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Gravitational Wave Signal from Preheating

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Inflation

- Simple, universal predictions:
  - Nearly scale-invariant, nearly Gaussian curvature perturbations
  - Nearly scale-invariant gravitational wave spectrum
  - Great for testing the paradigm,
    bad for understanding in detail
- Little knowledge about details
  - Any set of observable values compatible with a wide range of models

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End of Inflation

- Energy transfer from inflation to Standard Model fields
- Details depend sensitively on microscopic dynamics:
  - Perturbative decay
  - Preheating - Parametric resonance (Kofman et al 1994)
  - Tachyonic preheating - Symmetry breaking (Felder et al 2000)
- Observable signatures?
  - Relics (particles, strings etc.)
  - Curvature perturbations (Chambers&AR 2007)
  - Gravitational waves (Khlebnikov&Tkachev 1997)

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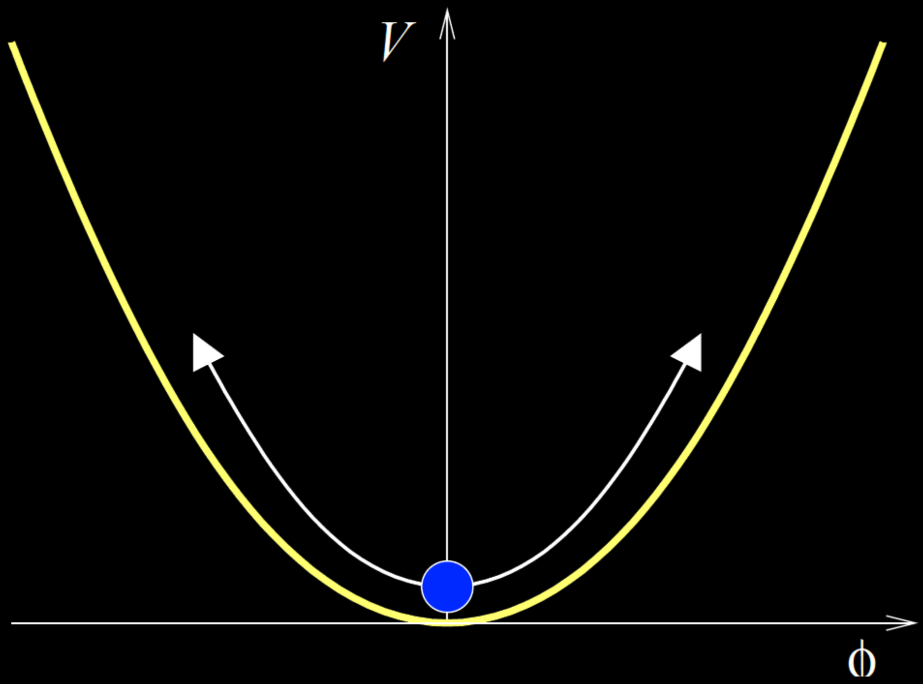
Preheating

- After inflation, inflaton oscillates about its minimum
- Coupling to other fields
  -> Parametric resonance (Kofman, Linde & Starobinsky 1994)



Inflaton Oscillations

- Toy model: Inflaton coupled to another scalar

$$V = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{1}{2}M^2\chi^2$$

- End of inflation: $\chi \approx 0$, $\phi \sim M_{\mathrm{Pl}}$
- Classical equation of motion

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\vec{\nabla}^2\phi + \frac{\partial V}{\partial \phi} = 0$$

- Approximate solution

$$\phi(t) = M_{\mathrm{Pl}} a^{-3/2} \sin mt$$

Oscillations of Other Scalar

- Equation of motion

$$\ddot{\chi}_k + 3H\dot{\chi}_k - \frac{k^2}{a^2}\chi_k + g^2\phi(t)^2\chi_k + M^2\chi_k = 0$$

- For slowly varying a, Mathieu equation

$$\chi_k{}''(z) + (A_k - 2q\cos 2z)\chi_k(z) = 0,$$

where $q = \frac{g^2\Phi^2}{4m^2}$, $A_k = 2q + \frac{k^2}{a^2m^2} + \frac{M^2}{m^2}$

- Floquet theorem: Solutions of the form

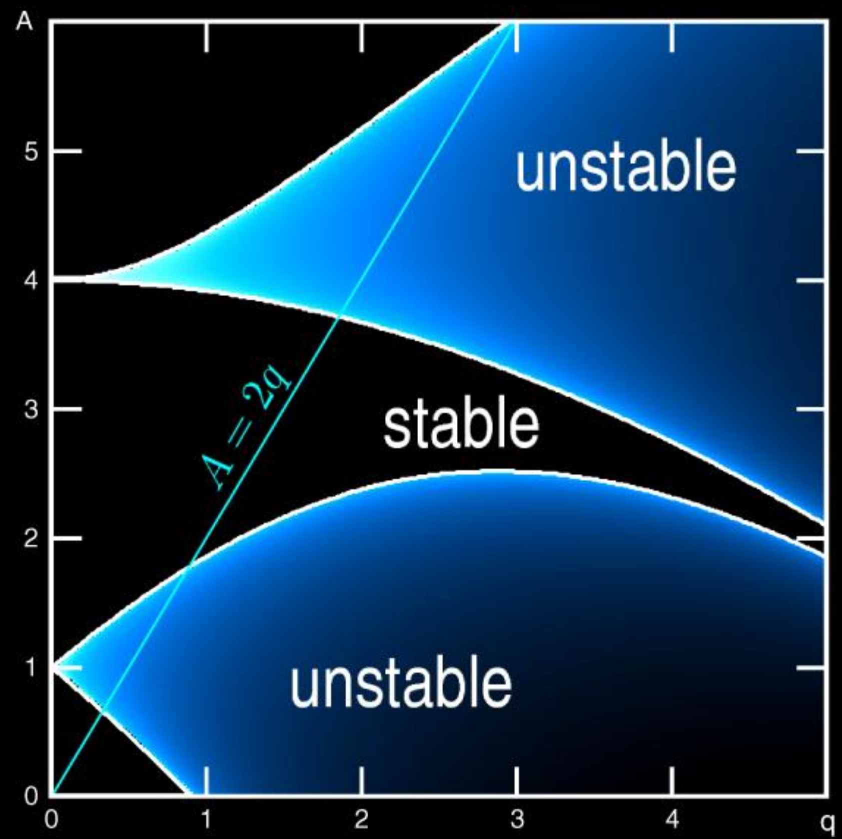
$$\chi_k(z) = f_k(z)e^{\mu_k t}$$

with periodic $f_k(z)$ and constant Floquet index μ_k

Instability Bands

$$\chi_k(z) = f_k(z) e^{\mu_k t}$$

- Imaginary μ_k -> Stable
- Real μ_k -> Unstable (exp growing)
- A_k and q decrease: Move through instability bands
- Narrow ($q \lesssim 1$) vs broad ($q \gtrsim 1$) resonance



