Anomalies, Chern-Simons Terms, and Black Hole Entropy

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Based on arXiv:1311.2940, 1407.6364, 1505.02816

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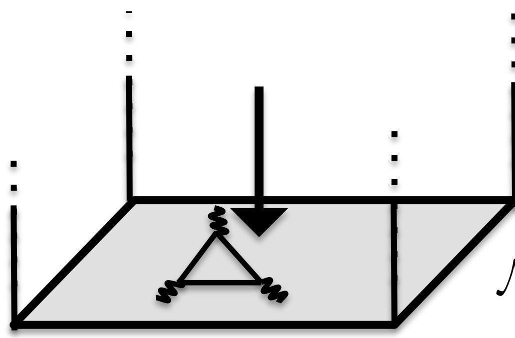
Iberian String 2015 @ University of Salamanca, Salamanca, May 29th, 2015

Anomalies at Zero Temperature

Beautiful mathematical structure associated with anomalies
(we consider Global Anomalies in Even-dim. QFT)

Anomaly Inflow Mechanism [Callan-Harvey]



Anomalies are classified by Anomaly Polynomials

$$\mathbf{P}_{anom}(\mathbf{F} = d\mathbf{A}, \mathbf{R} = d\mathbf{\Gamma} + \mathbf{\Gamma}\wedge\mathbf{\Gamma}) \equiv d\mathbf{I}_{CS}$$

<u>example</u>

	2d gravitational anomaly	4d mixed anomaly
CS term	$tr\left(\mathbf{\Gamma}\wedge\mathbf{R} - \frac{1}{3}\mathbf{\Gamma}\wedge\mathbf{\Gamma}\wedge\mathbf{\Gamma}\right)$	$\mathbf{A}\wedge tr(\mathbf{R}\wedge\mathbf{R})$
Anomaly Polynomial	$tr(\mathbf{R}\wedge\mathbf{R})$	$\mathbf{F}\wedge tr(\mathbf{R}\wedge\mathbf{R})$

Anomalies at Finite Temperature

Anomaly-Induced Transport

[Son-Surowka, Bhattacharyya et.al. Erdmenger et.al., Torabian-Yee, …]

In hydrodynamic limit, anomalies generate new transports

Replacement Rule

[Loganayagam, Loganayagam-Surowka Jensen-Logayanagam-Yarom, …]

Anomaly-induced transport coefficients are determined by anomaly polynomial

Stress-Energy Tensor $$(T_{\alpha\beta})_{anom} = -n\mathfrak{F}[\mu, T](u_\alpha V_\beta + u_\beta V_\alpha) + \cdots$$

U(1) current $$(J_\alpha)_{anom} = -\frac{\partial \mathfrak{F}}{\partial \mu} V_\alpha + \cdots$$

Entropy current $$(J^S_\alpha)_{anom} = -\frac{\partial \mathfrak{F}}{\partial T} V_\alpha + \cdots$$

with
$$V^\mu = \epsilon^{\mu\nu\rho_1\cdots\rho_{2n-2}} u_\nu (\partial_{\rho_1} u_{\rho_2}) \cdots (\partial_{\rho_{2n-3}} u_{\rho_{2n-2}})$$
$$\mathfrak{F}[T, \mu] = \mathbf{P}_{anom}(\mathbf{F} \to \mu, tr(\mathbf{R}^{2k}) \to 2(2\pi T)^{2k})$$

Anomaly polynomials classify anomaly-induced transports

Gauge/Gravity Duality

Setup on CFT Side

Fluid with non-trivial anomaly-induced transports

→ U(1) charged rotating (conformal) fluid in 2n-dim.

Setup on Gravity Side

Theory

- (2n+1)-d Einstein-Maxwell-Chern-Simons theory with negative cosmological const.
- CS Terms: U(1), gravitational, mixed

→ Same as anomaly inflow mechanism

Configuration

U(1) charged rotating black hole (BH) on AdS_{2n+1}

Cf. some quantities in 5d case
[Landsteiner-Megias-Melgar-Pena-Benitez, Karzheev-Yee,…]

Why Gravity Side ?

