3-Dimensional Structure of the Nucleon with QCD

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Fundamental Interactions .be

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Outline:

- Why do we need **TMD pdfs** to approach the $3D$-world of hadrons?
- Beyond the collinear approaximation: **multi-scale** problems in hadronic processes
- Specific issues in the **TMD theory**: gauge invariance, path dependence, universality, factorisation
- **Path-dependence**: can one make any use of it?
  Stokes-Mandelstam gTMD: fully gauge-invariant, maximally path-dependent
- **Current activity** in the TMD community

QCD Description of Proton-Proton Collisions

- **Fundamental Properties and Concepts of QCD**
  - Confinement: fundamental building blocks of QCD – quarks and gluons – do not exist as free particles
  - Running coupling: the strong coupling α_s changes with the characteristic energy
  - Asymptotic freedom: at small distance the quarks and gluons are (almost) free particles and the perturbative approach is applicable
  - Factorisation: enables the separation of large- [essentially nonperturbative] and small-distance [perturbative hard scattering matrix elements] contributions
  - Parton distribution functions [pdfs]: accumulate information about intrinsic structure of hadrons

Parton Distribution Functions

pdfs must be

- gauge-invariant
- universal
- renormalizable
- Wilson lines: save gauge invariance; jeopardise universality; complicate renormalizability
- Factorisation scale is arbitrary: transition from one scale to another (different experiments have different characteristic scales) by means of evolution equations
- Evolution: DGLAP, BFKL, CCFM... TMD; development of dedicated Monte-Carlo needed [in progress]

Practical Implementation of the QCD Factorisation Approach

- 3-dimensional pdfs contain the information about the intrinsic longitudinal and two-dimensional transverse momenta of the quarks and gluons, are called unintegrated or

Transverse-Momentum Dependent
(TMD)

- Current and future experimental **facilities**
  - LHC: unpolarised processes with sensitivity to polarized gluon distributions; testing resummation algorithms; Higgs, jet and heavy flavour production
  - Jefferson Lab: one third of approved experiments for 12 GeV Upgrade are devoted to the 3D structure of the nucleon (TMD and GPD)
  - Electron-Ion Collider: large-x regime, high luminosity, broad TMD program; spin effects

Transverse-Momentum Dependent pdfs

- **Inclusive** processes → **collinear factorisation**: one or less hadron detected; e.g., DIS, electron-positrion annihilation to hadrons
- **Semi-inclusive** processes → **TMD factorisation**: two or more hadrons in the initial or final state detected; e.g., Drell-Yan, SIDIS, hadron-hadron to jets, Higgs and heavy-flavour production
- **Collinear factorisation:** longitudinal momenta of the patrons are intrinsic, transverse momenta can be created by perturbative radiation effects (parton showers)
- **TMD factorisation:** a unifying QCD-based framework with both mechanisms of the transverse-momentum creation taken into account–intrinsic (essentially non-perturbative) and perturbative radiation

TMD Phenomenology: Identification of the benchmark observables

- **Polarised** processes: spin asymmetries [TMD in full glory]
- **Low-q_T** heavy particle spectra: vector and Higgs bosons, heavy flavours
- **High-energy** limit: $s \to \infty$ for fixed momentum transfer

Definitions of TMD/updfs

- TMD factorisation $\to$ operator definition
- updfs via evolution/resummation: DGLAP, BFKL, CCFM
- Effective theories: SCET
- High-energy/small-x: Balitsky, Kovchegov
- Looking for the a more general approach...

Operator structure of TMD

- 'Standard' approach:

  factorisation in a convenient gauge $\to$ gauge-dependent pdfs $\to$ gluon resummation $\to$ gauge-invariant pdfs with Wilson lines, path-dependence as prescribed by the factorisation

- Alternative approach:

  generic gauge-invariant path-dependent object $\to$ evolution in the coordinate space to fit the factorisation scheme $\to$ operator structure related to a given pdf $\to$ gauge-invariant pdf with Wilson lines, path-dependence as prescribed by the factorisation

