Hydrodynamics of p-wave Superconductors

(with I. Salazar on arXiv:1409.6357, JHEP11 (2014) 047)

Raúl E. Arias

Centro Atómico Bariloche
Argentina

Iberian Strings 2015

Outline

- Motivation
- Relativistic Hydrodynamics
- p-wave Holographic Superconductors
- Results
- Conclusions and Future

Motivation

- Hydrodynamics modes of p-wave superconductor at strong coupling
- Second sound
- Universal late time behavior — Gauntlett et. al '13
  - Described by poles of G_R that lie closest to the real axis

Hydrodynamics

- Formulated in terms of few relevant fields, their eoms and constitutive relations

- Equation of motions $\leftrightarrow$ Continuity equations

$$
\begin{aligned}
\partial_\mu T^{\mu\nu} &= 0, \\
\partial_\mu J^\mu &= 0
\end{aligned}
$$

- Constitutive equations relate the currents with the characteristic parameters (T, u^ν, μ)

- The key equation

$$\delta < A(t,\mathbf{x}) >= -i\int_{-\infty}^{t} dt' < [A(t',\mathbf{x}),\delta H(t')] >$$

- If perturbation is $\delta H(t) = \int d^d x j(t,\mathbf{x}) O(\mathbf{x})$ it reduces to the convolution of the source with the retarded two point Green function

$$\delta < A(t,\mathbf{x}) >= -\int dt' d^d x' G_R(t-t',\mathbf{x}-\mathbf{x}') j(t',\mathbf{x}')$$

- Poles of the retarded function give the dispersion relation
- Retarded Green functions can be computed using holography
- Quasinormal frequencies can be interpreted as poles of G^R Starinets '02

- Minimal Lagrangian ⇒ Einstein-Maxwell ($SU(2)$) Gubser '08

$$\kappa^2 \mathcal{L} = R - 2\Lambda - \frac{1}{4} Tr(F_{\mu\nu} F^{\mu\nu})$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_{ym} \epsilon^{abc} A^b_\mu A^c_\nu$$

- Plan: look for a solution for A^a_μ that break a $U(1)$ subgroup of the gauge symmetry ⇔ A^a_μ condense outside the horizon.

- Conceptually, a charged superconducting layer develops outside the horizon due to the interplay electric repulsion vs gravitational potential

- Ansatz in the probe limit ($\frac{g_{ym}}{\kappa} \to \infty$)

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2), \qquad f(r) = \frac{r^2}{L^2} - \frac{M}{r}$$

$$A = \phi(r)\tau^3 dt + \omega(r)\tau^1 dx$$

- $\omega \neq 0 \Rightarrow$ breaks the $U(1)$ gauge symm. associated to rotations around τ^3.
- We must write the eom for the Maxwell field and see the behavior at $r \to r_h, \infty$

$$\begin{cases} \phi = \mu + \frac{\rho}{r}, & r \to \infty; \\ \phi = \phi_1^h(r - r_h), & r \to r_h. \end{cases} , \qquad \begin{cases} \omega = \frac{W_1^b}{r}, & r \to \infty; \\ \omega = \omega_0^h + \omega_2^h(r - r_h)^2, & r \to r_h. \end{cases}$$

- Note that $W_0^b = 0 \Leftrightarrow$ no source for J_x^1 in the field theory $\Leftrightarrow$ SSB.

- We are going to study (holographically) the hydrodynamics modes that comes from $\partial_\mu j^\mu = 0$ in the field theory
- The gauge field fluctuations

$$A^a_\mu \to A^a_\mu + \delta A^a_i, \qquad i = t, x, y$$

$$\delta A^a_\mu = e^{ik_i x^i - i\omega t} a^a_\mu(r)$$

- Different choices: $k_y = 0, k_x \neq 0$ (longitudinal) or $k_x = 0, k_y \neq 0$ (transversal)
- First we are going to study the unbroken phase to count the number of modes and then we will analyze the broken phase

Hydrodynamic modes

Unbroken phase $w(r) = 0$

- Transverse and parallel modes coincide.
- The $a^{1,2}$ sector gives the Goldstone mode (sound modes when $T < T_c$) and Diffusive + pseudo-diffusive modes.
- The a^3 sector gives one more diffusive mode.
- Now we know that we should observe five modes in the broken phase.

