Causality constraints on the graviton 3-point coupling

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Iberian Strings 2015

Salamanca, May 29, 2015

Xian Camanho, JE, Juan Maldacena, Alexander Zhiboedov, arXiv:1407.5597

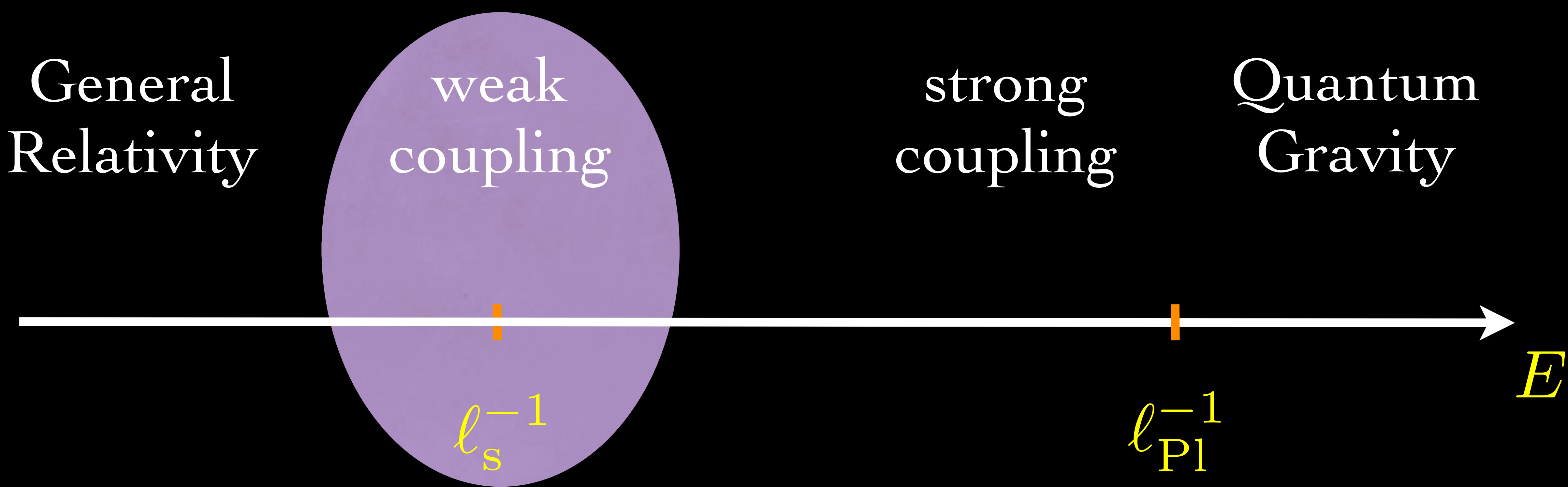
Local, Lorentz invariant Lagrangians may be inconsistent

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 2006

Causality, unitarity or analyticity of the S-matrix not guaranteed

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{c_1}{\Lambda^4}(F_{\mu\nu}F^{\mu\nu})^2 + \frac{c_2}{\Lambda^4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \ldots$$

We want to scrutinize the status of gravity by weakly coupled modifications to the Einstein tensor



From the point of view of the Lagrangian

$$\mathcal{L} = R + \sum_{m=1} \zeta_m \, \mathcal{P}_m(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma})$$

