FLAVOUR PHYSICS

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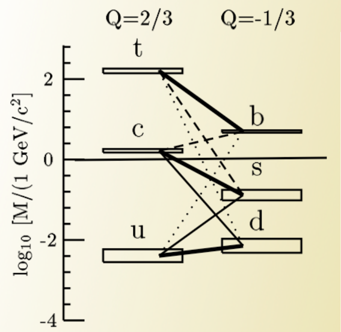
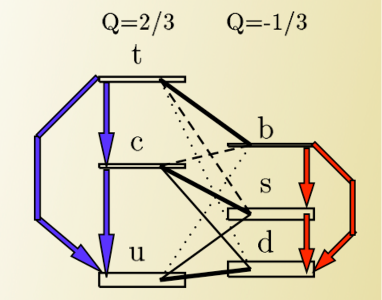
Theory Challenges for LHC Physics

Dubna, 20.07. - 30.07.2015

Preliminary Remarks

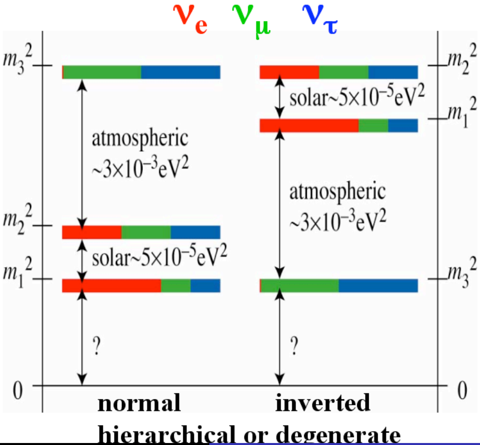
- Flavour Physics:
  Transitions between different kinds of Quarks





- Its all about weak interactions ...
- Strong interactions as a “background”

- Likewise for Leptons, but
- no strong interactions here
- Neutrinos hard to detect
  $\rightarrow$ Flavour Identification



More reading ...

- R. Fleischer: **Flavour Physics and CP Violation**
  Lectures given at European School of High-Energy Physics 2005
  hep-ph/0608010 $\rightarrow$ Non-leptonics and CP
- A. Buras: **Flavor physics and CP violation**
  Lectures given at European School of High-Energy Physics 2004
  hep-ph/0505175 $\rightarrow$ rare FCNC decays
- A. Buras: **Minimal flavor violation**
  Lectures given at 43rd Cracow School of Theoretical Physics 2003
  hep-ph/0310208 $\rightarrow$ MFV and New Physics

- Y. Nir: **Probing new physics with flavor physics**
  Lectures given at 2nd Joint Fermilab-CERN Hadron Collider Physics Summer School 2007
  arXiv:0708.1872 [hep-ph] $\rightarrow$ Mainly New Physics
- A. Bevan, B. Golob, T. Mannel, S. Prell, B. Yabsley (eds.) **The Physics of the *B* Factories**
  Eur.Phys.J. **C74** (2014) 3026, (926 pages)

Outline of the course

- Lecture 1: Flavour in the Standard Model
- Lecture 2: Theoretical Tools and Phenomenology
- Lecture 3: Flavour beyond the Standard Model

Flavour Physics 1
Flavour in the Standard Model

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Theory Challenges for LHC Physics

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Outline of Lecture 1

Gauge Structure of the Standard Model

I assume a few things to be known:

- The Standard Model is a gauge theory based on
  $SU(3)_{QCD} \otimes SU(2)_{Weak} \otimes U(1)_{Hypercharge}$
- Eight gluons, three weak gauge bosons, one photon
- Matter (quarks and leptons):
  Multiplets of the gauge group $\to$ Quantum numbers
- Spontaneous Symmetry Breaking:
  Introduction of scalar fields
- Massless Goldstone Modes:
  Higgs Mechanism:
  $\phi \to$ longitudinal modes of gauge bosons: $\phi \sim \partial_\mu W^\mu$

Matter Fields: Quarks

- Left Handed Quarks:
  $SU(3)_C$ Triplets, $SU(2)_L$ Doublets

$$Q_1 = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \; Q_2 = \begin{pmatrix} c_L \\ s_L \end{pmatrix} \; Q_3 = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

