

# On-Shell Diagrammatics and the Perturbative Structure of Planar Gauge Theories

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P.B. – *to appear*

# (Very) minimal bibliography

On-shell diagrams and Grassmannian for  $\mathcal{N} = 4$  SYM:

- Arkani-Hamed et al. – arXiv:1212.5605 [hep-th]

Forward amplitudes ( $\mathcal{N} \geq 1$ ) and all-loop RR ( $\mathcal{N} = 4$ ):

- Caron-Huot – arXiv:1007.3224 [hep-th]
- Arkani-Hamed et al. – arXiv:1008.2958 [hep-th]

Mom-twistor/on-shell diagrams/Grassmannian regularisation:

- Bourjaily, Caron-Huot, Trnka – arXiv:1303.4734 [hep-th]
- Ferro, Lukowski, Staudacher – arXiv:1407.6736 [hep-th]
- P.B., Conde, Gordo – arXiv:1411.7987 [hep-th]

Multivariate residues and unitarity:

- Sogaard, Zhang – arXiv:1310.6006 [hep-th]

Reviews:

- Elvang, Huang – arXiv:1308.1697 [hep-th]
- P.B. – arXiv:1312.5583 [hep-th]

# How do we understand perturbative field theory?

Textbook formulation: Lagrangian  $\longleftrightarrow$  Feynman diagrammatics.

Manifest properties:

- Poincaré invariance
- Unitarity
- Locality of the interactions

Drawbacks:

- Field redefinitions
- Gauge redundancies
- Gauge invariance broken at intermediate stages
- Further structures and symmetries hidden

# Constructive approach to perturbation theory

Object of investigation: Scattering amplitudes  $\mathcal{M}_n$

$$p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}, \quad \langle i, j \rangle = \epsilon^{ab} \lambda_a^{(i)} \lambda_b^{(j)} \quad [i, j] = \epsilon^{\dot{a}\dot{b}} \tilde{\lambda}_{\dot{a}}^{(i)} \tilde{\lambda}_{\dot{b}}^{(j)}$$

$$P_{ij}^2 \equiv (p^{(i)} + p^{(j)})^2 = \langle i, j \rangle [i, j]$$

# Constructive approach to perturbation theory

Object of investigation: Scattering amplitudes  $\mathcal{M}_n$

$$\text{Poincaré invariance} \left\{ \begin{array}{l} \text{Momentum conservation:} \\ \mathcal{M}_n = \delta^{(2 \times 2)} \left( \sum_{i=1}^n \lambda^{(i)} \tilde{\lambda}^{(i)} \right) \mathcal{M}_n \left( \{ \lambda^{(i)}, \tilde{\lambda}^{(i)} ; h_i \} \right) \\ \\ \text{Little group covariance:} \\ \hat{\mathcal{H}}^{(i)} \mathcal{M}_n = -2h_i \mathcal{M}_n \end{array} \right.$$

# Constructive approach to perturbation theory

Object of investigation: Scattering amplitudes  $\mathcal{M}_n$

Analytic function

Poincaré invariance

$$\left\{ \begin{array}{l} \text{Momentum conservation:} \\ \mathcal{M}_n = \delta^{(2 \times 2)} \left( \sum_{i=1}^n \lambda^{(i)} \tilde{\lambda}^{(i)} \right) M_n \left( \{ \lambda^{(i)}, \tilde{\lambda}^{(i)} ; h_i \} \right) \\ \\ \text{Little group covariance:} \\ \hat{\mathcal{H}}^{(i)} \mathcal{M}_n = -2h_i \mathcal{M}_n \end{array} \right.$$


# Constructive approach to perturbation theory

Object of investigation: Scattering amplitudes  $\mathcal{M}_n$

Analytic function

Super-Poincaré:



$$\mathcal{M}_n = \delta^{(2 \times 2)} \left( \sum_{i=1}^n \lambda^{(i)} \tilde{\lambda}^{(i)} \right) \delta^{(2 \times \mathcal{N})} \left( \sum_{i=1}^n \lambda^{(i)} \tilde{\eta}^{(i)} \right) M_n(\{\lambda^{(i)}, \tilde{\lambda}^{(i)}; \tilde{\eta}^{(i)}\})$$

Grassmann coherent states ( $\mathcal{N} < 4$ ):

$$|\lambda, \tilde{\lambda}; \tilde{\eta}, +\rangle = \sum_{s=0}^{\mathcal{N}} \frac{1}{s!} \left( \prod_{r=0}^s \tilde{\eta}^{lr} \right) |\lambda, \tilde{\lambda}; +(1-s/2)\rangle_{l_1 \dots l_s}$$

$$|\lambda, \tilde{\lambda}; \tilde{\eta}, -\rangle = \sum_{s=0}^{\mathcal{N}} \frac{1}{s!} \left( \prod_{r=0}^s \tilde{\eta}^{lr} \right) |\lambda, \tilde{\lambda}; -(1-(\mathcal{N}-s)/2)\rangle_{l_1 \dots l_s}$$