(Image: Frolov, JCAP 2008)

Preheating

- Exponentially growing χ_k:
  Rapid non-perturbative particle production
- Lasts until mode moves out of resonance band or dynamics becomes non-linear
- Non-equilibrium dynamics
  - Need numerical lattice simulations (Khlebnikov&Tkachev 1996)
  - Gravitational wave production (Khlebnikov&Tkachev 1997)
- Followed by turbulence, thermalisation

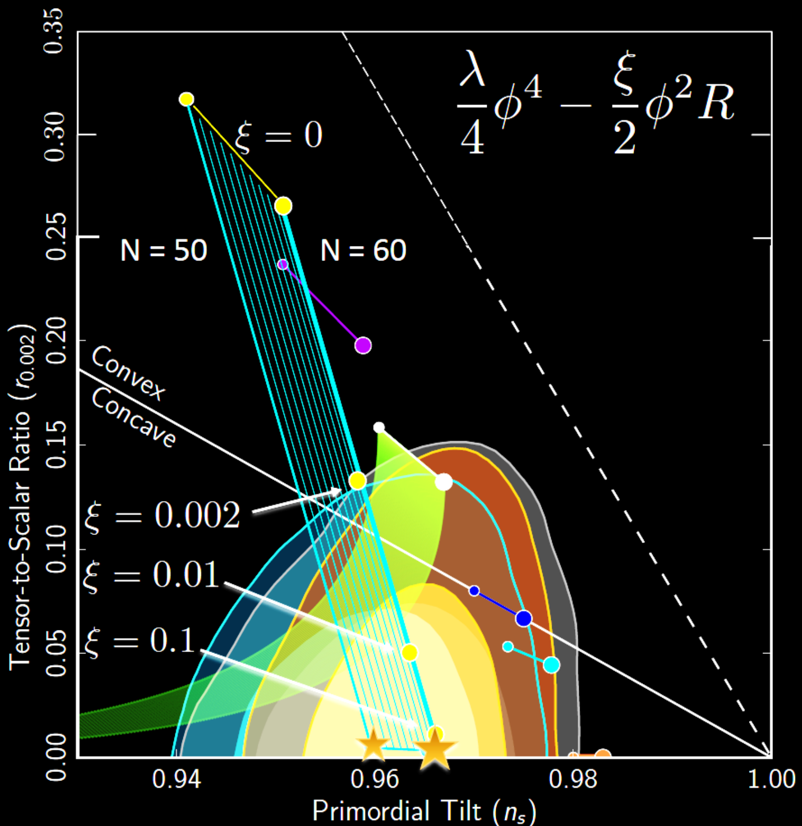
Massless Preheating

- Massless fields

$$V = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$$

- Compatible with observations if non-minimal coupling

$$\xi \gtrsim 0.005$$



(Image: Linde 2014)

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Massless Preheating

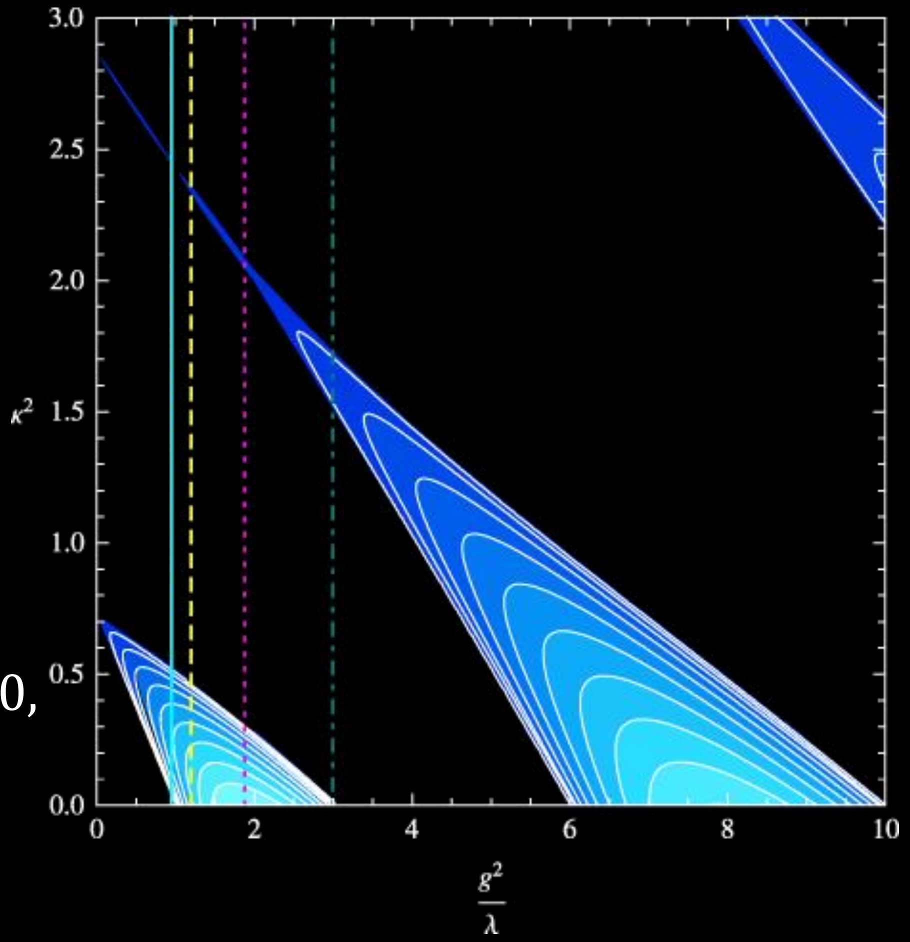
- Massless fields

$$V = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$$

- Convenient: No dimensionless parameters
- Lamé equation

$$\tilde{\chi}_k'' + \left(\kappa^2 + \frac{g^2}{\lambda}\mathrm{cn}^2\left(\tau; \frac{1}{\sqrt{2}}\right)\right)\tilde{\chi}_k = 0,$$