Path-dependent correlation functions: main issues

$$\mathcal{F}(k)_\gamma = \mathrm{F.T.}\ \langle h|\ \bar{\Psi}(z)\ \mathcal{W}_\gamma[z,0]\ \Psi(0)\ |h\rangle$$

Gauge invariance is guaranteed by the **Wilson line**

$$\mathcal{W}_\gamma = \mathcal{P}\ \exp\left[\pm ig\int_0^z d\zeta^\mu \mathcal{A}_\mu(\zeta)\right]_\gamma$$

Issues:

- Gauge invariance → complicated **structure of the Wilson lines**
- Path dependence → **universality** is geopardized
- Singularities → problems with **renormalization**
- Factorization → **evolution**

Beyond the tree approximation: Why divergences?

$$\langle h|_H \bar{\Psi}_H(z)\ \mathcal{W}_\gamma[z^-, z_\perp; 0^-, 0_\perp]\Psi_H(0)|h\rangle_H$$

$$\rightarrow$$

$$\langle h|\bar{\Psi}(z)\ \mathcal{W}_\gamma[z^-, z_\perp; 0^-, 0_\perp]\Psi(0)\ \mathbf{S}_{\rm int}|h\rangle$$

$$\mathbf{S}_{\rm int} = \int d^4x \mathcal{L}_{\rm int}^{\rm QCD}(x)$$

$\rightarrow$ Perturbative expansion, Feynman graphs etc.

Path-dependence in the collinear (integrated) PDF

Longitudinal **momentum fraction:**

$$xk^+ = P^+$$

$$\mathcal{F}(x) = \frac{1}{2} \int \frac{dz^-}{2\pi} \ \mathrm{e}^{-ik^+z^-} \langle h|\bar{\psi}(z^-)\gamma^+\psi(0^-)|h\rangle$$

Gauge transofmations:

$$\psi(x) \to U(x)\psi(x)$$
$$\bar{\psi}(x) \to \bar{\psi}(x)U^\dagger(x)$$

Gauge invariance and path-dependence of bi-local operator products

Generic bi-local product:

$$\Delta(y,x) = \bar{\psi}(y)\psi(x)$$

$$\Delta(y,x) \to \bar{\psi}(y)U^{\dagger}(y)U(x)\psi(x)$$

Problem: to find a 'transporter'

$$T_{[y,x]}\psi(x) \to U(y)[T_{[y,x]}\psi(x)]$$

Bi-local product supplied with the **transporter** is gauge invariant:

$$\bar{\psi}(y)T_{[y,x]}\psi(x) \to$$
$$\bar{\psi}(y)U^{\dagger}(y)U(y)[T_{[y,x]}\psi(x)] = \bar{\psi}(y)T_{[y,x]}\psi(x)$$

Parallel transport equation

$$\frac{d}{dt}\ T_{[y,x]} = \pm ig\mathcal{A}_\gamma(t)T_{[y,x]}$$

Path-dependence:

$$z \in \gamma$$

$$dz_\mu = \dot{\gamma}_\mu(t)dt,\ z(0) = x,\ z(t) = y$$

$$\mathcal{A}_\gamma(t) = A_\mu[z(t)]\ \dot{\gamma}_\mu(t)$$

Parallel transport equation: Solution

Integral form:

$$T_{[y,x]} - T_{[x,x]} = T(t) - T(0) = \int_0^t \mathcal{A}_\gamma(t_1) T(t_1) dt_1$$

Perturbative expansion:

$$T_{[y,x]}(t) = T^{(0)} + g\, T^{(1)} + g^2\, T^{(2)} + ... + g^n\, T^{(n)} + ...$$

Initial condition:

$$T(0) = T_{[x,x]} = T^{(0)}$$

Parallel transport equation: Solution

Leading order term:

$$T^{(1)}(t) = \left[\int_0^t \mathcal{A}_\gamma(t_1) dt_1\right] T^{(0)}$$

$$T^{(2)}(t) = \int_0^t \mathcal{A}_\gamma(t_1) T(t_1) dt_1$$

$$= \left[\int_0^t \mathcal{A}_\gamma(t_1) \int_0^{t_1} \mathcal{A}_\gamma(t_2) dt_1 dt_2\right] T^{(0)}$$

$$T^{(2)}(t) = \frac{1}{2}\left[\mathcal{P} \int_0^t \int_0^t \mathcal{A}_\gamma(t_1) \mathcal{A}_\gamma(t_2) dt_1 dt_2\right] T^{(0)}$$