Hydrodynamic modes

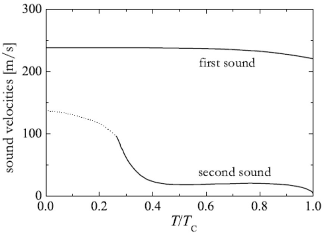
k_x direction

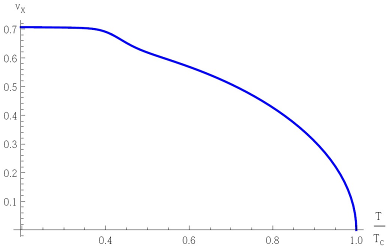
Broken phase

- 6 coupled equations involving a_x, a_t. We found three modes, two sound modes and one diffusive

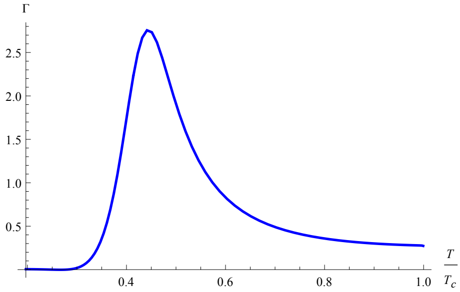
$$\omega(k_x) = \pm v_s k_x - i\Gamma k_x^2$$

- The second sound velocity





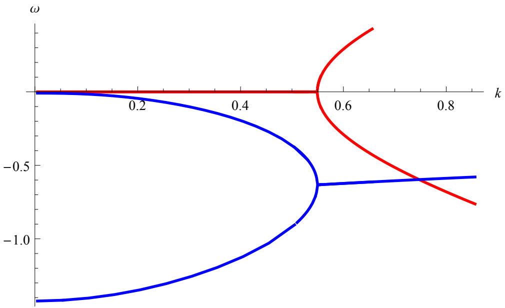
- The second sound attenuation



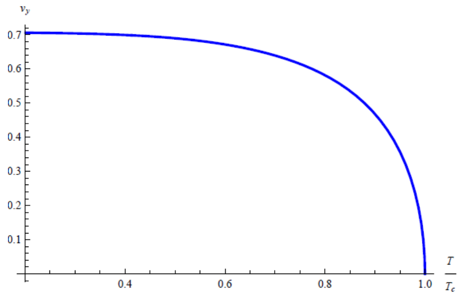
Hydrodynamic modes
k_x direction
Broken phase

- The three coupled equations for a_y give a diffusive mode

$$\omega(k_x) = -iDk_x^2 - i\gamma_x(T)$$

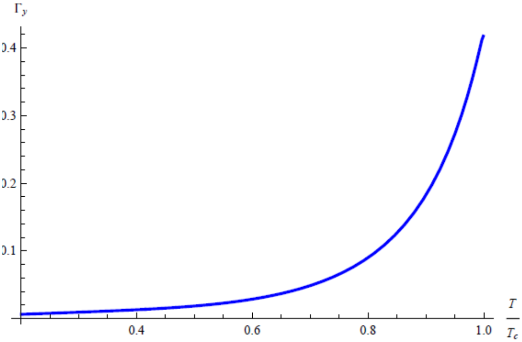


- There are 4 equations coupling $a_x^{1,2}, a_t^3, a_y^3$ that give the sound and diffusive modes
- The second sound velocity



Hydrodynamic modes
k_y direction
Broken phase

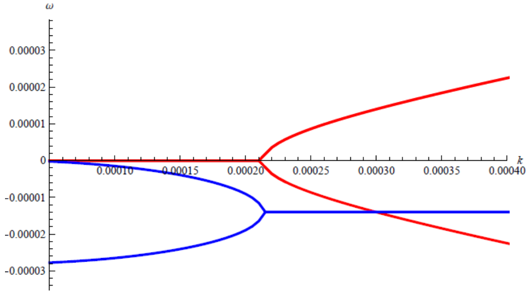
- The second sound attenuation



Hydrodynamic modes
k_y direction
Broken phase

- The remaining equations gives a diffusive and a pseudo-diffusive modes

$$\omega(k_y) = -iDk_y^2 - i\gamma_y(T)$$



Summary

- We compute the poles of the retarded Green function i.e the dispersion spectrum for the hydrodynamic behavior of a $p-$ wave superconductor at strong coupling
- It was analyzed the behavior of transversal and longitudinal hydro modes as function of T
- Future: Brane configurations, non-relativistic holographic SC, alternative quantization.