Heavy quark spectroscopy and prediction of bottom baryon masses

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I discuss several recent highly accurate theoretical predictions for masses of baryons containing the b quark, as well as an effective supersymmetry between heavy quark baryons and mesons. I also suggest some possibilities for observing exotic hadrons containing heavy quarks.

1. Introduction

QCD describes hadrons as valence quarks in a sea of gluons and $\bar{q}q$ pairs. At distances above $\sim 1\ \mathrm{GeV}^{-1}$ quarks acquire an effective *constituent mass* due to chiral symmetry breaking. A hadron can then be thought of as a bound state of constituent quarks. In the simplest approximation the hadron mass M is then given by the sum of the masses of its constituent quarks m_i, first written down by Sakharov and Zeldovich:

$$M = \sum_i m_i \tag{1}$$

the binding and kinetic energies are swallowed by the constituent quarks masses.

The first and most important correction comes from the color hyper-fine (HF) chromo-magnetic interaction,

$$M = \sum_i m_i + V_{i<j}^{HF(QCD)} \tag{2}$$

$$V_{ij}^{HF(QCD)} = v_0\, (\vec{\lambda}_i \cdot \vec{\lambda}_j)\, \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i m_j}\, \langle \psi | \delta(r_i - r_j) | \psi \rangle$$

where v_0 gives the overall strength of the HF interaction, $\vec{\lambda}_{i,j}$ are the $SU(3)$ color matrices, $\sigma_{i,j}$ are the quark spin operators and $|\psi\rangle$ is the hadron wave function. This is a contact spin-spin interaction, analogous to the EM hyperfine interaction, which is a product of the magnetic moments,

$$V_{ij}^{HF(QED)} \propto \vec{\mu}_i \cdot \vec{\mu}_j = e^2\, \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i m_j} \tag{3}$$

in QCD, the $SU(3)_c$ generators take place of the electric charge.

From eq. (3) many very accurate results have been obtained for the masses of the ground-state hadrons. Nevertheless, several caveats are in order. First, this is a low-energy phenomenological model, still awaiting a rigorous derivation from QCD. It is far from providing a complete description of the hadronic spectrum, but it provides excellent predictions for mass splittings and magnetic moments.

The crucial assumptions of the model are:

(a) HF interaction is considered as a perturbation which does not change the wave function
(b) effective masses of quarks are the same inside mesons and baryons
(c) there are no 3-body effects.

2. Quark masses

As the first example of the application of eq. (3) we can obtain the $m_c - m_s$ quark mass difference from the $\Lambda_c - \Lambda$ baryon mass difference:

$$\begin{aligned} &M(\Lambda_c) - M(\Lambda) = \\ &= (m_u + m_d + m_c + V_{ud}^{HF} + V_{uc}^{HF} + V_{dc}^{HF}) \qquad (4) \\ &- (m_u + m_d + m_s + V_{ud}^{HF} + V_{us}^{HF} + V_{ds}^{HF}) \\ &= m_c - m_s \end{aligned}$$

where the light-quark HF interaction terms V_{ud}^{HF} cancel between the two expressions and the HF interaction terms between the heavy and light quarks vanish: $V_{us}^{HF} = V_{ds}^{HF} = V_{uc}^{HF} = V_{dc}^{HF} = 0$, since the u and d light quarks are coupled to a spin-zero diquark and the HF interaction couples to the spin.

A second example shows how we can extract the ratio of the constituent quark masses from the ratio of the the hyperfine splittings in the corresponding mesons. The hyperfine splitting between K^* and K mesons is given by

$$M(K^*) - M(K) =$$
$$= v_0 \frac{\vec{\lambda}_u \cdot \vec{\lambda}_s}{m_u m_s} \left[(\vec{\sigma}_u \cdot \vec{\sigma}_s)_{K^*} - (\vec{\sigma}_u \cdot \vec{\sigma}_s)_K \right] \langle \psi | \delta(r) | \psi \rangle$$
$$= 4v_0 \frac{\vec{\lambda}_u \cdot \vec{\lambda}_s}{m_u m_s} \langle \psi | \delta(r) | \psi \rangle, \qquad (5)$$

and similarly for hyperfine splitting between D^* and D with $s \to c$ everywhere. From (5) and its D analogue we then immediately obtain

$$\frac{M(K^*)-M(K)}{M(D^*)-M(D)} = \frac{4v_0 \dfrac{\vec{\lambda}_u \cdot \vec{\lambda}_s}{m_u m_s} \langle \psi | \delta(r) | \psi \rangle}{4v_0 \dfrac{\vec{\lambda}_u \cdot \vec{\lambda}_c}{m_u m_c} \langle \psi | \delta(r) | \psi \rangle} \approx \frac{m_c}{m_s} \qquad (6)$$

Table I shows the quark mass differences obtained from mesons and baryons [1]. The mass difference between two quarks of different flavors denoted by i and j are seen to have the same value to a good approximation when they are bound to a "spectator" quark of a given flavor.

