_\mu = -\partial_\mu \Phi$

$$k^\mu k_\mu = 0 = -k^\mu \, \partial_\mu \Phi = -\frac{d\Phi}{d\lambda} \quad \rightarrow \quad \Phi = \text{const}$$

# Distance

- In an euclidian universe, $1/r$ comes from the **wave propagation equation**

$$\Box h_+ = \Box h_\times = 0 \qquad \text{with} \qquad \Box = \partial_r^2 - \partial_t^2$$

- In a non-euclidian universe $\Box = g^{\mu\nu} D_\mu D_\nu$

Schutz, 1986
Laguna, Larson, Spergel and Yunes, 2009

$$\frac{1}{r} \rightarrow \frac{1+z}{d_L}$$

Luminosity distance: $d_L \equiv \sqrt{\frac{\mathcal{L}}{4\pi\mathcal{F}}}$

The **luminosity distance** tells us how the energy emitted by the source is spread during propagation in a generic universe.

# At the observer

$$h_+ = \frac{2}{d_L}(1+z)^{5/3} M_c^{5/3} \Big(\pi f_O(\tau_O)\Big)^{2/3} (1+\cos^2\varphi)\cdot\cos\Phi_O(\tau_O)$$

$$h_\times = \frac{4}{d_L}(1+z)^{5/3} M_c^{5/3} \Big(\pi f_O(\tau_O)\Big)^{2/3} \cos\varphi\cdot\sin\Phi_O(\tau_O)$$

Redshifted chirp mass: $\mathcal{M}_c = (1+z)M_c$

$$\frac{df_S}{d\tau_S} = \frac{96}{5}\pi^{8/3}\left(\frac{GM_c}{c^3}\right)^{5/3} f_S^{11/3}$$

$$f_S = (1+z) f_O$$

$$d\tau_S = \frac{d\tau_O}{1+z}$$

$$\frac{df_O}{d\tau_O} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{5/3} f_O^{11/3}$$

assuming a constant redshift

# At the observer

$$h_+ = \frac{2}{d_L}\mathcal{M}_c^{5/3}\Big(\pi f_O(\tau_O)\Big)^{2/3}(1+\cos^2\varphi)\cdot\cos\Phi_O(\tau_O)$$

$$h_\times = \frac{4}{d_L}\mathcal{M}_c^{5/3}\Big(\pi f_O(\tau_O)\Big)^{2/3}\cos\varphi\cdot\sin\Phi_O(\tau_O)$$

Redshifted chirp mass: $\mathcal{M}_c = (1+z)M_c$

$$\frac{df_S}{d\tau_S} = \frac{96}{5}\pi^{8/3}\left(\frac{GM_c}{c^3}\right)^{5/3} f_S^{11/3}$$

$$f_S = (1+z)f_O$$

$$d\tau_S = \frac{d\tau_O}{1+z}$$

$$\frac{df_O}{d\tau_O} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{5/3} f_O^{11/3}$$

assuming a constant redshift

# At the observer

Schutz, 1986

$$h_+ = \frac{2}{d_L}\mathcal{M}_c^{5/3}\Big(\pi f_O(\tau_O)\Big)^{2/3}(1+\cos^2\varphi)\cdot\cos\Phi_O(\tau_O)$$

$$h_\times = \frac{4}{d_L}\mathcal{M}_c^{5/3}\Big(\pi f_O(\tau_O)\Big)^{2/3}\cos\varphi\cdot\sin\Phi_O(\tau_O)$$

Redshifted chirp mass: $\mathcal{M}_c = (1+z)M_c$

$$\frac{df_S}{d\tau_S} = \frac{96}{5}\pi^{8/3}\left(\frac{GM_c}{c^3}\right)^{5/3} f_S^{11/3}$$

$$f_S = (1+z)f_O$$

$$d\tau_S = \frac{d\tau_O}{1+z}$$

$$\frac{df_O}{d\tau_O} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{5/3} f_O^{11/3}$$

assuming a constant redshift

# At the observer

Schutz, 1986

$$h_+ = \frac{2}{d_L}\mathcal{M}_c^{5/3}\Big(\pi f_O(\tau_O)\Big)^{2/3}(1+\cos^2\varphi)\cdot\cos\Phi_O(\tau_O)$$

$$h_\times = \frac{4}{d_L}\mathcal{M}_c^{5/3}\Big(\pi f_O(\tau_O)\Big)^{2/3}\cos\varphi\cdot\sin\Phi_O(\tau_O)$$

