

Overview of beauty production models

I. Schienbein
LPSC Grenoble/Univ. Grenoble Alpes



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Outline

- FFNS
- ZM-VFNS
- GM-VFNS
- FONLL
- NLO Monte Carlo generators
- [k_T factorization]
- [Double parton scattering]
- [Diffractive production]

Theoretical approaches:
Fixed Flavor Number Scheme
(FFNS)

FFNS/Fixed Order

Factorization formula for inclusive heavy quark (Q) production:

$$d\sigma^Q \simeq \sum_{a,b} f_a^A \otimes f_b^B \otimes d\tilde{\sigma}_{ab \rightarrow Q+X}$$

sum over all possible
partonic subprocesses
NO heavy quark PDF

Calculable short distance cross section;
log(pT/m) terms kept in **fixed order**

FFNS/Fixed Order

Factorization formula for inclusive heavy quark (Q) production:

$$d\sigma^Q \simeq \sum_{a,b} f_a^A \otimes f_b^B \otimes d\tilde{\sigma}_{ab \rightarrow Q+X}$$

PDFs

sum over all possible
partonic subprocesses
NO heavy quark PDF

Calculable short distance cross section;
log(pT/m) terms kept in **fixed order**

Inclusive heavy-flavored hadron (H) production:

$$d\sigma^H = d\sigma^Q \otimes D_Q^H(z)$$

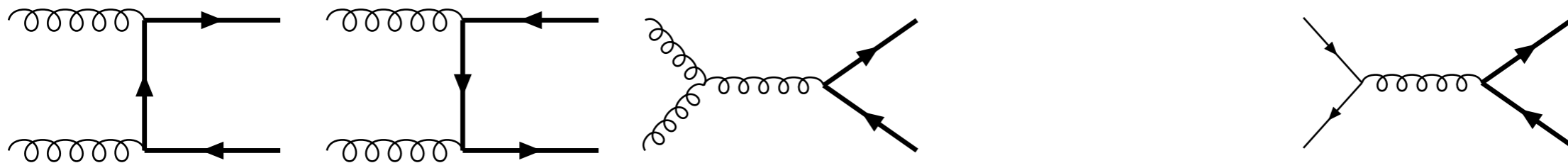
Convolution with a
scale-independent FF

- * non-perturbative
- * describes hadronization
- * not based on a fact. theorem

Leading Order (LO)

Leading order subprocesses:

1. $gg \rightarrow Q\bar{Q}$
2. $q\bar{q} \rightarrow Q\bar{Q}$ ($q = u, d, s$)



- The gg -channel is dominant at the LHC ($\sim 85\%$ at $\sqrt{S} = 14$ TeV).
- The total production cross section for **heavy quarks** is finite. The minimum virtuality of the t-channel propagator is m^2 . Sets the scale in α_s . Perturbation theory should be reliable.
- Note: For $m^2 \rightarrow 0$ total cross section would diverge.

[See M. Mangano, hep-ph/9711337; Textbook by Ellis, Stirling and Webber]

Next-to-leading Order (NLO)

Next-to-leading order (NLO) subprocesses:

1. $gg \rightarrow Q\bar{Q}g$
2. $q\bar{q} \rightarrow Q\bar{Q}g$ ($q = u, d, s$)
3. $gq \rightarrow Q\bar{Q}q, g\bar{q} \rightarrow Q\bar{Q}\bar{q}$ [new at NLO]
4. Virtual corrections to $gg \rightarrow Q\bar{Q}$ and $q\bar{q} \rightarrow Q\bar{Q}$

NLO corrections for σ_{tot} and differential cross sections $d\sigma/dp_T dy$ known since long:

- Nason, Dawson, Ellis, NPB303(1988)607; Beenakker, Kuif, van Neerven, Smith, PRD40(1989)54 [σ_{tot}]
- NDE, NPB327(1989)49; (E)B335(1990)260; Beenakker *et al.*, NPB351(1991)507 [$d\sigma/dp_T dy$]

Well tested by recalculations and zero-mass limit:

- Bojak, Stratmann, PRD67(2003)034010 [$d\sigma/dp_T dy$ (un)polarized]
- Kniehl, Kramer, Spiesberger, IS, PRD71(2005)014018 [$m \rightarrow 0$ limit of diff. x-sec]
- Czakon, Mitov, NPB824(2010)111 [σ_{tot} , fully analytic]

Next-to-leading Order (NLO)

- Fixed order NLO calculation also useful to obtain predictions of heavy quark correlations!

Mangano, Nason, Ridolfi ('92)

Next-to-next-to-leading Order (NNLO)

Channels: $q\bar{q}$, gg , qg

- Two-loop virtual most difficult

$$M_2^{(0)} + M_2^{(1)} + M_2^{(2)}$$

- Analytic approach: Bonciani, Ferroglia, Gehrmann, Maitre, Studerus, von Manteuffel ('08-'10)
- Numeric approach: Czakon, Mitov et al.

- Virtual + Real

Dittmaier, Uwer, Weinzierl ('08)

$$M_3^{(0)} + M_3^{(1)}$$

- Subtraction method for IR singularities in double real

Czakon ('10-'11)

$$M_4^{(0)}$$

Next-to-next-to-leading Order (NNLO)

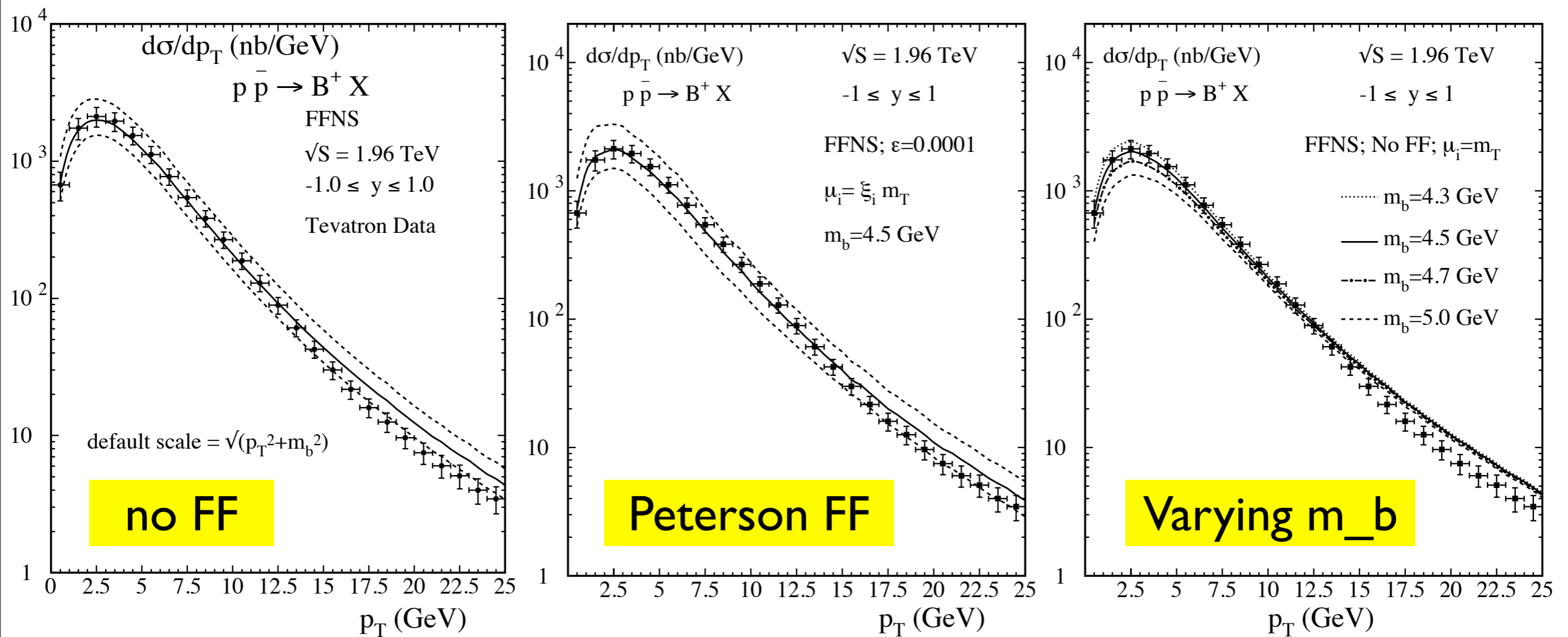
- Available now for top pair production!
- Total cross section Czakon, Mitov, PRL 110(2013)252004
- Differential distributions Czakon, Mitov, arXiv:1411.3007
- Analytic approach not yet complete
[Bonciani et al.]

Very large scale uncertainties at NLO in c,b production

NNLO will be crucial to make progress!

Some NLO results for B-meson production

NLO FFNS works very well for p_T up to roughly 5m



Remarks:

- Fixed order theory in reasonable agreement with Tevatron data up to $p_T \simeq 5m_b$
- At $p_T \lesssim m_b$ factorization less obvious. Depends on definition of convolution variable z : $p_B = zp_b$ or $p_T^B = zp_T^b$ or $p_B^+ = zp_b^+$ or $\vec{p}_B = z\vec{p}_b$
- Less hadronization effects than originally believed:
 ϵ -parameter small corresponding to a hard fragmentation function.
Harder FF \rightarrow harder p_T -spectrum
- Larger $\alpha_s(M_Z) \rightarrow$ harder p_T -spectrum
- Mass dependence important for $p_T \lesssim m$ (peak) $\rightarrow \sigma_{tot}$
- Only the 4th or 5th Mellin-moment of the FF is relevant for large p_T [M. Mangano]:
 $d\sigma^b/dp_T(b) \simeq A/p_T(b)^n$ with $n \simeq 4, \dots, 5$

$$d\sigma^B/dp_T(B) = \int dz/z D(z) d\sigma^b/dp_T(b)[p_T(b) = p_T(B)/z] = A/p_T(B)^n \times \int dz z^{n-1} D(z)$$

Theoretical approaches:
Zero Mass Variable Flavor Number Scheme
(ZM-VFNS)

Factorization formula for inclusive heavy quark (Q) production:

$$d\sigma^{H+X} \simeq \sum_{a,b,c} \int_0^1 dx_a \int_0^1 dx_b \int_0^1 dz f_a^A(x_a, \mu_F) f_b^B(x_b, \mu_F) d\hat{\sigma}_{ab \rightarrow c+X} D_c^H(z, \mu'_F) + \mathcal{O}(m^2/p_T^2)$$

- Same factorization formula as for inclusive production of pions and kaons
- Quark mass neglected in kinematics and the short distance cross section
- Allows to compute p_T spectrum for $p_T \gg m$
- Needs **scale-dependent** FFs of quarks and gluons into the observed heavy-flavored hadron (H)

List of subprocesses in the ZM-VFNS

Massless NLO calculation: [\[Aversa,Chiappetta,Greco,Guillet,NPB327\(1989\)105\]](#)

1. $gg \rightarrow qX$
2. $gg \rightarrow gX$
3. $qg \rightarrow gX$
4. $qg \rightarrow qX$
5. $q\bar{q} \rightarrow gX$
6. $q\bar{q} \rightarrow qX$
7. $qg \rightarrow \bar{q}X$
8. $qg \rightarrow \bar{q}'X$
9. $qg \rightarrow q'X$
10. $qq \rightarrow gX$
11. $qq \rightarrow qX$
12. $q\bar{q} \rightarrow q'X$
13. $q\bar{q}' \rightarrow gX$
14. $q\bar{q}' \rightarrow qX$
15. $qq' \rightarrow gX$
16. $qq' \rightarrow qX$

⊕ charge conjugated processes

Fragmentation functions

Approach I: Perturbative FFs (PFFs)

Caccciari, Greco,
Nason, Oleari, ...

$$D_i^H(z, \mu'_F) = D_i^Q(z, \mu'_F) \otimes D_Q^H(z)$$

PFF evolved with DGLAP;
short distance;
boundary condition calculable

Non-pert., scale-independent FF
describing hadronization of heavy
quark Q into heavy hadron H

Mellin-moments of $D_Q^H(z)$ determined from e^+e^- data

Approach II: treat FFs into H in the same way as FFs into pions or kaons

Binnewies, Kniehl, Kramer, ...