  $SU(2)_L$ will be gauged

- Right Handed Quarks:
  $SU(3)_C$ Triplets, $SU(2)_R$ Doublets

$$q_1 = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \; q_2 = \begin{pmatrix} c_R \\ s_R \end{pmatrix} \; q_3 = \begin{pmatrix} t_R \\ b_R \end{pmatrix}$$

  $SU(2)_R$ introduced “artificially”

Quantum Numbers

- Hypercharge

$$\boxed{Y = T_{3,R} + \frac{1}{2}(B - L)}$$

- Charge

$$\boxed{q = T_{3,L} + Y = T_{3,L} + T_{3,R} + \frac{1}{2}(B - L)}$$

Higgs Fields: Standard Model

- Single $SU(2)$ Doublett: Two Complex Fields

$$\Phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}$$

- Charge Conjugate Field is also an $SU(2)$ Doublett

$$\widetilde{\Phi} = (i\tau_2)\Phi^* = \begin{pmatrix} \phi_0^* \\ -\phi_- = -\phi_+^* \end{pmatrix}$$

- It is useful to gather these into a 2×2 matrix

$$H = \begin{pmatrix} \phi_0^* & \phi_+ \\ -\phi_- & \phi_0 \end{pmatrix}$$

- Transformation Properties: $L \in SU(2)_L$:

$$\Phi \to L\Phi \qquad \widetilde{\Phi} \to L\widetilde{\Phi}$$

- Transformation Properties: $R \in SU(2)_R$:

$$\begin{pmatrix} \phi_0 \\ \phi_- \end{pmatrix} \to R \begin{pmatrix} \phi_0 \\ \phi_- \end{pmatrix} \qquad \begin{pmatrix} \phi_+ \\ -\phi_0^* \end{pmatrix} \to R \begin{pmatrix} \phi_+ \\ -\phi_0^* \end{pmatrix}$$

- In total:

$$H \to LHR^\dagger \qquad (\text{remember } \; Q \to LQ \quad q \to Rq)$$

- Hypercharges

$$Y\Phi = -\Phi \qquad Y\widetilde{\Phi} = \widetilde{\Phi} \qquad YH = -HT_{3,R}$$

Gauge Interactions

- $SU(3)_{color}$ is gauged (not relevant for us now)
- $SU(2)_L$ is gauged Three W_a^μ Bosons
- Hypercharge is gauged One B^μ Boson
- Recipe: Replace the ordinary derivative in the kinetic terms by the covariant one

$$\begin{aligned}\partial^\mu \to D^\mu &= \partial^\mu - igT_{L,a}W_a^\mu - iYB^\mu \\ &\quad + \text{QCD interactions}\end{aligned}$$

- Weinberg rotation between W_3^μ and B^μ $\cdots$
- I assume you have heard the rest of the story ...
- This is not relevant for the phenomenon of masses and mixing !

Structure of the Standard Model

- Start out from an $SU(2)_L \times SU(2)_R$ symmetric case:
- Kinetic Term for Quarks and Higgs (i: Generation)

$$\mathcal{L}_{kin} = \sum_i \left[\bar{Q}_i \partial\!\!\!/ Q_i + \bar{q}_i \partial\!\!\!/ q_i \right] + \frac{1}{2} \mathrm{Tr} \left[(\partial_\mu H)^\dagger (\partial^\mu H) \right]$$

- Potential for the Higgs field

$$V = V(H) = V(\mathrm{Tr}\left[H^\dagger H \right])$$

- Interaction between Quarks and Higgs

$$\mathcal{L}_I = -\sum_{ij} y_{ij} \bar{Q}_i H q_j + \text{ h.c.}$$

- y_{ij} can be made diagonal: Any Matrix y can be diagonalized by a Bi-Unitary Transformation:

$$y = U^\dagger y_{diag} W$$

- Thus

$$\mathcal{L}_I = -\sum_{ijk} \bar{Q}_i (U^\dagger)_{ik} y_k W_{kj} H q_j + \text{ h.c.}$$

- Rotation of Q_i and q_j:

$$Q' = UQ \quad q' = Wq$$

- This has no effect on the kinetic term: $y_{ij} = y_i \delta_{ij}$ is the general case!

