Holographic charge localization at brane intersections

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arXiv 1505.05883 MA, Daniel Areán, Johanna Erdmenger, Javier M. Lizana

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MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
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Preliminaries

PROBE BRANES

Dp/Dq intersections with charge density and chemical potential.

- [hep-th/0611099]

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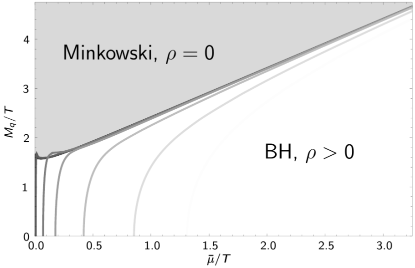

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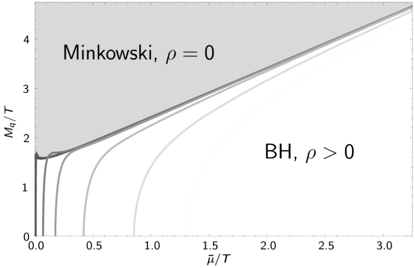

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+ "Minkoswki" or "BH".
+ Minkowski above given M.
+ $\rho \neq 0 \Rightarrow$ only BH are possible.
+ $\rho, M > 0$: BH above given μ.

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D3/D7 to holographically realize TI via m(x)

- [1007.3253]

  Hoyos, Jensen, Karch

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Key idea

m function of spatial coordinate

$$m(x) = M\left(\frac{2}{1+e^{-ax}} - 1\right)$$

$$m(\pm\infty) \to \pm M \qquad m(0) = 0$$

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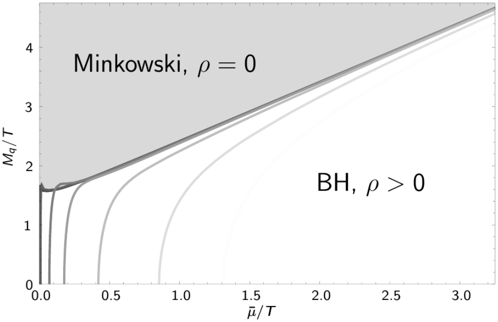

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Set-up

Geometry generated by stack of black D3-branes

$$ds^2 = \frac{L^2}{z^2}\left(-\frac{f(z)^2}{h(z)}\,dt^2 + h(z)\,d\vec{x}^2 + dz^2\right) + L^2\,d\Omega_5^2$$

with

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Embed D5-brane

	t	x	y	v	z	Ω_2	$\tilde{\Omega}_2$	θ
D3	×	×	×	×				
D5	×	×	×		×	×		

$$d\Omega_5^2 = d\theta^2 + \sin^2\theta\,d\Omega_2^2 + \cos^2\theta\,d\tilde{\Omega}_2^2$$

$$\cos\theta = \chi$$

Background

Embedding is determined by

$$A = \phi(z,x)dt \qquad \chi(z,x)$$

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IR behaviour (regularity)

$$\phi(z,x) = a^{(2)}(x)\,(1-z)^2 + \dots$$
$$\chi(z,x) = C^{(0)}(x) + f[C^{(0)}(x)]\,(1-z)^2 + \dots$$

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$$\phi(z,x) = \mu(x) - \rho(x)z + \ldots$$
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Resolution of the background

Numerical resolution of EOMs: Chebyshev + Newton-Raphson

$$\phi(1,x) = 0\,, \qquad \chi'(1,x) = 0$$

$$\chi'(0,x) = M\left(\frac{2}{1+e^{-ax}} - 1\right) \qquad \phi(0,x) = \mu$$

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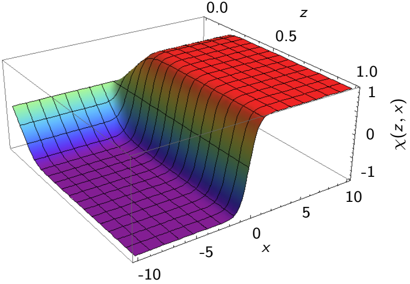