Redshifted chirp mass: $\mathcal{M}_c = (1+z)M_c$

$$\frac{df_S}{d\tau_S} = \frac{96}{5}\pi^{8/3}\left(\frac{GM_c}{c^3}\right)^{5/3} f_S^{11/3}$$

$$f_S = (1+z)f_O$$

$$d\tau_S = \frac{d\tau_O}{1+z}$$

$$\frac{df_O}{d\tau_O} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{5/3} f_O^{11/3}$$

assuming a constant redshift

# The effect of structures: redshift

$$ds^2 = -a^2(\eta)\Big[\big(1+2\Psi(\mathbf{x},\eta)\big)d\eta^2 + \big(1-2\Phi(\mathbf{x},\eta)\big)\delta_{ij}dx^i dx^j\Big]$$

Perturbations affect: ♦ the **redshift** → frequency $f_O = \frac{f_S}{1+z}$

mass $\mathcal{M}_c = (1+z)M_c$

♦ the luminosity **distance**

We solve the **null geodesic** equation at linear order

$$\frac{dk^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta}k^\alpha k^\beta = 0 \qquad\qquad 1+z = \frac{E_S}{E_O} = \frac{\left(g_{\mu\nu}k^\mu v^\nu\right)_S}{\left(g_{\mu\nu}k^\mu v^\nu\right)_O}$$

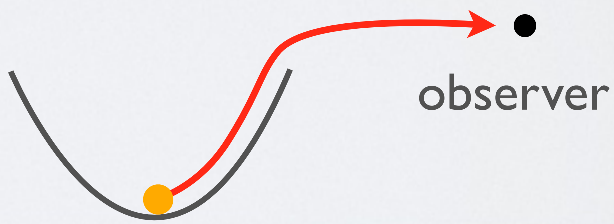
# The effect of structures: redshift

$$ds^2 = -a^2(\eta)\Big[\big(1+2\Psi(\mathbf{x},\eta)\big)d\eta^2 + \big(1-2\Phi(\mathbf{x},\eta)\big)\delta_{ij}dx^i dx^j\Big]$$

$$1+z = \frac{a_O}{a_S}\Big[1 + \mathbf{v}_S\cdot\mathbf{n} - \mathbf{v}_O\cdot\mathbf{n} + \Psi_O - \Psi_S - \int_0^{r_S} dr(\dot{\Phi}+\dot{\Psi})\Big]$$

Doppler Gravitational redshift Integrated Sachs-Wolfe

Gravitational redshift:



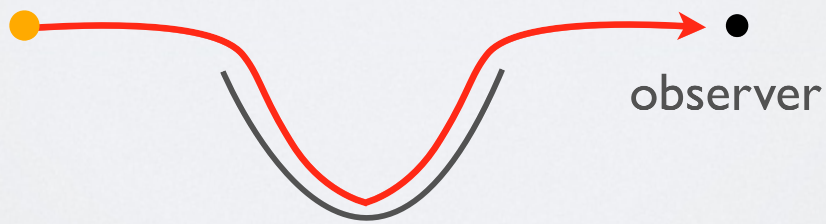
# The effect of structures: redshift

$$ds^2 = -a^2(\eta)\Big[\big(1+2\Psi(\mathbf{x},\eta)\big)d\eta^2 + \big(1-2\Phi(\mathbf{x},\eta)\big)\delta_{ij}dx^i dx^j\Big]$$