$$\mathcal{L}_I = -\sum_i y_i \bar{Q}_i H q_i + \text{ h.c.}$$

Sponaneous Symmetry Breaking

- The Higgs Potential is (Renormalizability):

$$V = \kappa \left(\mathrm{Tr} \left[H^\dagger H \right] \right) + \lambda \left(\mathrm{Tr} \left[H^\dagger H \right] \right)^2$$

- For $\kappa < 0$ we have SSB:
  H acquires a Vacuum Expectation Value (VEV)

$$\mathrm{Tr} \left[\langle H^\dagger \rangle \langle H \rangle \right] = -\frac{\kappa}{2\lambda} > 0$$

- Choice of the VEV

$$< \mathrm{Re}\phi_0 >= v \text{ or } < H >= v \, 1_{2\times 2}$$

- Three massless fields: ϕ_+, ϕ_-, Imϕ_0:
  Goldstone Bosons
- $\phi_0 \to v + \phi_0'$: One massive field
- Higgs Mechanism: The massless scalars become the longitudinal modes of the massive vector bosons:
  - * $\phi_\pm \sim \partial^\mu W_\mu^\pm$
  - * Im$\phi_0 \sim \partial^\mu Z_\mu$
- ϕ_0': Physical Higgs Boson

- The Quarks become massive:

$$\mathcal{L}_I = -\sum_i y_i v \bar{Q}_i q_i + \text{ h.c. } + \cdots$$

- We have $\bar{Q}_1 q_1 = \bar{u}_L u_R + \bar{d}_L d_R$ etc.
- Thus

$$\mathcal{L}_{mass} = -m_u(\bar{u}u + \bar{d}d) - m_c(\bar{c}c + \bar{s}s) - m_t(\bar{t}t + \bar{b}b)$$

- This is not (yet) what we want ...
- We still have too much symmetry!

Custodial $SU(2)$

- Symmetry of the Higgs Sector in the Standard Model:

$$SU(2)_L \otimes SU(2)_R \stackrel{SSB}{\longrightarrow} SU(2)_{L+R} = SU(2)_C$$

- Note that we cannot have explicit breaking of $SU(2)_R$ in the Higgs sector:

$$\mathrm{Tr}\left[H\tau_i H^\dagger\right] = 0$$

- $SU(2)_C$: Custodial Symmetry!
  $\rightarrow$ Extra Symmetry in the Higgs sector !
- This is more than needed: Only $U(1)_Y$ is needed
- $U(1)_Y$ will be related to the τ_3 direction of $SU(2)_R$

- Consequences of $SU(2)_C$:
  - Relation between charged and neutral currents: ρ parameter
  - Masses of $W^\pm$ and of Z^0 are equal
  - Up- and Down-type quark masses are equal in each family
  - No mixing occurs among the families
- $SU(2)_C$ is broken by:
  - Yukawa Couplings
  - Gauging only the Hypercharge

$$Y = T_3^{(R)} + \frac{1}{2}(B - L)$$

Breaking $SU(2)_C$: Yukawa Couplings

- Explicit breaking of $SU(2)_C$ by Yukawa Couplings:

$$\mathcal{L}'_I = -\sum_{ij} y'_{ij} \bar{Q}_i H (2T_{3,R}) q_j + \text{ h.c.}$$

- Effect of this term:
  - Introduces a splitting between up- and down quark masses
  - Introduces mixing between different families
  - Affects the ρ parameter
- Total Yukawa Coupling term:

$$\mathcal{L}_I + \mathcal{L}'_I = -\sum_{ij} \bar{Q}_i H (y_i \delta_{ij} + 2T_{3,R} y'_{ij}) q_j + \text{ h.c.}$$

Quark Mass Matrices

- Use the projections

$$P_\pm = \frac{1}{2} \pm T_{3,R} \qquad \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

- Up quark Yukawa couplings:

$$\mathcal{L}^u_{mass} = -\sum_{ij} \bar{Q}_i H(y_i \delta_{ij} + y'_{ij}) P_+ q_j + \text{ h.c.}$$

