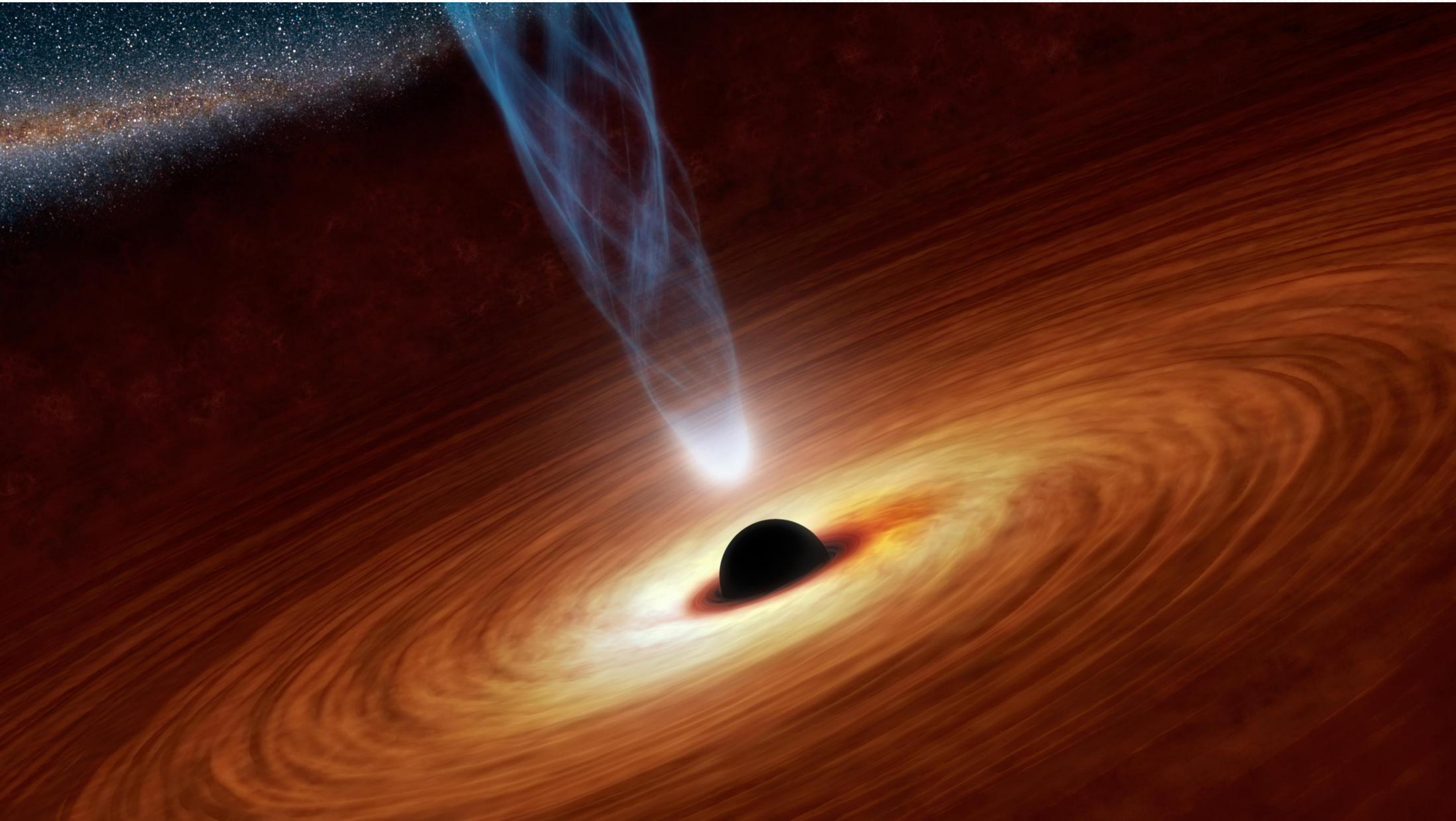
Quasinormal Modes of black holes for spin 3/2 fields

By: Gerhard Harmsen

Supervisor: Prof. Alan Cornell

What is a black hole?