On the other hand, Table I shows clearly that *constituent quark mass differences depend strongly on the flavor of the spectator quark.* For example, $m_s - m_d \approx 180$ MeV when the spectator is a light quark but the same mass difference is only about 90 MeV when the spectator is a b quark.

Since these are *effective masses*, we should not be surprised that their difference is affected by the environment, but the large size of the shift is quite surprising and its quantitative derivation from QCD is an outstanding challenge for theory.

Table 1
Quark mass differences from baryons and mesons

observable	baryons		mesons $J=1$		mesons $J=0$		Δm_{Bar} MeV	Δm_{Mes} MeV
	B_i	B_j	$\mathcal{V}_i$	$\mathcal{V}_j$	$\mathcal{P}_i$	$\mathcal{P}_j$		
$\langle m_s - m_u \rangle_d$	sud	uud	$s\bar{d}$	$u\bar{d}$	$s\bar{d}$	$u\bar{d}$	177	179
	Λ	N	K^*	ρ	K	π		
$\langle m_s - m_u \rangle_c$			$c\bar{s}$	$c\bar{u}$	$c\bar{s}$	$c\bar{u}$		103
			D_s^*	D_s^*	D_s	D_s		
$\langle m_s - m_u \rangle_b$			$b\bar{s}$	$b\bar{u}$	$b\bar{s}$	$b\bar{u}$		91
			B_s^*	B_s^*	B_s	B_s		
$\langle m_c - m_u \rangle_d$	cud	uud	$c\bar{d}$	$u\bar{d}$	$c\bar{d}$	$u\bar{d}$	1346	1360
	Λ_c	N	D^*	ρ	D	π		
$\langle m_c - m_u \rangle_c$			$c\bar{c}$	$u\bar{c}$	$c\bar{c}$	$u\bar{c}$		1095
			ψ	D^*	η_c	D		
$\langle m_c - m_s \rangle_d$	cud	sud	$c\bar{d}$	$s\bar{d}$	$c\bar{d}$	$s\bar{d}$	1169	1180
	Λ_c	Λ	D^*	K^*	D	K		
$\langle m_c - m_s \rangle_c$			$c\bar{c}$	$s\bar{c}$	$c\bar{c}$	$s\bar{c}$		991
			ψ	D_s^*	η_c	D_s		
$\langle m_b - m_u \rangle_d$	bud	uud	$b\bar{d}$	$u\bar{d}$	$b\bar{d}$	$u\bar{d}$	4685	4700
	Λ_b	N	B^*	ρ	B	π		
$\langle m_b - m_u \rangle_s$			$b\bar{s}$	$u\bar{s}$	$b\bar{s}$	$u\bar{s}$		4613
			B_s^*	K^*	B_s	K		
$\langle m_b - m_s \rangle_d$	bud	sud	$b\bar{d}$	$s\bar{d}$	$b\bar{d}$	$s\bar{d}$	4508	4521
	Λ_b	Λ	B^*	K^*	B	K		
$\langle m_b - m_c \rangle_d$	bud	sud	$b\bar{d}$	$c\bar{d}$	$b\bar{d}$	$c\bar{d}$	3339	3341
	Λ_b	Λ_c	B^*	D^*	B	D		
$\langle m_b - m_c \rangle_s$			$b\bar{s}$	$c\bar{s}$	$b\bar{s}$	$c\bar{s}$		3328
			B_s^*	D_s^*	B_s	D_s		

2.1. Color hyperfine splitting in baryons

As an example of hyperfine splitting in baryons, let us now discuss the HF splitting in the Σ (uds) baryons. Σ^* has spin $\frac{3}{2}$, so the u and d quarks must be in a state of relative spin 1. The Σ has isospin 1, so the wave function of u and d is symmetric in flavor. It is also symmetric in space, since in the ground state the quarks are in a relative S-wave. On the other hand, the u-d wave function is antisymmetric in color, since the two quarks must couple to a $\mathbf{3}^*$ of color to neutralize the color of the third quark. The u-d wave function must be antisymmetric in flavor $\times$ spin $\times$ space $\times$ color, so it follows it must be symmetric in spin, i.e. u and d are coupled to spin one. Since u and d are in spin 1 state in both Σ^* and Σ their HF interaction with each other cancels between the two and thus the u-d pair does not contribute to the $\Sigma^* - \Sigma$ HF splitting,

