Finite time calculations for hard parton production relevant to the QGP

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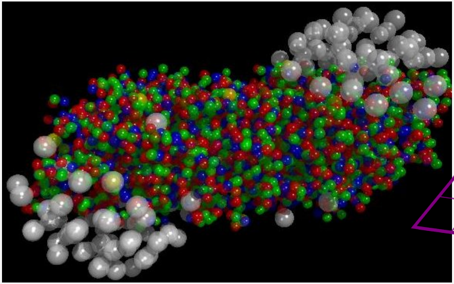
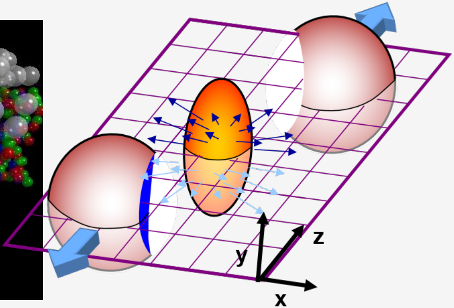
SA CERN

What is the Quark Gluon Plasma?

- A new Fundamental state of matter, formed in heavy ion collisions
- We think it behaves like a strongly coupled fluid





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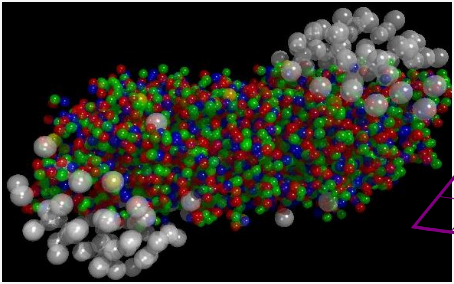

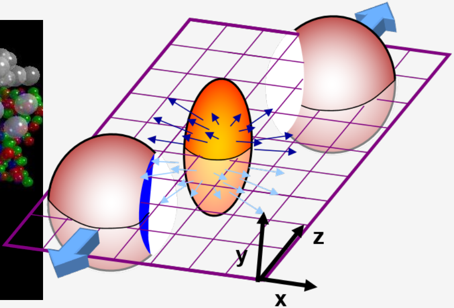

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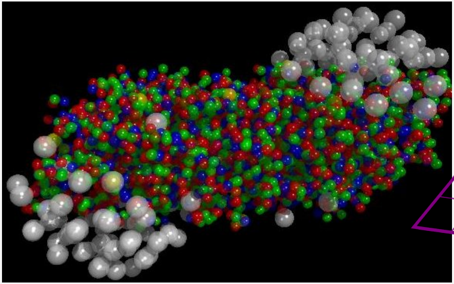

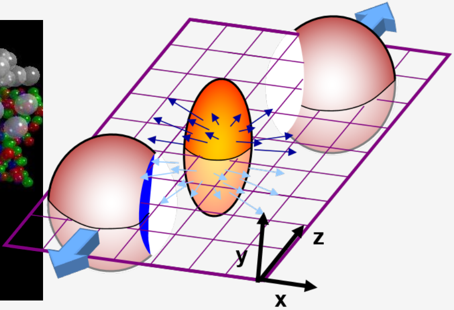

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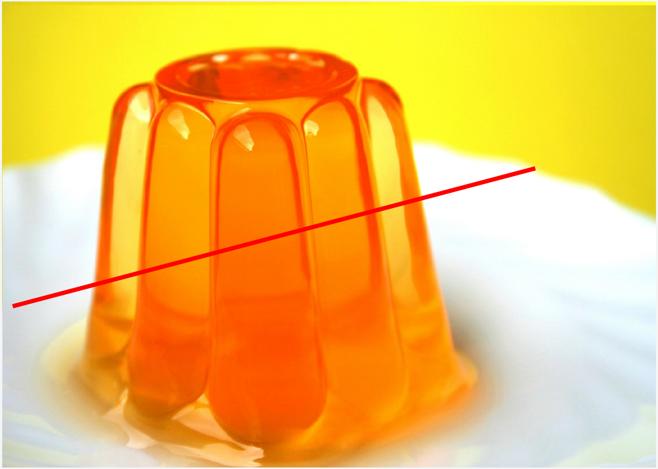