- Down quark Yukawa couplings:

$$\mathcal{L}^d_{mass} = -\sum_{ij} \bar{Q}_i H(y_i \delta_{ij} - y'_{ij}) P_- q_j + \text{ h.c.}$$

- $\to$ mass terms, once $\mathrm{Re}\phi_0 \to v$

- More compact notation

$$\mathcal{U}_{L/R} = \begin{bmatrix} u_{L/R} \\ c_{L/R} \\ t_{L/R} \end{bmatrix} \qquad \mathcal{D}_{L/R} = \begin{bmatrix} d_{L/R} \\ s_{L/R} \\ b_{L/R} \end{bmatrix}$$

- Mass Term for Up-type quarks

$$\mathcal{L}^u_{mass} = -v\, \bar{\mathcal{U}}_L Y^u \mathcal{U}_R + \text{ h.c.}$$

with $Y^u = (y + y')$

- Mass Term for down-type quarks

$$\mathcal{L}^d_{mass} = -v\, \bar{\mathcal{D}}_L Y^d \mathcal{D}_R + \text{ h.c.}$$

with $Y^d = (y - y')$

- Mass matrices:

$$\mathcal{M}^u = v\, Y^u \qquad \mathcal{M}^d = v\, Y^d$$

- In general non-diagonal: Diagonalization by a bi-unitary transformation:

$$\mathcal{M} = U^\dagger \mathcal{M}_{diag} W$$

- New basis for the quark fields

$$\mathcal{L}^u_{mass} = -\bar{\mathcal{U}}_L U^{u,\dagger} \mathcal{M}^u_{diag} W^u \mathcal{U}_R + \text{ h.c.}$$

and

$$\mathcal{L}^d_{mass} = -\bar{\mathcal{D}}_L U^{d,\dagger} \mathcal{M}^d_{diag} W^d \mathcal{D}_R + \text{ h.c.}$$

Quark Mixing: The CKM Matrix

- Effect of the basis transformation:
  - Mass matrices become diagonal
  - Interaction with $\mathrm{Re}\,\phi_0$ (= Physical Higgs Boson) becomes diagonal !
  - Interaction with $\mathrm{Im}\,\phi_0$ (= Z_0) becomes diagonal !

$$\begin{aligned} \mathcal{L}_{\mathrm{Re}\,\phi_0} &= -\mathrm{Re}\,\phi_0 [\mathcal{U}_L Y^u \mathcal{U}_R + \mathcal{D}_L Y^d \mathcal{D}_R] \\ \mathcal{L}_{\mathrm{Im}\,\phi_0} &= -\mathrm{Im}\,\phi_0 [\mathcal{U}_L Y^u \mathcal{U}_R - \mathcal{D}_L Y^d \mathcal{D}_R] \end{aligned}$$

- NO FLAVOUR CHANGING NEUTRAL CURRENTS (at tree level in the Standard Model)
- $\to$ GIM Mechanism

- Effect on the charged current ONLY: Interaction with ϕ_-:

$$\begin{aligned}
&\sum_{ij} \bar{Q}_i (y_i \delta_{ij} + y'_{ij}) \phi_- \tau_- P_+ q_j + \text{ h.c.} \\
&= \mathcal{D}_L Y^u \mathcal{U}_R \phi_- + \text{ h.c.} \\
&= \bar{\mathcal{D}}_L U^{d,\dagger} (U^d U^{u,\dagger}) Y^u_{diag} W^u \mathcal{U}_R \phi_- + \text{ h.c.}
\end{aligned}$$

- In the charged currents flavour mixing occurs!
- Parametrized through the Cabbibo-Kobayashi-Maskawa Matrix:

$$V_{CKM} = U^d U^{u,\dagger}$$

Properties of the CKM Matrix

- V_{CKM} is unitary (by our construction)
- Number of parameters for n families
  - Unitary $n \times n$ matrix: n^2 real parameters
  - Freedom to rephase the $2n$ quark fields: $2n - 1$ relative phases
- $n^2 - 2n + 1 = (n-1)^2$ real parameters
  - * $(n-1)(n-2)/2$ are phases
  - * $n(n-1)/2$ are angles
- Phases are sources of *CP* violation
- $n = 2$: One angle, no phase $\rightarrow$ no *CP* violation
- $n = 3$: Three angles, one phase
- $n = 4$: Six angles, three phases