where $\kappa^2 = k^2/\lambda\phi_{\mathrm{ini}}^2$



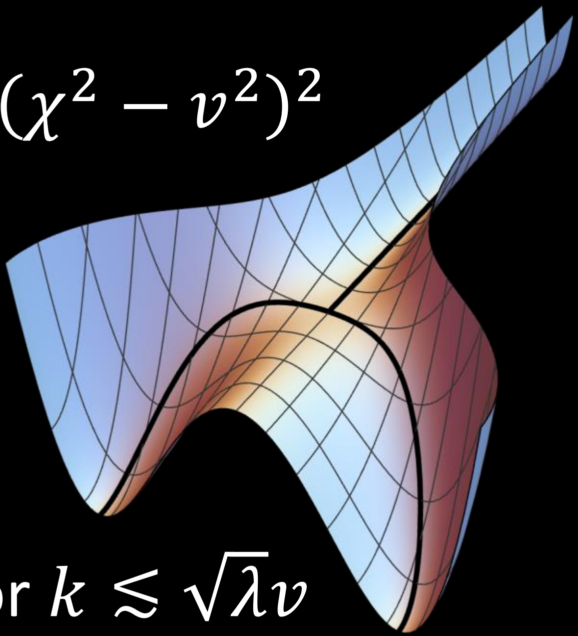
Tachyonic Preheating

- Hybrid inflation

$$V = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{1}{4}\lambda(\chi^2 - v^2)^2$$

- Symmetry breaking at

$$\phi = \phi_{\mathrm{c}} = \sqrt{\frac{\lambda}{g^2}}v$$



- Modes χ_k grow exponentially for $k \lesssim \sqrt{\lambda}v$

-> Non-equilibrium dynamics,
Gravitational waves

GW from Non-Equilibrium Fields

- Tensor perturbation h_{ij}:

$$ds^2 = dt^2 - a^2(t)\left(\delta_{ij} - h_{ij}(t,\vec{x})\right)dx^i dx^j$$

- Transverse $\partial_i h_{ij} = 0$, traceless $h_{ii} = 0$
- Linearised e.o.m

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\vec{\nabla}^2 h_{ij} = \frac{16\pi}{M_{\rm Pl}^2 a^2}\Pi_{ij}^{\rm TT},$$

with TT anisotropic stress tensor

$$\Pi_{ij}^{\rm TT}(\vec{k}) = \Lambda_{ij,lm}(\hat{k})\Pi_{ij}(\vec{k}),$$

$$\Pi_{ij}(\vec{x}) = \partial_i\chi\partial_j\chi + \partial_i\phi\partial_j\phi$$

GW from Non-Equilibrium Fields

- Energy density in gravitational waves

$$\frac{d\rho_{\mathrm{GW}}}{d\log k} = \frac{k^3}{(4\pi L)^3 G}\int \frac{d\Omega_{\hat{k}}}{4\pi}\left|\dot{h}_{ij}(\vec{k})\right|^2$$

- GW energy fraction today

$$\Omega_{\mathrm{GW}}h^2 = \Omega_{\mathrm{r}}h^2\left(\frac{\rho_{GW}}{\rho_{\mathrm{tot}}}\right)\left(\frac{g_0}{g_*}\right)^{1/3} \sim 10^{-5}\left(\frac{\rho_{GW}}{\rho_{\mathrm{tot}}}\right)$$

- Peak frequency

$$f_* \sim 6\times 10^{10}\mathrm{Hz}\frac{k_*}{\sqrt{M_{\mathrm{Pl}}H_{\mathrm{inf}}}}$$

Rough Estimates (Amin et al. 2014)

- Characteristic wave number $k_* \sim H/\beta, \beta < 1$
  -> Peak frequency

$$f_* \sim 10^{11}\mathrm{Hz}\,\beta^{-1}\frac{\rho_{\mathrm{inf}}^{1/4}}{M_{\mathrm{Pl}}}$$

- Energy fraction in anisotropic stresses $\delta_\Pi < 1$
  -> Maximum amplitude