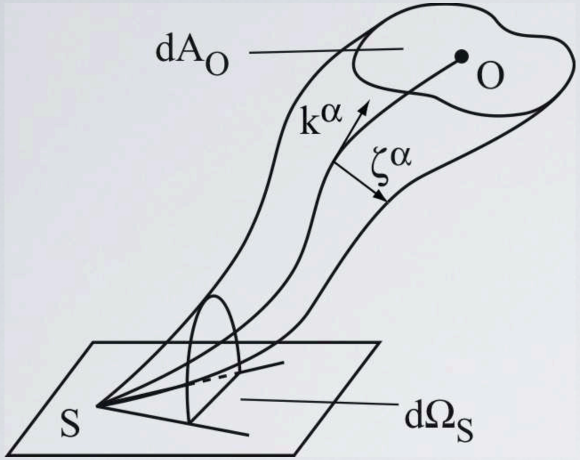
$$1+z = \frac{a_O}{a_S}\Big[1 + \mathbf{v}_S\cdot\mathbf{n} - \mathbf{v}_O\cdot\mathbf{n} + \Psi_O - \Psi_S - \int_0^{r_S} dr(\dot\Phi + \dot\Psi)\Big]$$

Doppler — Gravitational redshift — Integrated Sachs-Wolfe

ISW: integrated along the trajectory, sensitive to dark energy.



# The effect of structures: distance




$$d_L = \sqrt{\frac{\mathcal{L}}{4\pi\mathcal{F}}} = (1+z)\sqrt{\frac{dA_O}{d\Omega_S}}$$

$$\frac{D^2\xi^\alpha(\lambda)}{D\lambda^2} = R^\alpha{}_{\beta\mu\nu}k^\beta k^\mu \xi^\nu$$

$k^\alpha$ momentum

$\xi^\alpha$ connection vector

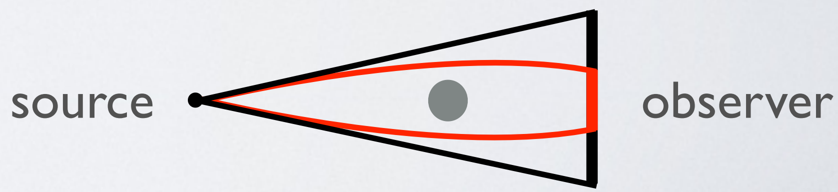
In a homogeneous and isotropic **expanding** universe:

$$d_L(z_S) = (1+z_S)r_S = (1+z_S)\int_0^{z_S}\frac{dz}{H(z)}$$

# Luminosity distance perturbations

Bonvin, Durrer and Gasparini, 2006
Hui and Greene, 2006

$$
\begin{aligned}
d_L \quad = \quad & (1+z_S) r_S \Bigg\{ 1 + \frac{1}{\mathcal{H}_S r_S} \mathbf{v}_O \cdot \mathbf{n} + \left(1 - \frac{1}{\mathcal{H}_S r_S}\right) \mathbf{v}_S \cdot \mathbf{n} \qquad \text{Doppler} \\
& + \left(1 - \frac{1}{\mathcal{H}_S r_S}\right) \Psi_O - \left(2 - \frac{1}{\mathcal{H}_S r_S}\right) \Psi_S \qquad \text{Gravitational potential} \\
\text{Time delay} \quad & + \frac{2}{r_S} \int_0^{r_S} dr\, \Psi - 2 \left(1 - \frac{1}{\mathcal{H}_S r_S}\right) \int_0^{r_S} dr\, \dot{\Psi} \qquad \text{Integrated Sachs-Wolfe} \\
& - \int_0^{r_S} dr\, \frac{(r_S - r)}{r_S r} \Delta_\perp \Psi \Bigg\} \qquad \text{Lensing}
\end{aligned}
$$



# Impact for observations: redshift

The **frequency** and the redshifted chirp **mass** are perturbed.