CKM Basics

- Three Euler angles θ_{ij}

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix}, \quad U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

- Single phase δ: $U_\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix}$.
- PDG CKM Parametrization:

$$V_{\mathrm{CKM}} = U_{23} U_\delta^\dagger U_{13} U_\delta U_{12}$$

- Large Phases in $V_{ub} = |V_{ub}| e^{-i\gamma} = s_{13} e^{-i\delta_{13}}$ and $V_{td} = |V_{td}| e^{i\beta}$

CKM Unitarity Relations

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Off diagonal zeros of $V_{CKM}^\dagger V_{CKM} = 1 = V_{CKM} V_{CKM}^\dagger$
- $V_{CKM}^\dagger V_{CKM} = 1 : \begin{cases} V_{ub} V_{ud}^* + V_{cb} V_{cd}^* + V_{tb} V_{td}^* = 0 \\ V_{ub} V_{us}^* + V_{cb} V_{cs}^* + V_{tb} V_{ts}^* = 0 \\ V_{us} V_{ud}^* + V_{cs} V_{cd}^* + V_{ts} V_{td}^* = 0 \end{cases}$
- $V_{CKM} V_{CKM}^\dagger = 1 : \begin{cases} V_{ud} V_{td}^* + V_{us} V_{ts}^* + V_{ub} V_{tb}^* = 0 \\ V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0 \\ V_{cd} V_{td}^* + V_{cs} V_{ts}^* + V_{cb} V_{tb}^* = 0 \end{cases}$

Wolfenstein Parametrization of CKM

- Diagonal CKM matrix elements are almost unity
- CKM matrix elements decrease as we move off the diagonal
- Wolfenstein Parametrization:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix}$$

- Expansion in $\lambda \approx 0.22$ up to λ^3
- A, ρ, η of order unity

Unitarity Triangle(s)

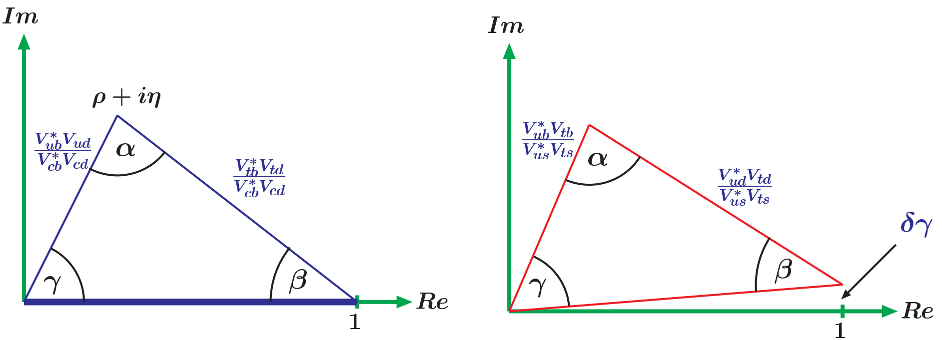
- The unitarity relations:
  Sum of three complex numbers = 0
- Triangles in the complex plane
- Only two out of the six unitarity relations involve terms of the same order in λ:

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$
$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0$$

- Both correspond to

$$A\lambda^3(\rho + i\eta - 1 + 1 - \rho - i\eta) = 0$$

- This is THE unitarity triangle ...



- Definition of the CKM angles α, β and γ
- To leading order Wolfenstein:

$$V_{ub} = |V_{ub}| e^{-i\gamma} \quad V_{tb} = |V_{tb}| e^{-i\beta}$$

  all other CKM matrix elements are real.
- $\delta\gamma$ is order λ^5

- Aerea of the Triangle(s): Measure of CP Violation
- Invariant measure of CP violation:

$$\mathrm{Im}\Delta = \mathrm{Im}\, V_{ud} V_{td}^* V_{tb} V_{ub}^* = c_{12} s_{12} c_{13}^2 s_{13} s_{23} c_{23} \sin \delta_{13}$$