$$\Omega_{\mathrm{GW}}(f_*)h^2 \sim 10^{-5}\delta_\Pi^2\beta^2$$

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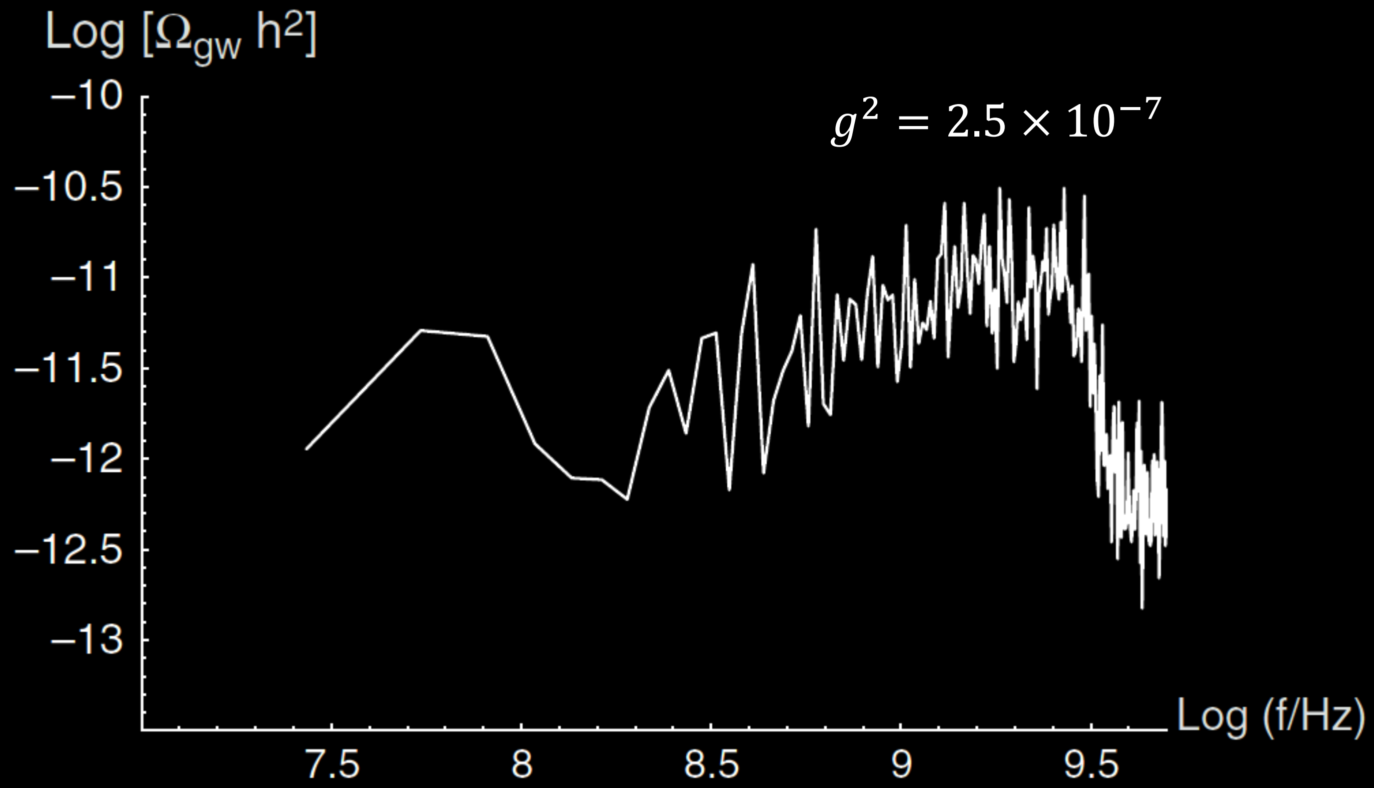
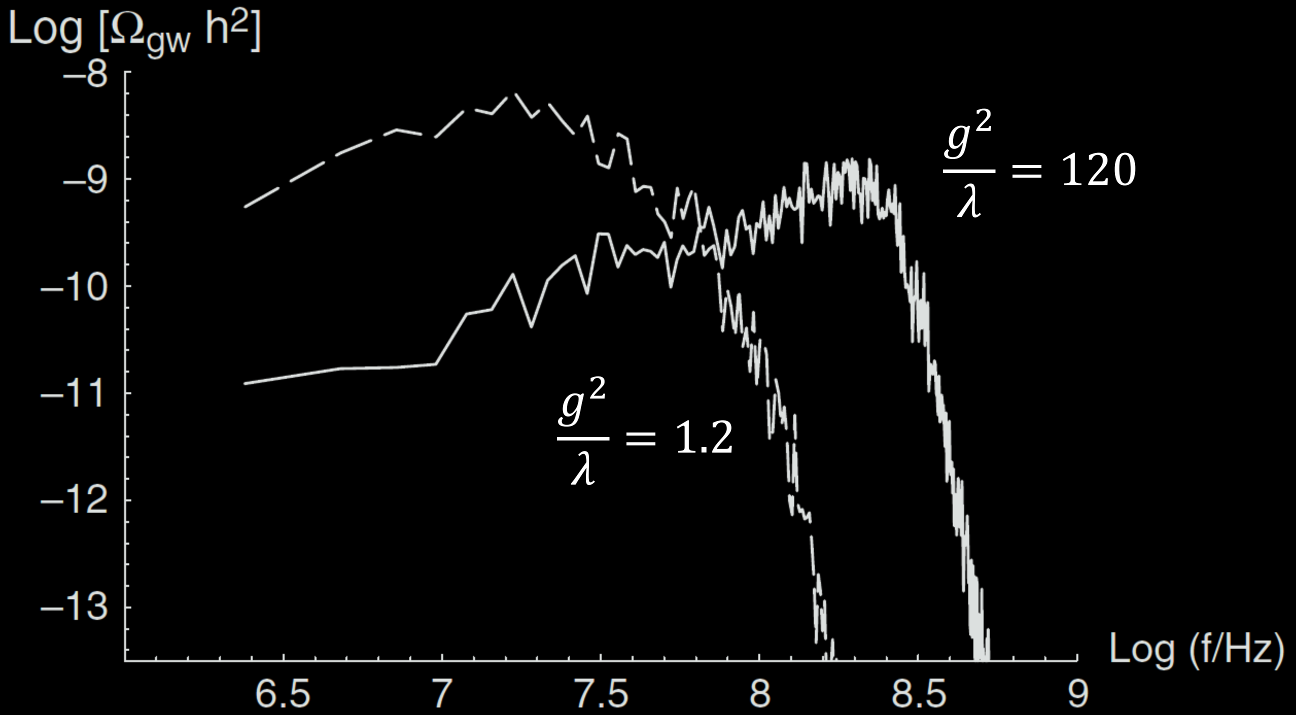
GW from Quadratic Preheating