Non-pert. boundary conditions $D_i^H(z, m)$ from fit to e^+e^- data;
Determine FFs directly in x-space; evolved with DGLAP

PFF approach

Cacciari, Nason, PRL89(2002)122003

Determine FF from N=2 moment in PFF approach;
not from entire x-spectrum

$$D_N \equiv \int D(z) z^N \frac{dz}{z}$$

$$\frac{d\sigma}{dp_T} = \int dz d\hat{p}_T D(z) \frac{A}{\hat{p}_T^n} \delta(p_T - z\hat{p}_T) = \frac{A}{p_T^n} D_n$$

n~3,4,5

$\langle x_E^{N-1} \rangle$

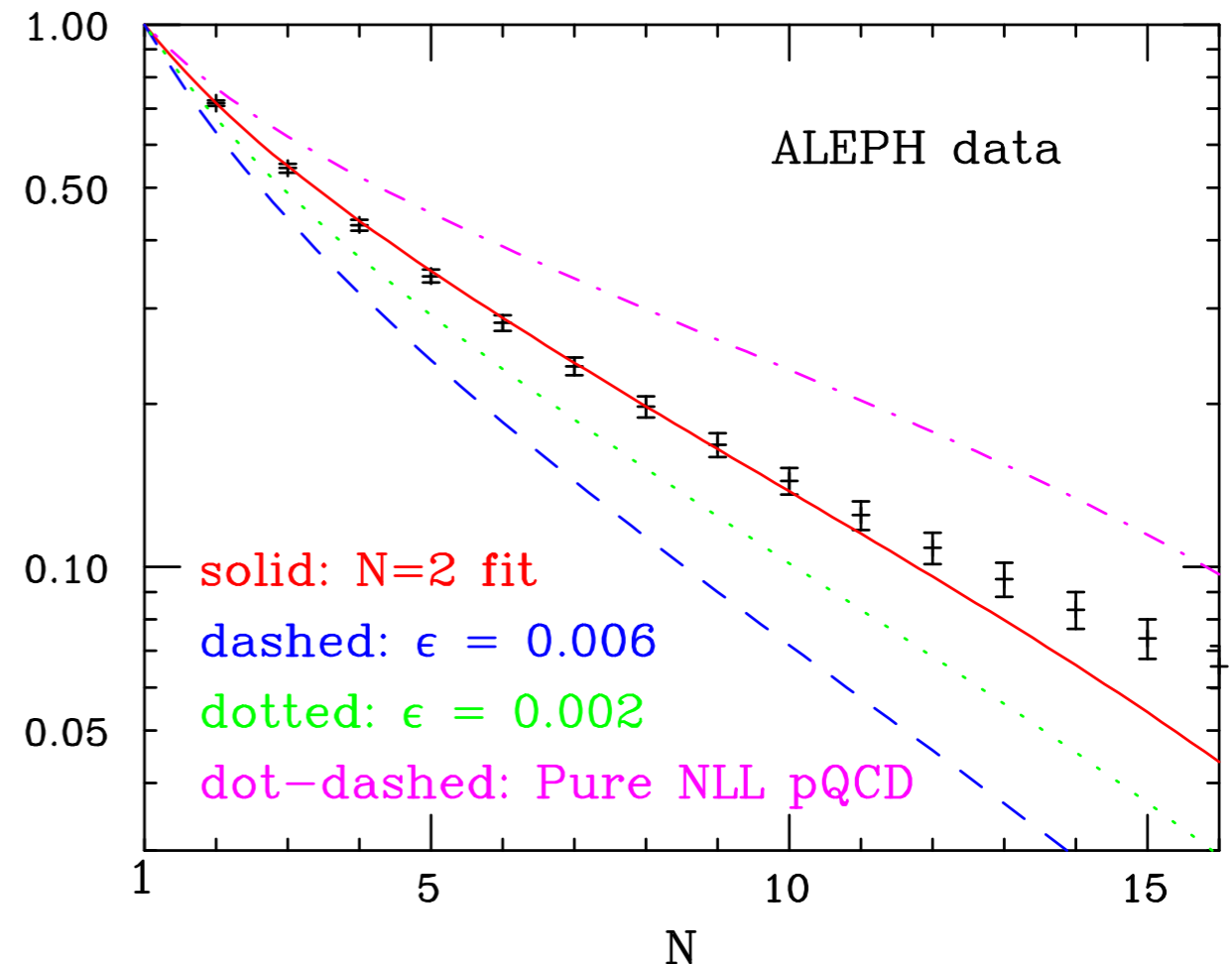
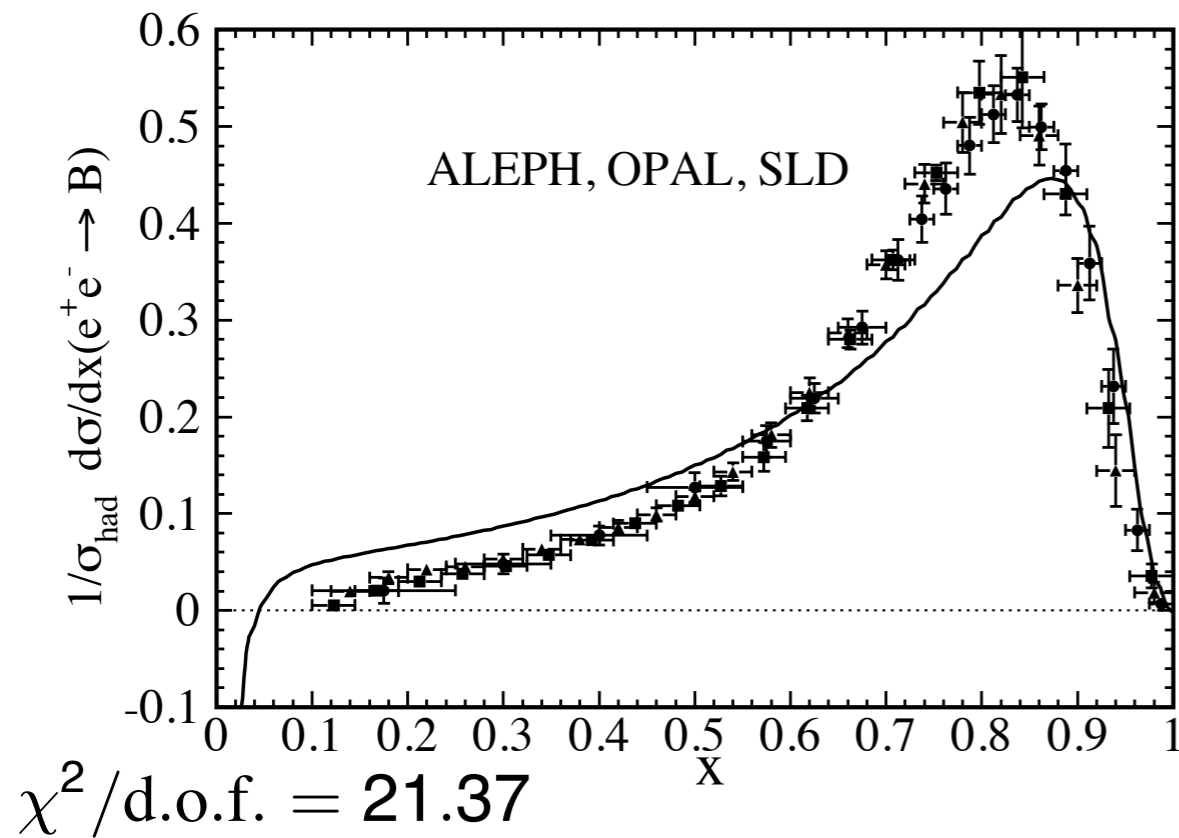


FIG. 1. Moments of the measured B meson fragmentation function, compared with the perturbative NLL calculation supplemented with different $D(z)$ non-perturbative fragmentation forms. The solid line is obtained using a one-parameter form fitted to the second moment.

FFs into B mesons [1] from LEP/SLC data [2]

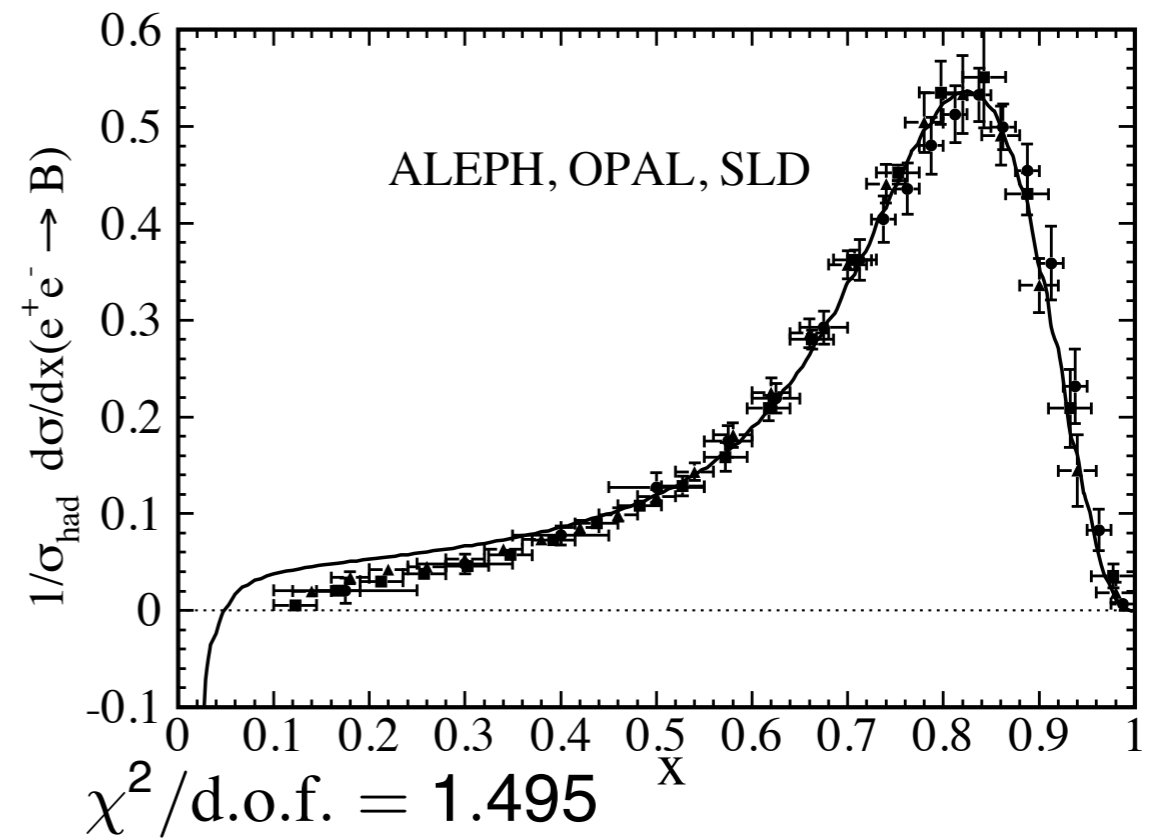
Petersen

$$D(x, \mu_0^2) = N \frac{x(1-x)^2}{[(1-x)^2 + \epsilon x]^2}$$



Kartvelishvili-Likhoded

$$D(x, \mu_0^2) = Nx^\alpha(1-x)^\beta$$



[1] Kniehl, Kramer, IS, Spiesberger, PRD77(2008)014011

[2] ALEPH, PLB512(2001)30; OPAL, EPJC29(2003)463; SLD, PRL84(2000)4300;