# Building blocks: 3-particle amplitudes

(Super)-Poincaré invariance:

- momentum conservation:  $(p^{(1)} + p^{(2)})^2 = (-p^{(3)})^2 = 0$

$$\langle i, j \rangle [i, j] = 0, \forall i \implies M_3 = M_3^{(1)}([i, j]) + M_3^{(2)}(\langle i, j \rangle)$$

- little group  $\implies$  functional form of  $M_3^{(k)}$ ,  $k = 1, 2$

# Building blocks: 3-particle amplitudes

(Super)-Poincaré invariance:

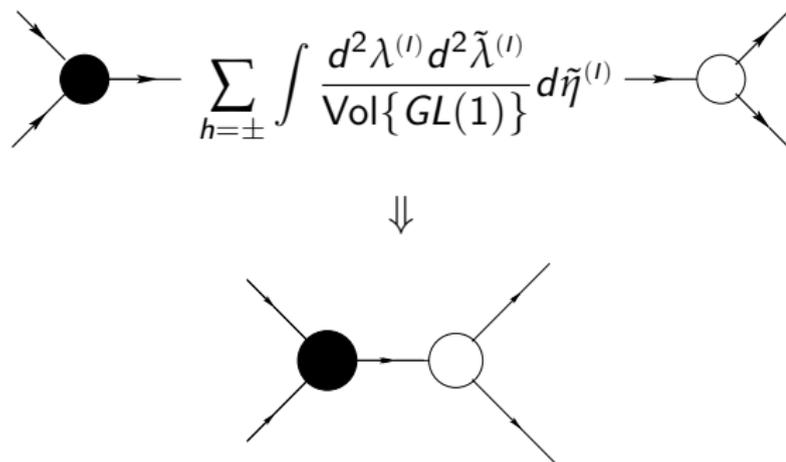
$$\begin{aligned} \mathcal{M}_3^{(1)}(1^+, 2^+, 3^-) &= \begin{array}{c} 1 \\ \swarrow \\ \circ \\ \searrow \\ 2 \end{array} \begin{array}{c} \rightarrow \\ \leftarrow \\ 3 \end{array} = \\ &= \delta^{(2 \times 2)} \left( \sum_{i=1}^3 \lambda^{(i)} \tilde{\lambda}^{(i)} \right) \delta^{(1 \times \mathcal{N})} \left( \sum_{i=1}^3 [i+1, i-1] \tilde{\eta}^{(i)} \right) \frac{[1, 2]^{4-\mathcal{N}}}{[1, 2][2, 3][3, 1]} \end{aligned}$$

$$\begin{aligned} \mathcal{M}_3^{(2)}(1^-, 2^-, 3^+) &= \begin{array}{c} 1 \\ \swarrow \\ \bullet \\ \searrow \\ 2 \end{array} \begin{array}{c} \rightarrow \\ \rightarrow \\ 3 \end{array} = \\ &= \delta^{(2 \times 2)} \left( \sum_{i=1}^3 \lambda^{(i)} \tilde{\lambda}^{(i)} \right) \delta^{(2 \times \mathcal{N})} \left( \sum_{i=1}^3 \lambda^{(i)} \tilde{\eta}^{(i)} \right) \frac{\langle 1, 2 \rangle^{4-\mathcal{N}}}{\langle 1, 2 \rangle \langle 2, 3 \rangle \langle 3, 1 \rangle} \end{aligned}$$

# Decorated on-shell diagrammatics

On-shell rules:

- Gluing  $\longrightarrow$  On-shell processes

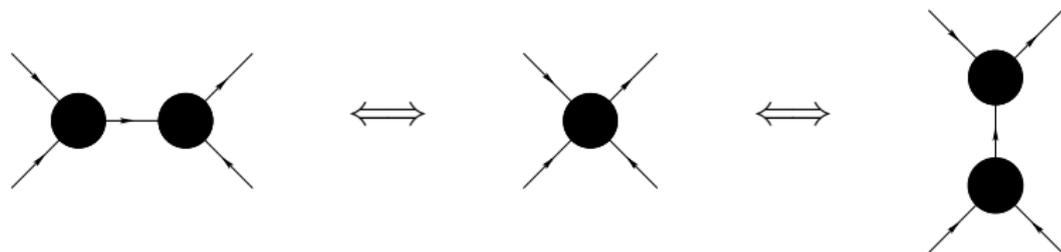


- More constraints than degrees of freedom: singularity
- Same number of constraints as dofs: localised diagram
- Less constraints than dofs: differential forms