1. Proof of replacement rule in QFT is formal

2. Interesting properties of CS terms :
   - Higher derivative terms
   - Non-covariant under diffeo and gauge trans.

$$\delta_\chi \mathbf{I}_{CS} = \mathcal{L}_\xi \mathbf{I}_{CS} + d(...) \qquad \chi = \{\xi : \text{diffeo}, \Lambda : \text{gauge}\}$$

while EoM is covariant

3. Basics are not well-understood :
   - Boundary stress tensor for general CS terms?
   - BH entropy with general CS term?
   - Charged rotating AdS BH in higher dim.?

Our Work : Anomaly Polynomial Plays Crucial Roles!

Implication to BH Microstates

<u>“Traditional” Microstate Counting for BH Entropy</u>

Map to CFT_2 entropy counting problem → Cardy Formula

(example) BTZ BH, (near) extremal BHs

<u>BH in higher-dim. AdS spacetime</u>

- Dual higher-dim. CFT does not have neither infinite dimensional symmetry nor modular invariance

  → Hard to compute entropy fully from higher dim. CFT

cf. supersymmetric cases in 4d and 6d [Komargodski et.al.]

<u>If Chern-Simons BH entropy = Replacement Rule …</u>

With replacement rule, we can compute anomaly-induced part of entropy for **higher-dim finite temp. BH from CFT!**

"BH Entropy is Noether Charge"

Wald Entropy Formula

[Wald, Lee-Wald, Iyer-Wald],
[Kang-Jacobson-Myers]

- BH entropy formula for general diff. invariant Lagrangian

($\delta_\chi L_{cov} = \mathcal{L}_\xi L_{cov} \quad \text{(example)} \; R_{abcd}R^{abcd}$)

$$S_{\text{Wald}} = 2\pi \int_H \epsilon_{ab}\epsilon_{cd} \frac{\delta L_{cov}}{\delta R_{abcd}}$$

Differential Noether Charge

Killing vector → Conserved charge

$\xi = \partial_t + \Omega_H \partial_\phi \quad \rightarrow \quad d\not{\delta}\mathbf{Q}_{\text{Noether}} = 0$

$\not{\delta}(\ldots)$: linear combination of $\delta(\text{something})$

$$\rightarrow \underbrace{\int_\infty \not{\delta}\mathbf{Q}_{\text{Noether}}}_{\delta M + \Omega_H \delta J} = \underbrace{\int_H \not{\delta}\mathbf{Q}_{\text{Noether}}}_{T_H \delta S}$$

A Sketch of Derivation

Some Steps of Noether Procedure

Point 1 : Variation of Lagrangian

$$\delta \mathbf{L}(\phi) = \not{\delta}\mathbf{E} + d(\not{\delta}\mathbf{\Theta}) \qquad \not{\delta}(\ldots) : \text{linear combination of } \delta(\text{something})$$

Point 2 : Pre-symplectic current $\mathbf{\Omega}$

$$d(\mathbf{\Omega}(\delta_1\phi, \delta_2\phi)) = \delta_1(\not{\delta}_2\mathbf{E}) - \delta_2(\not{\delta}_1\mathbf{E})$$

$$\vdots$$

Point 3 : Differential Noether charge

$$d(\not{\delta}\mathbf{Q}_{Noether}) = \mathbf{\Omega}(\delta\phi, \delta_\chi\phi) + (\mathrm{on-shell\ vanishing\ terms})$$

Key Point of Wald Formalism

[Wald, Lee-Wald, Iyer-Wald]

$$d(\mathbf{\Omega}(\delta_1\phi, \delta_2\phi)) = \delta_1(\not{\delta}_2\mathbf{E}) - \delta_2(\not{\delta}_1\mathbf{E}) = d(-\delta_1(\not{\delta}_2\mathbf{\Theta}) + \delta_2(\not{\delta}_1\mathbf{\Theta}))$$

"Lagrangian-Based Prescription"

Extension to CS Term

<u>Generalization to CS Term</u>

- Minimum modification of Wald formalism to take into account

$$\delta_\chi \mathbf{I}_{CS} = \mathcal{L}_\xi \mathbf{I}_{CS} + \underline{d(\ldots)}$$

[Tachikawa]

- In 5d and higher, an appropriate coordinate & gauge need to be taken to get desirable results … ?????????