where $\mathcal{P}_m$ are polynomials of order m+1 in the curvatures

If the above Lagrangian is a Wilsonian effective Lagrangian

$$\zeta_m \sim \ell_{\mathrm{Pl}}^{2m}$$

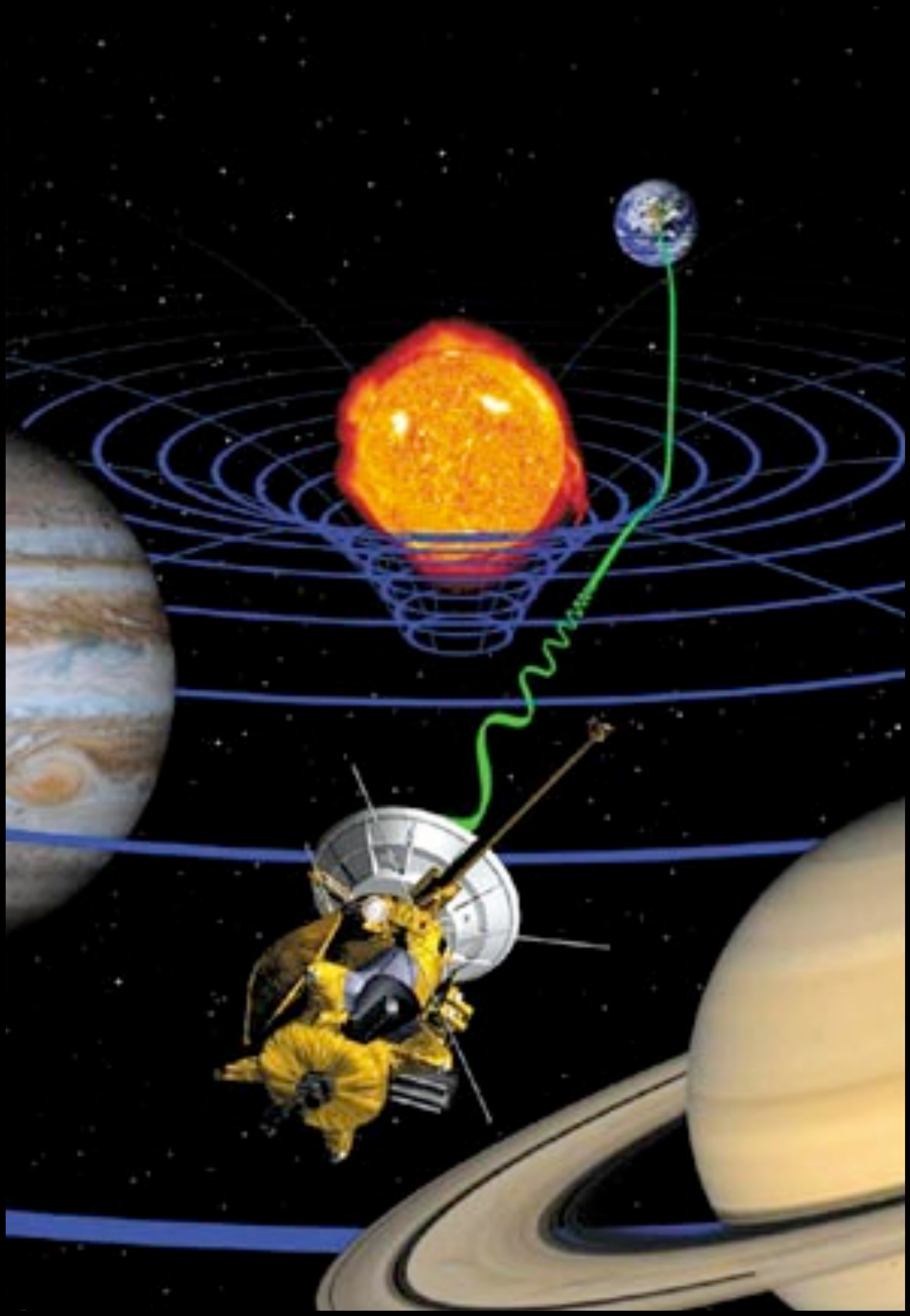
What we are willing to explore is

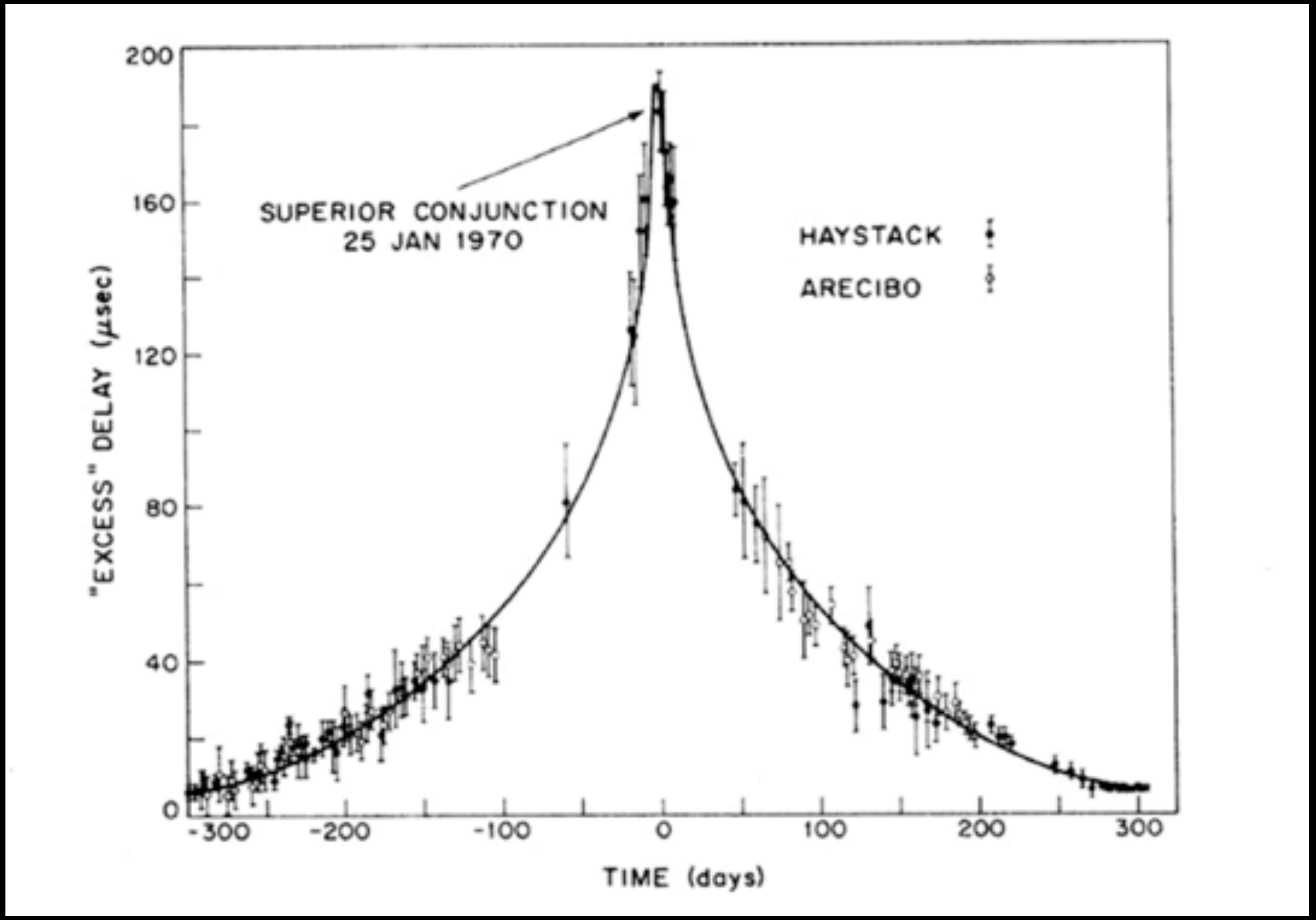
$$\zeta_m := \alpha_{2m}^m \sim \ell_{\mathrm{s}}^{2m} \qquad \alpha_{2m} \sim \ell_{\mathrm{s}}^2$$

with $\ell_{\mathrm{s}} \gg \ell_{\mathrm{Pl}}$ (e.g. weakly coupled string theory)