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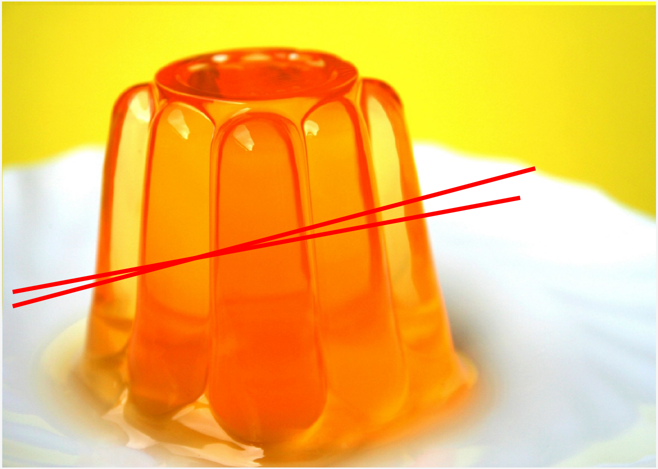

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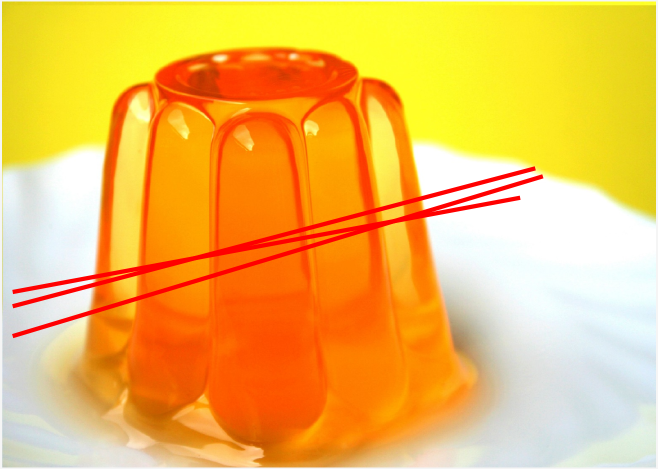

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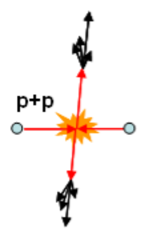

Figure: Proton-proton collision and resulting Jet

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The expectation value of the ϕ field, given an initial state of a single ψ particle at $t = -\infty$.

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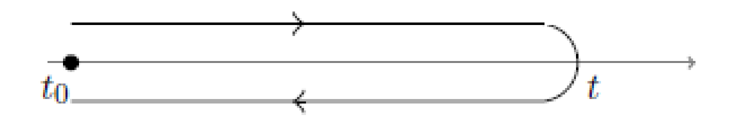

Figure: Schwinger-Keldysh Contour

Toy Problem: Find $\langle \phi(x) \rangle$ emitted from a static "quark" in scalar QCD

$$\boxed{\langle \phi(x) \rangle = -ig \int d^4z\, D_R(x-z) \langle \mathsf{in} | \psi(z) \psi(z) | \mathsf{in} \rangle}$$

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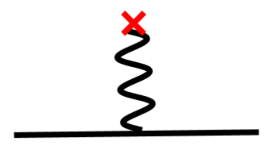

Figure: Leading order diagram for $\langle \phi(x) \rangle$

$$\mathcal{L} = \frac{1}{2}(\partial\psi)^2 - \frac{1}{2}m_\psi^2\psi^2 + \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 - g\psi\phi\psi$$

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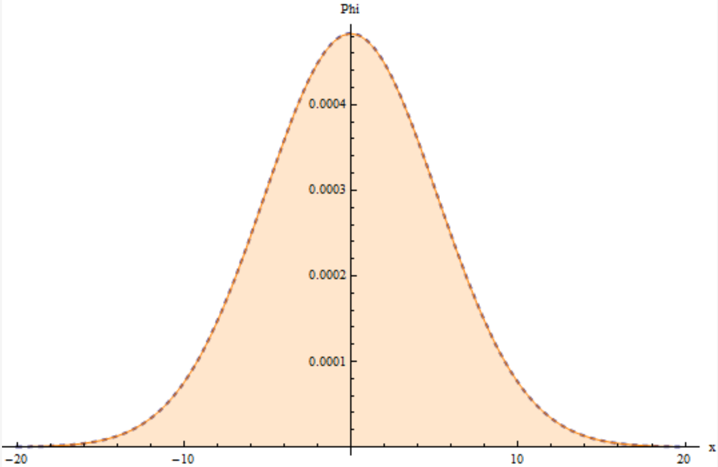