$$M(\Sigma^*) - M(\Sigma) = 6v_0 \frac{\vec{\lambda}_u \cdot \vec{\lambda}_s}{m_u m_s} \langle \psi | \delta(r_{rs}) | \psi \rangle \qquad (7)$$

we can then use eqs. (5) and (7) to compare the quark mass ratio obtained from mesons and baryons:

$$\left(\frac{m_c}{m_s}\right)_{Bar} = \frac{M_{\Sigma^*} - M_{\Sigma}}{M_{\Sigma_c^*} - M_{\Sigma_c}} = 2.84$$

$$\left(\frac{m_c}{m_s}\right)_{Mes} = \frac{M_{K^*} - M_{K}}{M_{D^*} - M_{D}} = 2.81 \tag{8}$$

$$\left(\frac{m_c}{m_u}\right)_{Bar} = \frac{M_{\Delta} - M_{p}}{M_{\Sigma_c^*} - M_{\Sigma_c}} = 4.36$$

$$\left(\frac{m_c}{m_u}\right)_{Mes} = \frac{M_{\rho} - M_{\pi}}{M_{D^*} - M_{D}} = 4.46 \tag{9}$$

We find the same value from mesons and baryons $\pm 2\%$.

The presence of a fourth flavor gives us the possibility of obtaining a new type of mass relation between mesons and baryons. The $\Sigma - \Lambda$ mass difference is believed to be due to the difference between the $u-d$ and $u-s$ hyperfine interactions. Similarly, the $\Sigma_c - \Lambda_c$ mass difference is believed to be due to the difference between the $u-d$ and $u-c$ hyperfine interactions. We therefore obtain the relation

$$\left(\frac{\frac{1}{m_u^2} - \frac{1}{m_u m_c}}{\frac{1}{m_u^2} - \frac{1}{m_u m_s}}\right)_{Bar} = \frac{M_{\Sigma_c} - M_{\Lambda_c}}{M_{\Sigma} - M_{\Lambda}} = 2.16$$

$$\left(\frac{\frac{1}{m_u^2} - \frac{1}{m_u m_c}}{\frac{1}{m_u^2} - \frac{1}{m_u m_s}}\right)_{Mes} = \frac{(M_\rho - M_\pi) - (M_{D^*} - M_D)}{(M_\rho - M_\pi) - (M_{K^*} - M_K)} = 2.10 \tag{10}$$

The meson and baryon relations agree to $\pm 3\%$.

We can now write down an analogous relation for hadrons containing the b quark instead of the s quark, obtaining the prediction for splitting between Σ_b and Λ_b :

$$\frac{M_{\Sigma_b} - M_{\Lambda_b}}{M_{\Sigma} - M_{\Lambda}} = \frac{(M_\rho - M_\pi) - (M_{B^*} - M_B)}{(M_\rho - M_\pi) - (M_{K^*} - M_K)} = 2.51 \tag{11}$$

yielding $M_{\Sigma_b} - M_{\Lambda_b} = 194\,\mathrm{MeV}$ [1,2].

This splitting was recently measured by CDF [3]. They obtained the masses of the Σ_b^- and Σ_b^+ from the decay $\Sigma_b \to \Lambda_b + \pi$ by measuring the corresponding mass differences in MeV

$$M(\Sigma_b^-) - M(\Lambda_b) = 195.5^{+1.0}_{-1.0}\,(\mathrm{stat.}) \pm 0.1\,(\mathrm{syst.})$$

$$M(\Sigma_b^+) - M(\Lambda_b) = 188.0^{+2.0}_{-2.3}\,(\mathrm{stat.}) \pm 0.1\,(\mathrm{syst.}) \tag{12}$$

with isospin-averaged mass difference $M(\Sigma_b) - M(\Lambda_b) = 192$ MeV, as shown in Fig. 1.

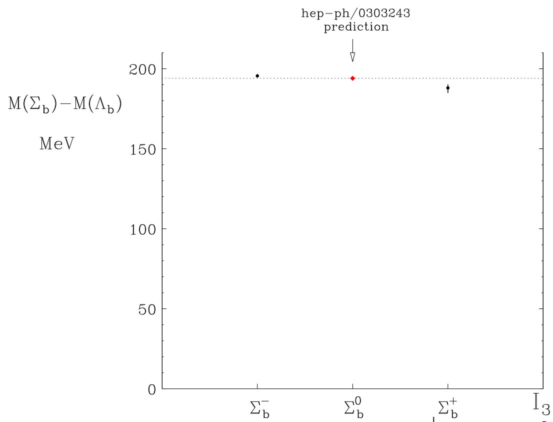

Figure 1. Experimental results for $\Sigma_b^\pm$ masses [3] vs the theoretical prediction in Ref. [1].