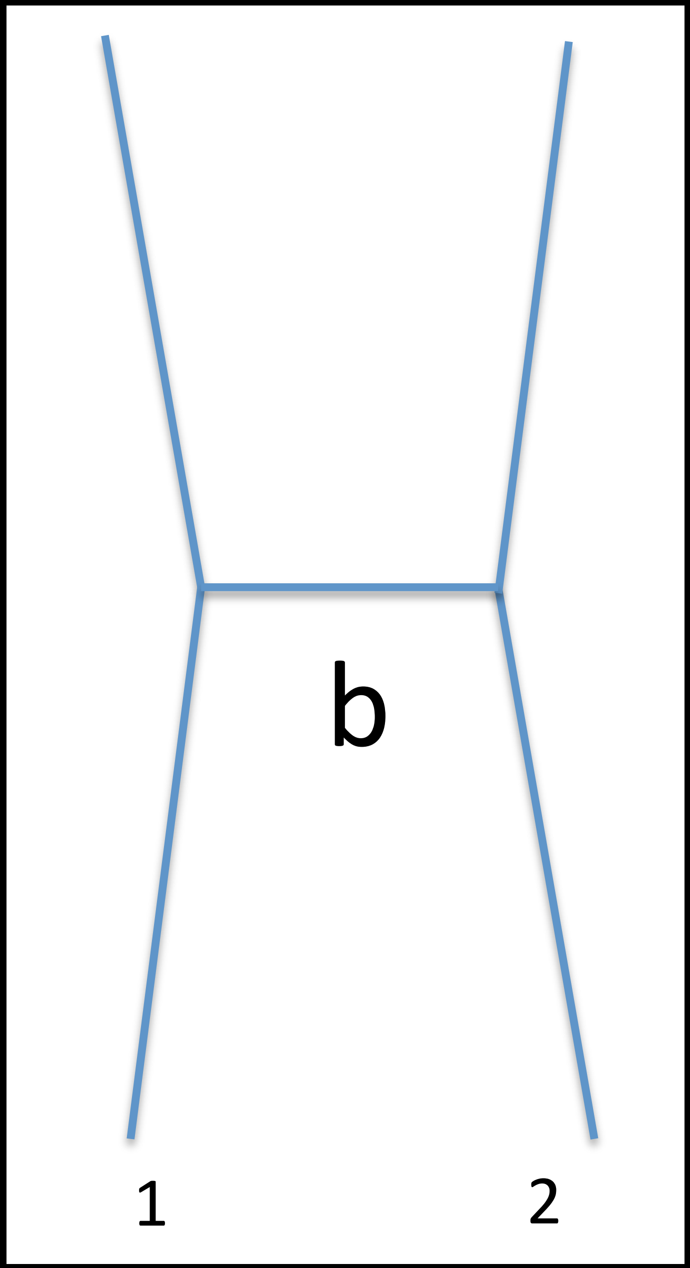
The fourth test of GR: Shapiro time delay

Shapiro, 1964





Shapiro delay: highly boosted particle



Its gravitational field is a plane wave

Dray, 't Hooft, 1985 't Hooft, 1987

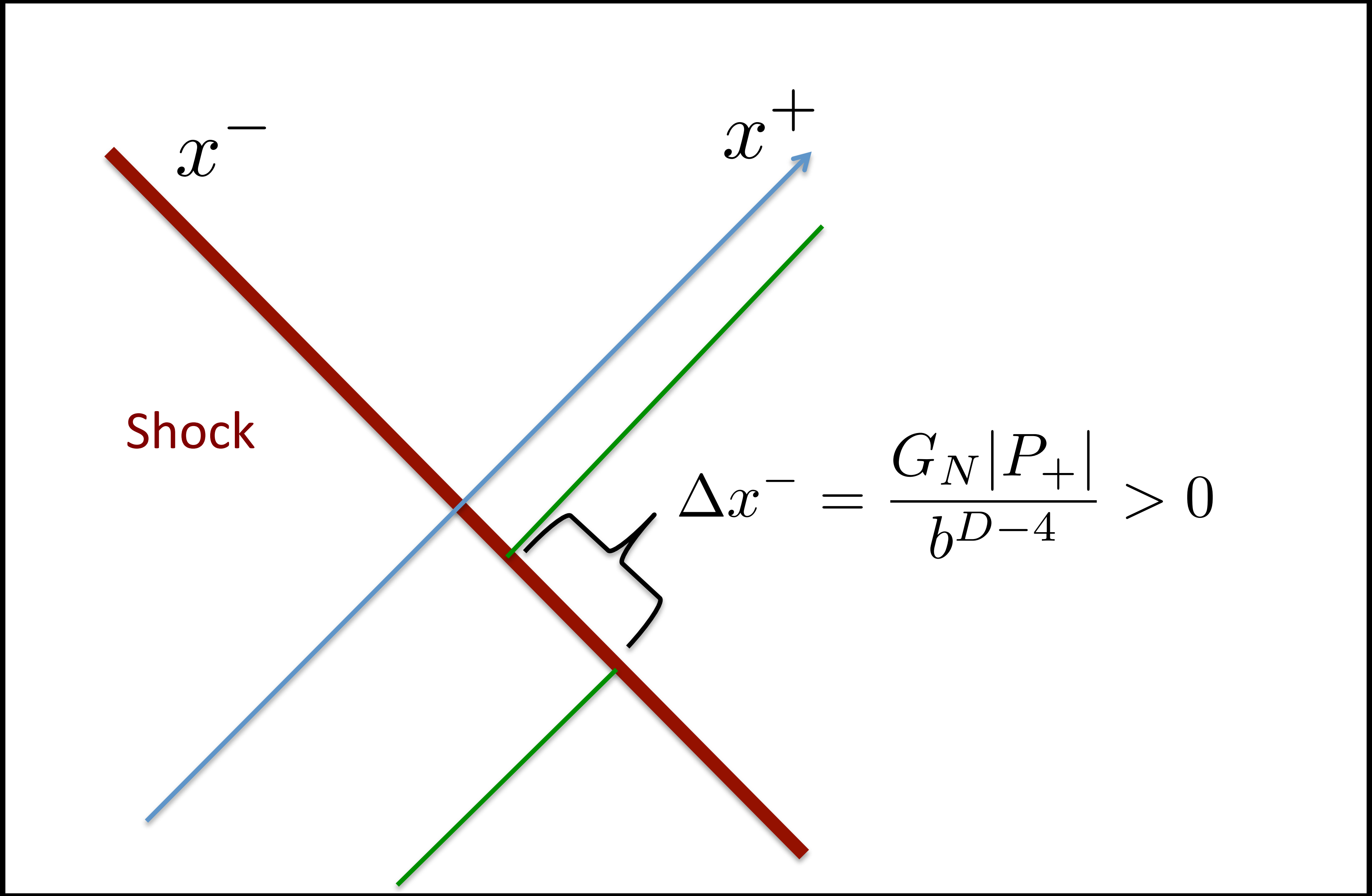
$$ds^2 = -du dv + h(u, x_i)\, du^2 + \sum_{i=1}^{D-2} (dx_i)^2$$