Figure: Large mass analytic and numerical results for $\langle\phi(x)\rangle$ in time (in 1+1 D)

Toy Problem: Find $\langle \phi(x) \rangle$ emitted from a moving "quark" in scalar QCD

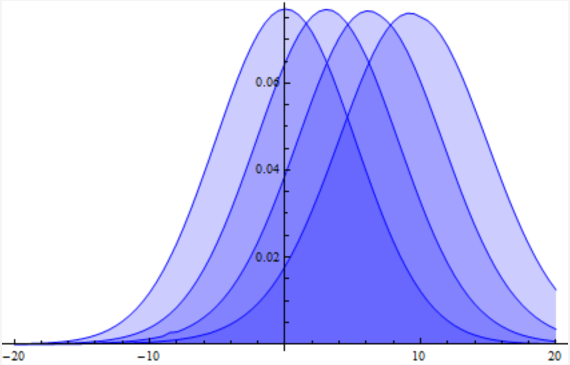

Figure: Evolution of $\langle \phi(x) \rangle$ in time (in 1+1 D)

Can we calculate $\langle T_{\mu\nu} \rangle$? Are there complications?

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Figure: Leading order diagrams for $\langle T_{\mu\nu} \rangle$

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu}(\tfrac{1}{2}\left(\partial\phi\right)^2 - \tfrac{1}{2} m_\phi^2 \phi^2 - \lambda \phi^4)$$

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When we tally these up, the answer is ∞!

Can we calculate $\langle T_{\mu\nu} \rangle$? Are there complications?

We need to include an improvement term

$$\Theta_{\mu\nu} = T_{\mu\nu} - \frac{1}{4}\frac{(n-2)}{(n-1)}\left(\partial_\mu \partial_\nu - g_{\mu\nu}\right)\phi^2$$

Callan, Coleman, Jackiw. ANNALS OF PHYSICS: 59, 42-73 (1970)

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$\langle \Theta_{\mu\nu} \rangle \sim$ finite stuff

$$+\lambda(k_\mu k_\nu - g_{\mu\nu} k^2) \int_0^1 dx \left(\frac{1}{6} - x(1-x)\right) \ln\left(\frac{m^2 - k^2 x(1-x)}{\mu}\right)$$

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So we can make sense of the Energy Momentum Tensor in QFT to leading order.

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- Specifying only the initial state does not break angular symmetry
- We want to refine our result by specifying the final state

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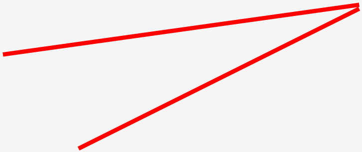

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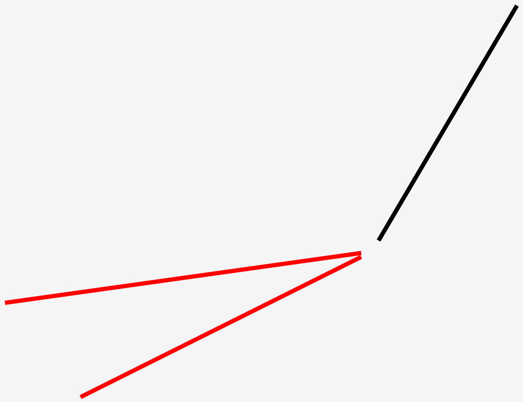