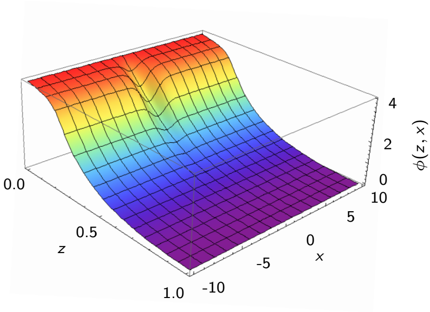

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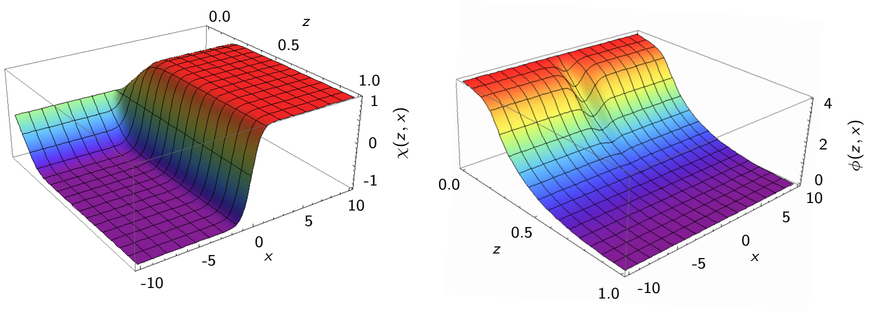

$$\dot{\chi}(z,\pm L) = 0\,, \qquad \dot{\phi}(z,\pm L) = 0$$

Results for background

Can read charge density

$$\rho(x) = -\partial_z \phi(z, x)|_{z=0}$$

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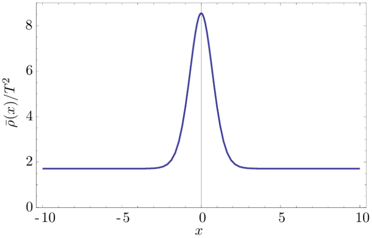

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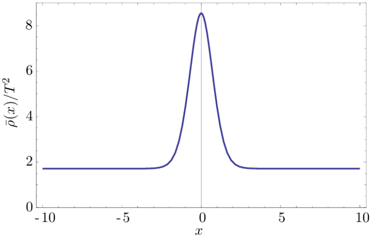

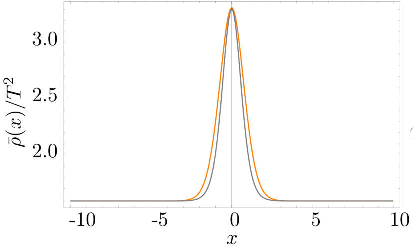

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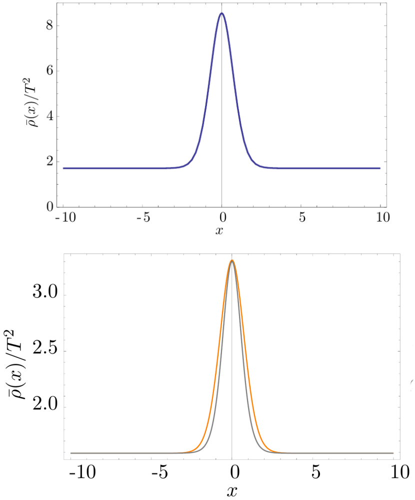
Results

- D3/D5 intersection realises (1+1)-dim interface.
- $\rho(x)$ is localised at the interface.
- Enhancement bigger close to phase transition.
- Not exactly as for homogeneous case.