$$T_{uu} = -P_u \delta(u) \delta^{D-2}(\vec{x})$$

$$h(u, x_i) = \frac{4\Gamma(\frac{D-4}{2})}{\pi^{\frac{D-4}{2}}} \delta(u) \frac{G|P_u|}{r^{D-4}}$$

$$v = v_{\text{new}} + \frac{4\Gamma(\frac{D-4}{2})}{\pi^{\frac{D-4}{2}}} \frac{G|P_u|}{b^{D-4}} \theta(u)$$

$$\Delta v = v_{\text{After crossing}} - v_{\text{Before crossing}} = \frac{4\Gamma(\frac{D-4}{2})}{\pi^{\frac{D-4}{2}}} \frac{G|P_u|}{b^{D-4}} > 0$$



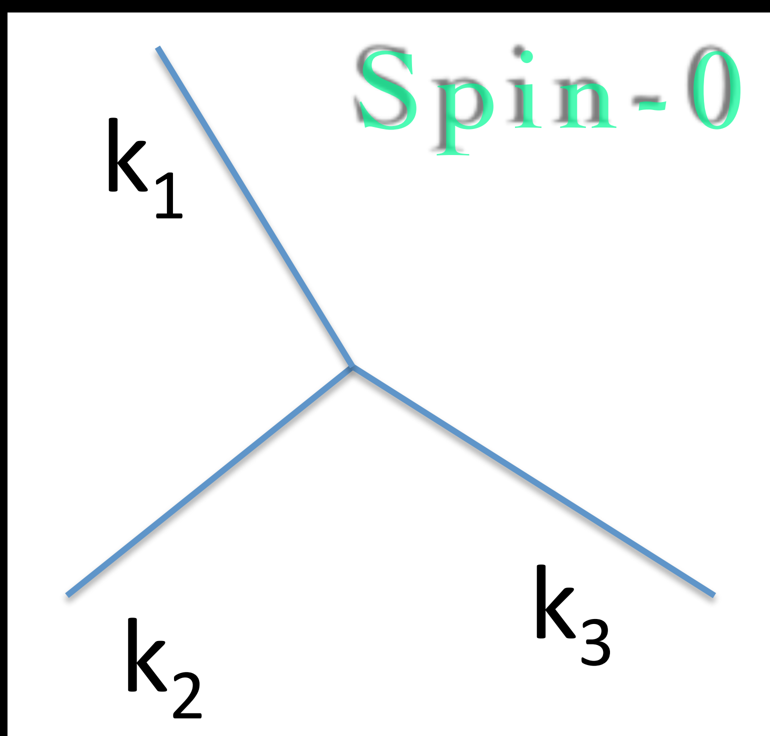
Gravity always slows you down: we will impose this as a defining principle for any theory of gravity

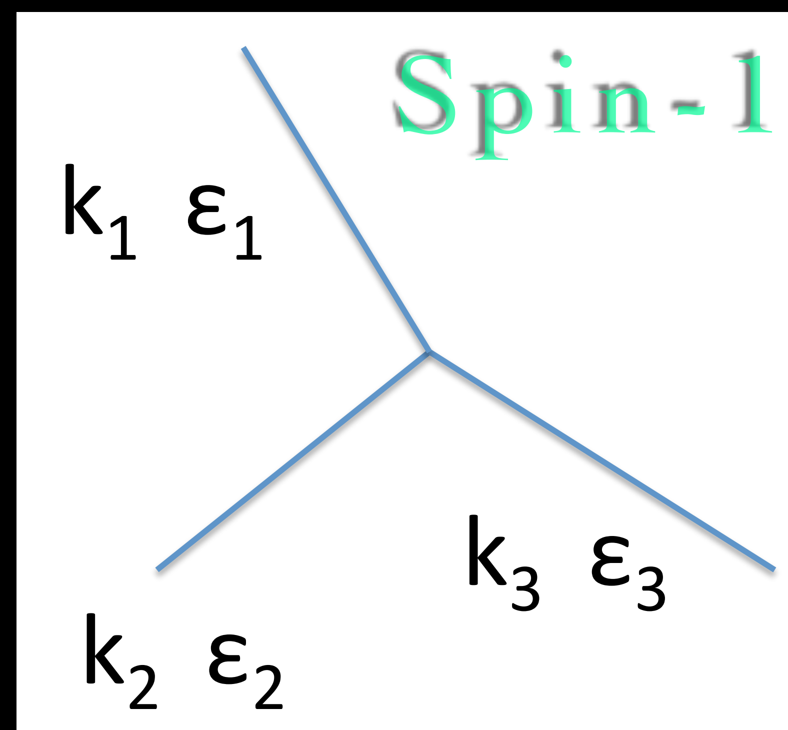
Consider asymptotically flat geometries: scattering amplitudes