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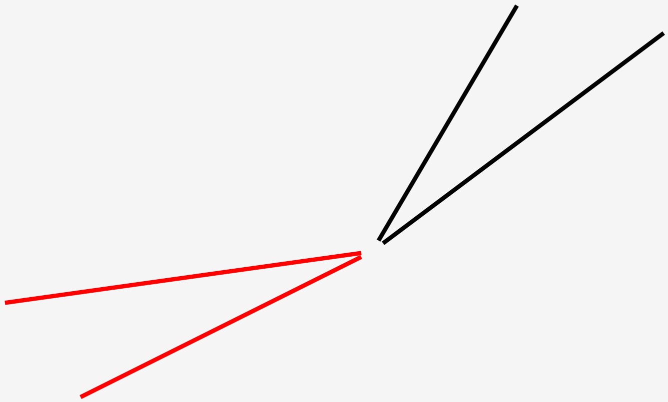

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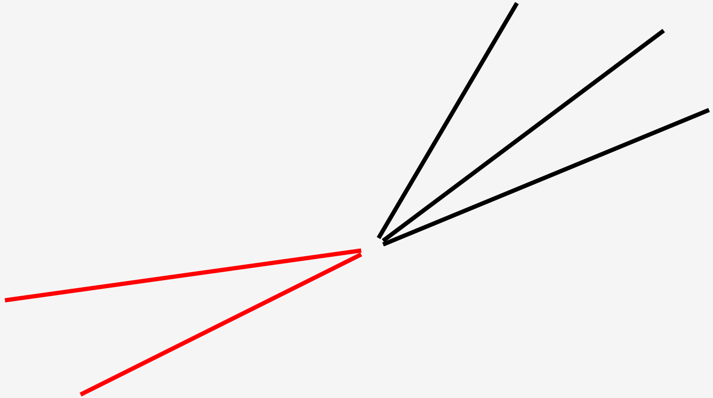

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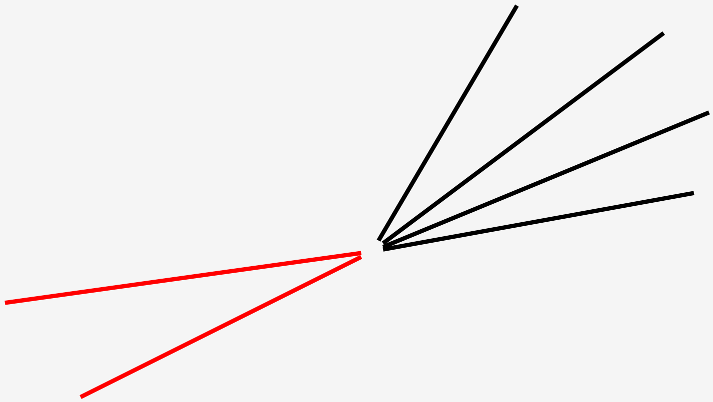

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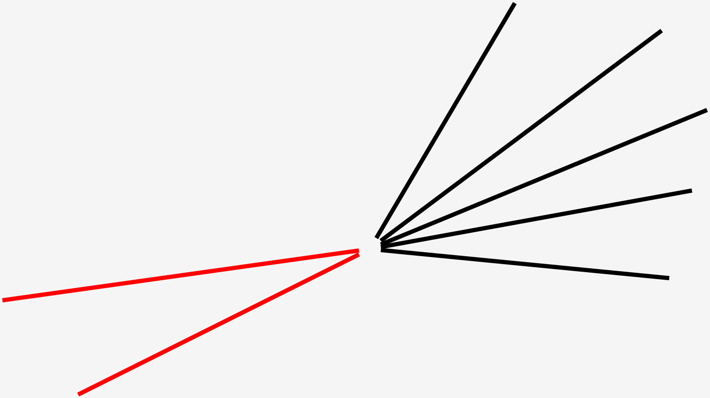

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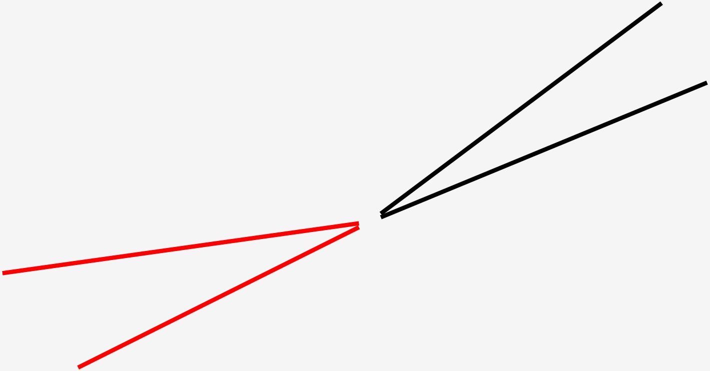

