Towards four-loop gauge coupling beta-functions in the SM

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SM RG functions and threshold corrections

- **Q**: What we need if we interested in SM behaviour at $Q^2 \sim M_{\rm Pl}$?
- **A**: We need two main ingridients:

1.Beta-functions

- Evolution of all SM couplings
- Calculated in $\overline{MS}$ scheme
- Just constants mass and momentum independent

2.Threshold corrections

- Initial values for evolution
- Connect $\overline{MS}$ running couplings with parameters extractable from experiment

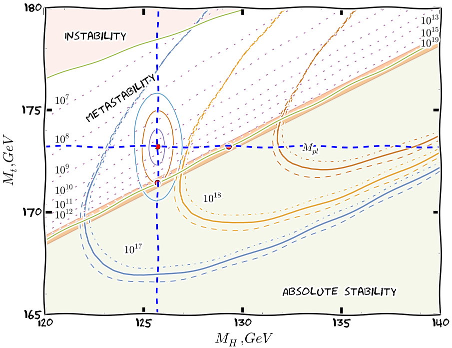
PDG 20XX		Fixed μ_0		
$M_b, M_W, M_Z, M_H, M_t, G_F$	$\longrightarrow$	$g_i(\mu_0), y_i(\mu_0), \lambda(\mu_0)$ in $\overline{MS}$ scheme	$\longrightarrow$	Evolve from μ_0 to scale μ

SM vacuum stability analysis at NNLO

- Three loop beta-functions for gauge Yukawa and self-coupling

  [Mihaila,Salomon,Steinhauser'12;Bednyakov,AP,Velizhanin'12,13;Chetyrkin,Zoller'13]
- Two loop full $\mathcal{O}(\alpha^2)$ threshold corrections

  [Buttazzo et al.'13;Kniehl,AP,Veretin'15]



RGE from propagator type integrals

- Three-loop experience
  - Massles propagators: gauge couplings, field renormalization constants

    using `FORM` based `MINCER` package
  - Three-loop massive boubles: Yukawa and Higgs self-coupling

    using `FORM` based `MATAD` package
- Four-loop experience
  - QCD beta function

    massive vacuum integrals available, possible to calculate all types of renormalization constants
  - Massles propagators. Difficult to prepare reduction. Easy to formulate the problem

    Independent tool for two-point Green functions renormalization constants calculation

Setup

- Model file tested at lower loop calculations
- DIANA/QGRAF [Nogueira'93;Fleischer,Tentyukov'99]
  Diagram generation
- Prepared set of mapings to 3 auxiliary topos
  each topo with 11 denominators and 3 irreducible numerators
- Reduction
  - LiteRed
    IBP rules preparation [Lee'12]
  - FIRE5, C++ version
    Integral reduction [Smirnov'14]
  - Master integrals
    4-loop propagators [Baikov,Chetyrkin'10;Lee,Smirnovs'11]

4 loop QCD β-function and renormalization constants

- IRR with auxiliary mass

  fully massive four-loop tadpoles, possible to calculate all renormalization constants

  - From Z_g, Z_c, Z_{ccg} [Larin,Vermaseren,Ritbergen'97]

$$Z_{a_g} = \frac{Z_{ccg}^2}{Z_c^2 Z_g}$$

  - From Z_g, Z_q, Z_{qqg} [Czakon'04]

$$Z_{a_g} = \frac{Z_{qqg}^2}{Z_q^2 Z_g}$$

- Using 3-loop massles integrals [Chetyrkin'04]
  - From Z_c, Z_{ccg} and already known β_{a_g}

    Impossible to calculate Z_g, but independent calculation of other RCs

$$Z_g = \frac{Z_{ccg}^2}{Z_c^2 Z_{a_g}}$$

Renormalization constants in background field gauge

- Split gauge fields $V = \tilde{V} + \hat{V}$ in
  - quantum $\tilde{V} = (\tilde{G}, \tilde{W}, \tilde{B}, \dots)$ and
  - background $\hat{V} = (\hat{G}, \hat{W}, \hat{B}, \dots)$
- Background felds do not propagate
- Modified Feynman rules needed
- QED like connection between renormalization constants

$$Z_{a_{g_i}} = 1/Z_{\hat{V}_i}, \qquad Z_{\xi_i} = Z_{\tilde{V}_i}$$

- Only need to calculate two-point functions
- Multiplicative renormalization using $a_{\text{bare}} = Z_a a_{\text{ren}}$

$$\Gamma^{(l)}_{\text{ren}} = Z^{(l)}_{\Gamma} \left[1 + \Gamma^{(1)}_{\text{bare}}(a_{\text{bare}}) + \Gamma^{(2)}_{\text{bare}}(a_{\text{bare}}) + \cdots + \Gamma^{(l)}_{\text{bare}}(a_{\text{bare}})\right]$$