# Decorated on-shell diagrammatics

## On-shell rules

- Mergers



- BCFW bridges

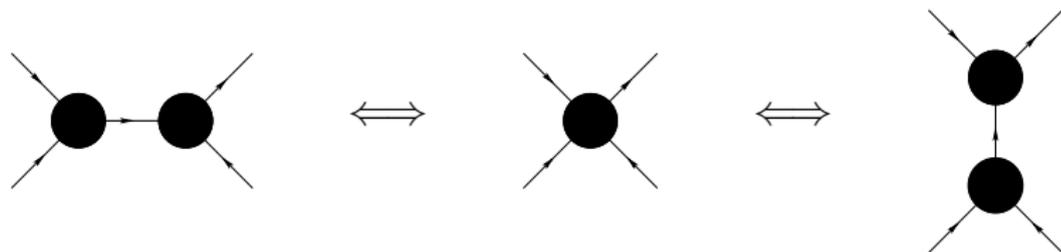
$$\mathcal{M}_n = \text{diagram} \Rightarrow \text{diagram} = dz \mu(z) \mathcal{M}_n(z)$$

The diagram on the left is a shaded circle with four external lines. The diagram on the right is a shaded circle with four external lines, connected to a white circle and a black circle by lines forming a triangle.

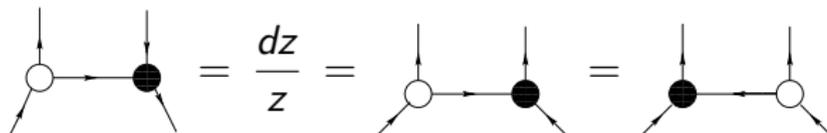
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## On-shell rules

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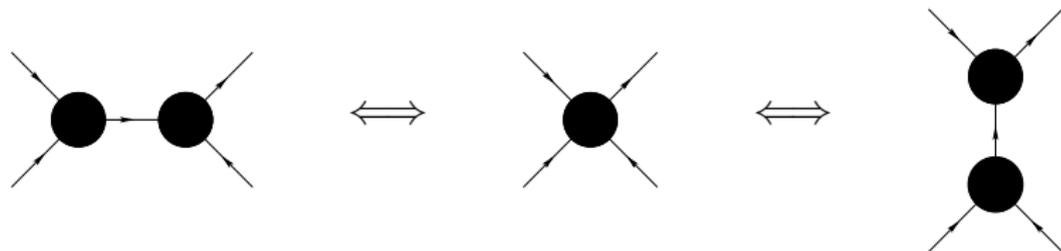
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# Decorated on-shell diagrammatics

## On-shell rules

- Mergers



- BCFW bridges

$$\text{Diagram 1} + \text{Diagram 2} \sim \frac{dz}{z} + dz z^{3-\mathcal{N}}$$

# Decorated on-shell diagrammatics

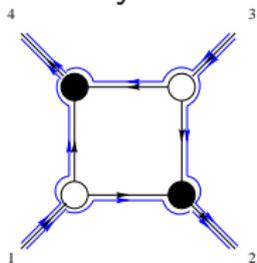
## On-shell rules

- Helicity flows

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## On-shell rules

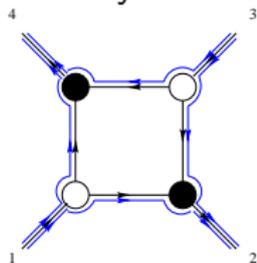
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## On-shell rules

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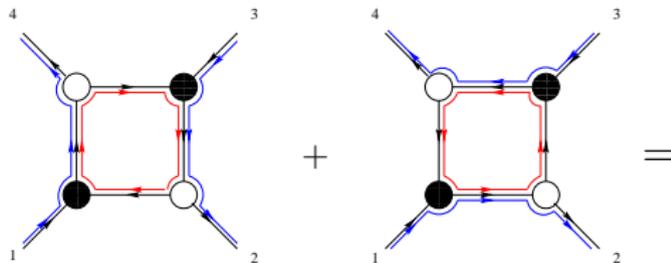