PRD65(2002)092006

Theoretical approaches:
General Mass Variable Flavor Number Scheme
(GM-VFNS)

Factorization Formula:

[1]

$$d\sigma(p\bar{p} \rightarrow D^* X) = \sum_{i,j,k} \int dx_1 dx_2 dz f_i^p(x_1) f_j^{\bar{p}}(x_2) \times \\ d\hat{\sigma}(ij \rightarrow kX) D_k^{D^*}(z) + \mathcal{O}(\alpha_s^{n+1}, (\frac{\Lambda}{Q})^p)$$

Q : hard scale, $p = 1, 2$

-
- $d\hat{\sigma}(\mu_F, \mu'_F, \alpha_s(\mu_R), \frac{m_h}{p_T})$: hard scattering cross sections free of long-distance physics $\rightarrow m_h$ kept
 - PDFs $f_i^p(x_1, \mu_F), f_j^{\bar{p}}(x_2, \mu_F)$: $i, j = g, q, c$ [$q = u, d, s$]
 - FFs $D_k^{D^*}(z, \mu'_F)$: $k = g, q, c$

\Rightarrow need short distance coefficients including heavy quark masses

[1] J. Collins, 'Hard-scattering factorization with heavy quarks: A general treatment',
PRD58(1998)094002

List of subprocesses in the GM-VFNS

Only light lines

- ① $gg \rightarrow qX$
- ② $gg \rightarrow gX$
- ③ $qg \rightarrow gX$
- ④ $qg \rightarrow qX$
- ⑤ $q\bar{q} \rightarrow gX$
- ⑥ $q\bar{q} \rightarrow qX$
- ⑦ $qg \rightarrow \bar{q}X$
- ⑧ $qg \rightarrow \bar{q}'X$
- ⑨ $qg \rightarrow q'X$
- ⑩ $qq \rightarrow gX$
- ⑪ $qq \rightarrow qX$
- ⑫ $q\bar{q} \rightarrow q'X$
- ⑬ $q\bar{q}' \rightarrow gX$
- ⑭ $q\bar{q}' \rightarrow qX$
- ⑮ $qq' \rightarrow gX$
- ⑯ $qq' \rightarrow qX$

Heavy quark initiated ($m_Q = 0$)

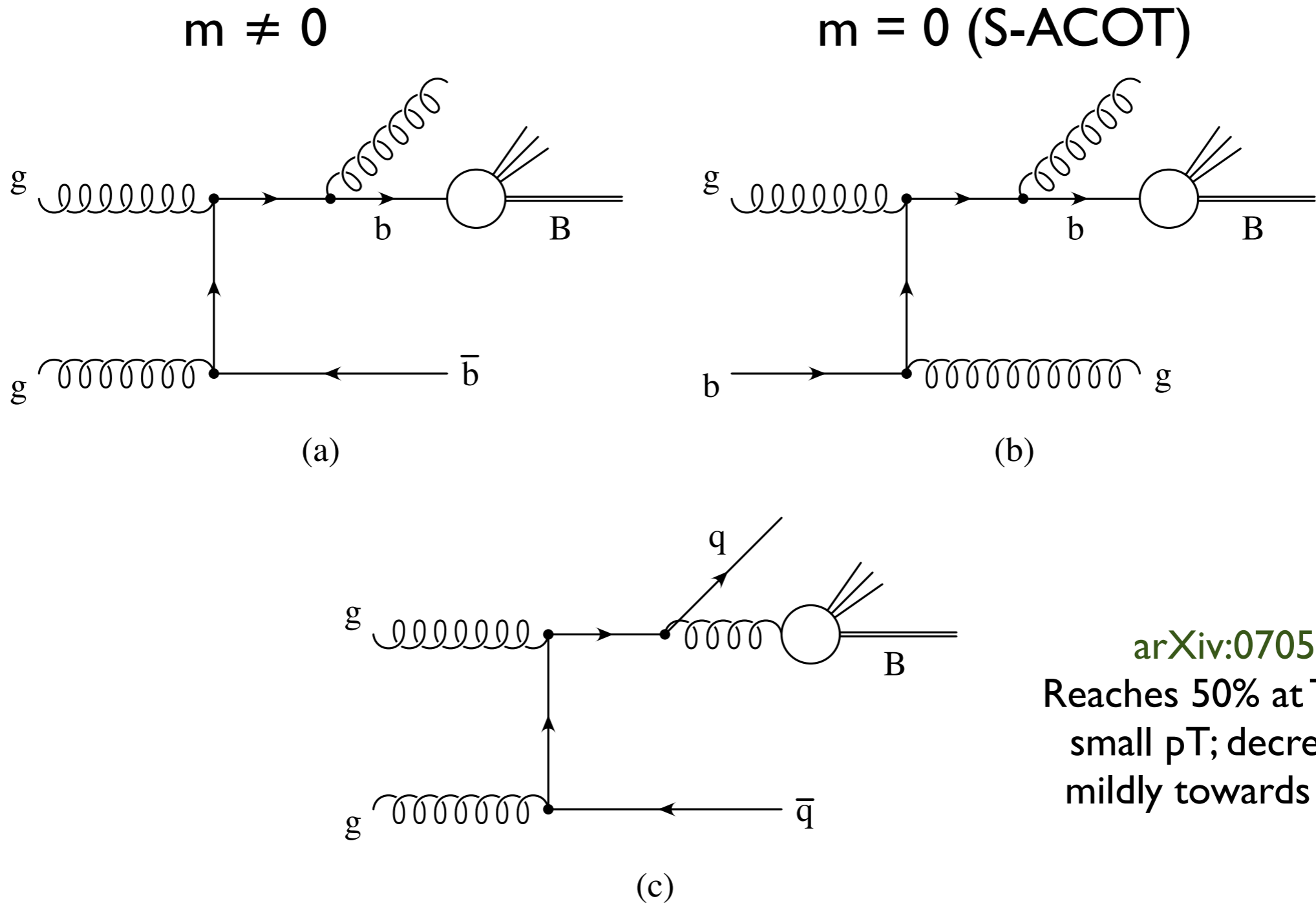
- ① -
- ② -
- ③ $Qg \rightarrow gX$
- ④ $Qg \rightarrow QX$
- ⑤ $Q\bar{Q} \rightarrow gX$
- ⑥ $Q\bar{Q} \rightarrow QX$
- ⑦ $Qg \rightarrow \bar{Q}X$
- ⑧ $Qg \rightarrow \bar{q}X$
- ⑨ $Qg \rightarrow qX$
- ⑩ $QQ \rightarrow gX$
- ⑪ $QQ \rightarrow QX$
- ⑫ $Q\bar{Q} \rightarrow qX$
- ⑬ $Q\bar{q} \rightarrow gX, q\bar{Q} \rightarrow gX$
- ⑭ $Q\bar{q} \rightarrow QX, q\bar{Q} \rightarrow qX$
- ⑮ $Qq \rightarrow gX, qQ \rightarrow gX$
- ⑯ $Qq \rightarrow QX, qQ \rightarrow qX$

Mass effects: $m_Q \neq 0$

- ① $gg \rightarrow QX$
- ② -
- ③ -
- ④ -
- ⑤ -
- ⑥ -
- ⑦ -
- ⑧ $qg \rightarrow \bar{Q}X$
- ⑨ $qg \rightarrow QX$
- ⑩ -
- ⑪ -
- ⑫ $q\bar{q} \rightarrow QX$
- ⑬ -
- ⑭ -
- ⑮ -
- ⑯ -

⊕ charge conjugated processes

Example diagrams



[arXiv:0705.4392](https://arxiv.org/abs/0705.4392)
 Reaches 50% at Tevatron at
 small p_T ; decreases only
 mildly towards larger p_T

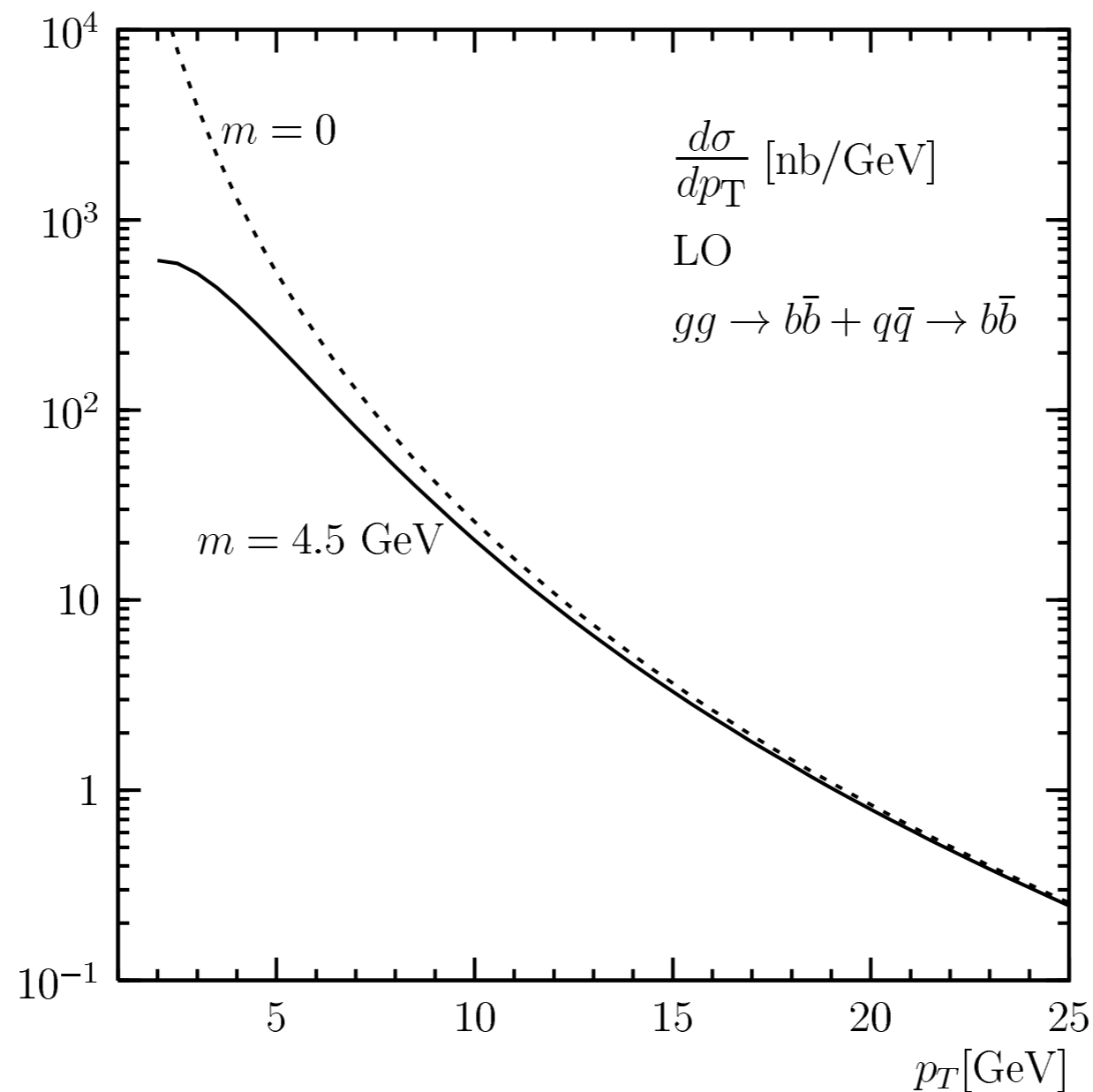
FIG. 2: Examples of Feynman diagrams leading to contributions of (a) class (i), (b) class (ii), and (c) class (iii).