There is also the prediction for the spin splittings, good to 5%

$$M(\Sigma_b^*) - M(\Sigma_b) = \tag{13}$$
$$= \frac{M(B^*) - M(B)}{M(K^*) - M(K)} \cdot [M(\Sigma^*) - M(\Sigma)] = 22\,\mathrm{MeV}$$

to be compared with 21 MeV from the isospin-average of CDF measurements [3].

The relation (11) is based on the assumption that the qq and $q\bar{q}$ interactions have the same flavor dependence. This automatically follows from the assumption that both hyperfine interactions are inversely proportional to the products of the same quark masses. But all that is needed here is the weaker assumption of same flavor dependence,

$$\frac{V_{hyp}(q_i \bar{q}_j)}{V_{hyp}(q_i \bar{q}_k)} = \frac{V_{hyp}(q_i q_j)}{V_{hyp}(q_i q_k)} \tag{14}$$

This yields [1]

$$\frac{M_{\Sigma_b} - M_{\Lambda_b}}{(M_\rho - M_\pi) - (M_{B^*} - M_B)} = 0.32 \approx$$

$$\approx \frac{M_{\Sigma_c} - M_{\Lambda_c}}{(M_\rho - M_\pi) - (M_{D^*} - M_D)} = 0.33 \approx \qquad (15)$$

$$\approx \frac{M_\Sigma - M_\Lambda}{(M_\rho - M_\pi) - (M_{K^*} - M_K)} = 0.325 \qquad (16)$$

The baryon-meson ratios are seen to be independent of the flavor f.

The challenge is to understand how and under what assumptions one can derive from QCD the very simple model of hadronic structure at low energies which leads to such accurate predictions.

3. Effective Meson-Baryon SUSY

Some of the results described above can be understood [2] by observing that in the hadronic spectrum there is an approximate effective supersymmetry between mesons and baryons related by replacing a light antiquark by a light diquark.

This supersymmetry transformation goes beyond the simple constituent quark model. It assumes only a valence quark of flavor i with a model independent structure bound to "light quark brown muck color antitriplet" of model-independent structure carrying the quantum numbers of a light antiquark or a light diquark. Since it assumes no model for the valence quark, nor the brown muck antitriplet coupled to the valence quark, it holds also for the quark-parton model in which the valence is carried by a current quark and the rest of the hadron is a complicated mixture of quarks and antiquarks.

This light quark supersymmetry transformation, denoted here by T^S_{LS}, connects a meson denoted by $|\mathcal{M}(\bar{q}Q_i)\rangle$ and a baryon denoted by $|\mathcal{B}([qq]_S Q_i)\rangle$ both containing the same valence quark of some fixed flavor Q_i, $i = (u, s, c, b)$ and a light color-antitriplet "brown muck" state with the flavor and baryon quantum numbers respectively of an antiquark $\bar{q}$ (u or d) and two light quarks coupled to a diquark of spin S.

$$T^S_{LS}|\mathcal{M}(\bar{q}Q_i)\rangle \equiv |\mathcal{B}([qq]_S Q_i)\rangle \qquad (17)$$

The mass difference between the meson and baryon related by this T^S_{LS} transformation has been shown [4] to be independent of the quark flavor i for all four flavors (u, s, c, b) when the contribution of the hyperfine interaction energies is removed. For the two cases of spin-zero[4] $S = 0$ and spin-one $S = 1$ diquarks,

$$\begin{aligned} M(N) - \tilde{M}(\rho) &= 323 \text{ MeV} \approx \\ \approx M(\Lambda) - \tilde{M}(K^*) &= 321 \text{ MeV} \approx \\ \approx M(\Lambda_c) - \tilde{M}(D^*) &= 312 \text{ MeV} \approx \\ \approx M(\Lambda_b) - \tilde{M}(B^*) &= 310 \text{ MeV} \end{aligned} \qquad (18)$$

$$\begin{aligned} \tilde{M}(\Delta) - \tilde{M}(\rho) &= 517.56 \text{ MeV} \approx \\ \approx \tilde{M}(\Sigma) - \tilde{M}(K^*) &= 526.43 \text{ MeV} \approx \\ \approx \tilde{M}(\Sigma_c) - \tilde{M}(D^*) &= 523.95 \text{ MeV} \approx \\ \approx \tilde{M}(\Sigma_b) - \tilde{M}(B^*) &= 512.45 \text{ MeV} \end{aligned} \qquad (19)$$