Let's analyze 3-point vertices

No kinematic invariants:

$$(k_1 + k_2)^2 = k_3^2 = 0$$





$$\epsilon_i \cdot k_i = 0$$

$$\epsilon_i \approx \epsilon_i + k_i$$

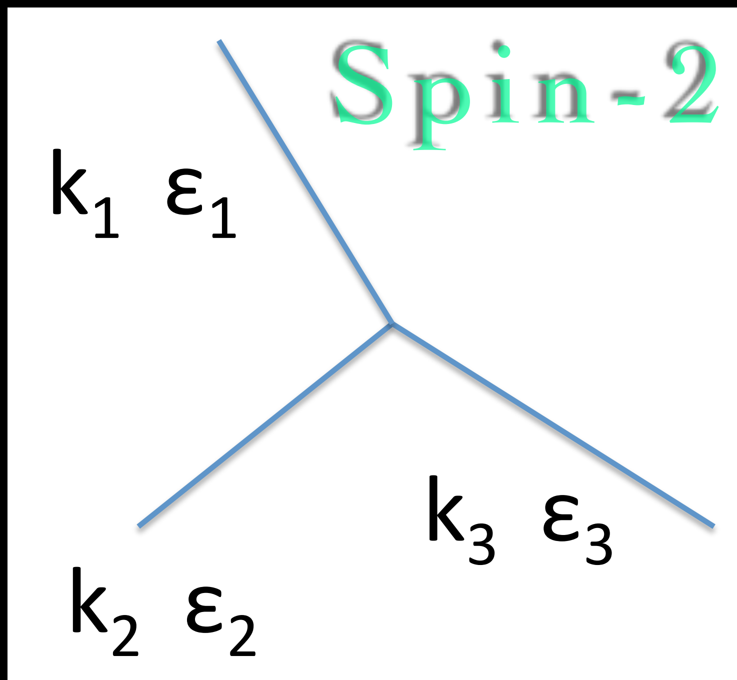
Single coupling

$$\sqrt{G}$$

$$\mathcal{A}_0 = \epsilon_1 \cdot \epsilon_2 \; \epsilon_3 \cdot k_1 + \text{cyclic}$$

(Yang-Mills)

$$\mathcal{A}_2 = \epsilon_1 \cdot k_2 \; \epsilon_2 \cdot k_3 \; \epsilon_3 \cdot k_1$$



$$\epsilon_{\mu\nu} = \epsilon_\mu \epsilon_\nu$$

There are three possible structures:

$$\mathcal{G}_0 = \mathcal{A}_0\, \mathcal{A}_0 \qquad \frac{1}{G_\mathrm{N}} \int d^4x\ R$$

$$\mathcal{G}_2 = \mathcal{A}_0\, \mathcal{A}_2 \qquad \frac{\alpha_2}{G_\mathrm{N}} \int d^4x\ R^2_{\mu\nu\lambda\sigma}$$

absent for D=4 or maximal SUSY

$$\mathcal{G}_4 = \mathcal{A}_2\, \mathcal{A}_2 \qquad \frac{\alpha_4^2}{G_\mathrm{N}} \int d^4x\ R^3_{\mu\nu\lambda\sigma}$$

absent for SUSY

Claim: The latter two structures lead to causality violation

In the quadratic theory, the time delay depends on the helicity

$$\alpha_2 = \lambda \ell_s^2$$

Tensor channel $\qquad h_{ij} b^j = 0$

$$\Delta v = \left[1 - \lambda (D-4) \frac{\ell_s^2}{b^2}\right] \frac{G_N |P_u|}{b^{D-4}}$$