Limiting cases

- **GM-VFNS** → **ZM-VFNS** for $p_T \gg m$
(this is the case by construction)
- **GM-VFNS** → **FFNS** for $p_T \sim m$
(formally this can be shown; numerically problematic in the S-ACOT scheme)

The GM-VFNS at low p_T

LO: $m=0$ case diverges at $p_T=0$



Problem: current implementation in S-ACOT scheme
 $b+g$ channel with $m=0$ diverges at small p_T !

The GM-VFNS at low p_T

Problem can be solved by suitable scale choice

arXiv:1502.01001

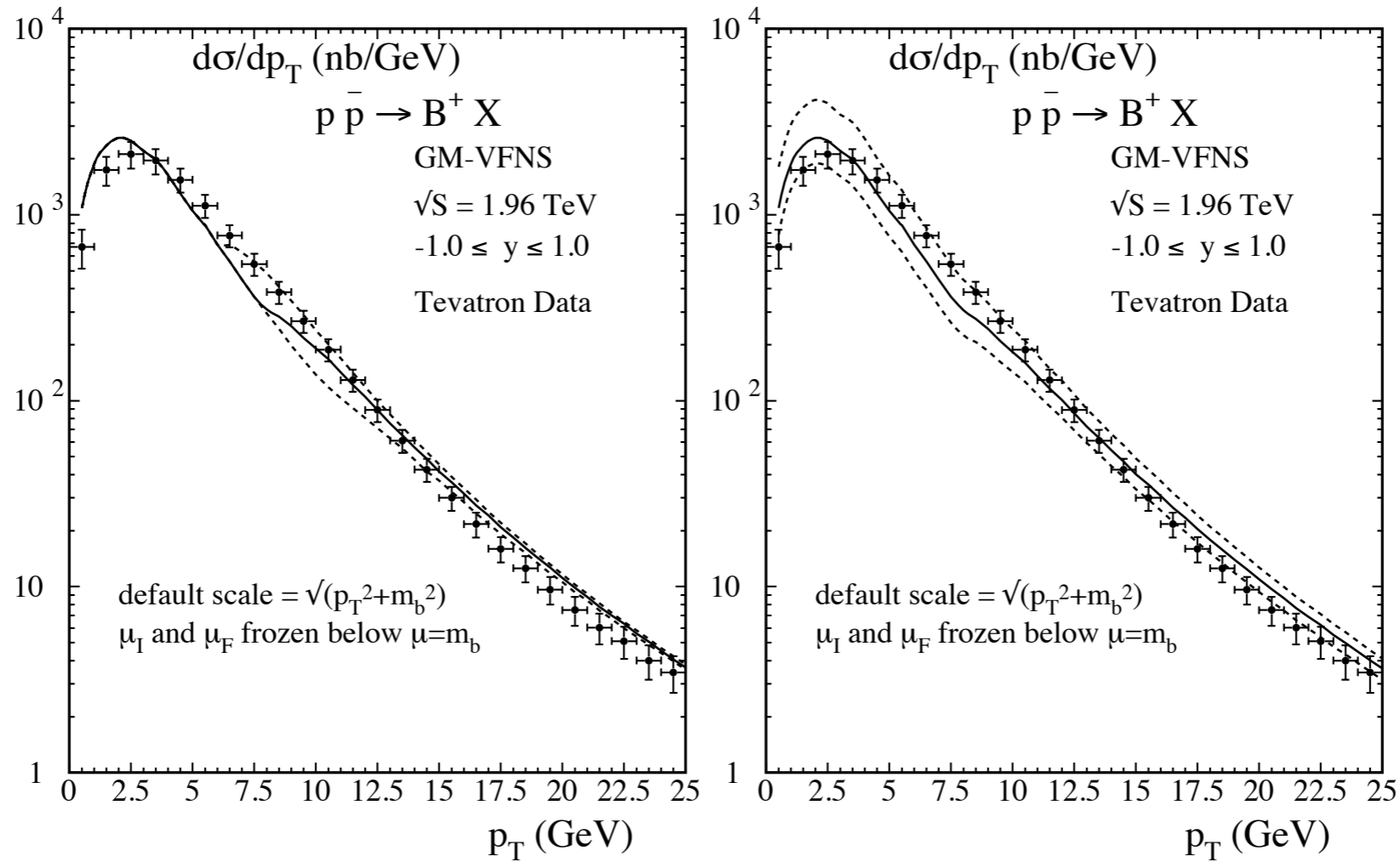


Figure 5: $d\sigma/dp_T$ for $p\bar{p} \rightarrow B^+ + X$ at $\sqrt{S} = 1.96$ TeV, $|y| < 1.0$, in the GM-VFNS (data from CDF [6]). Left panel: $\xi_R = 1$, $\xi_I = 0.5$ and $\xi_F = 0.5$ (full curve), $\xi_F = 0.6$ (upper dashed curve), $\xi_F = 0.4$ (lower dashed curve). Right panel: $\xi_i = (1, 0.5, 0.5)$ for the central curve; upper curve: $\xi_R = 0.5$, lower curve: $\xi_R = 2$.

The GM-VFNS at low p_T

Comparison with LHCb data

arXiv:1502.01001

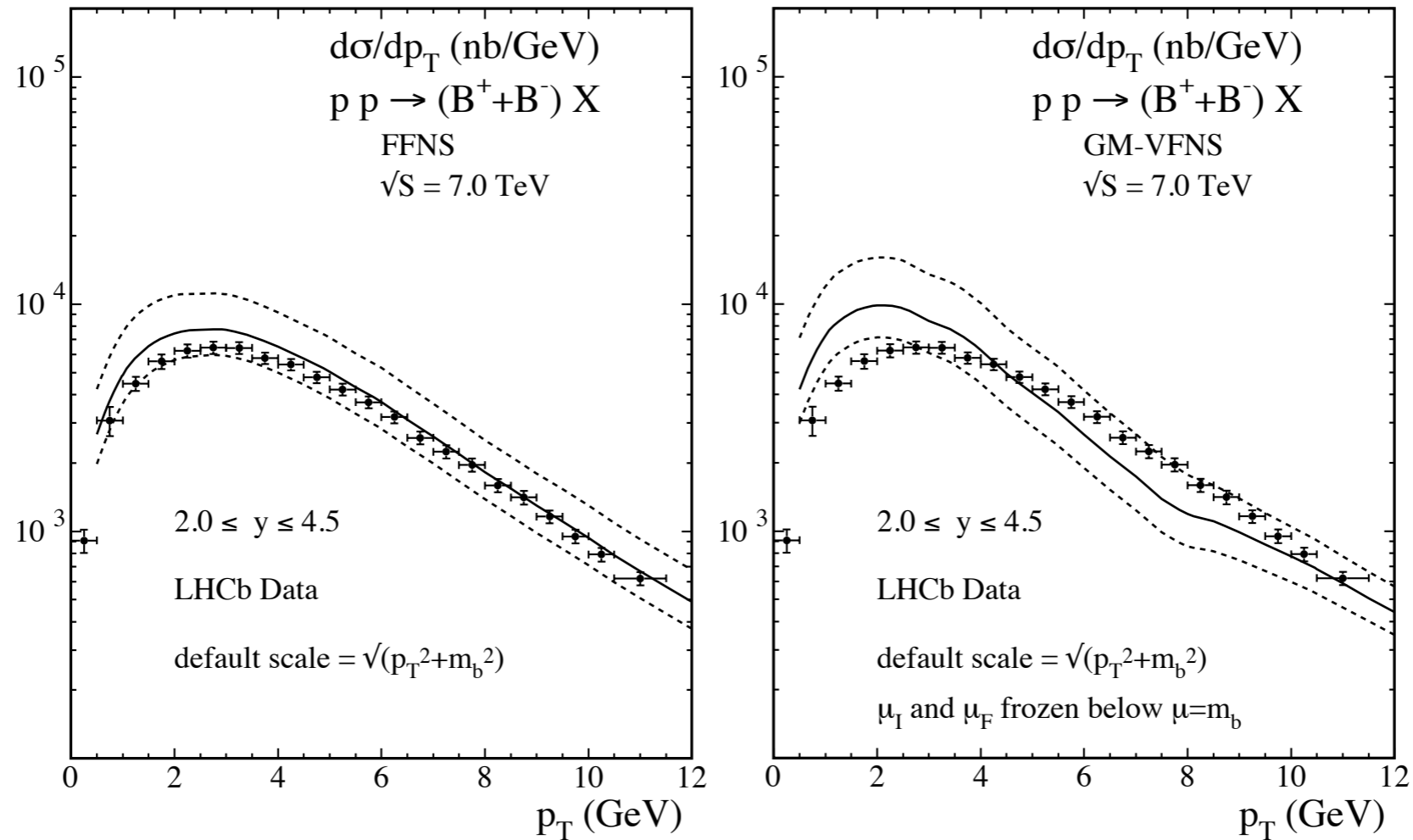


Figure 6: $d\sigma/dp_T$ for $pp \rightarrow B^+ + B^- + X$ at $\sqrt{S} = 7$ TeV with $2.0 < y < 4.5$, compared with results from the FFNS (left) and the GM-VFNS (right). $\xi_{R,I,F} = (1, 0.5, 0.5)$. The error band is obtained from variations by factors 2 up and down (maximum: $\xi_R = 0.5$, minimum: $\xi_R = 2$). The factorization scale parameters are frozen below $\mu_{I,F} = m_b$. Data points are taken from [15].