where

$$\tilde{M}(V_i) \equiv \frac{3M_{\mathcal{V}_i} + M_{\mathcal{P}_i}}{4}; \qquad (20)$$

are the weighted averages of vector and pseudoscalar meson masses, denoted respectively by $M_{\mathcal{V}_i}$ and $M_{\mathcal{P}_i}$, which cancel their hyperfine contribution, and

$$\tilde{M}(\Sigma_i) \equiv \frac{2M_{\Sigma^*_i} + M_{\Sigma_i}}{3}; \tilde{M}(\Delta) \equiv \frac{2M_\Delta + M_N}{3} \qquad (21)$$

are the analogous weighted averages of baryon masses which cancel the hyperfine contribution between the diquark and the additional quark.

4. Magnetic Moments of Heavy Quark Baryons

In Λ, Λ_c and Λ_b baryons the light quarks are coupled to spin zero. Therefore the magnetic moments of these baryons are determined by the magnetic moments of the s, c and b quarks, respectively. The latter are proportional to the chromomagnetic moments which determine the hyperfine splitting in baryon spectra. We can use this fact to predict the Λ_c and Λ_b baryon magnetic moments by relating them to the hyperfine splittings in the same way as given in the original prediction [5] of the Λ magnetic moment,

$$\mu_\Lambda = -\frac{\mu_p}{3} \cdot \frac{M_{\Sigma^*} - M_\Sigma}{M_\Delta - M_N} = -0.61 \, \text{n.m.} \quad (\text{EXP} = -0.61 \, \text{n.m.}) \qquad (22)$$

We obtain

$$\mu_{\Lambda_c} = -2\mu_\Lambda \cdot \frac{M_{\Sigma_c^*} - M_{\Sigma_c}}{M_{\Sigma^*} - M_\Sigma} = \quad 0.43 \quad \text{n.m.} \qquad (23)$$

$$\mu_{\Lambda_b} = \quad \mu_\Lambda \cdot \frac{M_{\Sigma_b^*} - M_{\Sigma_b}}{M_{\Sigma^*} - M_\Sigma} = -0.067\,\text{n.m.} \qquad (24)$$

We hope these observables can be measured in foreseeable future and view the predictions (23) and (24) as a challenge for the experimental community.

5. Testing Confining Potentials Through Meson/Baryon HF Splitting Ratio

The ratio of color hyperfine splitting in mesons and baryons is a sensitive probe of the details of the confining potential. This is because this ratio depends only on the value of the wave function at the origin, which in turn is determined by the confining potential and by the ratio of quark masses, as can be readily seen from eqs. (5) and (7), together with the fact that the color quark-antiquark interaction in mesons is twice as strong as the quark-quark interaction in baryons, $(\vec{\lambda}_u \cdot \vec{\lambda}_s)_{meson} = 2(\vec{\lambda}_u \cdot \vec{\lambda}_s)_{baryon}$. We then have

$$\frac{M(K^*) - M(K)}{M(\Sigma^*) - M(\Sigma)} = \frac{4}{3}\,\frac{\langle\psi|\delta(\vec{r}_u - \vec{r}_s)|\psi\rangle_{meson}}{\langle\psi|\delta(\vec{r}_u - \vec{r}_s)|\psi\rangle_{baryon}} \qquad (25)$$

and analogous expressions with the s quark replaced by another heavy quark Q. From the experiment we have 3 data points for this ratio, with $Q = s, c, b$. We can then compute the ratio (25) for 5 different representative confining potentials and compare with experiment. The 5 potentials are

- harmonic oscillator
- Coulomb interaction
- linear potential
- linear + Coulomb, i.e. Cornell potential
- logarithmic

The results are shown in Fig. 2 and Table 2 [6].

For all potentials which contain one coupling constant the coupling strength cancels in the meson-baryon ratio. The Cornell potential which is a combination of a Coulomb and linear potential contains two couplings, one of which cancels in the meson-baryon ratio. The remaining coupling is denoted by k. The gray band corresponds to the range of values $0.2 < k < 0.5$ of the Cornell potential. The crosses correspond to $k = 0.28$ which is the value previously used to fit the charmonium data. Clearly the Cornell potential with $k = 0.28$ provides the best fit to the experiment.