- Maximal possible value $\delta_{\max} = \frac{1}{6\sqrt{3}} \sim 0.1$
- CP Violation is a small effect:
  Measured value $\delta_{\exp} \sim 0.0001$
- CP Violation vanishes in case of degeneracies: (Jarlskog)

$$\begin{aligned} J &= \mathrm{Det}([M_u\,,\,M_d]) \\ &= 2i\mathrm{Im}\Delta (m_u - m_c)(m_u - m_t)(m_c - m_t) \\ &\quad \times (m_d - m_s)(m_d - m_b)(m_s - m_b) \end{aligned}$$

Leptons in the Standard Model

- If the neutrinos are massless:
  - Only left handed neutrinos couple
  - Right handed neutrinos do not have any $SU(2)_L \times U(1)_Y$ quantum numbers
  - No mixing in the lepton sector
- Recent evidence for neutrino mixing:
  - Right handed components couple through the mass term
  - Mixing in the Lepton Sector
- It could be just a copy of the quark sector, but it may be different due to the properties of the right-handed neutrino

Multiplets and Quantum Numbers

- Left Handed Leptons: $SU(2)_L$ Doublets

$$L_1 = \begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix} \; L_2 = \begin{pmatrix} \nu_{\mu,L} \\ \mu_L \end{pmatrix} \; L_3 = \begin{pmatrix} \nu_{\tau,L} \\ \tau_L \end{pmatrix}$$

- Right Handed Leptons: $SU(2)_R$ Doublets

$$\ell_1 = \begin{pmatrix} \nu_{e,R} \\ e_R \end{pmatrix} \; \ell_2 = \begin{pmatrix} \nu_{\mu,R} \\ \mu_R \end{pmatrix} \; \ell_3 = \begin{pmatrix} \nu_{\tau,R} \\ \tau_R \end{pmatrix}$$

- Charge and Hypercharge

$$Y = T_{3,R} + \frac{1}{2}(B - L) = T_{3,R} - \frac{1}{2} \quad q = T_{3,L} + Y$$

- Y (and q) project the lower component: Right handed Neutrinos: No charge, no Hypercharge

Majorana Fermions

- A “neutral” fermion can have a Majorana mass
- Charged fermions $\Leftrightarrow$ complex scalar fields
- Majorana fermion: “Real (= neutral) fermion”
- Definition of “complex conjugation” in this case: Charge Conjugation:

$$\psi \to \psi^c = C\bar{\psi}^T \quad C = i\gamma_2\gamma_0 = \left(\begin{array}{cc} 0 & -i\sigma_2 \\ -i\sigma_2 & 0 \end{array} \right)$$

- Properties of C

$$-C = C^{-1} = C^T = C^\dagger$$

- Majorana fermion: $\psi_{Majorana} = \psi^c_{Majorana}$
  (Just as $\phi^* = \phi$ for a real scalar field)

Majorana Mass Terms

- Mass term for a Majorana fermion: The charge conjugate of a right handed fermion is left handed.
- Possible mass term

$$\mathcal{L}_{MM} = -\frac{1}{2} M \left(\bar{\nu}_R (\nu_R^c)_L + h.c. \right)$$

- Only for fields without $U(1)$ quantum numbers
- In the SM: only for the right handed neutrinos !
- Remarks:
  - The Majorana mass of the right handed neutrinos is NOT due to the Higgs mechanism.
  - Thus this majorana mass can be “large”
  - Natural explanation of the small neutrino masses: see-saw mechanism

See Saw Mechanism

- Simplification: One family: ν_L and ν_R
- Total Mass term: Dirac and Majorana mass

$$\begin{aligned} \mathcal{L}_{mass} &= -m(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L) \\ &\quad -\frac{1}{2} M(\nu_R^T C \nu_R + \bar{\nu}_R C \bar{\nu}_R^T) \end{aligned}$$

- We use

$$\overline{(\nu_R^c)}_L (\nu_L^c)_R = \bar{\nu}_L \bar{\nu}_R$$

and the properties of the C matrix ...

$$\mathcal{L}_{mass} = -\frac{1}{2} \left(\bar{\nu}_L \; \overline{(\nu_R^c)}_L \right) \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} (\nu_L^c)_R \\ \nu_R \end{pmatrix} + h.c.$$

