Using a Classical Gluon Cascade to study the Equilibration of a Gluon-Plasma

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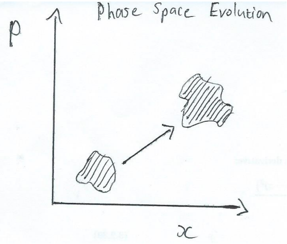
Matter in thermal equilibrium

State of matter described
by T, p, and V.
e.g. For ideal gas: $pV = Nk_B T$



Matter NOT in thermal equilibrium

No definition of T and p. Need to go back to the microscopic picture using the relativistic Boltzmann equation

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}}{E}\nabla\right) f(\mathbf{x}, \mathbf{p}) = C[f],$$

where $f(\mathbf{x}, \mathbf{p})$ is particle density in phase space, $C[f]$ is the collision term.

Boltzmann Eq. for $f(\mathbf{x}, \mathbf{p}) = \frac{\Delta N}{(2\pi)^3 \Delta x^3 \Delta p^3}$, not the individual particles.
No general analytic solution - solve numerically

Monte Carlo cascade simulation

The idea is to solve the Boltzmann equation by deriving the collision probability directly from the collision term.

$$C[f] = \frac{1}{2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{1}{\nu} \int \frac{d^3 p_1'}{(2\pi)^3 2E_1'} \frac{d^3 p_2'}{(2\pi)^3 2E_2'} f_1' f_2' \times |M_{1'2' \to 12}|^2 (2\pi)^4 \delta^{(4)}(p_1' + p_2' - p_1 - p_2)$$
$$- \frac{1}{2E_1} \times \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{1}{\nu} \int \frac{d^3 p_1'}{(2\pi)^3 2E_1'} \frac{d^3 p_2'}{(2\pi)^3 2E_2'} f_1 f_2 \times |M_{12 \to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_1' - p_2')$$

Background cont.

Obtain the Collision Probability

Collision rate per unit phase space for incoming particles p_1 and p_2 with $\Delta^3 p_1$ and $\Delta^3 p_2$:

$$\frac{\Delta N_{coll}}{\Delta t \frac{1}{(2\pi)^3}\Delta^3 x \Delta^3 p_1} = \frac{1}{2E_1}\frac{\Delta^3 p_2}{(2\pi)^3 2E_2} f_1 f_2 s\sigma \qquad f_j = \frac{\Delta N_j}{\frac{1}{(2\pi)^3}\Delta^3 x \Delta^3 p_j}$$

σ is cross-section

$$P_{coll} := \boxed{\frac{N_{coll}}{\Delta N_1 \Delta N_2} = v_{rel}\sigma(s)\frac{\Delta t}{\Delta^3 x}}$$

With pair's relative velocity $v_{rel} = \frac{s}{E_1 E_2}$.

Cross-Section

For Elastic $gg \to gg$ Collisions

$$\frac{d\sigma^{gg\to gg}}{dt} = \frac{9\pi\alpha_s}{2s^2}\left(3 - \frac{tu}{s^2} - \frac{su}{t^2} - \frac{st}{u^2}\right)$$

Mandelstam t is related to s via $t = -\frac{s}{2}(1-\cos\theta')$, with θ' being the scattering angle in CoM.

Small angle scatterings favoured
$\Rightarrow$ small t
Cross-section approximated as:

$$\frac{d\sigma}{dt} \approx \frac{9\pi\alpha_s}{t^2}$$

Cross-Section cont.

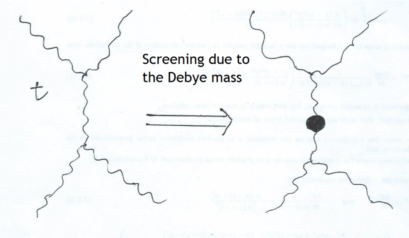
Infrared Divergence

Attempting to integrate to find the total cross-section gives

$$\sigma(s) = \int_{-s}^{0} \frac{9\pi\alpha_s}{t^2}\, dt = \infty$$

Example of infrared divergence.

Need to regularise the cross-section using screening effects due to the Debye mass

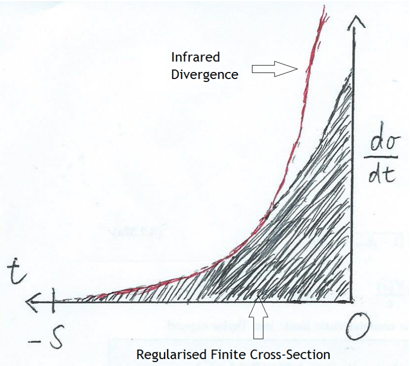


Cross-section cont.