From QCD to SM gauge beta-functions

- **Starting point:** comparision with QCD beta function

  Calculation from $Z_{\hat{G}}$, coincide with known result
- In SM more complicated set of feynman rules
- **Try more complicated model**: QCD with additional fermions in adjoint representation (gluino)

  Result available from independent calculation
- **Find renormalisation constants** in SM with vanishing EW couplings for parameters:

$$a_i = \left(\frac{g_s^2}{16\pi^2}, \frac{y_t^2}{16\pi^2}, \frac{\lambda}{16\pi^2}, \xi_G \right)$$

From QCD to SM gauge beta-functions

- **Starting point:** comparision with QCD beta function **OK**

  Calculation from $Z_{\hat{G}}$, coincide with known result

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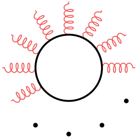
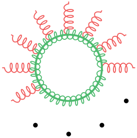
QCD with fermions in adjoint representation

- At diagram generation level simply replace, where $(F^a)_{bc} = f^{abc}$


$= n_f\,\mathrm{tr}[T^{a_1}\ldots T^{a_n}] \rightarrow$

$= n_g\,\mathrm{tr}[F^{a_1}\ldots F^{a_n}]$

- QCD with adjoint fermion has only <u>12</u> independent color structures

  in $g \rightarrow g$ 4-loop diagrams: 5 for n_f, 5 for n_f^2 and 2 for n_f^3

- each keeping track of QCD color structures($n_g = 0$), except one
  - $\mathbf{n_f}$: $C_F^3 T_F,\ C_A C_F^2 T_F,\ C_A^2 C_F T_F,\ C_A^3 T_F,\ d_F^{abcd} d_A^{abcd}$
  - $\mathbf{n_f^2}$: $C_A^2 T_F^2,\ C_A C_F T_F^2,\ \mathbf{C_F^2 T_F^2},\ d_F^{abcd} d_F^{abcd}$
  - $\mathbf{n_f^3}$: $C_A T_F^3,\ C_F T_F^3$

- Only <u>5</u> color structures at three loops [Clavelli,Coulter,Surguladze'96]

  3 for n_f and 2 for n_f^2

Why reduced model instead of full SM?

- Ten times less diagrams to calculate for $\hat{g} \to \hat{g}$

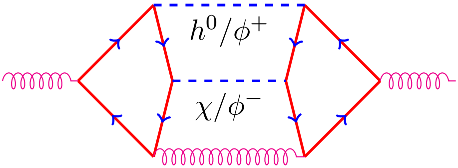
SM, $g_1 = g_2 = 0$	47531
Full SM	438211

- No problems with γ_5 in $d \neq 4$
- Model for general field theory with gauge, Yukawa and self-interaction

Dealing with γ_5

- Only single type of diagram with its nonplanar version contribute

  Only diagrams with two traces containig γ_5 contracted give nontrivial contribution



- All of them have only single pole $1/\epsilon$

  higher poles are absent

- Divergent part do not affected by relation

$$\mathrm{tr}(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5) = -4i\epsilon_{\mu\nu\rho\sigma} + \mathcal{O}(\epsilon)$$

- safe to use $D = 4$ relation

$$\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\delta} = -\mathcal{T}^{[\mu\nu\rho\sigma]}_{[\alpha\beta\gamma\delta]}, \qquad \mathcal{T}^{\mu\nu\rho\sigma}_{\alpha\beta\gamma\delta} = \delta^\mu_\alpha\delta^\nu_\beta\delta^\rho_\gamma\delta^\sigma_\delta\,,$$

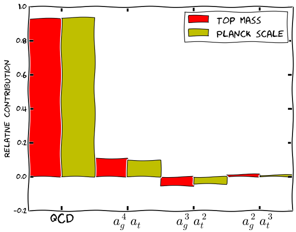
SM gauge coupling beta-function in $g_1 = g_2 = 0$ limit

- Beta function

$$\beta = \frac{d\, a_g}{d\, \log \mu^2} = h^2\beta_0 + h^3\beta_1 + h^4\beta_2 + h^5\beta_3$$

- h counts a_i powers

$$a_g = \frac{g^2}{16\pi^2}, a_t = \frac{y_t^2}{16\pi^2}, a_\lambda = \frac{\lambda}{16\pi^2}$$



4-loop non pure QCD part

$$\beta_3 = \beta_3^{\text{QCD}} + a_g^4 a_t \left[T_F C_F^2 \left(-\frac{3}{2} + 36\zeta_3 \right) + T_F C_A C_F \left(-\frac{523}{36} + 18\zeta_3 \right) - \frac{985}{18} T_F C_A^2 + \frac{322}{9} T_F^2 C_F n_G + \frac{218}{9} T_F^2 C_A n_G \right] + a_g^3 a_t^2 \left[T_F^2 \left(\frac{38}{3} - 8\zeta_3 \right) + T_F C_F \left(\frac{117}{4} - 36\zeta_3 \right) + \frac{111}{2} T_F C_A \right] + a_g^2 a_t^3 T_F \left(-\frac{423}{8} - 3\zeta_3 \right) - 15 a_g^2 a_t^2 a_\lambda T_F + 18 a_g^2 a_t a_\lambda^2 T_F$$