$$= \mathcal{M}_4^{\text{tree}}(1^-, 2^+, 3^-, 4^+)$$

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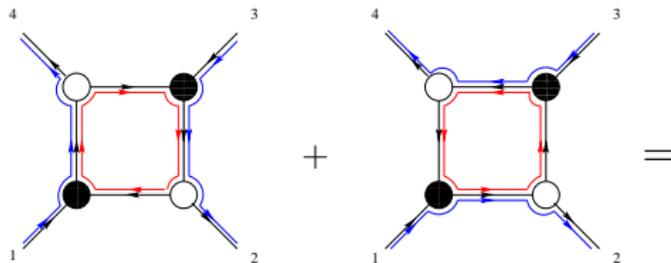
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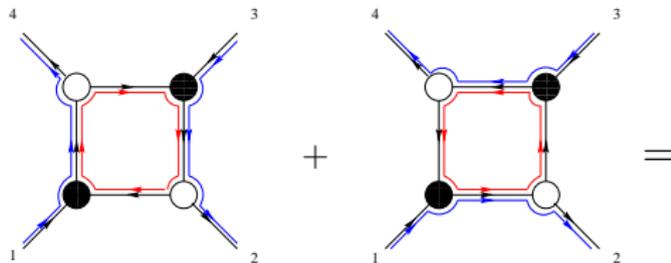


$$= \mathcal{M}_4^{\text{tree}}(1^-, 2^+, 3^-, 4^+) \left[ \left( -\frac{s}{u} \right)^{4-\mathcal{N}} + \left( -\frac{t}{u} \right)^{4-\mathcal{N}} \right]$$

# Decorated on-shell diagrammatics

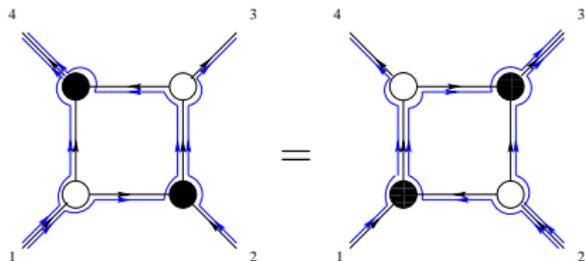
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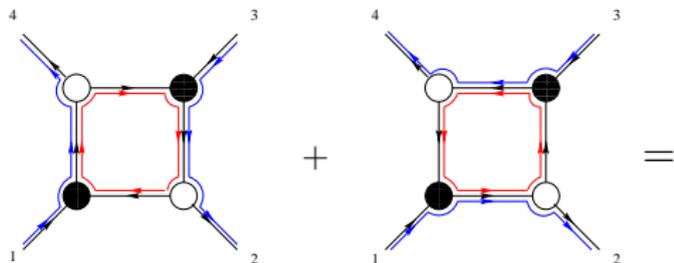
- Square moves



# Decorated on-shell diagrammatics

## On-shell rules

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$$= \mathcal{M}_4^{\text{tree}}(1^-, 2^+, 3^-, 4^+) \left[ \left(-\frac{s}{u}\right)^{4-\mathcal{N}} + \left(-\frac{t}{u}\right)^{4-\mathcal{N}} \right]$$

- Square moves

$$= \mathcal{M}_4^{\text{tree}}(1^-, 2^-, 3^+, 4^+)$$

# Scattering amplitudes as on-shell forms

General structure of the perturbative expansion

$$\mathcal{M}_n(\{\lambda^{(i)}, \tilde{\lambda}^{(i)}; h_i; \tilde{\eta}\}) = \sum_{L=0}^{\infty} \mathcal{M}_n^{(L)}(\{\lambda^{(i)}, \tilde{\lambda}^{(i)}; h_i; \tilde{\eta}\}, \{z_l\}) \bigwedge_{l=1}^{4L} dz_l,$$

- $L = 0$ : fully localised diagrams  $\iff$  Tree level amplitudes
- $L \geq 1$ :  $4L$ -differential forms  $\iff$   $L$ -loop *integrand*

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Inequivalent on-shell diagrams with  $4L$  dofs unfixed

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# Scattering amplitudes as on-shell forms

*Singularity equation:*

$$\partial \left[ \text{Diagram} \right] = \sum_{L,R} \text{Diagram} + \sum_k \text{Diagram}$$

The diagram on the left is a shaded circle with four external lines. The first sum is over diagrams with two shaded circles labeled L and R connected by a red line, each with two external lines. The second sum is over diagrams with a shaded circle with a red loop on one of its external lines, labeled k and k+1.

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BCFW bridging as integration of the *singularity* equation

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*Singularity equation:*

$$\partial \left[ \text{Diagram} \right] = \sum_{L,R} \text{Diagram} + \sum_k \text{Diagram}$$

The diagram on the left is a shaded circle with four external lines. The first sum is over pairs of lines (L, R) connected by a red line. The second sum is over a shaded circle with a red loop on one of its lines, labeled with indices k and k+1.