- If we don't know the chirp mass, we measure it through:

$$\frac{df_O}{d\tau_O} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{5/3} f_O^{11/3}$$

  and then we plug the **perturbed** mass into the polarisations

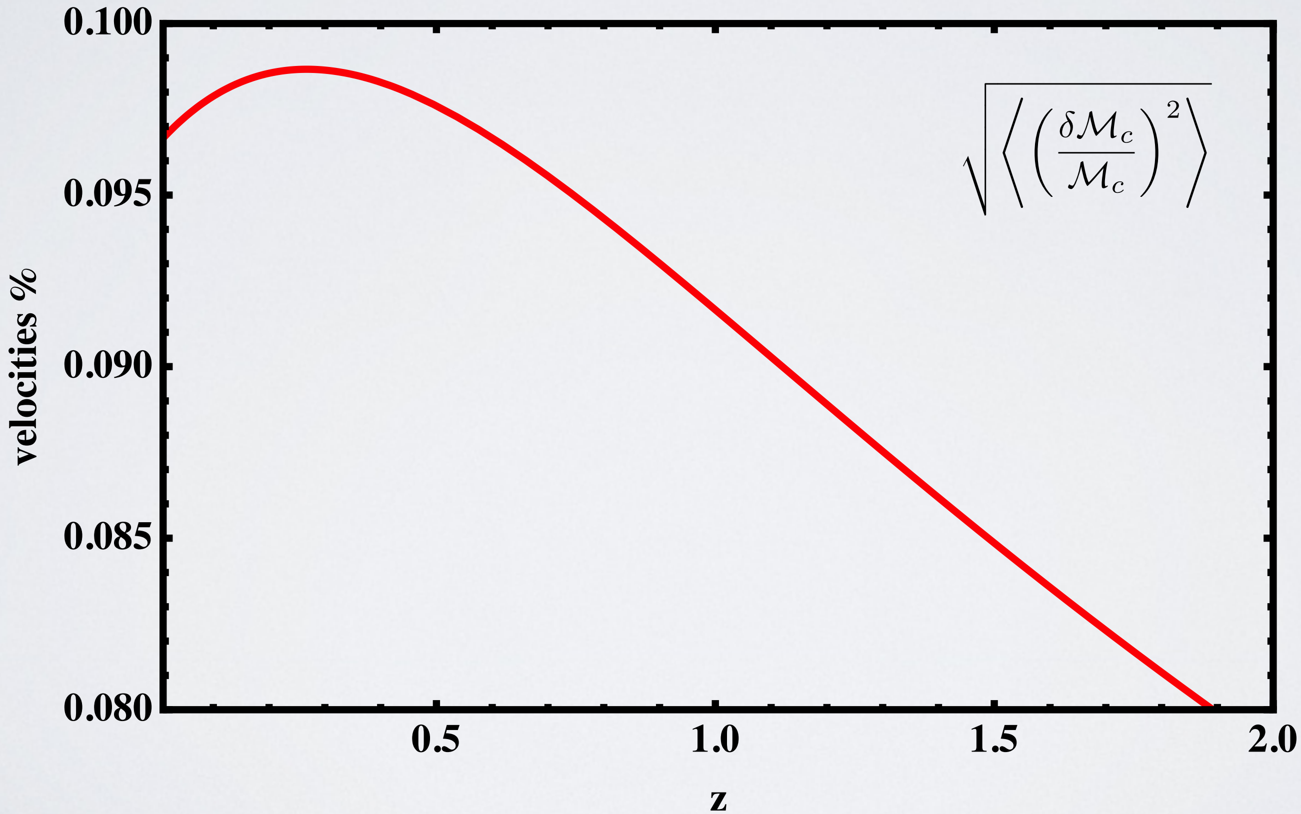
$$h_+ = \frac{2}{d_L}\mathcal{M}_c^{5/3}\Big(\pi f_O(\tau_O)\Big)^{2/3}(1+\cos^2\varphi)\cdot\cos\Phi_O(\tau_O)$$

- If we know the chirp mass (e.g. neutron stars binaries), and we look at close-by binaries, for which the expansion is negligible, then the perturbations induce an **error** on the measured mass.