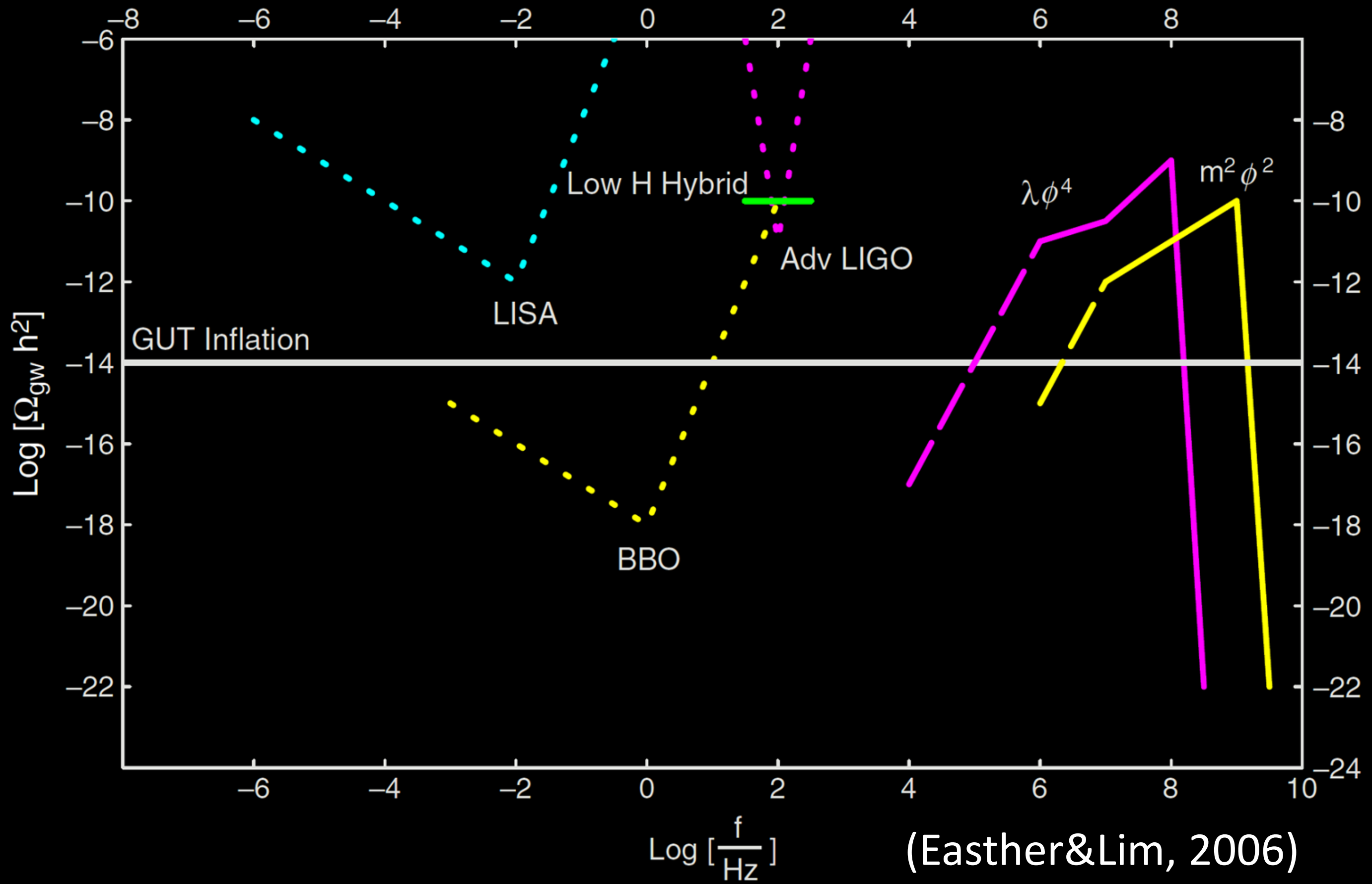
(Easther&Lim, 2006)

GW from Massless Preheating



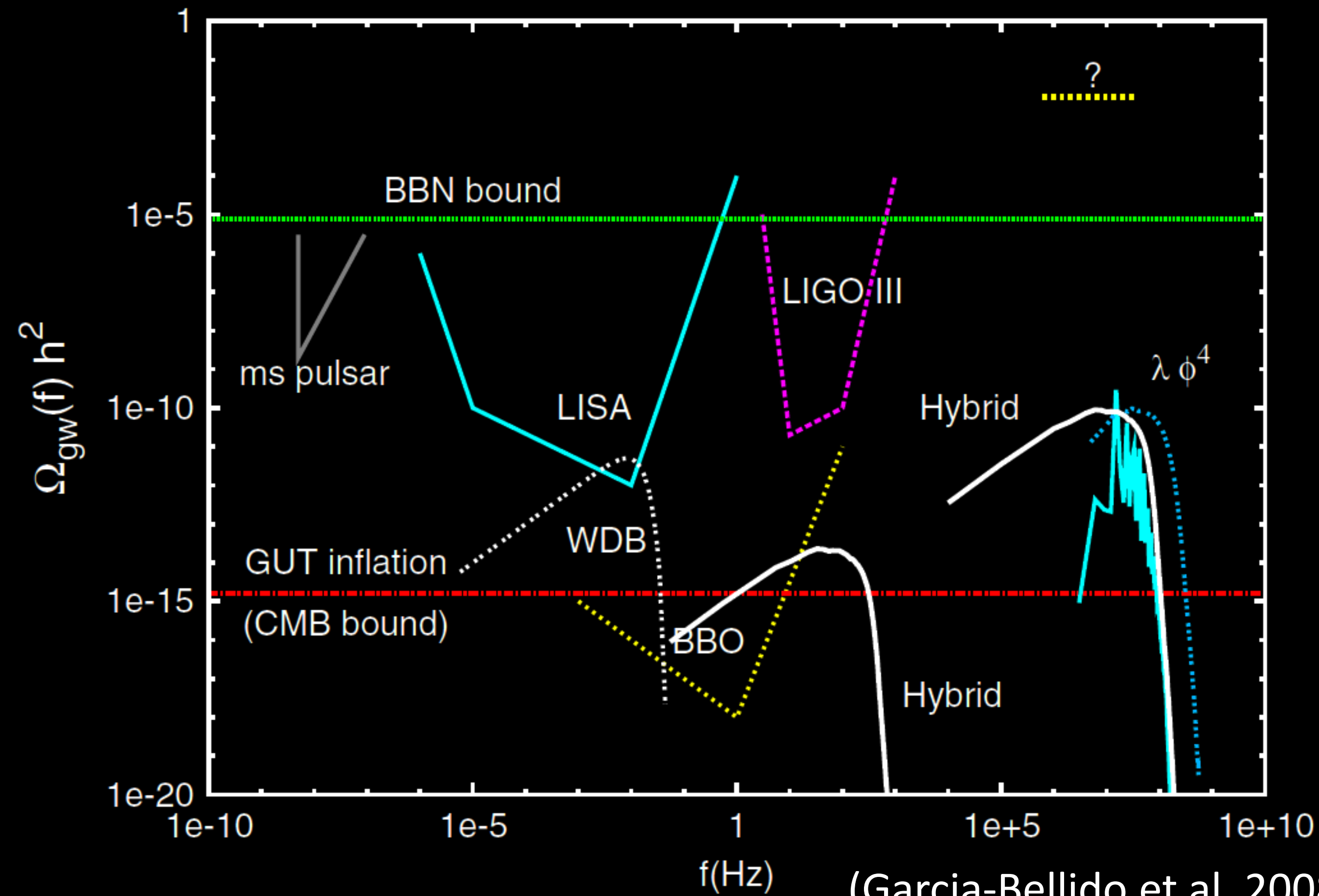
(Easther&Lim, 2006)