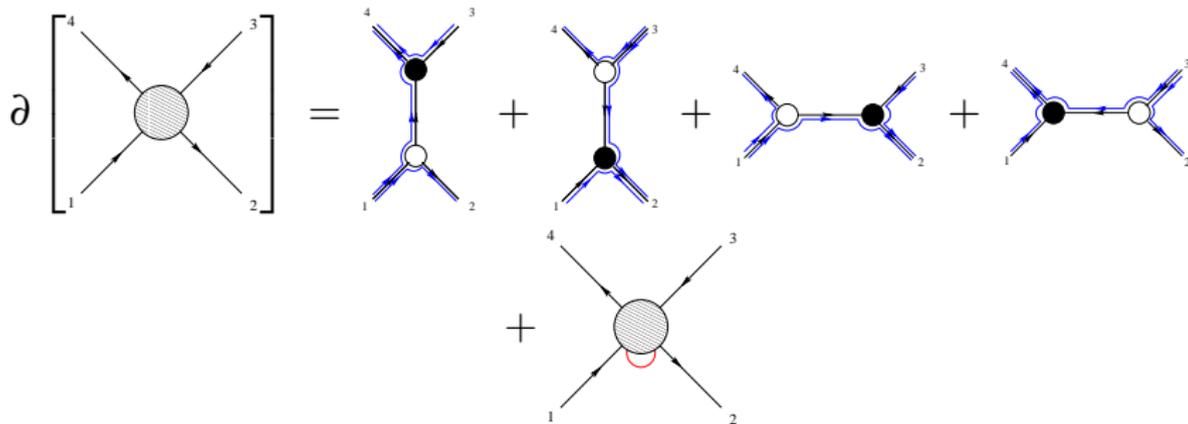
BCFW bridging as integration of the *singularity equation*

$$\text{Diagram} = \sum_{k \in \mathcal{P}} \mathcal{I}_k \text{Diagram} + \mathcal{J}_k \text{Diagram}$$

The diagram on the left is a shaded circle with four external lines. The first sum is over a shaded circle with a red line connecting two lines, labeled with indices i and i+1. The second sum is over a shaded circle with a red loop on one of its lines, labeled with indices i and i+1.

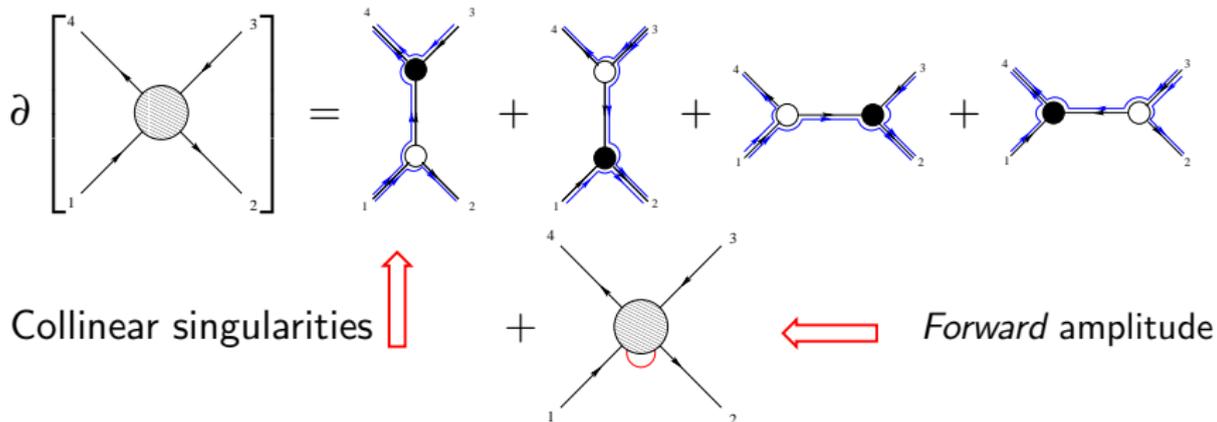
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*Singularity* equation:

$$\partial \left[ \begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \quad 3 \end{array} \right] = \begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \quad 3 \end{array} + \begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \quad 3 \end{array} + \begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \quad 3 \end{array} + \begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \quad 3 \end{array} + \begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \quad 3 \end{array}$$

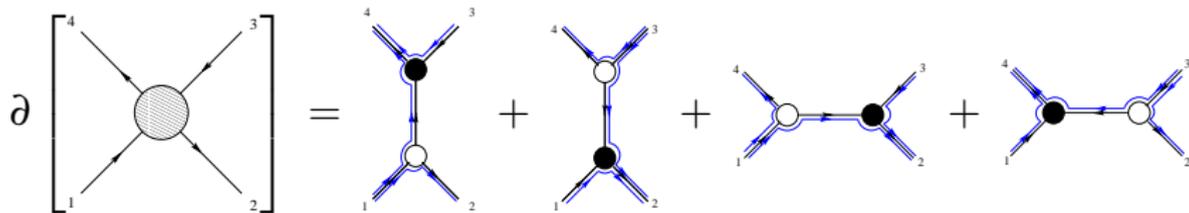
Collinear singularities  $\uparrow$   $+$   $\left[ \begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \quad 3 \end{array} \right]$   $\leftarrow$  Forward amplitude