Parallel transport equation: Solution

Path-ordering:

$$\mathcal{P}\mathcal{A}_\gamma(t_1)\mathcal{A}_\gamma(t_2)$$
$$= \theta(t_1 - t_2)\ \mathcal{A}_\gamma(t_1)\mathcal{A}_\gamma(t_2) + \theta(t_2 - t_1)\ \mathcal{A}_\gamma(t_2)\mathcal{A}_\gamma(t_1)$$

$$T^{(n)}(t) = \frac{1}{n!}\ \mathcal{P}\int_0^t ... \int_0^t [\mathcal{A}_\gamma(t_1)...\mathcal{A}_\gamma(t_n)dt_1 dt_2 ... dt_n]\ \ T^{(0)}$$

$$T(t) = \sum_{n=0} g^n \frac{1}{n!}\ \mathcal{P}\int_0^t ... \int_0^t [\mathcal{A}_\gamma(t_1)...\mathcal{A}_\gamma(t_n)dt_1 dt_2 dt_n]\ T^{(0)}$$
$$\equiv \mathcal{P}\mathrm{exp}\left[g \int_0^t \mathcal{A}_\gamma(t')dt'\right]\ T^{(0)}$$

Parallel transport equation: Wilson line

$$T^{(0)} = T_{[x,x]} = 1$$

$$T_{[y,x]} = \mathcal{P}\exp\left[\pm ig \int_x^y A_\mu[z] dz_\mu\right]_\gamma$$

Parallel transporter is a Wilson line:

$$T_{[y,x]} = \mathcal{W}_\gamma[y, x]$$

Gluon TMD: Several Operator Definitions

@ [Mulders, Rodrigues (2001); Collins (2011)]

- Highly **gauge-dependent correlator** for a hadron h with a momentum P and spin S

$$\mathbf{G}^{\mu\nu}_{\rm g-d}(k;P,S) = \int d^4z\ \mathrm{e}^{-ikz}\ \langle h|\ \mathcal{A}^\mu(z)\mathcal{A}^\nu(0)\ |h\rangle$$

- Generic **gauge-invariant correlator**

$$\mathbf{G}^{\mu\nu|\rho\sigma}(k;P,S) = \int d^4z\ \mathrm{e}^{-ikz}\ \langle h|\ \mathcal{F}^{\mu\nu}(z)\ \mathcal{W}_\gamma\ \mathcal{F}^{\rho\sigma}(0)\ |h\rangle$$

$$\mathcal{F}^{\mu\nu} = \partial^\mu\mathcal{A}^\nu - \partial^\nu\mathcal{A}^\mu - ig[\mathcal{A}^\mu,\mathcal{A}^\nu]$$

Wilson line (system of lines) $\mathcal{W}_\gamma$ in the adjoint representation

$$\mathcal{F}_{\mu\nu} = F^a_{\mu\nu}T^a$$

Gluon TMD: Several Operator Definitions

@ [Mulders, Rodrigues (2001); Collins (2011)]

Gluon TMD from the generic **gauge-invariant correlator**

$$\mathbf{G}^{\mu\nu|\rho\sigma}(k; P, S) =$$

$$\int d^4 z \ \mathrm{e}^{-ikz} \ \langle h| \ \mathcal{F}^{\mu\nu}(z) \ \mathcal{W}_\gamma \ \mathcal{F}^{\rho\sigma}(0) \ |h\rangle$$

$$\to$$

$$\mathbf{G}^{ij}(x, k_\perp; P, S) \sim \int dk^- \ \mathbf{G}^{+i|+j}(k; P, S) =$$

$$\int dz^- d^2 z_\perp \ \mathrm{e}^{-ikz} \ \langle h| \ \mathcal{F}^{+i}(z) \ \mathcal{W}_\gamma \ \mathcal{F}^{+j}(0) \ |h\rangle$$