GW from Preheating



(Easther&Lim, 2006)

GW from Tachyonic Preheating



(Garcia-Bellido et al, 2008)

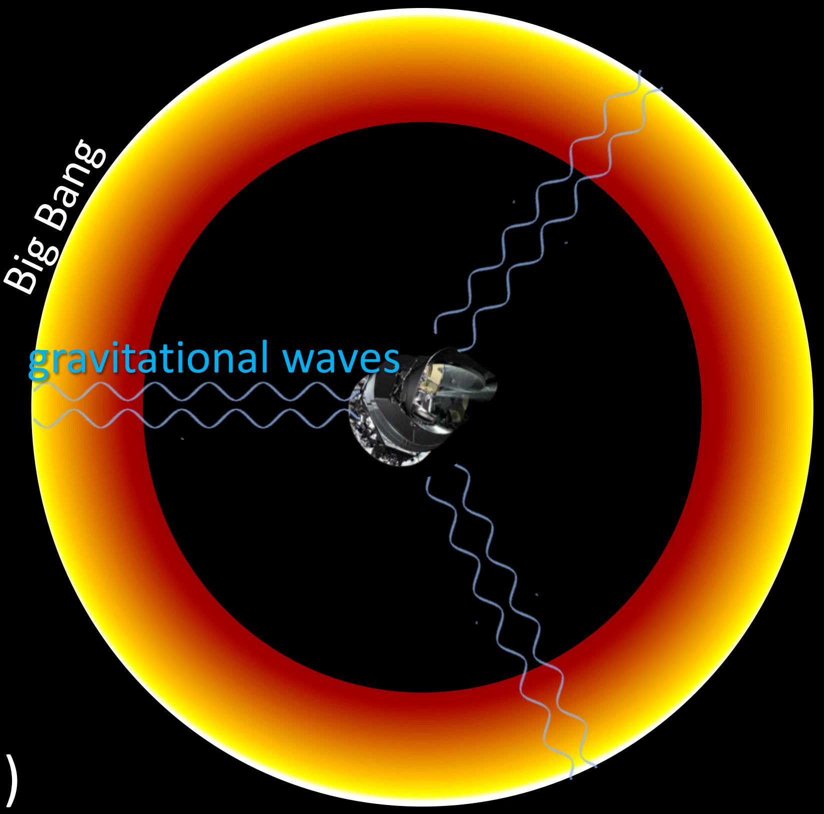
$$\Omega_{\mathrm{GW}} h^2 \propto v^2$$

Anisotropic GW Background

- Light scalar χ:
  - Scale-invariant fluctuation
- Each direction on the sky:
  - Different χ
- Influences preheating
  - GW amplitude

$$\Omega_{\mathrm{GW}} = \Omega_{\mathrm{GW}}(\chi)$$

  - Different amplitude in different directions (Bethke, Figueroa & AR 2013)



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Anisotropy in Massless Preheating

- Massless preheating

$$V(\phi,\chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2 \quad \text{with} \quad \frac{g^2}{\lambda} = 2$$

- Simulate for a range of χ centred around average $\bar{\chi}$
  - Knowing statistics of χ, we can obtain statistics of Ω_{GW}
  - Calculate amplitude on large angular scales
- Strength of signal depends on average $\bar{\chi}$