BCFW bridging as integration of the *singularity* equation



# Scattering amplitudes as on-shell forms

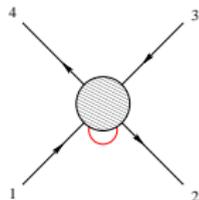
*Singularity equation:*



Collinear singularities



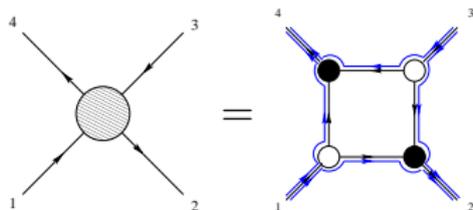
+



*Forward amplitude*

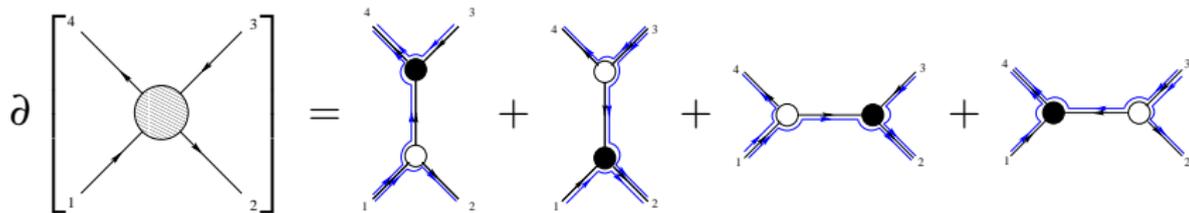
BCFW bridging as integration of the *singularity equation*

Tree-level



# Scattering amplitudes as on-shell forms

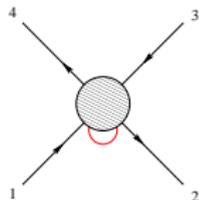
*Singularity equation:*



Collinear singularities



+

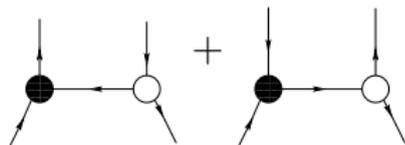


Forward amplitude

BCFW bridging as integration of the *singularity equation*

Tree-level

BCFW bridge 2:

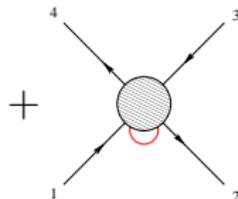


# Scattering amplitudes as on-shell forms

*Singularity* equation:

$$\partial \left[ \begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \end{array} \right] = \begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \end{array} + \begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \end{array} + \begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \end{array} + \begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \end{array} + \begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \end{array}$$

Collinear singularities



Forward amplitude

BCFW bridging as integration of the *singularity* equation

Tree-level

$$\begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \end{array} \stackrel{?}{=} \begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \end{array} + \begin{array}{c} 4 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 2 \end{array}$$

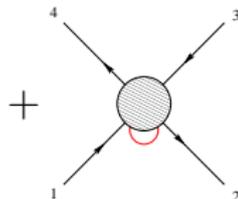


# Scattering amplitudes as on-shell forms

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Collinear singularities



Forward amplitude

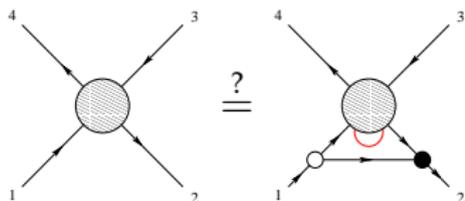
BCFW bridging as integration of the *singularity equation*

Tree-level

$$\Delta = \mathcal{M}_4^{\text{tree}}(-, +, -, +) \left[ \varepsilon_{\mathcal{N},3}(4 - \mathcal{N}) \frac{st}{u^2} - \delta_{\mathcal{N},0} 2 \frac{s^2 t^2}{u^4} \right]$$

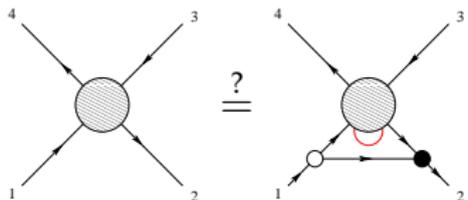
# Scattering amplitudes as on-shell forms

One loop:



# Scattering amplitudes as on-shell forms

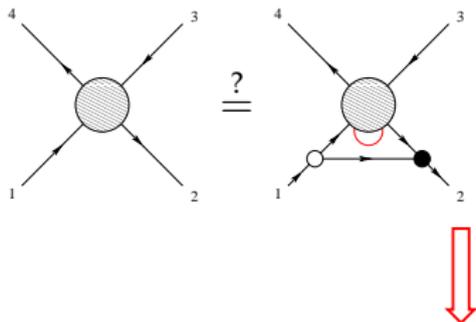
One loop:



tree level 6-pt  
forward amplitude

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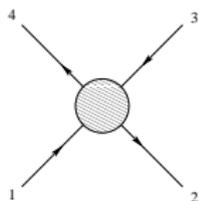
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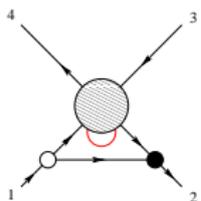
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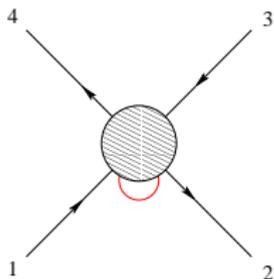
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$\stackrel{?}{=}$

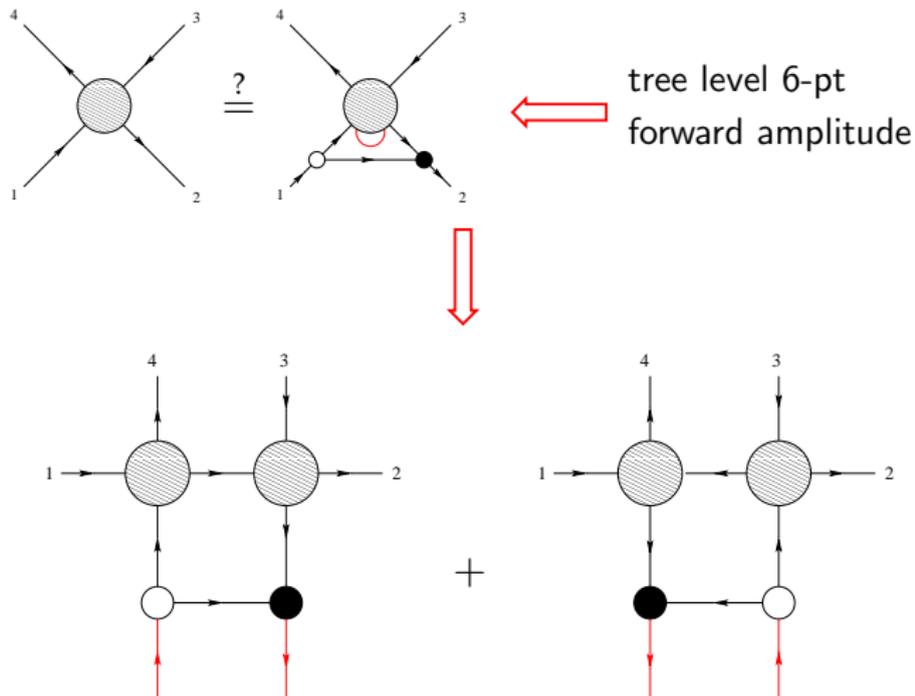


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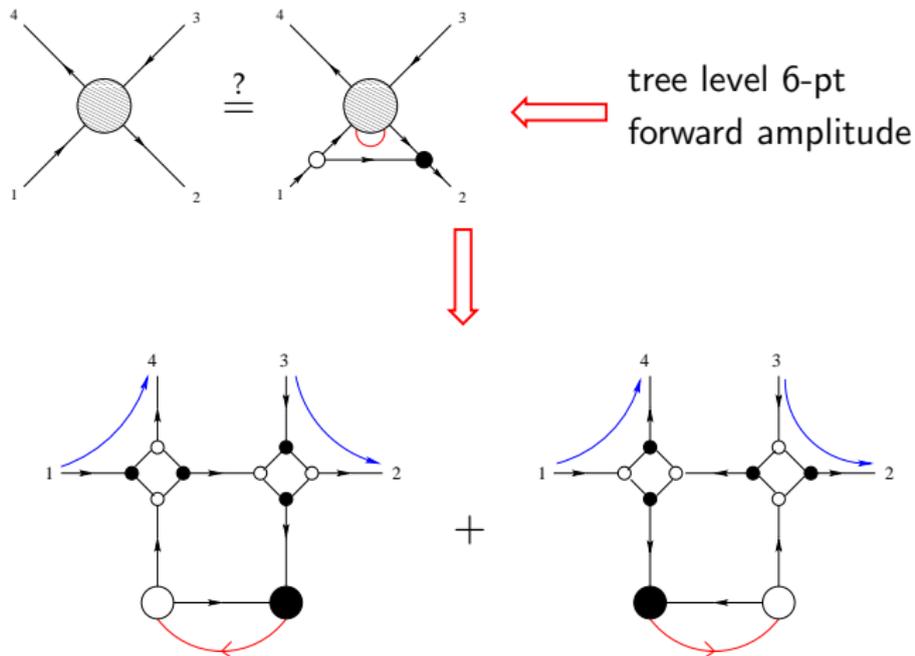
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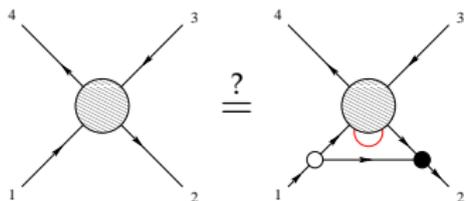
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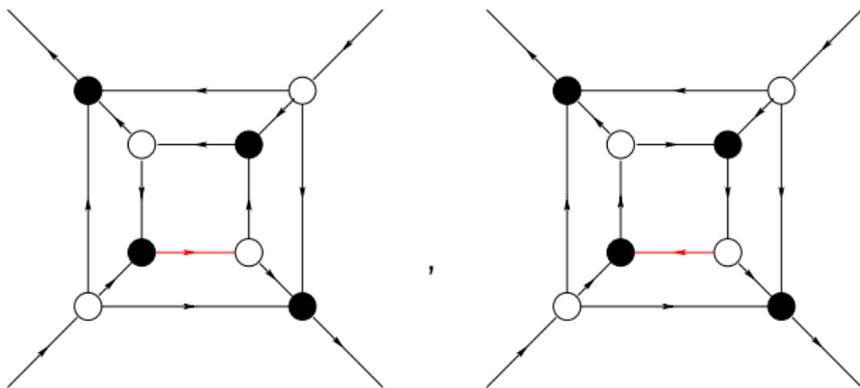


# Scattering amplitudes as on-shell forms

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# Scattering amplitudes as on-shell forms

$$\begin{aligned}
 \tilde{\mathcal{M}}_4(\zeta) = & \text{Diagram 1} + \text{Diagram 2} = \\
 = & \bigwedge_{i=1}^4 \frac{d\zeta_i}{\zeta_i} \frac{\left(-\frac{s}{u}\right)^{4-\mathcal{N}} (1-\zeta_1)^{4-\mathcal{N}} (1-\zeta_3)^{4-\mathcal{N}} \left(-\frac{t}{u}\right)^{4-\mathcal{N}} (1-\zeta_2)^{4-\mathcal{N}} (1-\zeta_4)^{4-\mathcal{N}}}{\left[-\frac{s}{u}(1-\zeta_1)(1-\zeta_3) - \frac{t}{u}(1-\zeta_2)(1-\zeta_4)\right]^{4-\mathcal{N}}}