GM-VFNS: Comparison with ATLAS data

arXiv:1502.01001

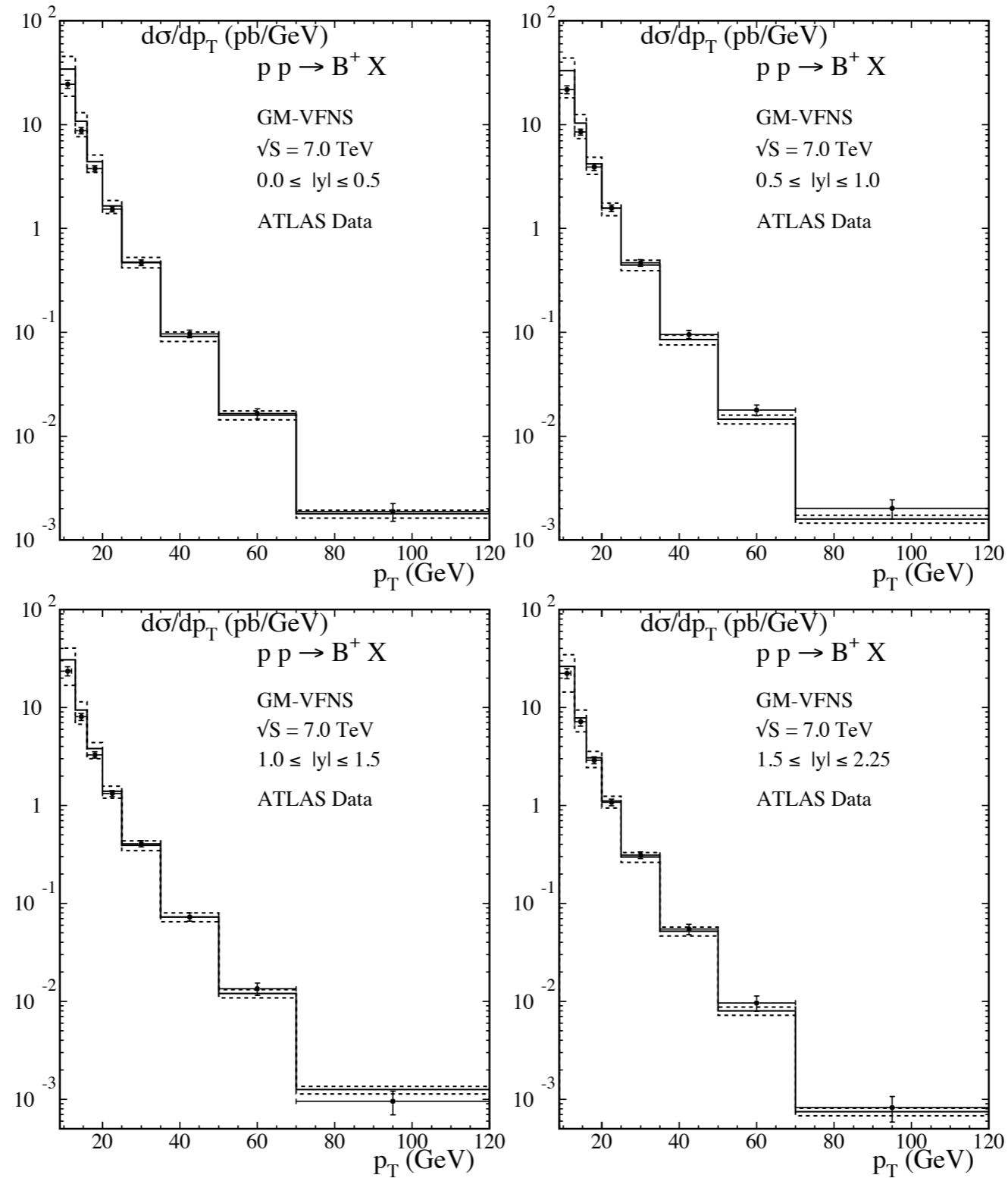


Figure 9: $pp \rightarrow B^+ + X$ at $\sqrt{S} = 7$ TeV in the GM-VFNS compared with data from ATLAS [13]. $\mu_{I,F}$ are frozen below m_b and $\xi_i = (1, 1, 1)$.

GM-VFNS

- FFs in x -space in the BKK approach
- Heavy-quark initiated contributions ($Q+g \rightarrow Q+X, \dots$) get very large at small p_T in the massless case:
 - (i) switch off heavy-quark PDF sufficiently quickly
 - OR
 - (ii) calculate these subprocesses with mass
- Error bands: μ_R , μ_F , $\mu_{F'}$ varied independently
- Predictions for D and B prod. at Tevatron, RHIC, LHC:
[arXiv:1502.01001](#), [1202.0439](#), [1109.2472](#), [0901.4130](#), [0705.4392](#),
[hep-ph/0508129](#), [ph/0502194](#), [ph/0410289](#)
- Predictions including D-decay and B-decay:
[arXiv:1310.2924](#), [1212.4356](#)

Theoretical approaches:
Fixed Order plus Next-to-Leading Logarithms
(FONLL)

FONLL=FO+NLL [1]

$$\text{FONLL} = \text{FO} + (\text{RS} - \text{FOM0})G(m, p_T)$$

FO: Fixed Order; FOM0: Massless limit of FO; RS: Resummed

$$G(m, p_T) = \frac{p_T^2}{p_T^2 + 25m^2} \simeq \begin{cases} 0.04 & : p_T = m \\ 0.25 & : p_T = 3m \\ 0.50 & : p_T = 5m \\ 0.66 & : p_T = 7m \\ 0.80 & : p_T = 10m \end{cases}$$

$$\Rightarrow \text{FONLL} = \begin{cases} \text{FO} & : p_T \lesssim 3m \\ \text{RS} & : p_T \gtrsim 10m \end{cases}$$

[1] Cacciari, Greco, Nason, JHEP05(1998)007

FONLL

- FFs in N-space in the PFF approach

- RS-FOM0 gets very large at small p_T :

$$G(m, p_T) = p_T^2 / (p_T^2 + a^2 m^2) \text{ with } \mathbf{a=5}$$

needed to suppress this contribution sufficiently rapidly

- Central scale choice for FO, RS, FOM0: m_T
- Error bands: $\mu_F = \mu_F'$ (only two scales varied)
- Predictions for LHC7 in [arXiv:1205.6344](#)

Termes in the perturbation series

Termes in the perturbation series

$$L = \ln(m/p_T)$$
$$a = \alpha_s/(2\pi)$$

Resummed



Fixed Order →

	LL	NLL	NNLL	...
LO	1			
NLO	aL	a		
NNLO	(aL) ²	a(aL)	a ²	
...

FFNS/Fixed Order NLO

Resummed



	LL	NLL	NNLL	...
LO $m \neq 0$	1			
NLO $m \neq 0$	aL	a		
NNLO	$(aL)^2$	$a(aL)$	a^2	
...

Fixed Order →

ZM-VFNS/Resummed NLO

Resummed



Fixed Order →

	LL $m=0$	NLL $m=0$	NNLL	...
LO	I			
NLO	aL	a		
NNLO	$(aL)^2$	$a(aL)$	a^2	
...

GM-VFNS/FONLL (NLO+NLL)

Resummed



Fixed Order →

	LL	NLL	NNLL	...
LO	$l_{m \neq 0}$			
NLO	$aL_{m \neq 0}$	$a_{m \neq 0}$		
NNLO	$(aL)_{m=0}^2$	$a(aL)_{m=0}$	a^2	
...	$\dots_{m=0}$	$\dots_{m=0}$	\dots	\dots

NLO Monte Carlo generators: MC@NLO and POWHEG

NLO MC generators

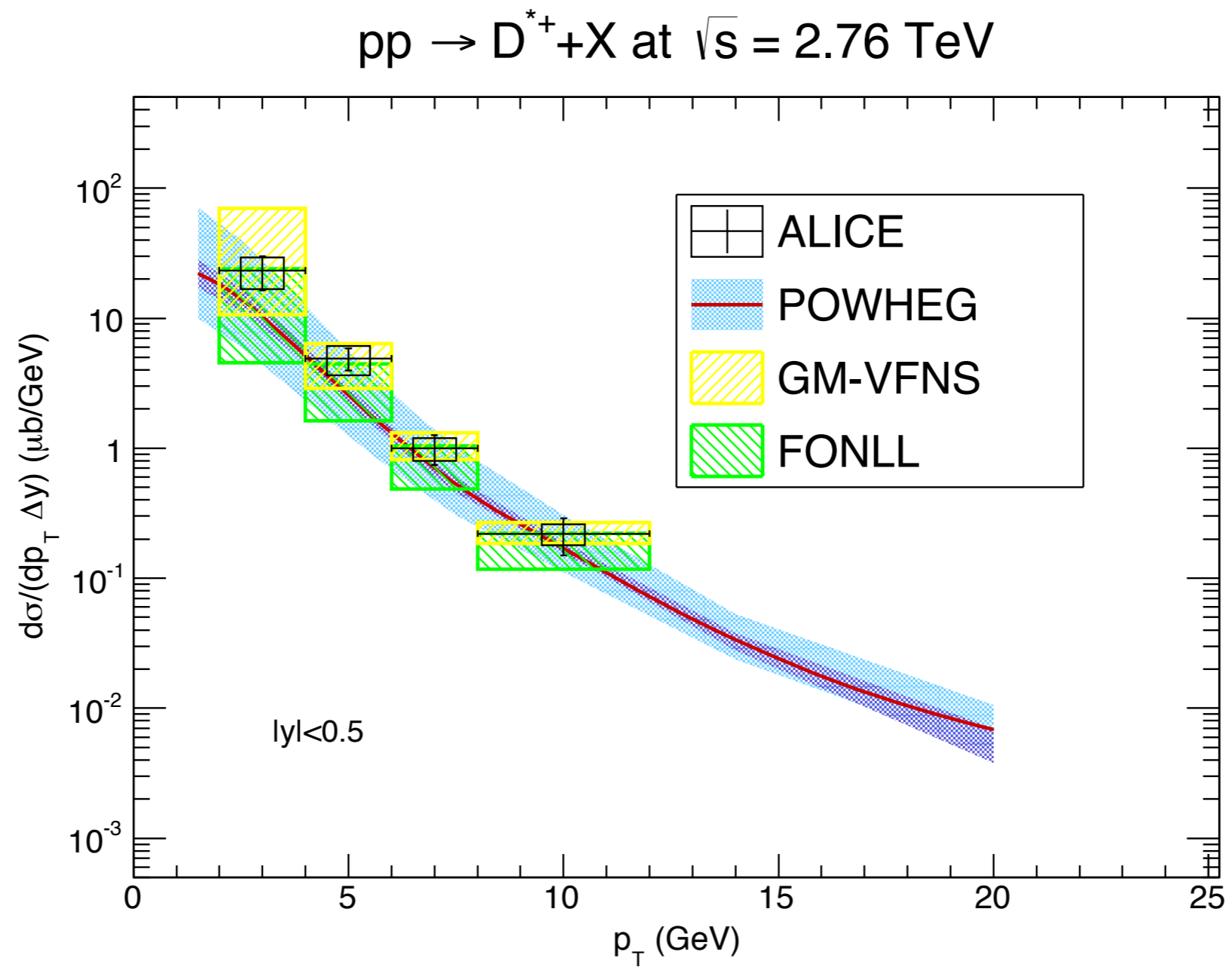
- MC@NLO, POWHEG: [hep-ph/0305252](https://arxiv.org/abs/hep-ph/0305252), [arXiv:0707.3088](https://arxiv.org/abs/0707.3088)
consistent matching of NLO matrix elements with parton showers (PS)
- Flexible simulation of hadronic final state
(PS, hadronization, detector effects)

Note: FONLL and GM-VFNS only one-particle inclusive observables

- High accuracy: NLO+LL*
(FONLL and GM-VFNS have NLO+NLL accuracy)
- Simulation of hadronic final state involves tuning;
NOT a pure theory prediction!