Equations of motion in the loop space

@ [Polyakov (1979); Makeenko, Migdal (1979, 1981); Kazakov, Kostov (1980); Brandt et al. (1981, 1982)]

- **Wilson loops** as the (fundamental) gauge-invariant degrees of freedom:

$$\langle \mathcal{W}_\gamma \rangle = \langle \mathcal{P}_\gamma \exp \oint_\gamma \, d\zeta_\mu \mathcal{A}^\mu(\zeta) \rangle$$

or

$$\langle \mathcal{W}^n_{\gamma_1, \dots \gamma_n} \rangle = \langle \mathcal{T} \mathcal{W}_{\gamma_1} \cdots \mathcal{W}_{\gamma_n} \rangle$$

- The Wilson functionals obey the **Makeenko-Migdal** loop equations:

$$\partial^\nu \, \frac{\delta}{\delta \sigma_{\mu\nu}(x)} \, \langle \mathcal{W}^1_\gamma \rangle = N_c g^2 \, \oint_\gamma \, dz^\mu \, \delta^{(4)}(x - z) \langle \mathcal{W}^2_{\gamma_{xz}\gamma_{zx}} \rangle$$

Loop space and differential operators

- Area derivative:

$$\frac{\delta}{\delta \sigma_{\mu\nu}(z)} \langle \mathcal{W}_\gamma \rangle = \lim_{|\delta\sigma_{\mu\nu}(z)| \to 0} \frac{\langle \mathcal{W}_{\gamma \delta \gamma_x} \rangle - \langle \mathcal{W}_\gamma \rangle}{|\delta \sigma_{\mu\nu}(z)|}$$

- Path derivative:

$$\partial_\mu \langle \mathcal{W}_\gamma \rangle = \lim_{|\delta z_\mu| \to 0} \frac{\langle \mathcal{W}_{\delta z_\mu^{-1} \gamma \delta z_\mu} \rangle - \langle \mathcal{W}_\gamma \rangle}{|\delta z_\mu|}$$

- Differential operators in the loop space $\to$ evolution of the Wilson loops in the coordinate representation $=$ equations of motion

Stokes-Mandelstam Gluon TMD

- Non-Abelian Stokes' theorem

  @ [Arefeva (1980) etc.]

$$\mathcal{P}_\gamma \exp \oint_\gamma \, d\zeta_\mu \mathcal{A}^\mu(\zeta) = \mathcal{P}_\gamma \mathcal{P}_\sigma \exp \int_\sigma \, d\sigma_{\mu\nu}(\zeta) \mathcal{F}^{\mu\nu}(\zeta)$$

- Mandelstam formula

  @ [Mandelstam (1968)]

$$\frac{\delta}{\delta\sigma_{\mu\nu}(x)} \mathcal{P}_\gamma \exp \oint_\gamma \, d\zeta_\mu \mathcal{A}^\mu(\zeta) = \mathcal{P}_\gamma \mathcal{F}^{\mu\nu}(x) \exp \oint_\gamma \, d\zeta_\mu \mathcal{A}^\mu(\zeta)$$

Stokes-Mandelstam Gluon TMD

- Stokes-Mandelstam definition

$$\tilde{\mathbf{G}}^{\mu\nu|\rho\sigma}(z;P,S) = \frac{\delta}{\delta\sigma_{\mu\nu}(z)}\frac{\delta}{\delta\sigma_{\mu\nu}(0)}\langle h|\mathcal{W}_{\gamma[z,0]}\ |h\rangle =$$

$$\frac{\delta}{\delta\sigma_{\mu\nu}(z)}\frac{\delta}{\delta\sigma_{\mu\nu}(0)}\sum_X \langle h|\mathcal{W}'_{\gamma[z]}|X\rangle\langle X|\mathcal{W}'_{\gamma[0]}\ |h\rangle$$

- Non-Abelian exponentiation

$$\langle W_{\gamma[z,0]}\rangle = \exp\left[\sum a_n W^{(n)}\right]\ ,\ W^{(n)} = \text{hadronic correlators}$$