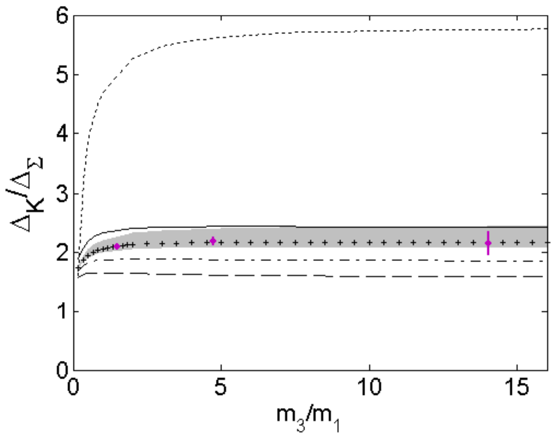

Figure 2. Ratio of the hyperfine splittings in mesons and baryons, as function of the quark mass ratio. Shaded region: Cornell potential for $0.2 < k < 0.5$; crosses: Cornell, $k = 0.28$; long dashes: harmonic oscillator; short dashes: Coulomb; dot-dash: linear; continuous: logarithmic; thick dots: experimental data.

Table 2
Ratio of the hyperfine splittings in mesons and baryons, for different potentials.

	Δ_K/Δ_Σ	$\Delta_D/\Delta_{\Sigma_c}$	$\Delta_B/\Delta_{\Sigma_b}$
m_Q/m_q	1.33	4.75	14
EXP	2.08 ± 0.01	2.18 ± 0.08	2.15 ± 0.20
Harmonic	1.65	1.62	1.59
Coulomb	5.07 ± 0.08	5.62 ± 0.02	5.75 ± 0.01
Linear	1.88 ± 0.06	1.88 ± 0.08	1.86 ± 0.08
Log	2.38 ± 0.02	2.43 ± 0.02	2.43 ± 0.01
Cornell k=0.028	2.10 ± 0.05	2.16 ± 0.07	2.17 ± 0.08

6. Predicting the Mass of Ξ_Q Baryons

The Ξ_Q baryons quark content is Qsd or Qsu. The name Ξ_Q is a mnemonic for the fact that these states can be obtained from "ordinary" Ξ (ssd or ssu) by replacing one of the s quarks by a heavier quark $Q = c, b$. There is one important difference, however. In the ordinary Ξ, Fermi statistics dictates that two s quarks must couple to spin-1, while in the ground state of Ξ_Q the (sd) and (su) diquarks have spin zero.

Consequently, the Ξ_b mass is given by the expression:

$$\Xi_q = m_q + m_s + m_u - \frac{3v\langle\delta(r_{us})\rangle}{m_u m_s} \qquad (26)$$

The Ξ_b mass can thus be predicted using the known Ξ_c baryon mass as a starting point and adding the corrections due to mass differences and HF interactions:

$$\Xi_b = \Xi_c + (m_b - m_c) + \qquad (27)$$

$$- \frac{3v}{m_u m_s}\left(\langle\delta(r_{us})\rangle_{\Xi_b} - \langle\delta(r_{us})\rangle_{\Xi_c}\right) \qquad (28)$$

6.1. Estimating $(m_b - m_c)$

The mass difference $(m_b - m_c)$ can be obtained from experimental data using one of the following expressions:

- We can simply take the difference of the masses of the Λ_q baryons, ignoring the differences in the HF interaction:

$$m_b - m_c = \Lambda_b - \Lambda_c = 3333.2 \pm 1.2\ . \qquad (29)$$

- We can use the spin averaged masses of the Λ_q and Σ_q baryons:

$$m_b - m_c = \left(\frac{2\Sigma_b^* + \Sigma_b + \Lambda_b}{4} - \frac{2\Sigma_c^* + \Sigma_c + \Lambda_c}{4}\right) = 3330.4 \pm 1.8\ . \qquad (30)$$

- Since the Ξ_Q baryon contains a strange quark, and the effective constituent quark masses depend on the spectator quark, it might be better to use masses of mesons which contain both s and Q quarks:

$$m_b - m_c = \left(\frac{3B_s^* + B_s}{4} - \frac{3D_s^* + D_s}{4}\right) = 3324.6 \pm 1.4\ . \qquad (31)$$

6.2. Ξ_b Mass

The corresponding results for Ξ_b mass are summarized in Table 3.