What is a black hole?

- Schwarzschild metric

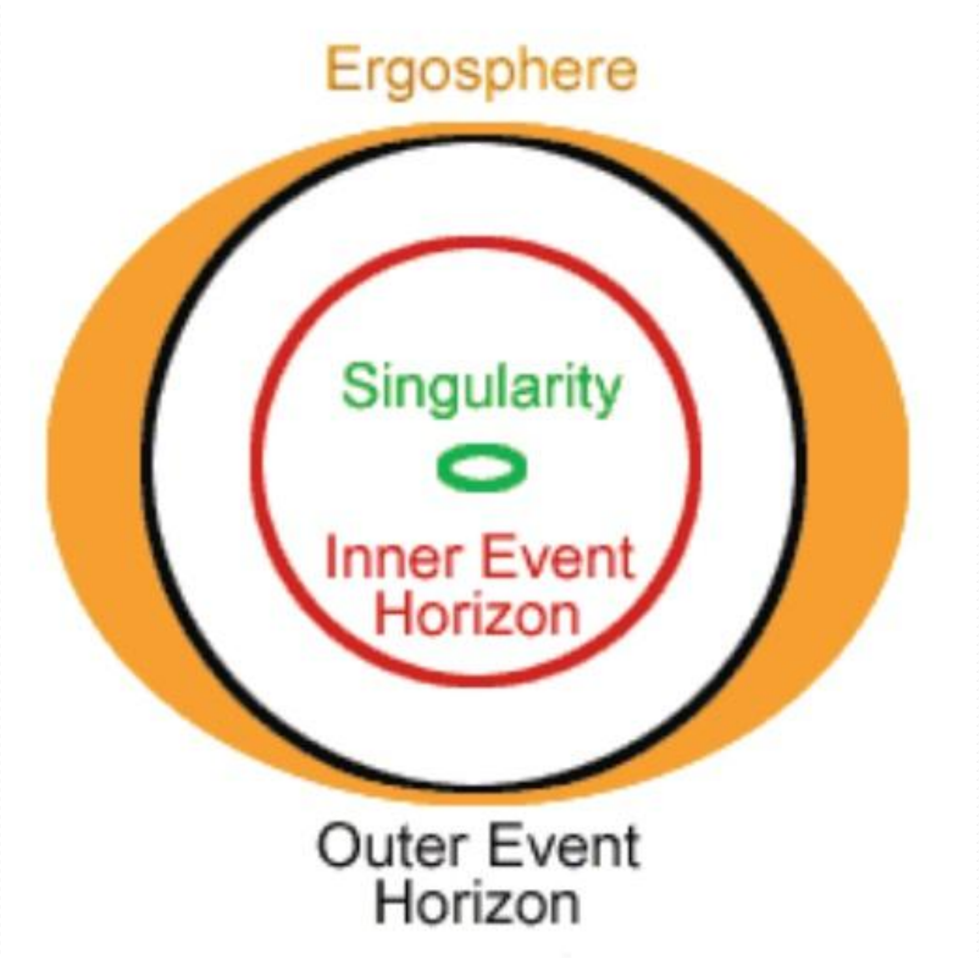
$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + (\sin\theta)^2 d\phi^2)$$

- Kerr metric

$$ds^2 = -\left(1 - \frac{2GMr}{r^2 + a^2cos^2\theta}\right)dt^2 + \left(\frac{r^2 + a^2cos^2\theta}{r^2 - 2GMr + a^2}\right)dr^2 + (r^2 + a^2cos^2\theta)d\theta^2$$

$$+\left(r^2 + a^2 + \frac{2GMra^2sin^2\theta}{r^2 + a^2cos^2\theta}\right)sin^2\theta d\phi^2 - \left(\frac{4GMrasin^2\theta}{r^2 + a^2cos^2\theta}\right)d\phi dt$$