# Error on the chirp mass

$$\mathcal{M}_c^{\text{meas.}} = \mathcal{M}_c^{N-N} + \delta\mathcal{M}_c$$

Dominant effect due to **velocities** $\dfrac{\delta\mathcal{M}_c}{\mathcal{M}_c} = \dfrac{\delta z}{(1+z)} = (\mathbf{v}_S - \mathbf{v}_O)\mathbf{n}$



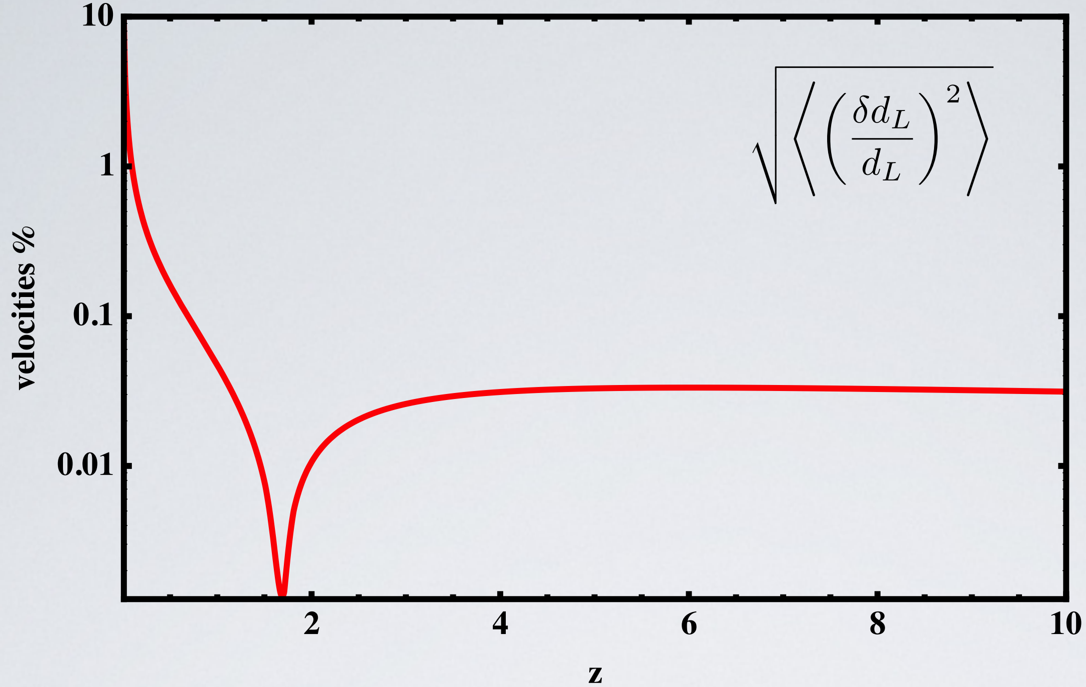
# Impact for observations: distance

- We measure the **perturbed** luminosity **distance**

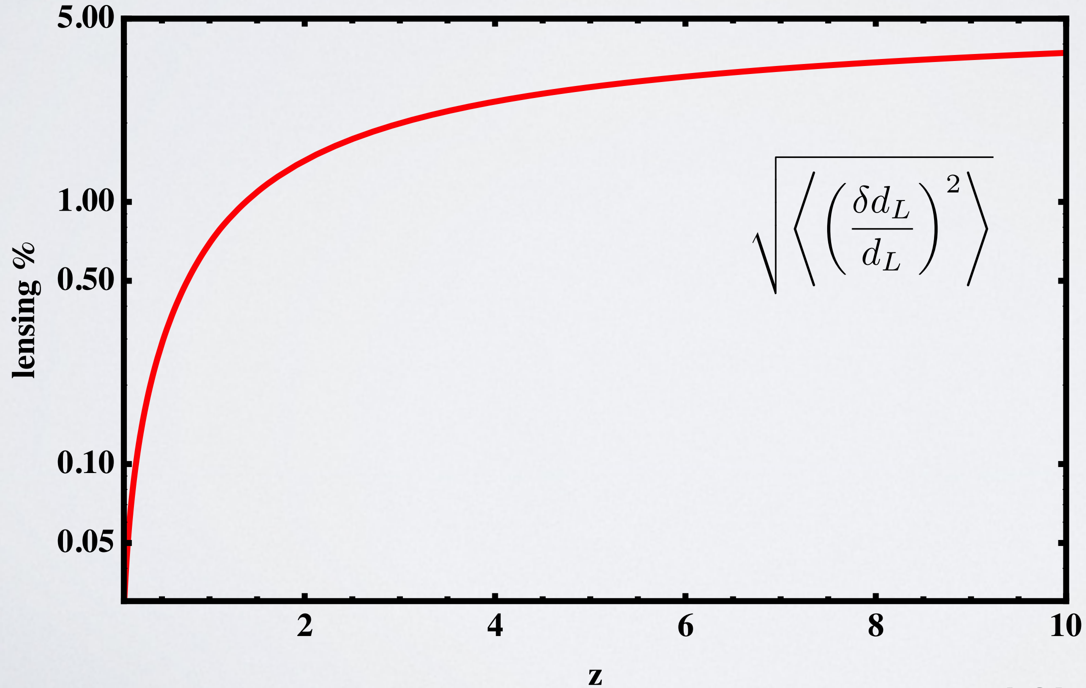
$$h_+ = \frac{2}{d_L}\mathcal{M}_c^{5/3}\Big(\pi f_O(\tau_O)\Big)^{2/3}(1+\cos^2\varphi)\cdot\cos\Phi_O(\tau_O)$$

- If we want to extract **cosmological parameters**, the perturbations will induce an error.

- Two dominant contributions:
  - At small redshift: peculiar **velocities**
  - At large redshift: **lensing** Holz and Linder, 2004




$$d_L = d_L^{\mathrm{FRW}} + \delta d_L$$



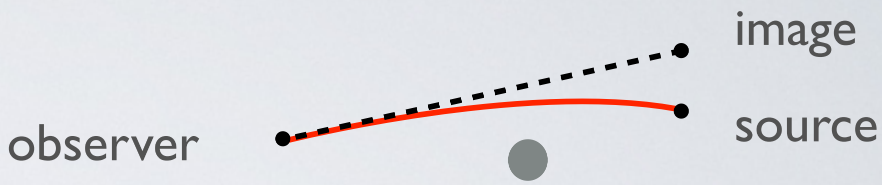