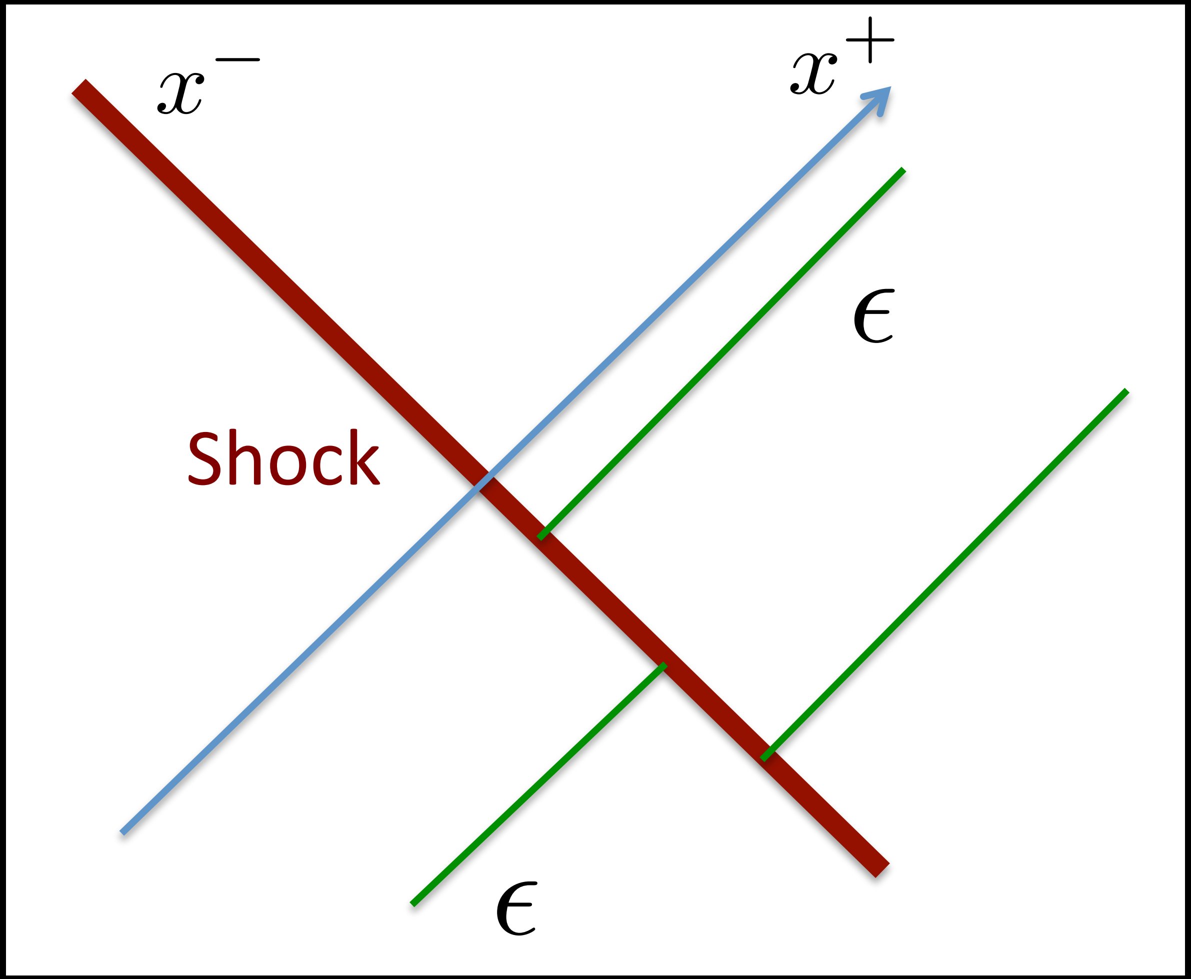
Vector channel $\qquad h_{ij} b^i b^j = 0$

$$\Delta v = \left[1 + \lambda (D-4)^2 \frac{\ell_s^2}{b^2}\right] \frac{G_N |P_u|}{b^{D-4}}$$

Scalar channel $\qquad h_{ij} b^i b^j \neq 0$

$$\Delta v = \left[1 + 2\lambda (D-4)^2 \frac{\ell_s^2}{b^2}\right] \frac{G_N |P_u|}{b^{D-4}}$$