Comparison with ALICE data

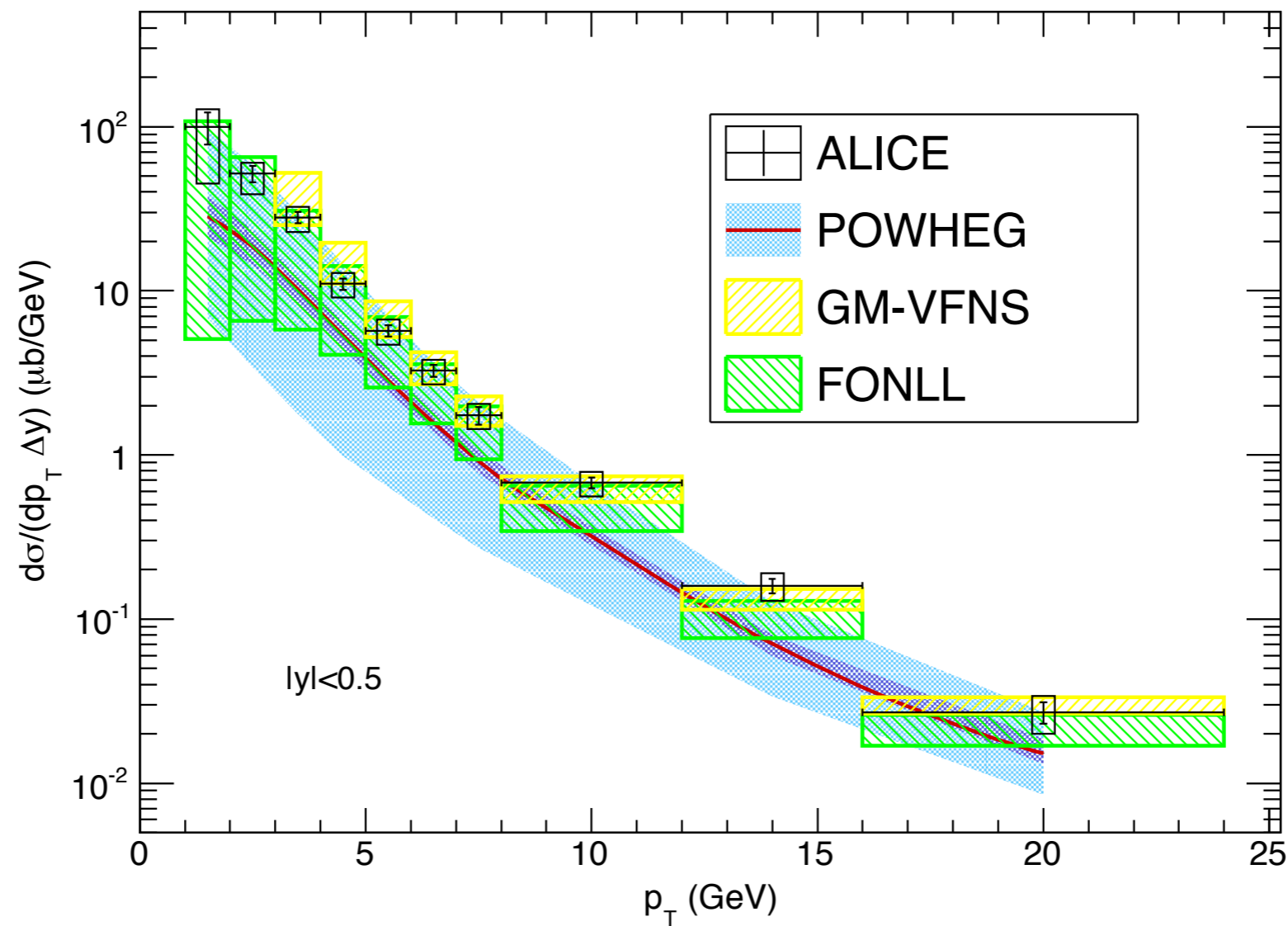
arXiv:1405.3083



Comparison with ALICE data

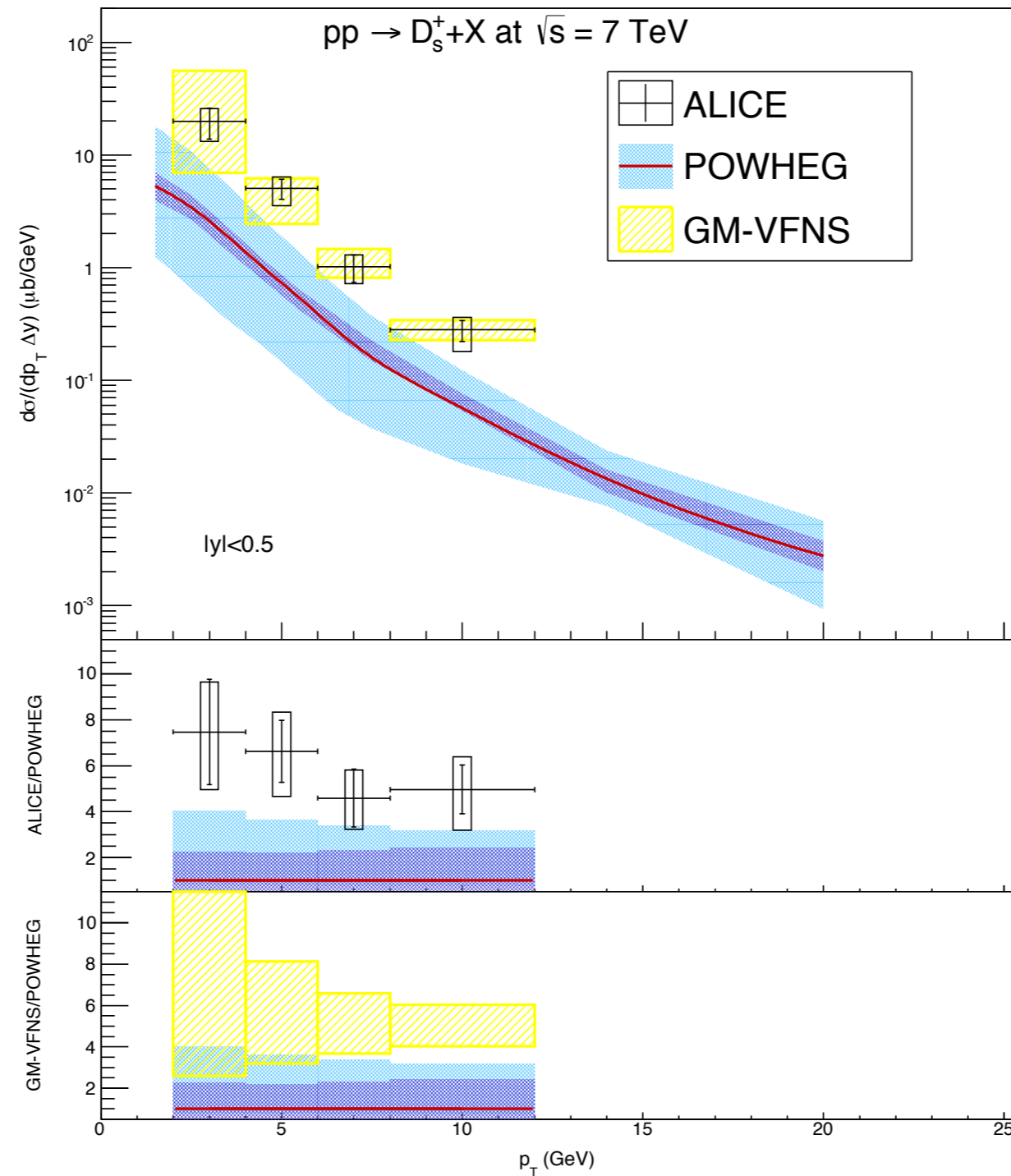
arXiv:1405.3083

$pp \rightarrow D^{*+} + X$ at $\sqrt{s} = 7$ TeV



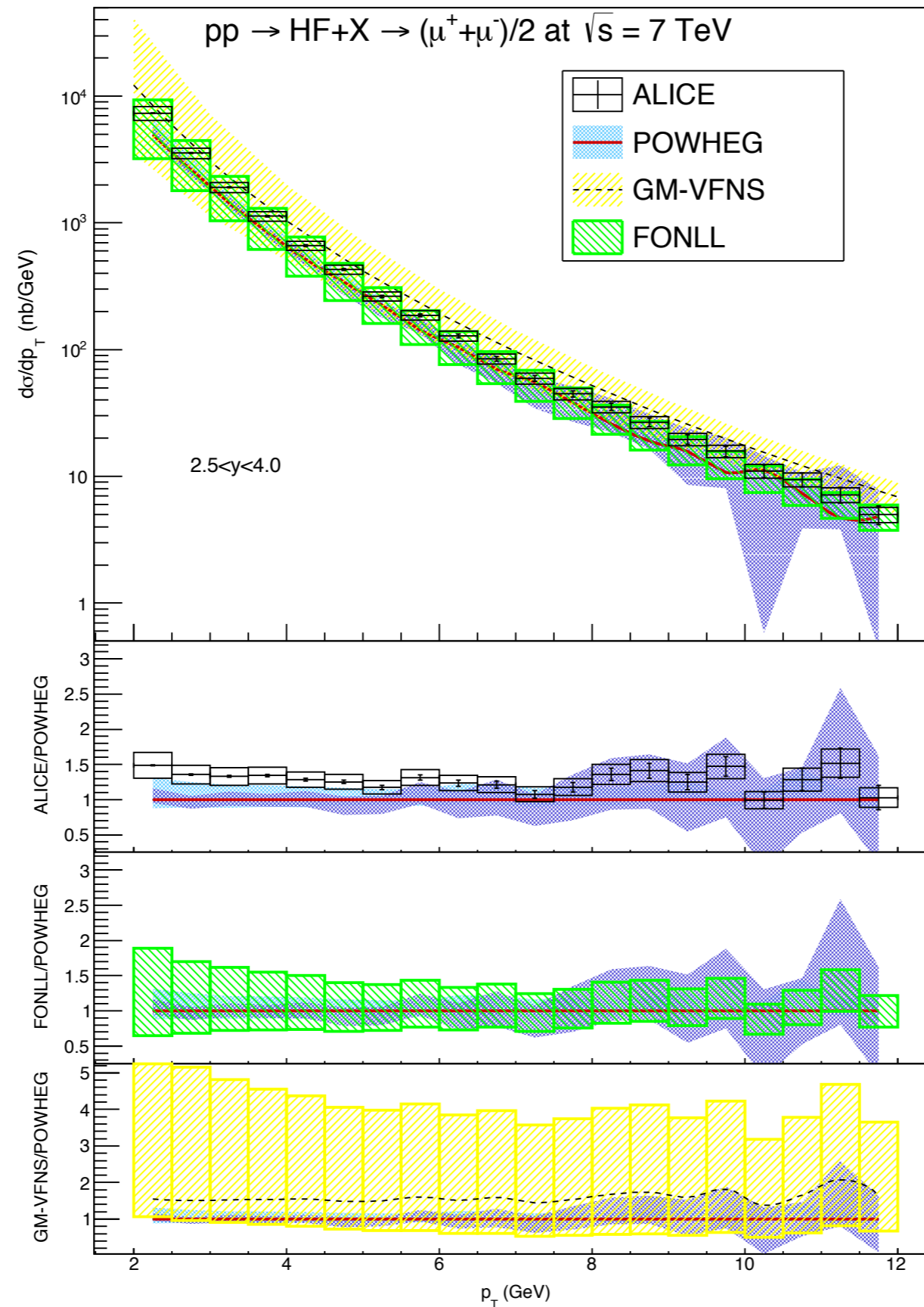
Comparison with ALICE data

arXiv:1405.3083



Comparison with ALICE data

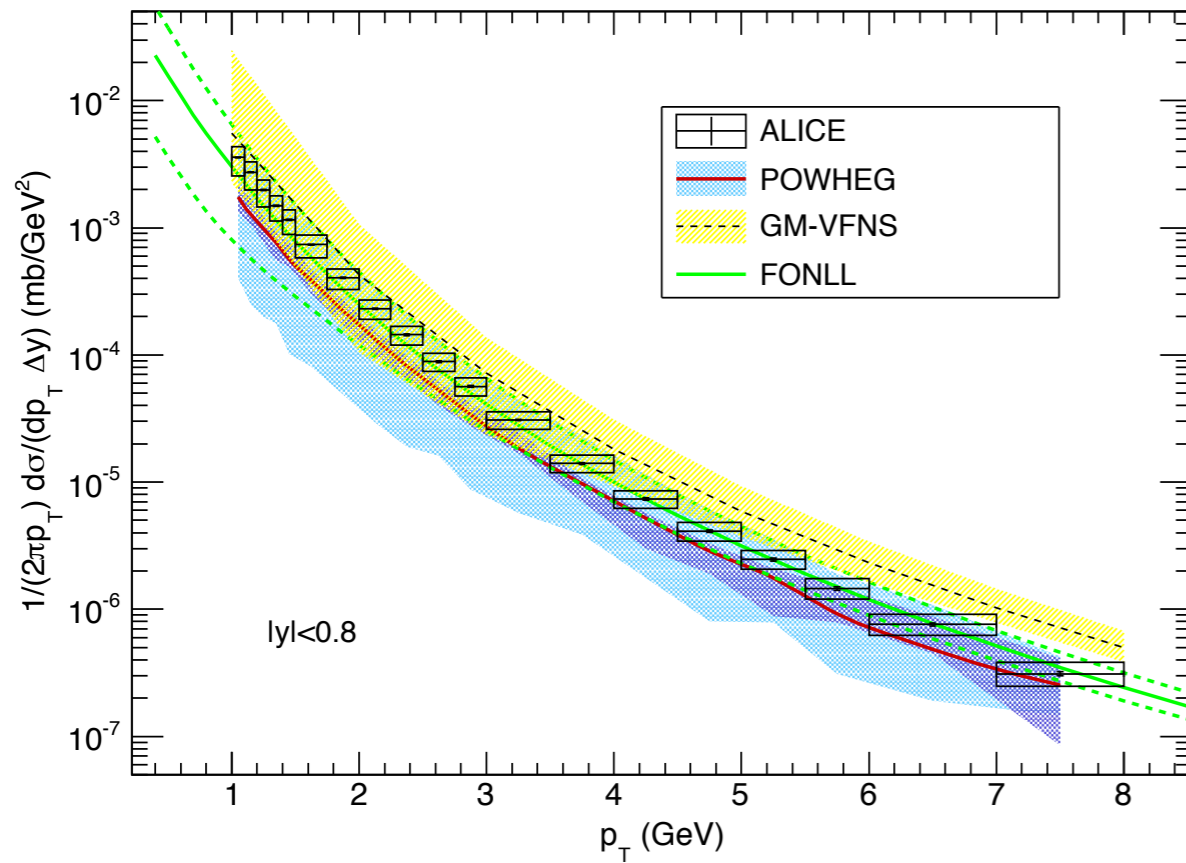
arXiv:1405.3083



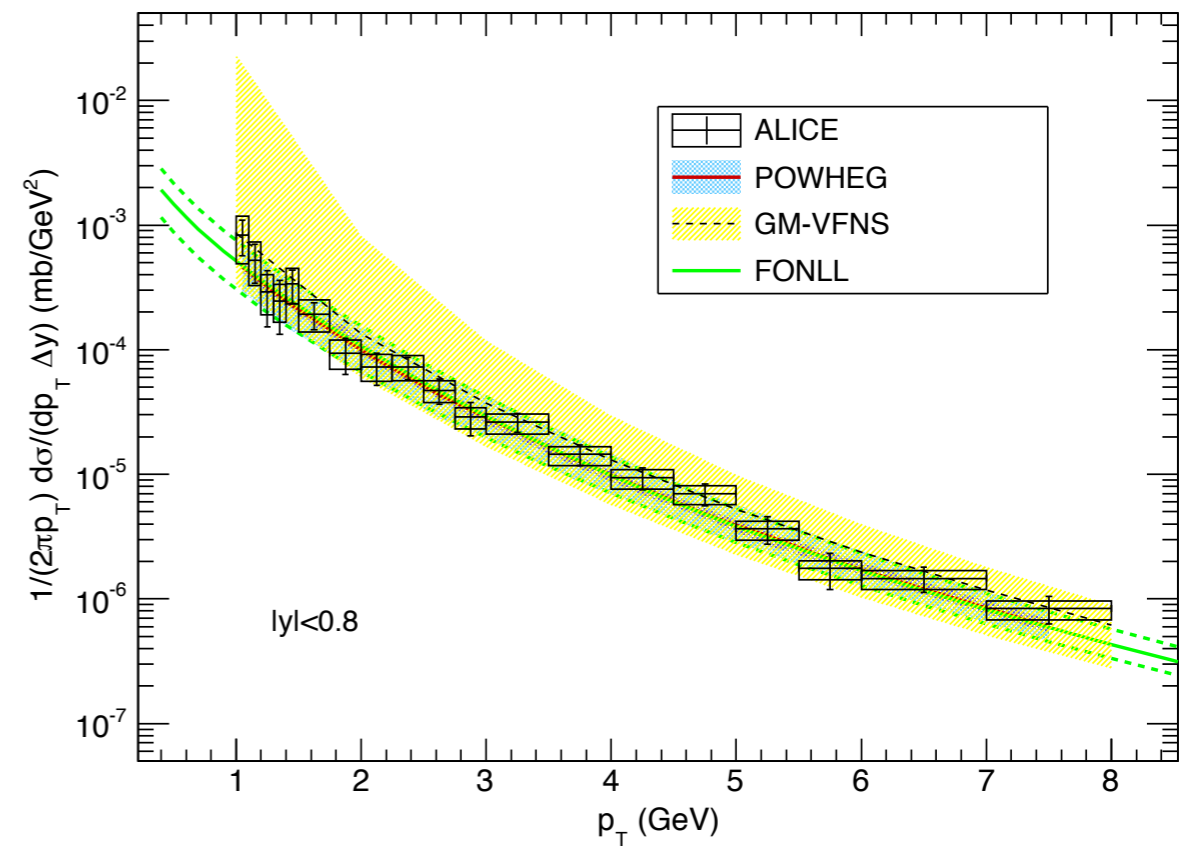
Comparison with ALICE data

arXiv:1405.3083

$pp \rightarrow c+X \rightarrow e^-+X$ at $\sqrt{s} = 7$ TeV



$pp \rightarrow b+X (\rightarrow c+X) \rightarrow e^-+X$ at $\sqrt{s} = 7$ TeV



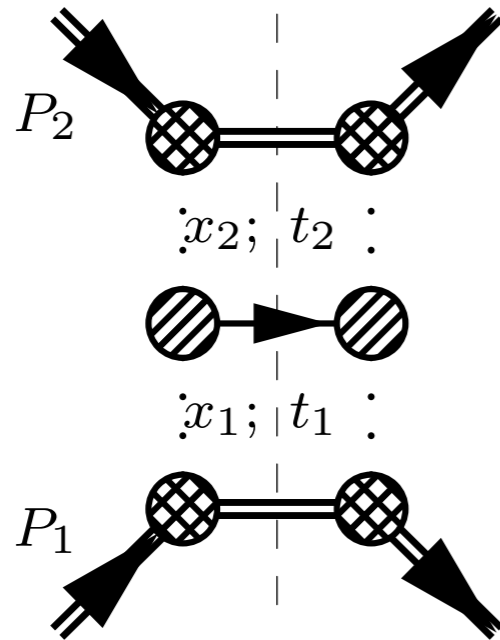
Theoretical approaches: k_T factorization

k_T factorization

k_T factorization: Gribov et al. '83; Collins et al. '91; Catani et al. '91

Central production, small $x \sim 0.01 \dots 0.001$

Factorization:



Factorization formula:

$$d\sigma = \int \frac{d^2 \mathbf{q}_{T1}}{\pi} \int \frac{dx_1}{x_1} \Phi(x_1, t_1, \mu_F) \times \\ \times \int \frac{d^2 \mathbf{q}_{T2}}{\pi} \int \frac{dx_2}{x_2} \Phi(x_2, t_2, \mu_F) d\hat{\sigma}_{PRA}$$

Where Φ - Unintegrated PDFs.