Table 3
Predictions for the Ξ_b mass with various confining potentials and methods of obtaining the quark mass difference $m_b - m_c$

$m_b - m_c =$	$\Lambda_b - \Lambda_c$ Eq. (29)	$\Sigma_b - \Sigma_c$ Eq. (31)	$B_s - D_s$ eq. (32)
No HF corr.	5803 ± 2	5800 ± 2	5794 ± 2
Linear	5801 ± 11	5798 ± 11	5792 ± 11
Coulomb	5778 ± 2	5776 ± 2	5770 ± 2
Cornell	5799 ± 7	5796 ± 7	5790 ± 7

On the basis of these results we predicted [7] $M(\Xi_b) = 5795 \pm 5$ MeV. Our paper was submitted on June 14, 2007. The next day CDF announced the result, following up on earlier but less precise D0 measurement. These results are summarized in Fig. 3 and in Table 4.

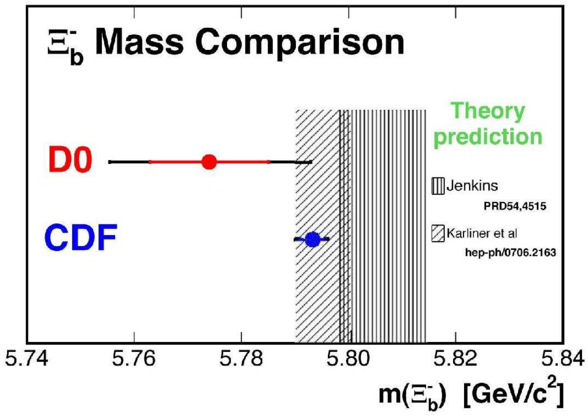

Figure 3. Experimental results for the Ξ_b mass compared with theoretical predictions.

Table 4
Measurements of Ξ_b mass in MeV at the Tevatron. Errors on mass are (statistical, systematic).

	D0 [8]	CDF [9]
Mass	$5774 \pm 11 \pm 15$	$5792.9 \pm 2.5 \pm 1.7$
Width	37 ± 8	~ 14
$\#\sigma$	5.5σ	7.8σ

6.3. Mass of Ξ_b' and Ξ_b^*

In the Ξ_b' baryon (bsd) the (sd) diquark has $S=1$ and the total spin $=1/2$. In the Ξ_b^* baryon (bsd) the (sd) diquark also has $S=1$ and the total spin $=3/2$.

The spin-averaged mass of these two states can be expressed as

$$\frac{2\Xi_q^*+\Xi_q'}{3}=m_q+m_s+m_u+\frac{v\langle\delta(r_{us})\rangle}{m_u m_s}\ , \qquad (32)$$

and as for the Ξ_b case, the following prediction can be given:

$$\frac{2\Xi_b^*+\Xi_b'}{3}=\frac{2\Xi_c^*+\Xi_c'}{3}+(m_b-m_c)+ \\ +\frac{2\Xi_c^*+\Xi_c'-3\Xi_c}{12}\left(\frac{\langle\delta(r_{us})\rangle_{\Xi_b}}{\langle\delta(r_{us})\rangle_{\Xi_c}}-1\right) \qquad (33)$$

The predictions obtained using the same methods described above are given in Table 5. Here the effect of the HF correction is negligible, so the difference between the spin averaged mass $(2\Xi_b^*+\Xi_b')/3$ and Ξ_b is roughly $150-160$ MeV.

Table 5
Predictions for the spin averaged Ξ_b' and Ξ_b^* masses with various confining potentials and methods of obtaining the quark mass difference m_b-m_c.

$m_b-m_c=$	$\Lambda_b-\Lambda_c$	$\Sigma_b-\Sigma_c$	B_s-D_s
	Eq. (29)	Eq. (31)	Eq. (32)
No HF corr.	5956 ± 3	5954 ± 3	5948 ± 3
Linear	5957 ± 4	5954 ± 4	5948 ± 4
Coulomb	5965 ± 3	5962 ± 3	5956 ± 3
Cornell	5958 ± 3	5955 ± 3	5949 ± 3

The $\Xi_b^*-\Xi_b'$ mass difference is more difficult to predict. It is small, due to the large m_b:

$$\Xi_b^*-\Xi_b'=3v\left(\frac{\langle\delta(r_{bs})\rangle}{m_b m_s}+\frac{\langle\delta(r_{bu})\rangle}{m_b m_u}\right) \qquad (34)$$

This expression is strongly dependent on the confinement model. In the results given in Table 6 we have used $m_s/m_u=1.5\pm0.1$, $m_b/m_c=2.95\pm0.2$.

In the context of Ξ_b' and Ξ_b^* masses it is worth mentioning two elegant relations among bottom baryons [10] which incorporate the effects of $SU(3)_f$ breaking:

Table 6
Predictions for $M(\Xi_b^*)-M(\Xi_b')$ with various confining potentials.