Fluctuations

Study linear response to electric field to analyse conductivities.

$$A_\mu = A_\mu(z,x) + a_\mu(z,x)e^{i\omega t}$$
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- $(a_x,\ a_t,\ c,\ a_z)$
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$$f_{ti} = (i\omega\, \mathfrak{e}_i)\, e^{i\omega\, t} \qquad \sigma_i(\omega, x) = \frac{j_i}{i\omega\, \mathfrak{e}_i} = \frac{f_{iz}}{f_{ti}}\bigg|_{z=0} \qquad (i = x, y)$$

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IR: infallling wave condition

UV: no mass fluctuations + homogeneous E + constraint

Conductivities

Use same numerical apparatus as for background.

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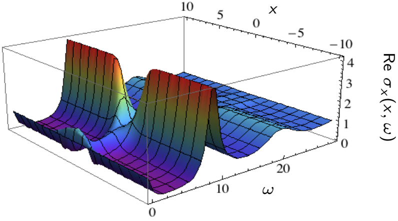

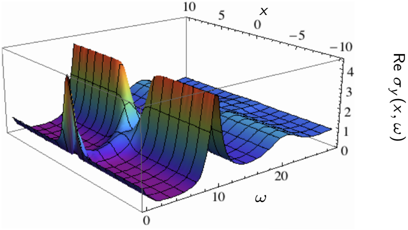

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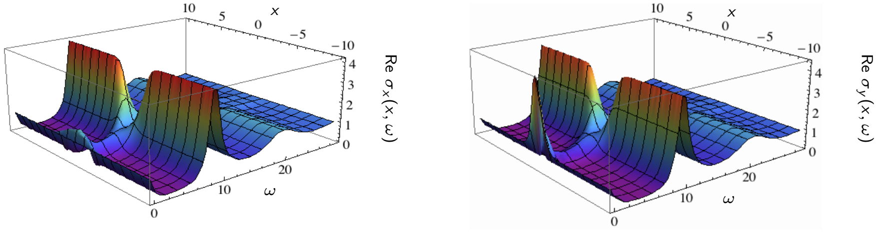

Results

- σ_x and σ_y look alike at high ω and away from interface.
- σ_x and σ_y differ at low ω and close to interface.
- σ_x is suppressed (feels charge density all over (mostly edges)).
- σ_y is enhanced (feels charge density at interface).

DC conductivity σ_x^{DC}

Can do the same as in

- [0809.3808] Iqbal, Liu
- [1103.6068] Ryu, Takayanagi, Ugajin

EOMs in DC limit ($\omega \to 0$)

$$d\zeta = \sqrt{h(z)/f(z)^2}\,dz$$
$$\partial_\zeta \left(\mathcal{F}(\zeta, x)\,\partial_\zeta a_x\right) = 0$$
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Constant expression

$$\sigma_x^{\mathrm{DC}} = \mathcal{F}(z=1, x)\, (1 + \partial_x \frac{a_t(0,x)}{i\omega})$$

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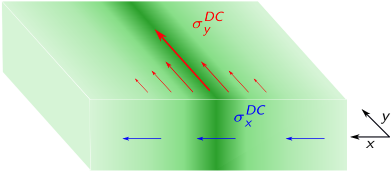
σ_x^{DC} from IR data

$$\sigma_x^{\rm DC} = \frac{2L}{\int_{-L}^{L} \frac{dx}{\mathcal{F}(1,x)}}\,,$$

Short vs long systems $\to$ different BCs

Conductivities



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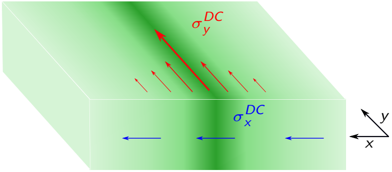

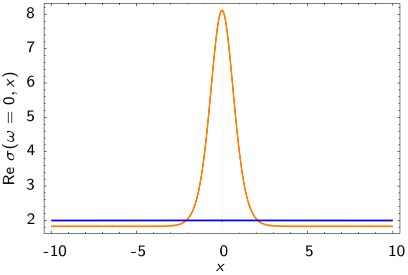

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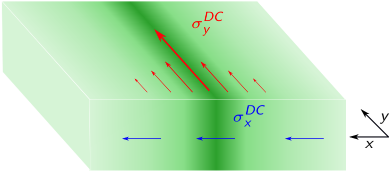