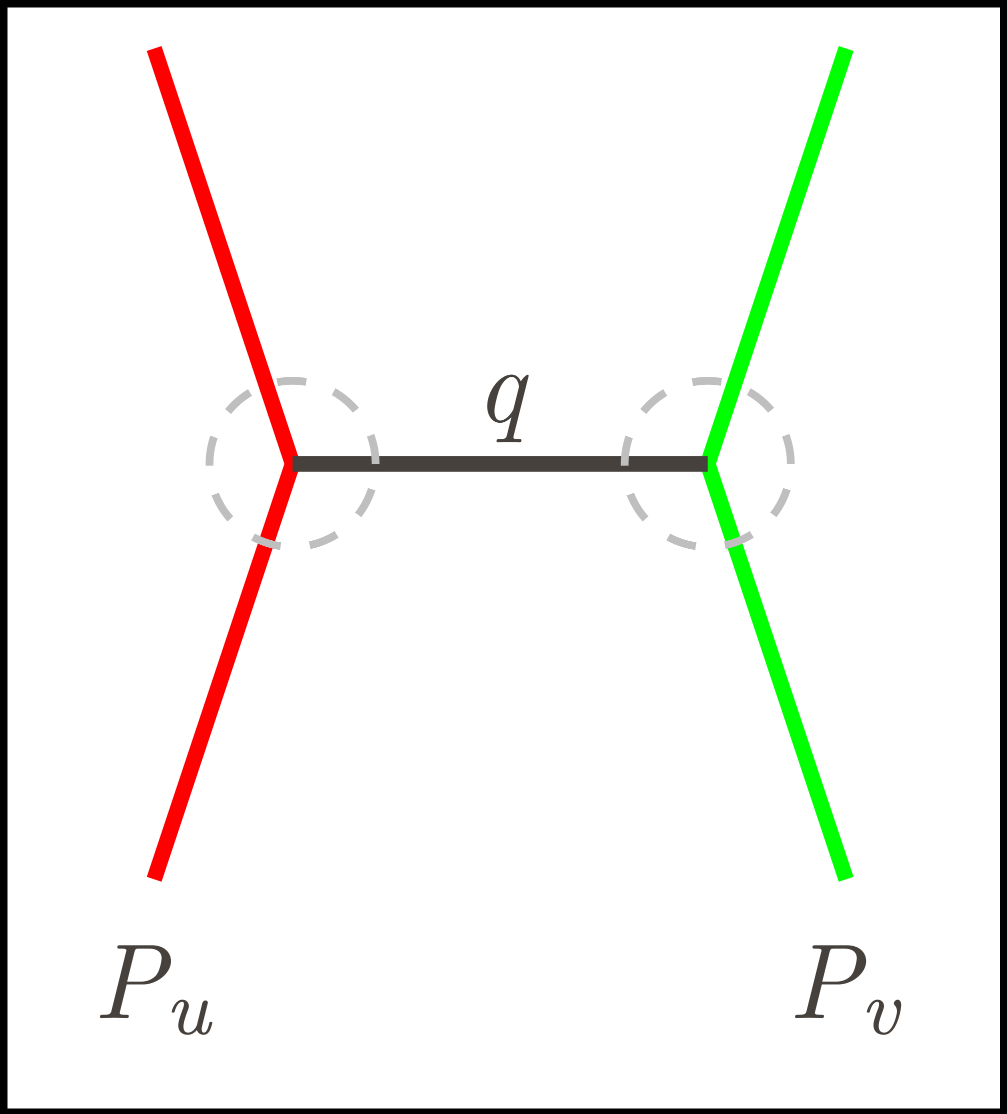
When higher dervatives are included



$$\Delta v = \left(1 \pm \frac{\alpha_2}{b^2} \pm \frac{\alpha_4^2}{b^4}\right) \frac{G_N |P_u|}{b^{D-4}}$$

Propagation faster than light as seen from infinity, violating asymptotic causality

Equivalent description in terms of tree-level 4-point amplitudes



Kinematic regime:

Forward or Eikonal limit

- s very large but small enough $\ell_s^{-2} \ll \ell_{\mathrm{Pl}}^{-2}$

$$s \simeq P_u P_v \equiv P_+ P_-$$

- $t/s \ll 1$
- fixed b

Eikonal phase shift (impact parameter representation)

$$\delta(s, \mathbf{b}) = \frac{1}{2s} \int \frac{d^{D-2}\mathbf{q}}{(2\pi)^{D-2}} \, e^{i\mathbf{b}\cdot\mathbf{q}} \mathcal{A}^{[4]}_{tree}(s, t \simeq -\mathbf{q}^2)$$

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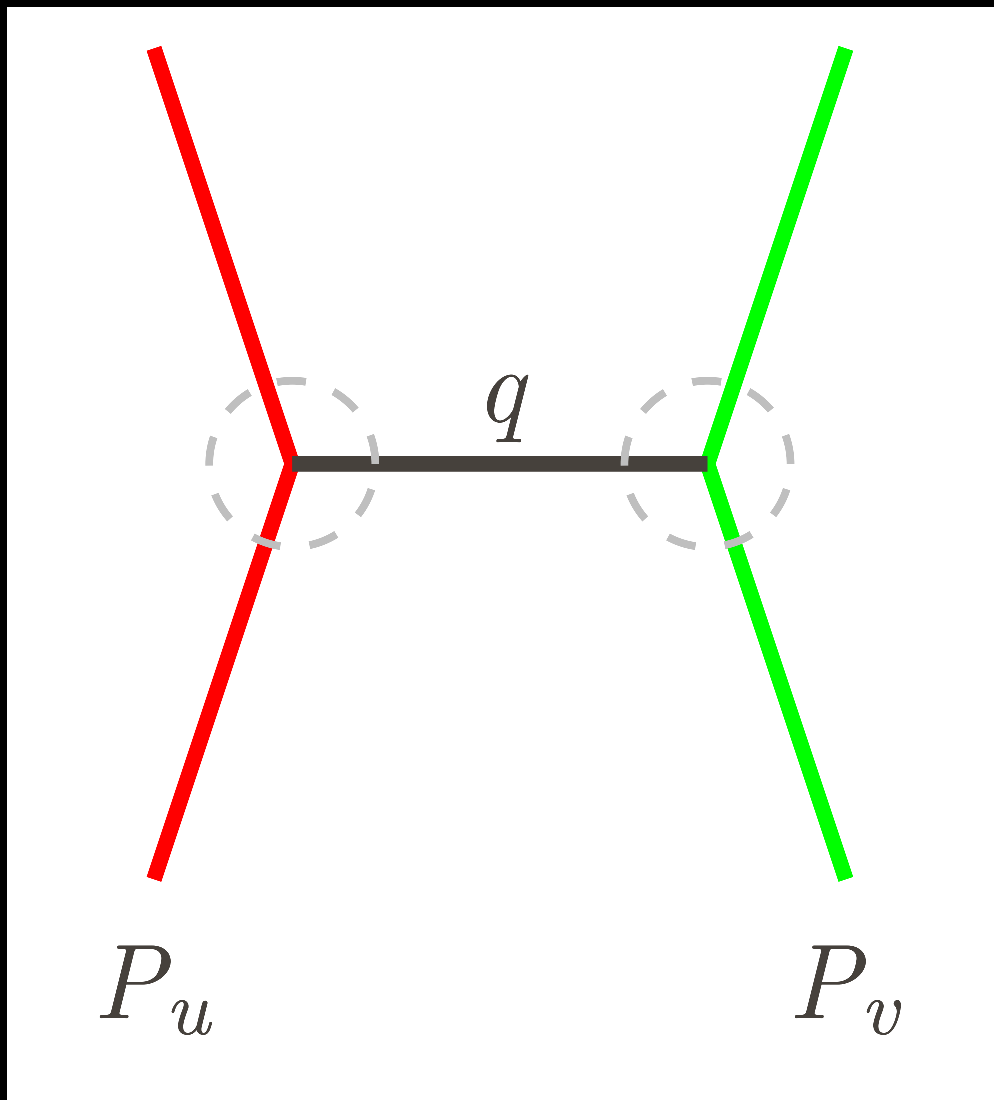