Partonic cross-section:

$$d\hat{\sigma}_{PRA} = \frac{(2\pi)^4}{2x_1 x_2 S} |\overline{\mathcal{M}}|^2_{PRA} \delta^{(4)}(P_{[i]} - P_{[f]}) \times \\ \times \prod_{j=[f]} \frac{d^3 \mathbf{p}_j}{(2\pi)^3 2p_j^0},$$

Normalization of the unPDF:

$$\int^{\mu^2} dt \Phi(x, t, \mu^2) = x f(x, \mu^2),$$

where $f(x, \mu^2)$ - collinear PDF, implies, that the *collinear limit* holds for the amplitude:

$$\int \frac{d\phi_1 d\phi_2}{(2\pi)^2} \lim_{t_{1,2} \rightarrow 0} |\overline{\mathcal{M}}|^2_{PRA} = |\overline{\mathcal{M}}|^2_{CPM}$$

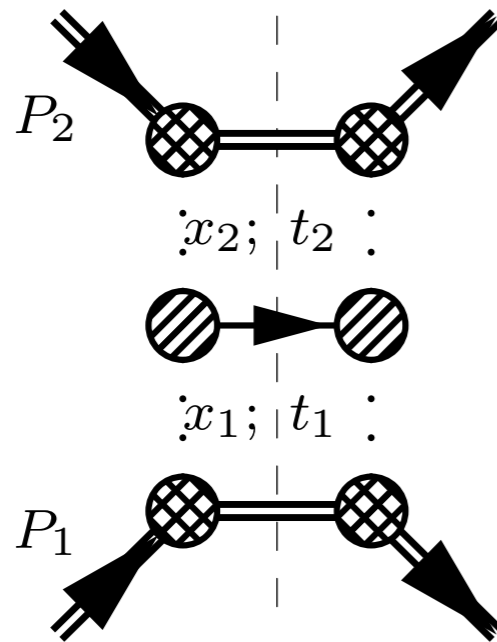
Application to D and B meson production in Parton Reggeization Approach (RPA):
Karpishkov, Nefedov, Saleev, Shipilova, ...

k_T factorization

k_T factorization: Gribov et al. '83; Collins et al. '91; Catani et al. '91

Central production, small $x \sim 0.01 \dots 0.001$

Factorization:



Factorization formula:

$$d\sigma = \int \frac{d^2 \mathbf{q}_{T1}}{\pi} \int \frac{dx_1}{x_1} \Phi(x_1, t_1, \mu_F) \times \\ \times \int \frac{d^2 \mathbf{q}_{T2}}{\pi} \int \frac{dx_2}{x_2} \Phi(x_2, t_2, \mu_F) d\hat{\sigma}_{PRA}$$

Where Φ - Unintegrated PDFs.