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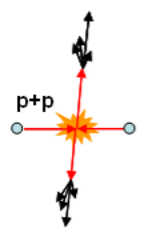

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$$E[\hat{O}(x)|\ |\text{in}\rangle, |\text{out}\rangle] = \frac{\langle\text{in}|\hat{\Theta}_M \hat{O}(x) \hat{\Theta}_M|\text{in}\rangle}{\langle\text{in}|\hat{\Theta}_M|\text{in}\rangle}$$

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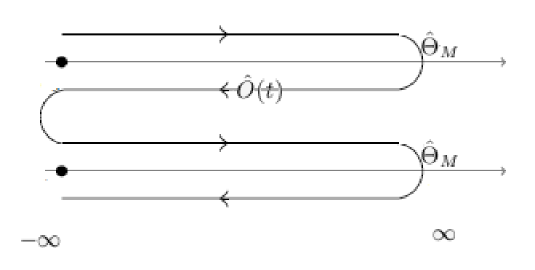

Figure: Time contour required for the conditional probability of an operator $\hat{O}(t)$

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The End.

Back up Slides

Data for QGP

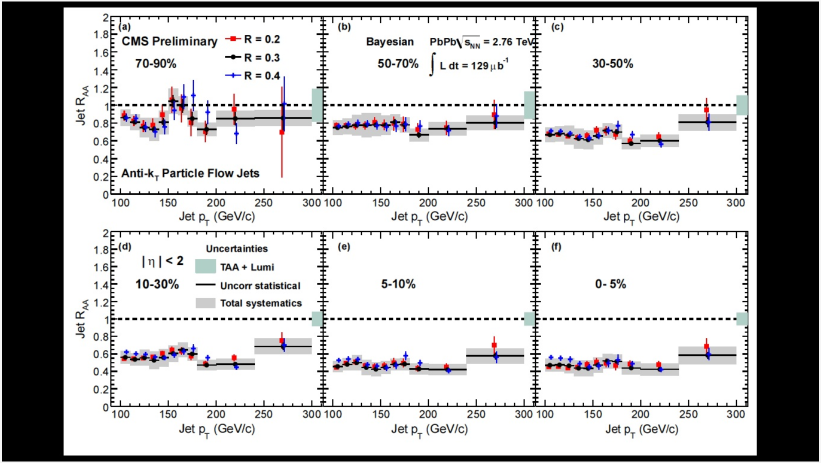

Figure: CMS Preliminary data for Jet R_{AA}. Taken from (1409.7545)

Conditional Expectation Value: Example

For $|\text{in}\rangle = |\psi\rangle$, $|\text{out}\rangle = |\psi'\rangle$ we find

$$E[\phi(x)|\ |\psi\rangle, |\psi'\rangle] = \langle\psi'|\phi(x)|\psi'\rangle + 2i\,\text{Im}\left(\frac{\langle\psi'|\phi(x)|\psi\rangle}{|\langle\psi'|\psi\rangle|}\right)$$

(At least to first order).

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(At least to first order).

This says that if $|\psi\rangle \neq |\psi^{'}\rangle$, the expectation of the field is complex.

Understanding $\langle\phi(x)\rangle$

We found a general expression for $\langle\phi(x)\rangle$ with $H_{\text{int}} = g\psi\phi\psi$

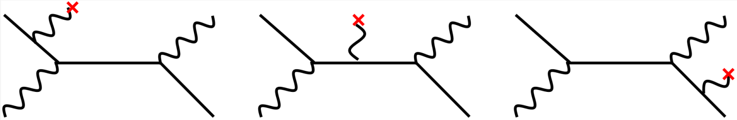

Figure: Typical Diagrams for $\langle\phi(x)\rangle$ for $|\text{in}\rangle = |\psi\phi\rangle$

$$\boxed{\langle\phi(x)\rangle = -ig\int d^4z\, D_R(x-z)\langle\text{in}|\psi(z)\psi(z)|\text{in}\rangle}$$

(Generalizes for arbitrary H_{int})