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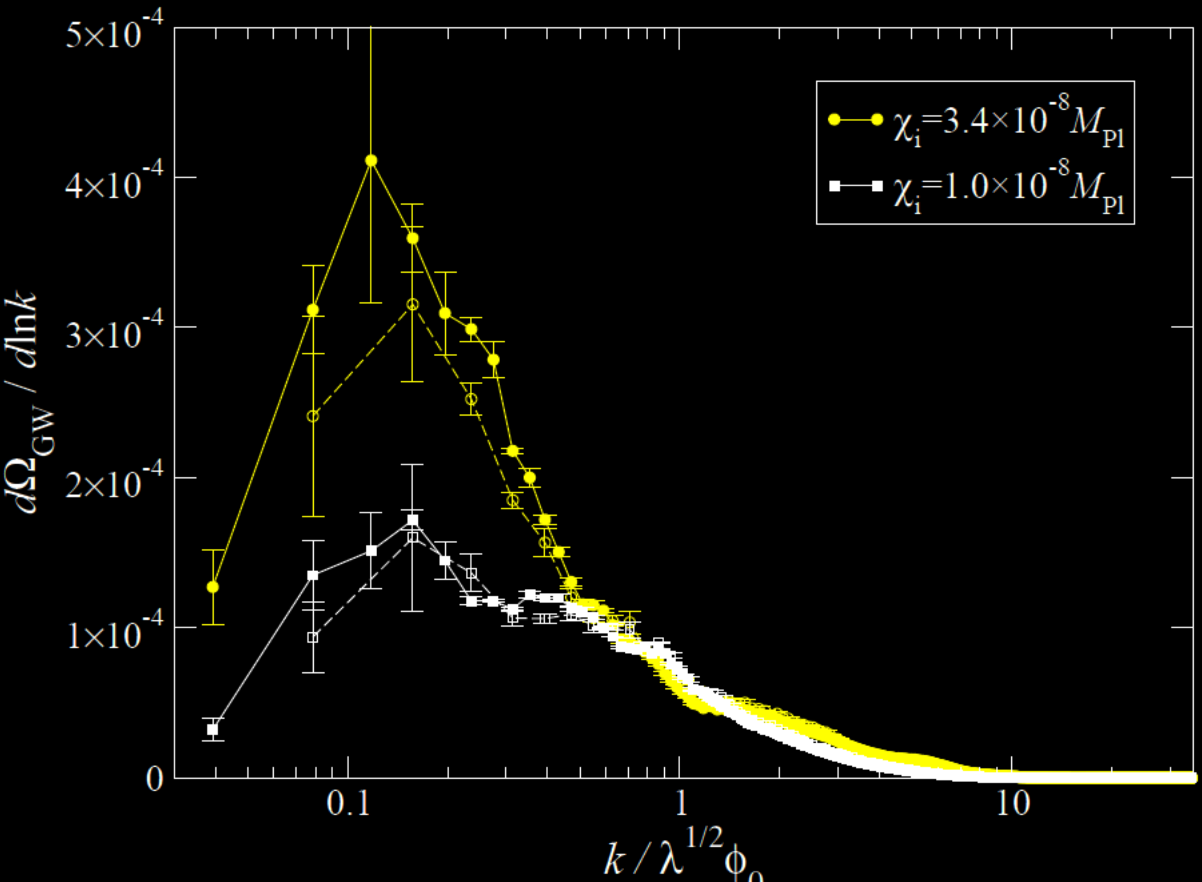
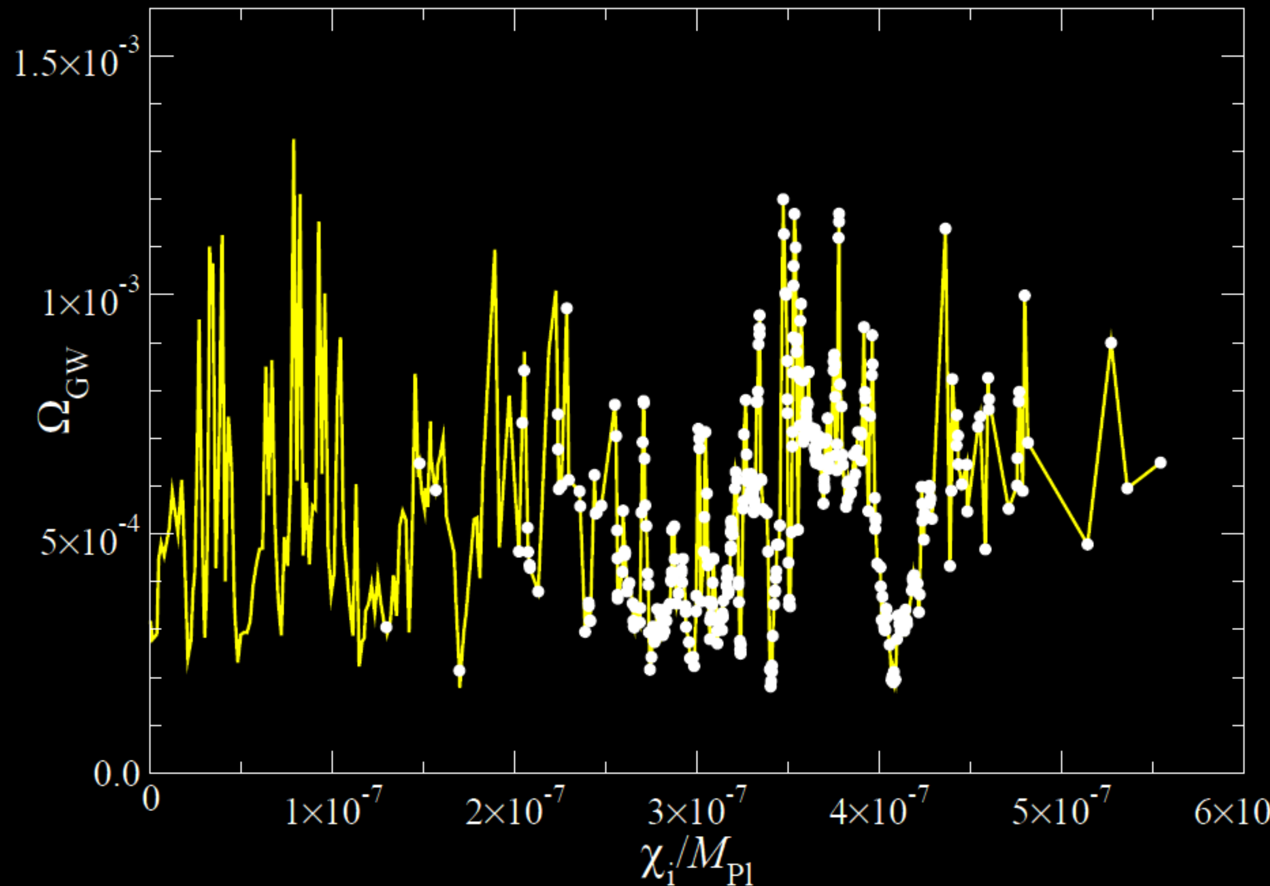
Dependence on Local χ Value