Include Screening Effects

Include debye mass and integrate

$$\sigma(s) = \int_{-s}^{0} \frac{9\pi\alpha_s}{(t - m_D^2)^2} dt$$
$$= 9\pi\alpha_s^2 \frac{s}{m_D^2(s + m_D^2)}$$

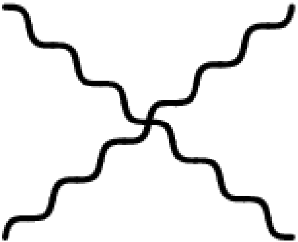


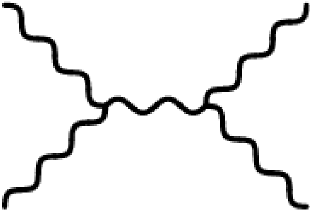
How do the gluons scatter?

Lorentz boost into CoM (easiest). Scatter momenta given by scattering angles in CoM

$$\mathbf{p}'_{i_{scattered}} = p'_i \left(\cos\theta'\hat{\mathbf{x}}' + \sin\theta' \sin\phi'\hat{\mathbf{y}}' + \sin\theta' \cos\phi'\hat{\mathbf{z}}'\right)$$

Afterwards, inverse boost back to Lab frame.

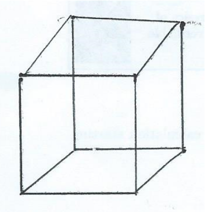




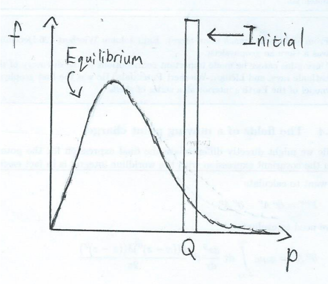
Simulation: Part 1

Overview

Study the thermalisation of a gluon-plasma in homogeneous box.



Start from isotropic momentum shell: $f_{ini}(p) = a\delta(Q - p)$ where the thickness of the shells is the coarse graining. We fix the equilibrium temperature through the parameters Q and a.



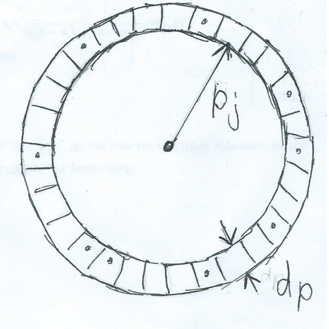
Simulation: Part 1

Finding the Entropy

Group the particles according to their energy. For a group j: G_j is number of states and N_j is number of particles. Then off-equilibrium

$$S = \sum_j (N_j \ln G_j - \ln N_j!)$$

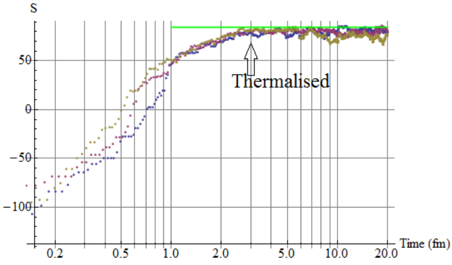
We have $f(p)$



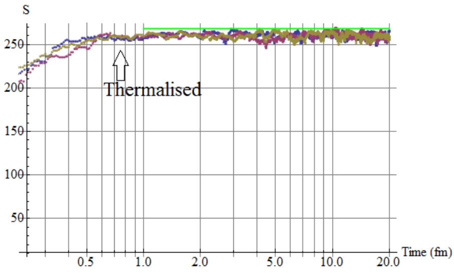
coarse graining dp

Simulation: Part 1 cont.

Time Evolution of the Entropy



$T_{eq} = 300$ MeV

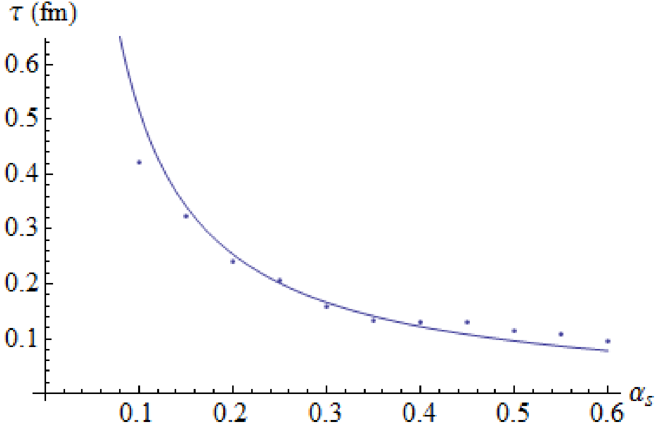


$T_{eq} = 500$ MeV

Simulation: Part 1 cont.