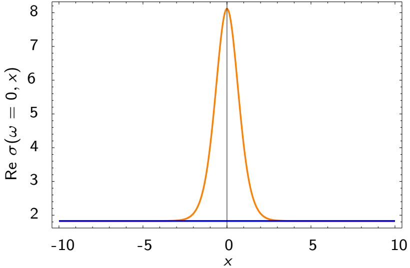

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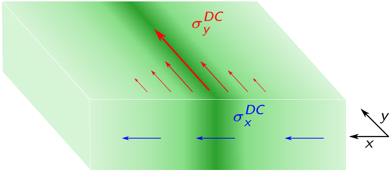

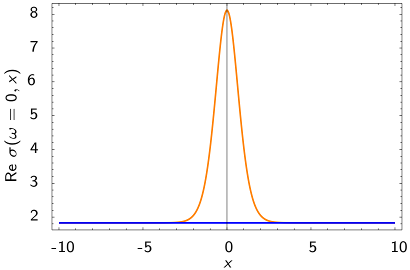

σ_x^{DC} sensible to system size

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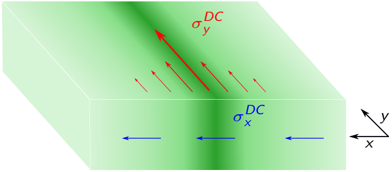

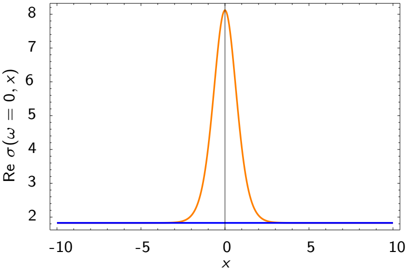

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σ_y^{DC} is also sensitive to inhomogeneity.

- Gradients in χ and ϕ: $\rho(x)$
- Fluctuation EOMs

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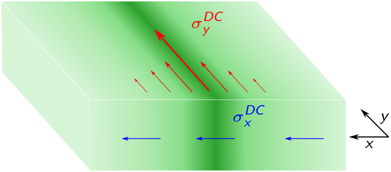

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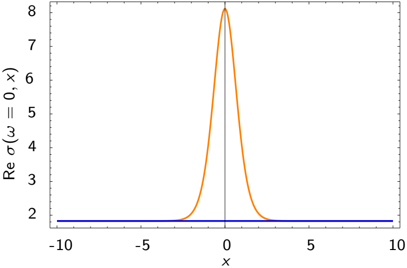

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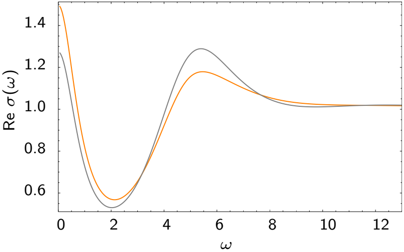

Mostly due to $\rho(x)$.
Sensitive to system size.

Conclusions and Outlook

CONCLUSIONS

- D3/D5 intersection in probe limit realises 1-dim charged interface.
- Charge density peaks at interface ($\rho_0 \sim 5\rho_L$).
- $\sigma_x(\omega, x) \sim \sigma_y(\omega, x)$ at $\omega, x \gg 0$. $\sigma_x(\omega, x) \neq \sigma_y(\omega, x)$ at $\omega, x \to 0$
- $\sigma_y(\omega, x)$ sensitive to ∂_x.
- Current conservation: constant σ_x^{DC}. IR background data.
  + $\sigma_x^{DC} \leftrightarrow \rho_L$, $\sigma_y^{DC} \leftrightarrow \rho(x)$.
  + At $x = 0$, $\sigma_y^{DC} \sim 4\,\sigma_x^{DC}$.
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FUTURE DIRECTIONS

- Turn on topological terms.
- Look for signatures of Fermi surface.
- Study effects of disorder.