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Eikonal phase shift (impact parameter representation)

$$\delta(s, \mathbf{b}) \sim \frac{1}{2s} \sum_I \mathcal{A}^{(3)}_{13I}(-i\partial_{\mathbf{b}}) \mathcal{A}^{(3)}_{I24}(-i\partial_{\mathbf{b}}) \log \mathbf{b}^2$$

If the particle along the t-channel is a massless graviton

$$\delta(s, \mathbf{b}) \sim -P_v \Delta v \qquad \text{(scales like } s\text{)}$$

time advance being encoded in the phase shift

The effect accumulates and builds up into a macroscopic effect

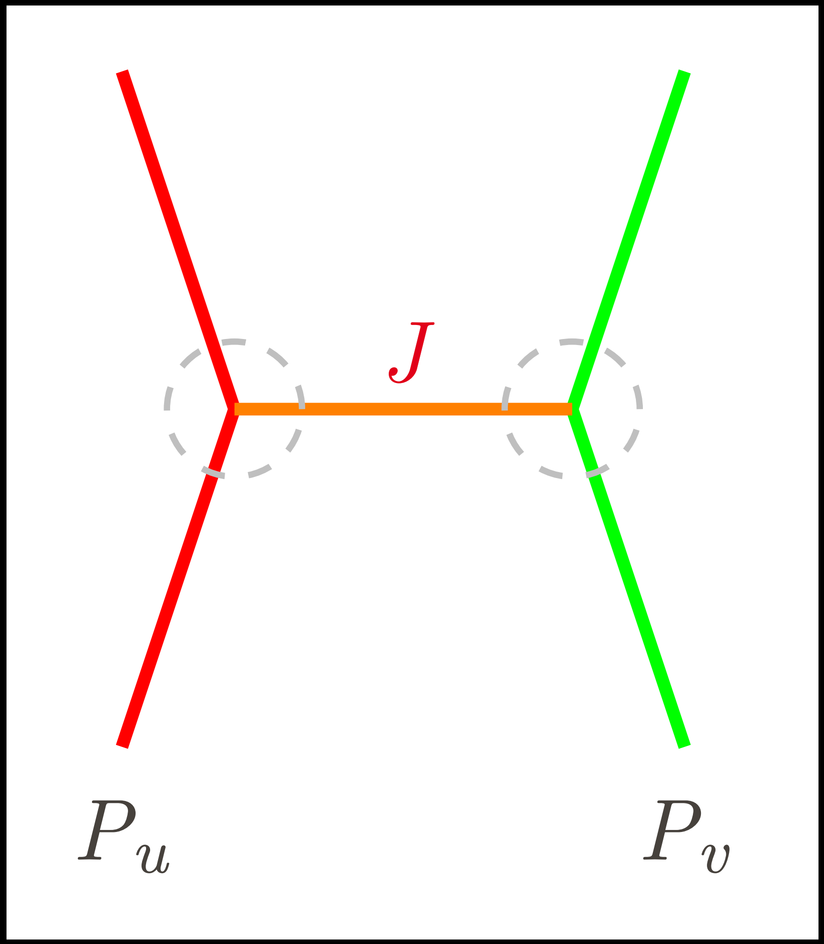
$$S(s, \mathbf{b}) \simeq e^{i\delta(s,\mathbf{b})}$$

Analytic & Bounded $|e^{i\delta(s,\mathbf{b})}| \leq 1$
in upper half plane

New degrees of freedom must appear at energies

$$E \sim \ell_{\mathrm{s}}^{-1}$$

Particles with spin J contribute like $\delta(s, \mathbf{b}) \sim s^{J-1}$



Spin 0 or 1 particles do not cure the problem

Massive spin 2 neither: no KK fixing!

Finite number of higher spin entails non-unitarity

Need an infinite number of higher spin particles $m_J \lesssim \ell_s^{-1}$ with delicate cancellations...

...which works in string theory!

D'Appollonio, Di Vecchia, Russo, Veneziano, 2015

What about the case of asymptotically dS geometries?

If primordial B-mode gravity waves are detected with $r \sim 0.12$ during inflation: $H \sim 1.94\ 10^{16}\mathrm{GeV}$

Gravitons' characteristic energies at inflation are $E \sim H$

They are weakly coupled: since the 2-point function is a direct measurement of the coupling

$$\langle hh \rangle \sim \frac{H^2}{M_{\mathrm{Pl}}^2} \sim g_{\mathrm{eff}}^2 \leq 10^{-6}$$

They can collide: signatures of the graviton 3-point vertex

Maldacena, 2003 Maldacena, Pimentel, 2011

$$\frac{\langle hhh \rangle_{R^3}}{\langle hhh \rangle_{\mathrm{E}}} \approx \alpha_4^2 H^4 \qquad\qquad \frac{\langle hhh \rangle}{\langle hh \rangle^{3/2}} \sim \frac{H}{M_{\mathrm{Pl}}}\left[F_{\mathrm{E}} + \alpha_4^2 H^4 F_2\right]$$

Non-Gaussianities in the spectrum of primordial fluctuations

Concluding remarks

Deviations of the graviton 3-point vertex from the Einstein value signal the existence of new higher spin particles

$$M_{\text{h.s.}}^2 \sim \frac{1}{\alpha_{2,4}} \sim \ell_{\text{s}}^{-2}$$

On-shell discussion: independent of field redefinition ambiguity

If such deviations were seen in the CMB then a structure like string theory must have existed during inflation

In AdS this appears as non-positivity of certain energy correlators

Camanho, Edelstein, Zhiboedov, in progress

Camanho, upcoming talk

Tighter constraints on CFTs having weakly coupled gravity duals