Thermalisation Time τ

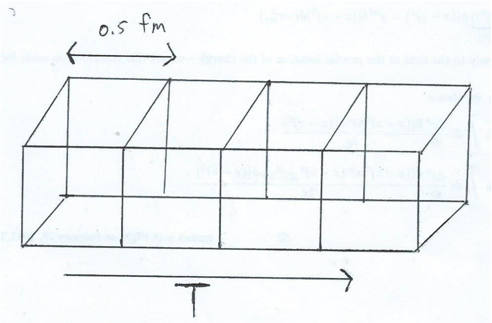
τ is found by fitting modelling the change in entropy as an exponential decay. It is found that it decreases for increasing $N_{particles}$ and T_{eq}. From cross-section we guess (naively) $\tau \propto 1/\alpha_s$.



Simulation: Part 2

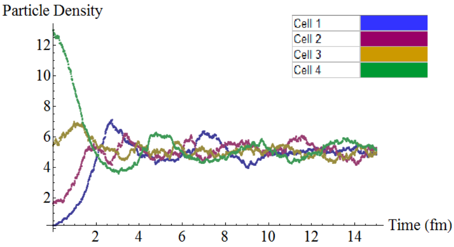
Overview

Partition the volume into cells.
Allow streaming in z-direction.
Linearly increasing temp. gradient.
Num. gluons and momenta determined by T.

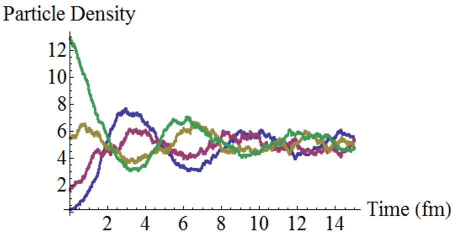


Simulation: Part 2 cont.

Particle Density per Cell



Free-Streaming



With Collisions

Find u^z from E-M tensor

$$T^{\alpha\beta} = \begin{pmatrix} T^{00} & 0 & 0 & T^{0z} \\ 0 & T^{xx} & 0 & 0 \\ 0 & 0 & T^{yy} & 0 \\ T^{0z} & 0 & 0 & T^{zz} \end{pmatrix},$$

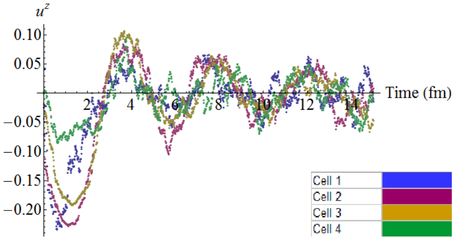
$$u^z = \frac{T^{0z}}{\epsilon + T^{zz}},$$

where ϵ is eigenvalue,

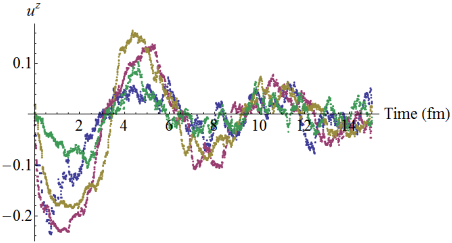
$$\epsilon = T^{tt} + \sqrt{(T^{tt} + T^{zz})^2 - (T^{0z})^2}.$$

Simulation: Part 2 cont.

Flow-Velocity in Each Cell



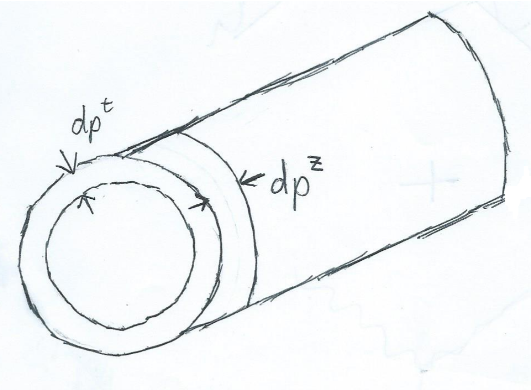
Free-Streaming



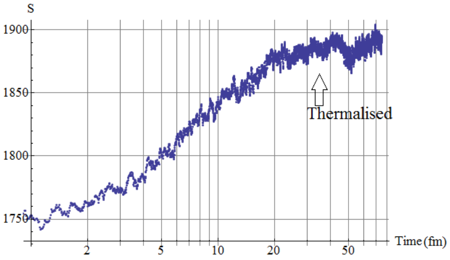
With Collisions

Simulation: Part 2 cont.

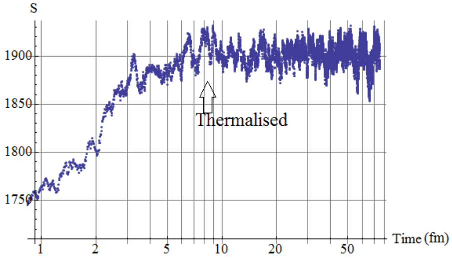
Need new groups



Now $f(p^z, p^t)$
with dp^z and dp^t are coarse grainings.



Free-Streaming



With Collisions

Thank you for listening

This is a Camel.