	$\Xi_b^*-\Xi_b'$
No HF correction	24 ± 2
Linear	28 ± 6
Coulomb	36 ± 7
Cornell	29 ± 6

$$\Sigma_b+\Omega_b-2\Xi_b'=0\ , \qquad (35)$$

$$(\Sigma_b^*-\Sigma_b)+(\Omega_b^*-\Omega_b)-2(\Xi_b^*-\Xi_b')=0\ , \qquad (36)$$

where isospin averaging is implicit.

7. Predictions for Other b Baryons

Using methods similar to those discussed in the previous section it is possible to make predictions for many other ground-state and excited baryons containing the b quark [11].

7.1. Ω_b

For the spin-averaged Ω_b mass we have

$$\tfrac{1}{3}[2M(\Omega_b^*)+M(\Omega_b)]= \qquad (37)$$
$$=\tfrac{1}{3}[2M(\Omega_c^*)+M(\Omega_c)]+(m_b-m_c)_{B_s-D_s}$$
$$=6068.9\pm2.4\text{ MeV}$$

For the HF splitting we obtain

$$M(\Omega_b^*)-M(\Omega_b)=$$
$$=(M(\Omega_c^*)-M(\Omega_c))\frac{m_c}{m_b}\frac{\langle\delta(r_{bs})\rangle_{\Omega_b}}{\langle\delta(r_{cs})\rangle_{\Omega_c}}= \qquad (38)$$
$$=30.7\pm1.3\text{ MeV}$$

leading to the following predictions:

Ω_b^*=6082.8±5.6 MeV; Ω_b=6052.1±5.6 MeV (39)

7.2. Comparison with Other Approaches

The sign in our prediction

$$M(\Sigma_b^*)-M(\Sigma_b)<M(\Omega_b^*)-M(\Omega_b) \qquad (40)$$

appears to be counterintuitive, since the color hyperfine interaction is inversely proportional to the quark mass. The expectation value of the interaction with the same wave function for Σ_b and

Ω_b violates our inequality. When wave function effects are included, the inequality is still violated if the potential is linear, but is satisfied in predictions which use the Cornell potential [6].

This reversed inequality is not predicted by other recent approaches [12–14] which all predict an Ω_b splitting smaller than a Σ_b splitting.

However the reversed inequality is also seen in the corresponding charm experimental data,

$$\begin{array}{ccc} M(\Sigma_c^*) - M(\Sigma_c) & < & M(\Omega_c^*) - M(\Omega_c) \\ 64.3 \pm 0.5\mathrm{MeV} & & 70.8 \pm 1.5\mathrm{MeV} \end{array} \quad (41)$$

This suggests that the sign of the $SU(3)$ symmetry breaking gives information about the form of the potential. It is of interest to follow this clue theoretically and experimentally.

7.3. Additional States

We have made predictions [7,11] for isospin splitting of Ξ_b, and for orbital excitations of of Λ_b and Ξ_b. Our results are summarized in Table 10 of the second paper in Ref. [11].

8. $\Upsilon(5S)\to\Upsilon(nS)\pi^+\pi^-$ and Tetraquarks

The Belle Collaboration has recently reported [15] anomalously large partial widths in $\Upsilon(1S)\,\pi^+\pi^-$ and $\Upsilon(2S)\,\pi^+\pi^-$ production at the $\Upsilon(5S)$, more than two orders of magnitude larger than the corresponding partial widths for $\Upsilon(4S)$, $\Upsilon(3S)$ or $\Upsilon(2S)$ decays.

We suggested [16] that the large partial widths of these channels might be due to their production by decays via an intermediate $T^{\pm}_{\bar{b}b}\pi^{\mp}$ state, where $T^{\pm}_{\bar{b}b}$ denotes an isovector charged tetraquark $\bar{b}bu\bar{d}$ or $\bar{b}b\bar{u}d$,

$$\Upsilon(nS) \;\to\; \pi^{\mp}\,T^{\pm}_{\bar{b}b} \;\to\; \Upsilon(mS)\,\pi^-\pi^+ \quad (42)$$

9. Open Questions

I close with a list of open questions which I consider the most interesting in this field:

- need to understand the XYZ states in the charm sector and their counterparts in the bottom sector, remembering that replacing charmed quark by bottom quark makes the binding stronger; this is an excellent challenge for experiment and theory
- general question of exotics in QCD
- baryons with two heavy quarks: ccq, bbq, bcq where $q = u, d$. So far the only positive experimental report is about doubly charmed baryons from SELEX, but the very large isospin splitting they reported between ccu and ccd is very hard to understand.

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