Partonic cross-section:

$$d\hat{\sigma}_{PRA} = \frac{(2\pi)^4}{2x_1 x_2 S} |\overline{\mathcal{M}}|^2_{PRA} \delta^{(4)}(P_{[i]} - P_{[f]}) \times \\ \times \prod_{j=[f]} \frac{d^3 \mathbf{p}_j}{(2\pi)^3 2p_j^0},$$

Normalization of the unPDF:

$$\int^{\mu^2} dt \Phi(x, t, \mu^2) = x f(x, \mu^2),$$

where $f(x, \mu^2)$ - collinear PDF, implies, that the *collinear limit* holds for the amplitude:

$$\int \frac{d\phi_1 d\phi_2}{(2\pi)^2} \lim_{t_{1,2} \rightarrow 0} |\overline{\mathcal{M}}|^2_{PRA} = |\overline{\mathcal{M}}|^2_{CPM}$$

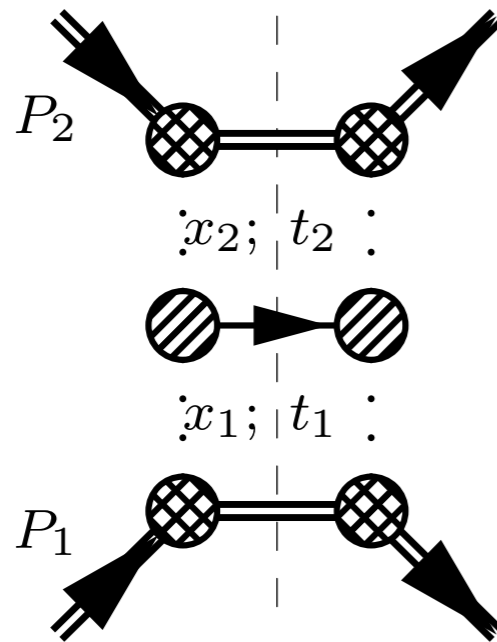
Application to D and B meson production in Parton Reggeization Approach (RPA):
Karpishkov, Nefedov, Saleev, Shipilova, ...

k_T factorization

k_T factorization: Gribov et al. '83; Collins et al. '91; Catani et al. '91

Central production, small $x \sim 0.01 \dots 0.001$

Factorization:



Factorization formula:

$$d\sigma = \int \frac{d^2 \mathbf{q}_{T1}}{\pi} \int \frac{dx_1}{x_1} \Phi(x_1, t_1, \mu_F) \times \\ \times \int \frac{d^2 \mathbf{q}_{T2}}{\pi} \int \frac{dx_2}{x_2} \Phi(x_2, t_2, \mu_F) d\hat{\sigma}_{PRA}$$

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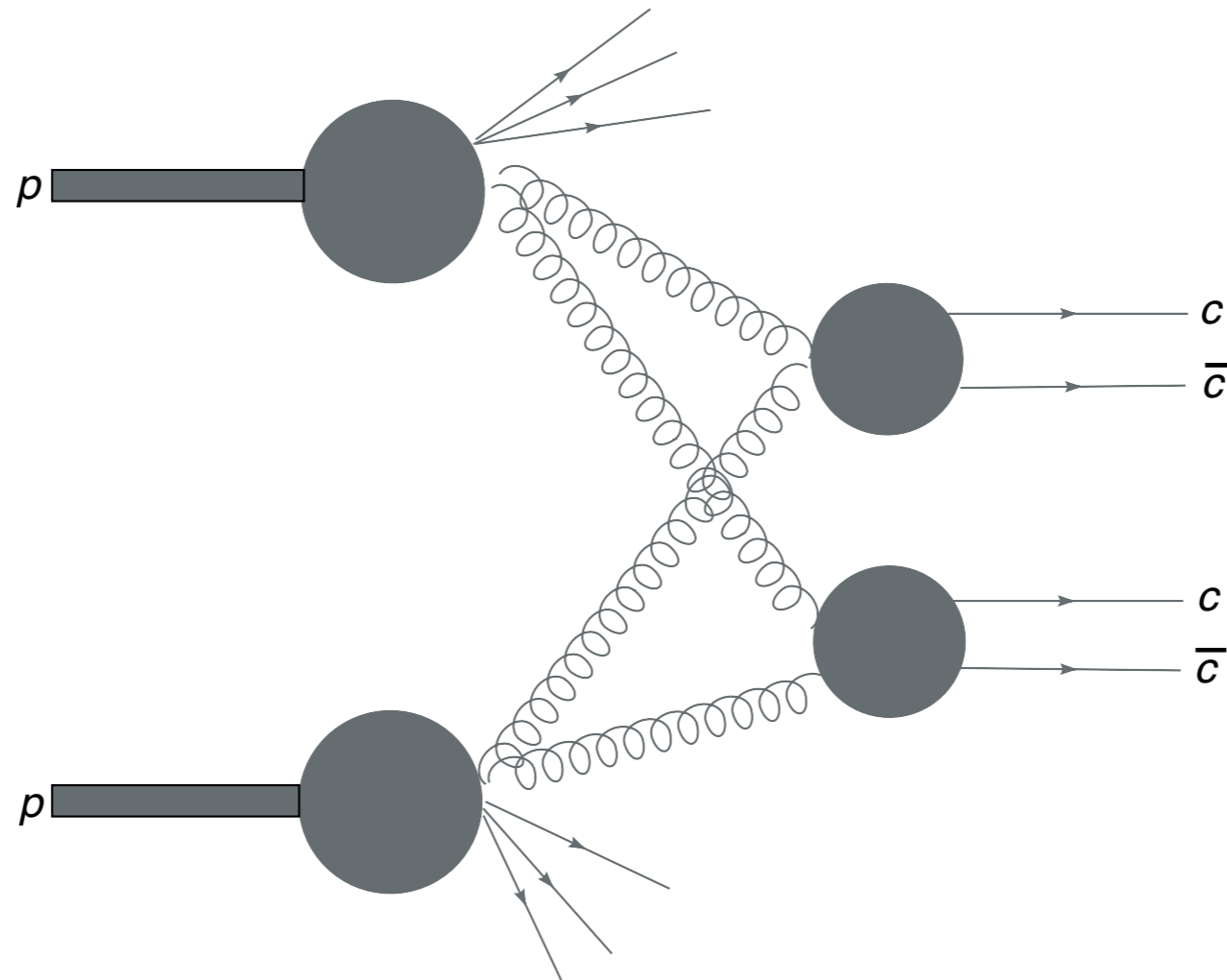
$$\int \frac{d\phi_1 d\phi_2}{(2\pi)^2} \lim_{t_{1,2} \rightarrow 0} |\overline{\mathcal{M}}|^2_{PRA} = |\overline{\mathcal{M}}|^2_{CPM}$$

Application to D and B meson production in Parton Reggeization Approach (RPA):
Karpishkov, Nefedov, Saleev, Shipilova, ...

Theoretical approaches:
Double parton scattering

Production of two $c\bar{c}$ pairs in double-parton scattering

Consider two hard (parton) scatterings



Not considered so far in the literature

Luszczak, Maciula, Szczurek, arXiv:1111.3255



Factorization of double parton scattering?

arXiv:1410.6664

- Correlations between the two momentum fractions, the transverse separation of partons and/or flavor
- Spin correlations between the partons
- Color correlations between the partons
- Interferences in fermion number
- Interferences in flavor

Formalism

$$d\sigma^{DPS} = \frac{1}{2\sigma_{\text{eff}}} F_{gg}(x_1, x_2, \mu_1^2, \mu_2^2) F_{gg}(x'_1, x'_2, \mu_1^2, \mu_2^2) d\sigma_{gg \rightarrow c\bar{c}}(x_1, x'_1, \mu_1^2) d\sigma_{gg \rightarrow c\bar{c}}(x_2, x'_2, \mu_2^2) dx_1 dx_2 dx'_1 dx'_2 .$$

$F_{gg}(x_1, x_2, \mu_1^2, \mu_2^2), F_{gg}(x'_1, x'_2, \mu_1^2, \mu_2^2)$
are called **double parton distributions**

dPDF are subjected to special **evolution equations**

single scale evolution: **Snigireev**

double scale evolution: **Ceccopieri, Gaunt-Stirling**



Formalism

Consider reaction: $pp \rightarrow c\bar{c}c\bar{c}X$

Modeling double-parton scattering

Factorized form:

If two hard scatterings are completely independent!
(very rough!)

$$\sigma^{DPS}(pp \rightarrow c\bar{c}c\bar{c}X) = \frac{1}{2\sigma_{\text{eff}}} \sigma^{SPS}(pp \rightarrow c\bar{c}X_1) \cdot \sigma^{SPS}(pp \rightarrow c\bar{c}X_2).$$

The simple formula can be generalized to include differential distributions

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} dy_3 dy_4 d^2p_{2t}} = \frac{1}{2\sigma_{\text{eff}}} \cdot \frac{d\sigma}{dy_1 dy_2 d^2p_{1t}} \cdot \frac{d\sigma}{dy_3 dy_4 d^2p_{2t}}.$$

σ_{eff} is a model parameter (12-15 mb)



Theoretical approaches: diffractive production

Diffractive production

Szczurek et al., arXiv:1412.3132

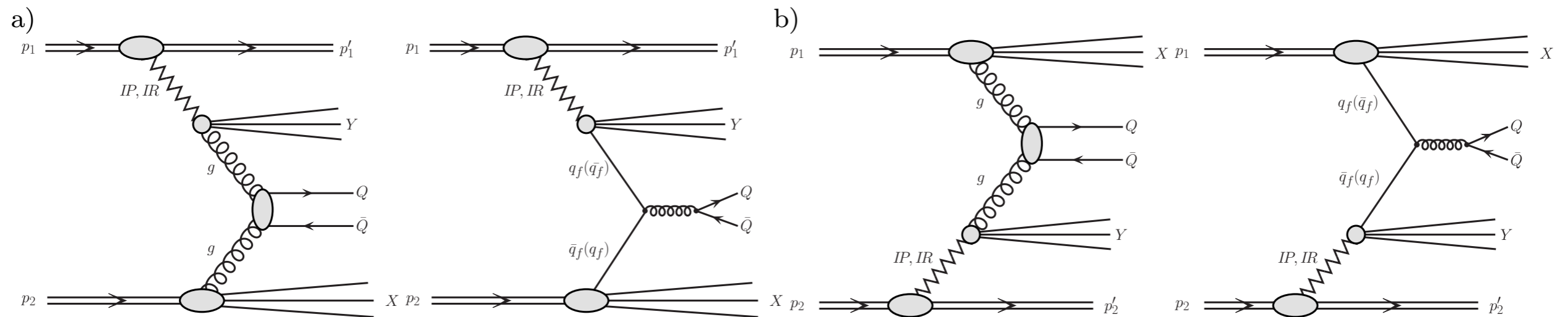


FIG. 1: The mechanisms of single-diffractive production of heavy quarks.

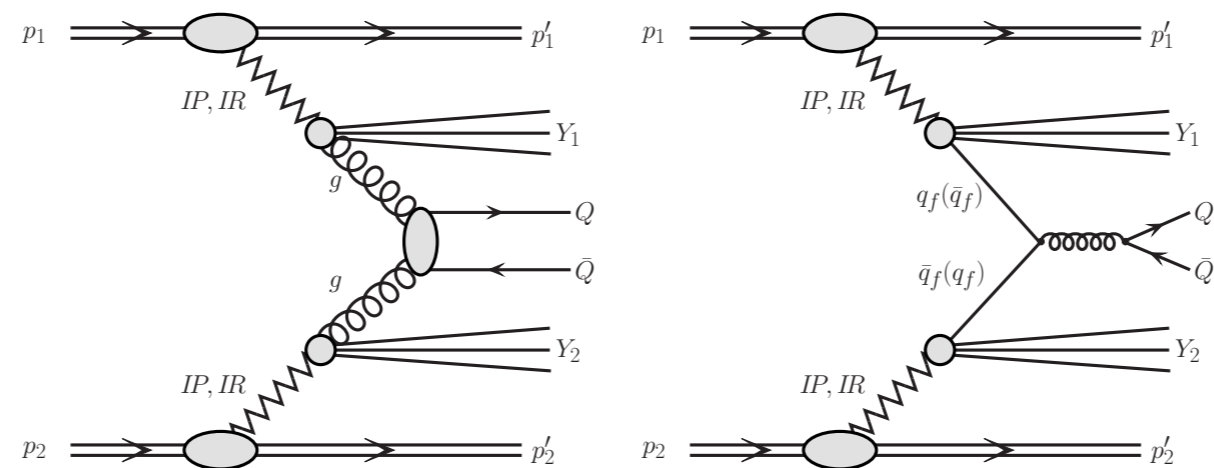


FIG. 2: The mechanisms of central-diffractive production of heavy quarks.

IV. SUMMARY

Summary

- Discussed different theoretical approaches to open heavy flavor hadroproduction
- GM-VFNS, FONLL, POWHEG in good agreement with data within large uncertainties!
- pA data for B meson production useful for constraining nuclear gluon PDF
- GM-VFNS at low p_T improved; more work