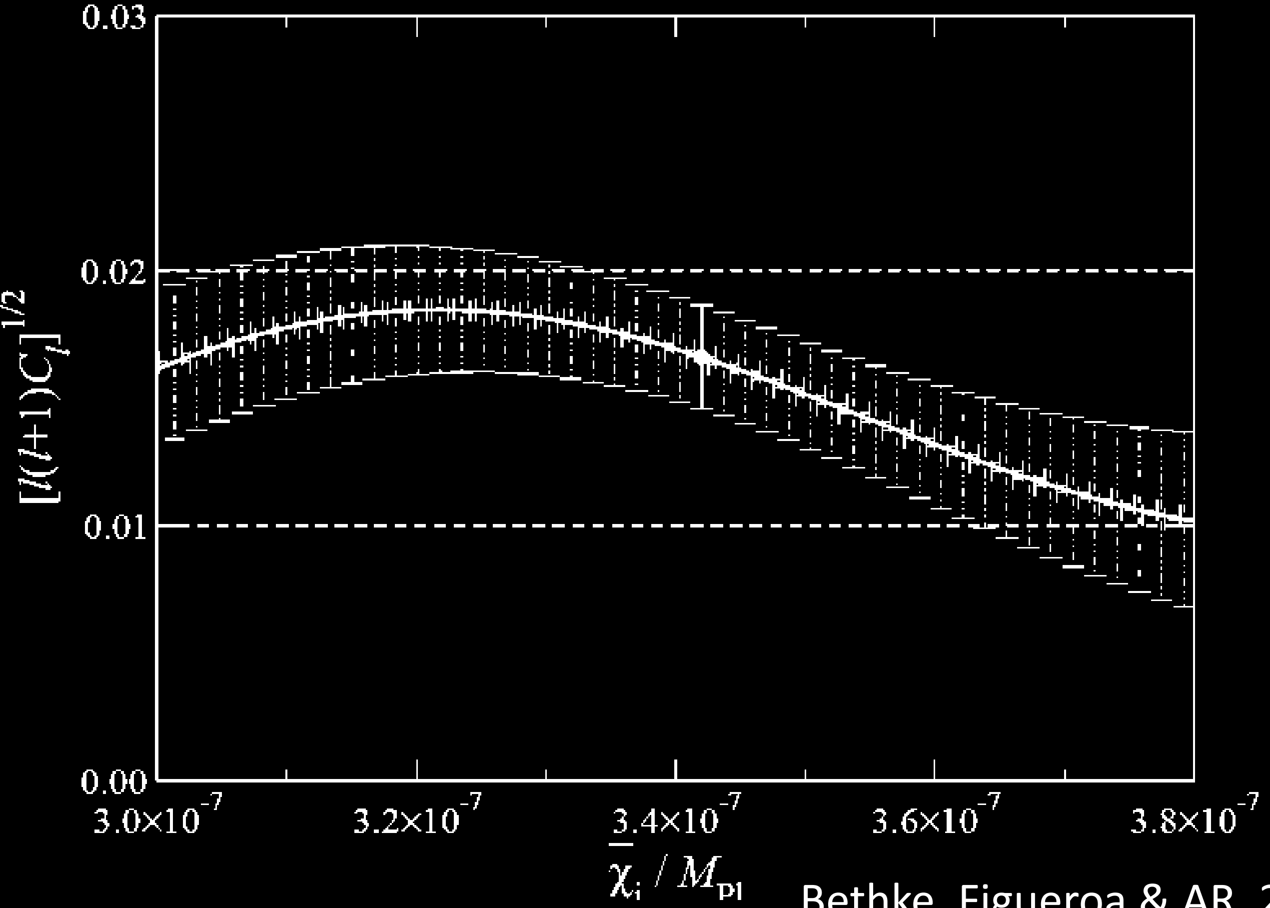
Bethke, Figueroa & AR, 2013

Dependence on Local χ Value



Bethke, Figueroa & AR, 2013

Amplitude of GW Anisotropy

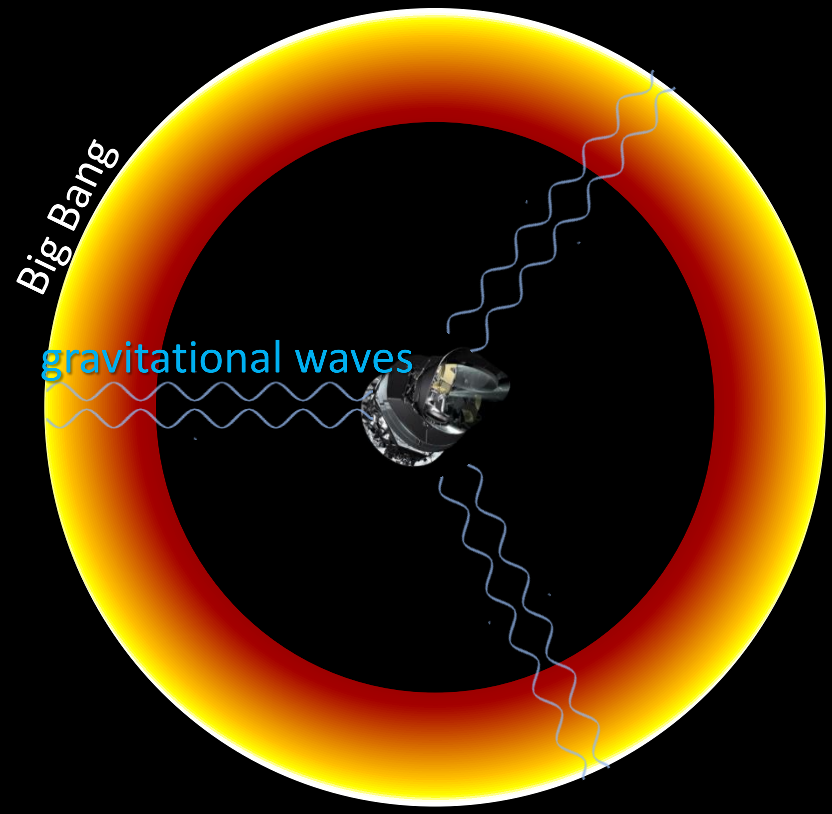


Bethke, Figueroa & AR, 2013

- ~1% amplitude – observable?

GW Background Anisotropy

- Need a light scalar whose value influences GW production locally
- Smoother dependence may lead to a stronger signal
- Phase transitions, defects?



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Summary: GW from Preheating

- In principle a way to distinguish between inflationary models:
  - Amplitude
  - Spectrum
  - Anisotropy
- Simplest models:
  - Frequency far too high for eLISA, $f_* \sim 100\ \mathrm{MHz}$
- eLISA frequencies need TeV-scale inflation
  - Typically suppresses amplitude by $\rho_{\mathrm{inf}}^{1/2}/M_{\mathrm{Pl}}^2$