[Bonora et. al.]

<u>Origin of Non-Covariance</u>

$$\not\delta \mathbf{\Theta} = \delta \mathbf{\Gamma}^b{}_a \underline{\left(\frac{\partial \mathbf{I}_{CS}}{\partial \mathbf{R}^a{}_b} \right)} + \delta \mathbf{A} \underline{\left(\frac{\partial \mathbf{I}_{CS}}{\partial \mathbf{F}} \right)} + \cdots \longrightarrow$$

Non-covariant

$\mathbf{\Omega}$ $\quad \not\delta \mathbf{Q}_{Noether}$

Chern-Simons BH Entropy

Manifestly Covariant Formalism

CS contribution to EoM $\sim$ derivatives of anomaly polynomials

$\longrightarrow$ $d(\mathbf{\Omega}(\delta_1\phi, \delta_2\phi)) = \delta_1(\not{\oint}_2\mathbf{E}) - \delta_2(\not{\oint}_1\mathbf{E})$

$\longrightarrow$ $\mathbf{\Omega}$ $\oint\mathbf{Q}_{Noether}$ $\sim$ derivatives of anomaly polynomial

$\longrightarrow$ Covariant !

"EoM-Based Prescription"

BH Entropy Formula for CS Term

For general anomaly polynomial $\mathbf{P}_{anom}(\mathbf{F}, \mathbf{R}) = \mathbf{F}^l \wedge \prod_i \mathrm{tr}\mathbf{R}^{2k_i}$

$$S_{CS} = \int_H \sum_{k=1}^{\infty} 8\pi k \mathbf{\Gamma}_N (d\mathbf{\Gamma}_N)^{2k-2} \frac{\partial \mathbf{P}_{anom}}{\partial \mathrm{tr}\mathbf{R}^{2k}} \qquad \mathbf{\Gamma}_N = \frac{1}{2}\epsilon_a{}^b \mathbf{\Gamma}^b{}_a$$

(Generalized) Tachikawa's Proposal Proved Covariantly!

Gravity Dual of Anomalous Fluid

- Fluid/gravity solution for charged rotation AdS BH in Einstein-Maxwell theory (up to 2nd order)
- Back reaction due to CS term

$$ds^2 = -2u_\mu dx^\mu dr + r^2[-f(r,m,q)u_\mu u_\nu + P_{\mu\nu}]dx^\mu dx^\nu + \cdots$$
$$+g_V(r,m,q)(u_\mu V_\nu + u_\nu V_\mu)dx^\mu dx^\nu + \cdots$$

- Evaluation of diff. Noether charge at horizon & boundary
  → Replacement rule reproduced!

…but anomaly-induced transports are higher-order:

$$V^\mu = \epsilon^{\mu\nu\rho_1\cdots\rho_{2n-2}} u_\nu (\partial_{\rho_1} u_{\rho_2}) \cdots (\partial_{\rho_{2n-3}} u_{\rho_{2n-2}})$$

Is 2nd order enough?
→ "Non-renormalization" proved for **any odd dim.** by using CS diff. Noether charge ∼ deriv. of anomaly polynomial !

Summary

Anomaly Polynomials Play Crucial Roles!

CS Terms are Nice Class of Higher-Derivative Terms!

- Manifestly covariant diff. Noether charge for CS term
  $\to$ BH entropy formula for CS terms

- Systematic study of holography for anomalies in the hydrodynamics limit.
  $\to$ Replacement rule reproduced