SM $\tilde{a}$-function and Weyl consistency conditions

- Motivated by Zamolodchikov c-function in 2d conformal theory
- Construct $\tilde{a}$-function in 4d SM [Jack,Osborn'90;Antipin et.al'13]
- Perturbative relations between different beta-function terms

$$\frac{\partial^2 \tilde{a}}{\partial g_i \partial g_j} = \frac{\partial}{\partial g_i}(\chi^{jk}\beta_k) + \mathcal{O}(g) = \frac{\partial}{\partial g_j}(\chi^{ik}\beta_k) + \mathcal{O}(g)$$

- At lowest order metric χ is diagonal

$$\chi = \mathrm{diag}\left(\frac{1}{a_1^2}\ ,\frac{3}{a_2^2}\ ,\frac{8}{a_3^2}\ ,\frac{2}{a_t}\ ,4\right)$$

- At higher orders non-diagonal terms of χ are involved and other additions

Connection between different orders in beta-functions

$$\beta_1 = 2a_1^2\left\{\cdots + \left(\frac{3}{4} + \frac{n_G}{2}\right)a_2 + \frac{22n_G}{9}a_3 + \cdots + a_t\left[-\frac{17}{12} - \ldots\right] + a_\lambda\left(\frac{3}{4}a_1 + \frac{3}{4}a_2 - \frac{3}{2}a_\lambda\right)\right\}$$

$$\beta_2 = 2a_2^2\left\{\cdots + \left(\frac{1}{4} + \frac{n_G}{6}\right)a_1 + \cdots + 2n_G a_3 \cdots + a_t\left[-\frac{3}{4} - \ldots\right] + a_\lambda\left(\frac{1}{4}a_1 + \frac{3}{4}a_2 - \frac{3}{2}a_\lambda\right)\right\}$$

$$\beta_3 = 2a_3^2\left\{\cdots + \frac{11n_G}{36}a_1 + \frac{3n_G}{4}a_2 + \cdots + a_t\left[-1 - \ldots\right]\right\}$$

$$\beta_t = 2a_t\left\{\frac{9}{4}a_t - 4a_3 - \frac{17}{24}a_1 - \frac{9}{8}a_2 + 3a_\lambda^2 - 6a_t a_\lambda - \ldots\right\}$$

$$\beta_\lambda = \frac{9}{16}a_2^2 - \frac{9}{2}a_\lambda a_2 + \frac{3}{16}a_1^2 - \frac{3}{2}a_\lambda a_1 + \frac{3}{8}a_1 a_2 + 12a_\lambda^2 + 6a_\lambda a_t - 3a_t^2$$

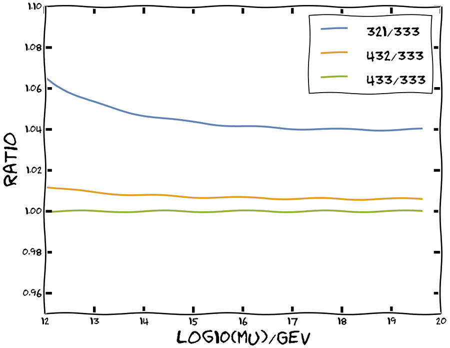
- Three-loop $\beta_g \to$ one-loop β_λ
- Two-loop $\beta_g \to$ one-loop β_y
- Two-loop $\beta_y \to$ one-loop β_λ

Application to SM power counting

[Antipin et al.'13] argued that proper expansion is not in loop orders of beta-functions (3-3-3), but in orders of $\tilde{a}$-function

- Sample term in $\tilde{a} = \cdots + a_2 a_t a_\lambda^2 + \ldots$ connected with:
  - $a_2 a_t a_\lambda$, 2-loop term in β_λ
  - $a_2 a_t a_\lambda^2$, 3 -loop term in β_t
  - $a_2^2 a_t a_\lambda^2$, 4-loop term in β_2
- With only three-loop beta-functions available we should use only three-loop β_g, two-loop β_y and one-loop β_λ (3-2-1)
- But now we equiped with β_g at four-loops and try (4-3-2) counting

λ running with different orders of PT



Simply think about widely used 3-3-3 running as 4-3-3 or 5-4-3 but with missed higher order terms

Conclusions

- Calculated four-loop QCD beta-function from different set of renormalization constants and coincide with known result
- Setup tested with new calculation of four-loop QCD beta-function with fermion in adjoint representation and compared with independent prediction
- Calculated four-loop gauge coupling beta-function in SM in $g_1 = g_2 = 0$ limit
- $\tilde{a}$-function motivated PT expansion tested with four-loop gauge beta-function