with non-linearities

$5-10\%$ error

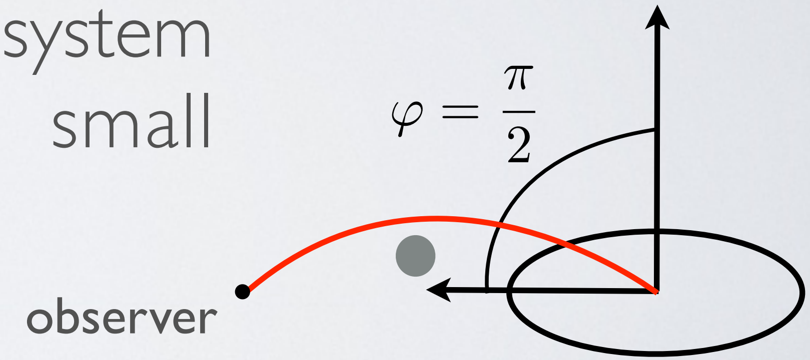
Holz and Hughes, 2005

# Other effects

- The deflection has an impact on the **position** of the source in the sky.




- The deflection has an impact on the **orientation** of the binary system with respect to us: even if we see the system edge-on, we will observe a small component $h_\times$



# Conclusion

- Large-scale structure affect both the **redshift** and the **luminosity distance.**

- The redshift perturbations change the observed **frequency** and the redshifted chirp **mass**. The effect is however negligible, less than 0.1 percent.

- The luminosity **distance** perturbations on the other hand can reach a few percents.

- At small redshift, the dominant contribution is due to **velocities**.

- At large redshift, the dominant contribution is due to gravitational **lensing**.