Understanding $\langle\phi(x)\rangle$

$$\langle\phi(x)\rangle = \frac{g}{m_\psi}\int d^3z \left(\frac{e^{-m_\phi|\vec{z}|}}{|\vec{z}|}\right)\frac{e^{-\frac{(\vec{x}-\vec{z})^2}{\alpha}}}{\sqrt{4\pi\alpha}^3}$$

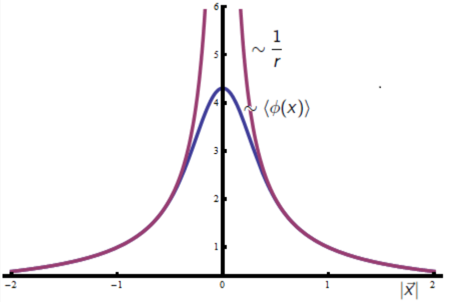

Figure: Analytic expression in 3+1 D, with comparison to $\frac{1}{r}$ potential.

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$$E[\hat{O}(x)|\ |\text{in}\rangle, |\text{out}\rangle] = \sum_i O_i P(|q_i\rangle|\ |\text{in}\rangle, |\text{out}\rangle)$$

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where $\hat{\Theta}_M = |\text{out}\rangle\langle\text{out}|$ is the projection operator built from the out states.

This result is actually a generalization of Baye's Theorem.

Toy Problem: Find $\langle\phi(x)\rangle$ emitted from a static "quark" in scalar QCD

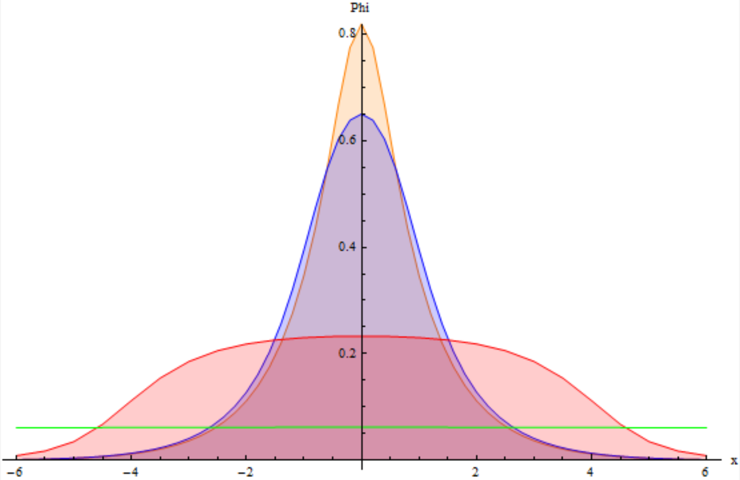

Figure: Evolution of $\langle\phi(x)\rangle$ in time (in 1+1 D)

How does the Stopping distance in AdS/CFT depend on the initial conditions?

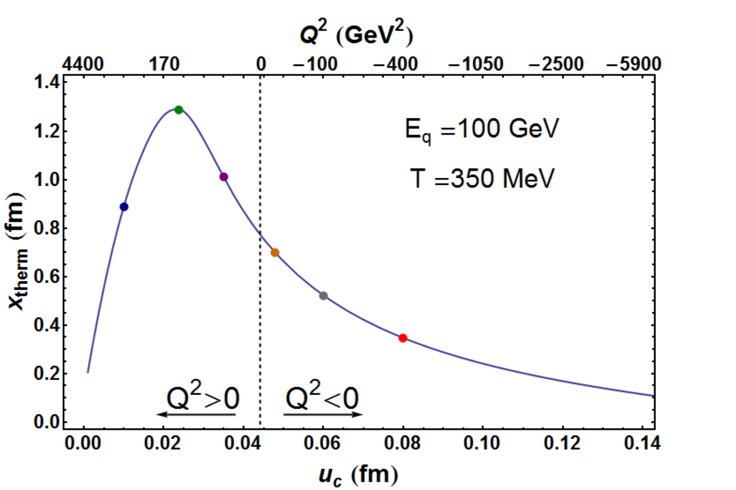

Figure: Stopping Distance as a function of radial u_c. R. Morad & W.A Horowitz (1409.7545)