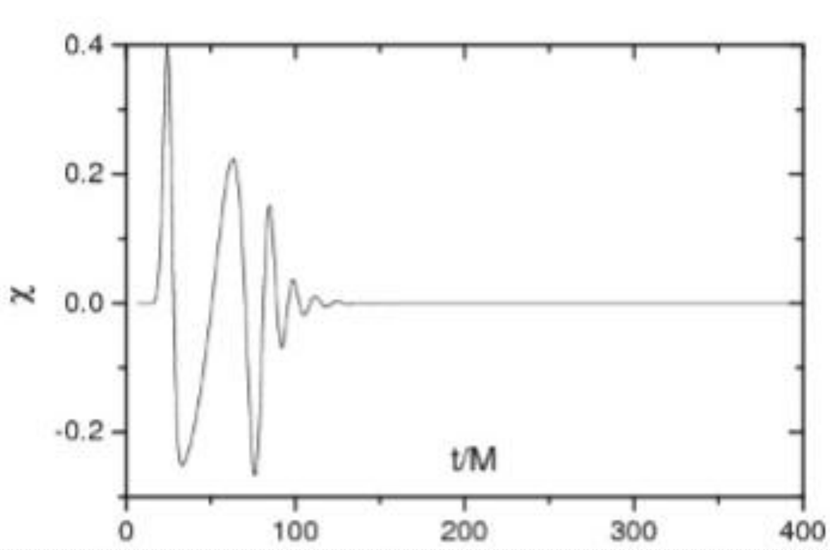
Kerr black hole



QuasiNormal Modes (QNM's)

- Allowed energy modes for a perturbed system

QNM's of black holes



- Graph showing the evolution of a gravitational wave packet in the background of a Schwarzschild black hole

- Allowed QNM's are dependant on characteristics of the black hole

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Previous works in our group

- Calculation of QNM’s for spin-1, ½ particles in Schwarzschild black hole backgrounds.

- Simulation of spin-1 and spin – ½ particles in black hole backgrounds.

Work in this project

- Use of WKB approximation to determine the allowed QNM's for spin- 3/2 fields

- Add on Warren Carlson's program to try and simulate these spin- 3/2 fields

WKB approximation

- Determine approximate solutions ODE's

- Assume wave function of the following form

$$\epsilon^2 \frac{d^2 y}{dx^2} = Q(x) y$$

- With $\epsilon \ll \mathbf{1}$

WKB Approximation

- Possible solution

$$y(x,\epsilon) = A(x,\epsilon)e^{\frac{iu(x)}{\epsilon}},$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(A' + A\frac{iu'}{\epsilon}\right)e^{\frac{iu(x)}{\epsilon}},$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \left(-\frac{(u')^2}{\epsilon^2} + \frac{i}{\epsilon}(Au'' + 2A'u') + A''\right)e^{\frac{iu(x)}{\epsilon}}.$$

WKB Approximation

- Solution of the form

$$\left(A''(x,\epsilon)\epsilon^2 + 2\epsilon A'(x,\epsilon) i u'^{(x)} + A(x,\epsilon)\left((iu''(x)\epsilon) + (iu'(x))^2\right)\right)$$
$$= Q(x)A(x,\epsilon)$$

- Let:

$$A(x,\epsilon) = A_0(x) + \epsilon A_1(x) + \epsilon^2 A_2(x) + \ \dots$$
$$A'(x,\epsilon) = A'_0(x) + \epsilon A'_1(x) + \epsilon^2 A'_2(x) + \ \dots$$
$$A''(x,\epsilon) = A''_0(x) + \epsilon A''_1(x) + \epsilon^2 A''_2(x) + \ \dots$$

WKB Approximation

- Gather in terms of powers of ϵ^0

$$A_0(x)(iu'(x))^2 = Q(x)A_0(x)$$

- Solution:

$$u(x) = \pm \int_{x_0}^{x} \sqrt{Q(k)}\, dk$$

Why WKB

- Doesn't tend to infinity for higher order perturbations
- Perturbations of black holes can be described by

$$\frac{d^2}{dx^2}\psi(x) = Q(x,\omega)\psi(x)$$

- Has been used for spin - ½ and spin - 1 fields

Extensions

- Calculate and simulate QNM's for black holes in AdS space.
- Study stability of rotating black holes in higher dimensional models

Thank you

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