- Diagonalization of the mass matrix:
  $\rightarrow$ Majorana mass eigenstates of the Neutrinos
  For $M \gg m$ we get

$$m_1 \approx \frac{m^2}{M} \quad m_2 \approx M$$

- One very heavy, practically right handed neutrino
- One very light, practically left handed neutrino
- At energies small compared to M:
  Majorana mass term for the left handed neutrino

$$\mathcal{L}_{mass} = -\frac{1}{2}\frac{m^2}{M}\left(\nu_L^T C \nu_L + \bar{\nu}_L C \bar{\nu}_L{}^T\right)$$

- Majorana mass is small if $M \gg m$

Right handed neutrinos in the Standard Model

- In case of three families: **Neutrino Mixing**
- Compact notation for the Leptons:

$$\mathcal{N}_{L/R} = \begin{bmatrix} \nu_{e,L/R} \\ \nu_{\mu,L/R} \\ \nu_{\tau,L/R} \end{bmatrix} \qquad \mathcal{E}_{L/R} = \begin{bmatrix} e_{L/R} \\ \mu_{L/R} \\ \tau_{L/R} \end{bmatrix}$$

- Dirac masses are generated by the Higgs mechanism: (as for the quarks)

$$\mathcal{L}_{DM}^{N} = -\mathcal{N}_L m^N \mathcal{N}_R + h.c.$$
$$\mathcal{L}_{DM}^{E} = -\mathcal{E}_L m^E \mathcal{E}_R + h.c.$$

- m^N: Dirac mass matrix for the neutrinos
- m^E: (Dirac) mass matrix for e, μ, τ

- Right handed neutrinos $\to$ Majorana mass term:

$$\mathcal{L}_{MM} = -\frac{1}{2}\left(N_R^T MCN_R + \bar{N}_R MC\bar{N}_R^T\right)$$

- M: (Symmetric) Majorana Mass Matrix
- This term is perfectly $SU(2)_L \otimes U(1)$ invariant
- Implementation of the see saw mechanism:
  Assume that all Eigenvalues of M are large
- Effective Theory at low energies:
  Only light, practically left handed neutrinos
- Effect of right handed neutrino:
  Majorana mass term for the light neutrinos

$$\mathcal{L}_{mass} = -\frac{1}{2}\left(N_L^T m^T M^{-1} mCN_L + \bar{N}_L m^T M^{-1} mC\bar{N}_L^T\right)$$

Lepton Mixing: PMNS Matrix

- Diagonalization of the Mass matrices:
  - Charged leptons:

$$m^E = U^\dagger m^E_{diag} W$$

  - Neutrinos: “Orthogonal” transformation:

$$m^T M^{-1} m = O^T m^\nu_{diag} O \text{ with } O^\dagger O = 1$$

- Again no Effect on neutral currents
- Charged Currents: Interaction with ϕ_+:

$$\frac{1}{v} \mathcal{N}_L m^E \mathcal{E}_R \phi_+ + \text{ h.c.}$$
$$= \frac{1}{v} \bar{\mathcal{N}}_L O^T (O^* U^\dagger) m^E_{diag} W \mathcal{E}_R \phi_+ + \text{ h.c.}$$

- A Mixing Matrix occurs:

$$V_{PMNS} = O^* U^\dagger$$

  Pontecorvo Maki Nakagawa Sakata Matrix
- V_{PMNS} is unitary like the CKM Matrix
- Left handed neutrinos are Majorana: No freedom to rephase these fields!
  - For n families: n^2 Parameters
  - Only n Relative phases free
  - $\longrightarrow n(n-1)$ Parameters
  - $n(n-1)/2$ are angles
  - $n(n-1)/2$ are phases: More sources for CP violation

- Almost like CKM: Three Euler angles θ_{ij}

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix}, \quad U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

- A Dirac Phase δ and two Majorana Phases α_1 and α_2

$$U_\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix} \quad U_\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{-i\alpha_2} \end{bmatrix}$$

- PMNS Parametrization: $V_{\mathrm{PMNS}} = U_{23} U_\delta^\dagger U_{13} U_\delta U_{12} U_\alpha$
- $\Theta_{23} \sim 45^\circ$ is “maximal” (atmospheric ν’s)
- $\Theta_{13} \sim 0$ is small (ν’s from reaktors)
- $\sin\Theta_{13} \sim 1/\sqrt{3}$ is large (solar ν’s)