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Leading singularity:  $\gamma_{\text{LS}} \equiv \{\zeta_i \in \hat{\mathbb{C}}^4 \mid \zeta_i = 0\}$

$$\oint_{\gamma_{\text{LS}}} \tilde{\mathcal{M}}_4(\zeta) = \text{[Diagram 3]} + \text{[Diagram 4]}$$

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Triangle coefficients:

$$\gamma_{\Delta} \equiv \{\zeta_i \in \hat{\mathbb{C}}^4 \mid \zeta_i = 0, i = 1, 3, \zeta_2 = 0, \zeta_4 = -u/t, \zeta_2 \succ \zeta_4\}$$

$$\oint_{\gamma_{\Delta}} \tilde{\mathcal{M}}_4(\zeta) = \Delta$$

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Extraction of physical info: Look at the poles

Double cut structure:

$$\gamma_0 \equiv \{\zeta_i \in \hat{\mathbb{C}}^4 \mid \zeta_i = 0, i = 1, 3\}$$

$$\oint_{\gamma_0} \tilde{\mathcal{M}}_4(\zeta) = \text{Diagram 1} + \text{Diagram 2}$$

# Conclusion

- On-shell diagrammatics as a starting point for a Lagrangian free definition of field theory
- Locality is lost for individual diagrams
- Symmetry and Structure of the theory more transparent.
- All-loop recursion relation holds for the integrand of  $\mathcal{N} \geq 1$  SYM in the planar sector (diagrammatic proof!)
- Provide a natural definition for the forward limit in  $\mathcal{N} = 0$  YM ← It needs to be checked
- Difference in structure between SYM and YM suggested to be due to the different structure in the tree-level boundary term (deeper understanding needed)
- It is lacking an on-shell interpretation for the all-plus helicity amplitudes