- Gauge invariance, Path dependence, Universality

Current Activity

Recent review:
"Transverse momentum dependent (TMD) parton distribution functions: status and prospects"
arXiv:1507.05267 [hep-ph]

Regular workshops 'Resummation, Evolution, Factorization' organized by Antwerp-DESY-Amsterdam-Oxford-Groningen:

- International Workshop REF2015 , 02 - 08 Nov 2015, DESY, Hamburg, **Germany** → NEW!
  https://indico.desy.de/conferenceDisplay.py?confId=12707
- International Meeting preREF2015, 01 - 03 Jun 2015, Amsterdam, **The Netherlands**
- International Workshop REF2014, 08 - 11 Dec 2014, Antwerp, **Belgium**
- TMD/uPDF Workshop, 23 - 24 Jun 2014, Antwerp, **Belgium**

Back-Up Slides

Evolution in the coordinate space: Abelian case

- Abelian exponentiation

$$\langle \mathcal{W}_\gamma \rangle = \langle \mathcal{P}_\gamma \exp \oint_\gamma d\zeta_\mu \mathcal{A}^\mu(\zeta) \rangle =$$

$$\exp\left[-\frac{g^2}{2} \oint_\gamma d\zeta_\mu \oint_\gamma d\zeta'_\nu \; D_{\mu\nu}(\zeta - \zeta')\right]$$

- Basic correlator

$$D_{\mu\nu}(\zeta - \zeta') = \langle A_\mu(\zeta) A_\nu(\zeta') \rangle$$

- Parameterization

$$D_{\mu\nu}(z) = g_\mu \nu D_1(z^2, P^2) + \partial_\mu \partial_\nu D_2(z^2, P^2)$$

$$+\{P_\mu \partial_\nu\} D_3(z^2, P^2) + P_\mu P_\nu D_4(z^2, P^2)$$

Evolution in the coordinate space: Abelian case

- Area derivative

$$\frac{\delta}{\delta\sigma_{\mu\nu}(z)}\langle\mathcal{W}_\gamma\rangle =$$

$$-\frac{g^2}{2}\left[\frac{\delta}{\delta\sigma_{\mu\nu}(z)}\oint_\gamma d\zeta_\mu \oint_\gamma d\zeta'_\nu\ D_{\mu\nu}(\zeta-\zeta')\right]\langle\mathcal{W}_\gamma\rangle$$

- Non-vanishing terms after taking the path-derivative ∂_ν

$$\sim \oint_\gamma d\zeta_\nu\ \partial^2\ D_1(z^2, P^2)$$

★ standard Makeenko-Migdal term

$$\sim \oint_\gamma d\zeta_\nu\ (P\partial)^2\ D_4(z^2, P^2)$$

★ hadron momentum-dependent term

Evolution in the coordinate space: Abelian case

- Shape evolution equation

$$\partial_\mu^z \frac{\delta}{\delta\sigma_{\mu\nu}(z)} \langle \mathcal{W}_\gamma \rangle =$$

$$-\frac{g^2}{2} \left[\oint_\gamma d\zeta_\nu \left(\partial^2 \ D_1(z^2, P^2) + (P\partial)^2 \ D_4(z^2, P^2) \right) \right] \langle \mathcal{W}_\gamma \rangle$$

- More complicated shape variations: Fréchet, Polyakov etc.

- Basic correlator contains all necessary information

$$D_{\mu\nu}(\zeta - \zeta') = \langle A_\mu(\zeta) A_\nu(\zeta') \rangle$$

Outlook:

- Gluon TMD distribution function can be formulated within fully gauge-invariant, generically path-dependent framework based on the loop space formalism in the coordinate representation
- This approach goes the other way round wrt to the standard one: one starts with a maximally path-dependent, fully gauge-invariant object, and then obtains a gluon TMD, which is adjustable to any specific factorisation scheme
- The main ingredients of this approach are the hadronic matrix elements of Wilson loops $\langle h | \mathcal{W}_\gamma | h \rangle$
- Non-Abelian exponentiation enables separation of the non-local path-dependence and local UV-divergent contributions and appropriate parameterisation of various gTMD functions
- The work in progress...