Maltoni et al '04

parameter	best fit	2σ	3σ	5σ
$\Delta m^2_{21}\ [10^{-5}\mathrm{eV}^2]$	6.9	6.0–8.4	5.4–9.5	2.1–28
$\Delta m^2_{31}\ [10^{-3}\mathrm{eV}^2]$	2.6	1.8–3.3	1.4–3.7	0.77–4.8
$\sin^2\theta_{12}$	0.30	0.25–0.36	0.23–0.39	0.17–0.48
$\sin^2\theta_{23}$	0.52	0.36–0.67	0.31–0.72	0.22–0.81
$\sin^2\theta_{13}$	0.006	≤ 0.035	≤ 0.054	≤ 0.11

$$V_{\mathrm{PMNS}} \sim \begin{bmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \end{bmatrix} \sim \begin{bmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{bmatrix}$$

- No Hierarchy !

Consequences of Lepton Mixing

- FCNC Processes in the leptonic Sector:

$$\tau \to \mu\gamma \quad \mu \to e\gamma \quad \tau \to eee \text{ etc.}$$
$$\nu_\tau \to \nu_e\gamma \quad \nu_\tau - \nu_e \text{ mixing}$$

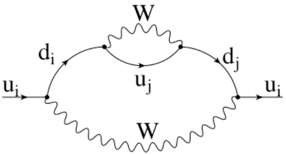
- Lepton Number Violation:
  Right handed Neutrinos are Majorana fermions:
  No conserved quantum number corresponding to the rephasing of the right handed neutrino fields
  Lepton number violation could feed via conserved $B - L$ into Baryon number violation
  Relation to the Baryon Asymmetry of the Universe ?

Peculiarities of SM Flavour Mixing

- Hierarchical structure of the CKM matrix
- Quark Mass spectrum ist widely spread
  $m_u \sim 10$ MeV to $m_t \sim 170$ GeV
- PMNS Matrix for lepton flavour mixing is not hierarchical
- Only the charged lepton masses are hierarchical
  $m_e \sim 0.5$ MeV to $m_\tau \sim 1772$ MeV
- Up-type leptons $\sim$ Neutrinos have very small masses
- (Enormous) Suppression of Flavour Changing Neutral Currents:
  $b \to s$, $c \to u$, $\tau \to \mu$, $\mu \to e$, $\nu_2 \to \nu_1$

Peculiarities of SM CP Violation

- Strong CP remains mysterious
- Flavour diagonal CP Violation is well hidden:
  e.g electric dipole moment of the neutron:
  At least three loops (Shabalin)



$$
\begin{aligned}
d_e &\sim e \frac{\alpha_s}{\pi} \frac{G_F^2}{(16\pi^2)^2} \frac{m_t^2}{M_W^2} \mathrm{Im}\Delta\, \mu^3 \\
&\sim 10^{-32} e\, cm \quad \text{with } \mu \sim 0.3\,\mathrm{GeV} \\
d_{\mathrm{exp}} &\leq 3.0 \times 10^{-26} e\, cm
\end{aligned}
$$

- Pattern of mixing and mixing induced CP violation determined by GIM: Tiny effects in the up quark sector
  - $\Delta C = 2$ is very small
  - Mixing with third generation is small: charm physics basically “two family”
  - $\to$ CP violation in charm is small in the SM
- Fully consistent with particle physics observations
- ... but inconsistent with matter-antimatter asymmetry

??? Many Open Questions ???

- Our Understanding of Flavour is unsatisfactory:
  - 22 (out of 27) free Parameters of the SM originate from the Yukawa Sector (including Lepton Mixing)
  - Why is the CKM Matrix hierarchical?
  - Why is CKM so different from the PMNS?
  - Why are the quark masses (except the top mass) so small compared with the electroweak VEV?
  - Why do we have three families?
- Why is CP Violation in Flavour-diagonal Processes not observed? (e.g. z.B. electric dipolmoments of electron and neutron)
- Where is the CP violation needed to explain the matter-antimatter asymmetry of the Universe?