QCD for Colliders — Simulation of Jets

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Karlsruhe Institute of Technology

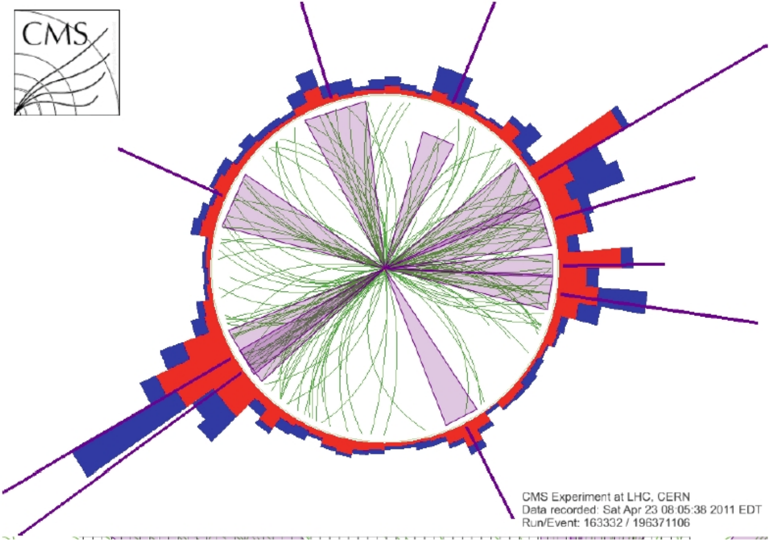
MCnet

PHYSICS AT THE TERA SCALE
Helmholtz Alliance

Motivation: jets

[Google Images]

Motivation: jets (at LHC of course)



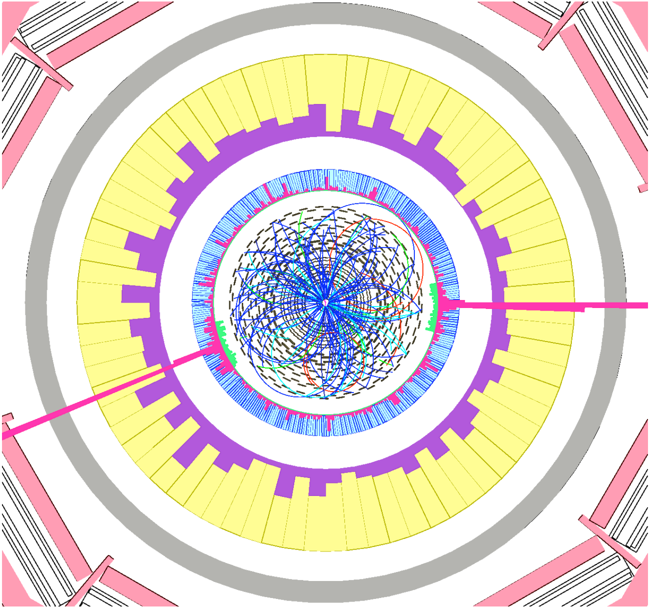
[CMS 2011]

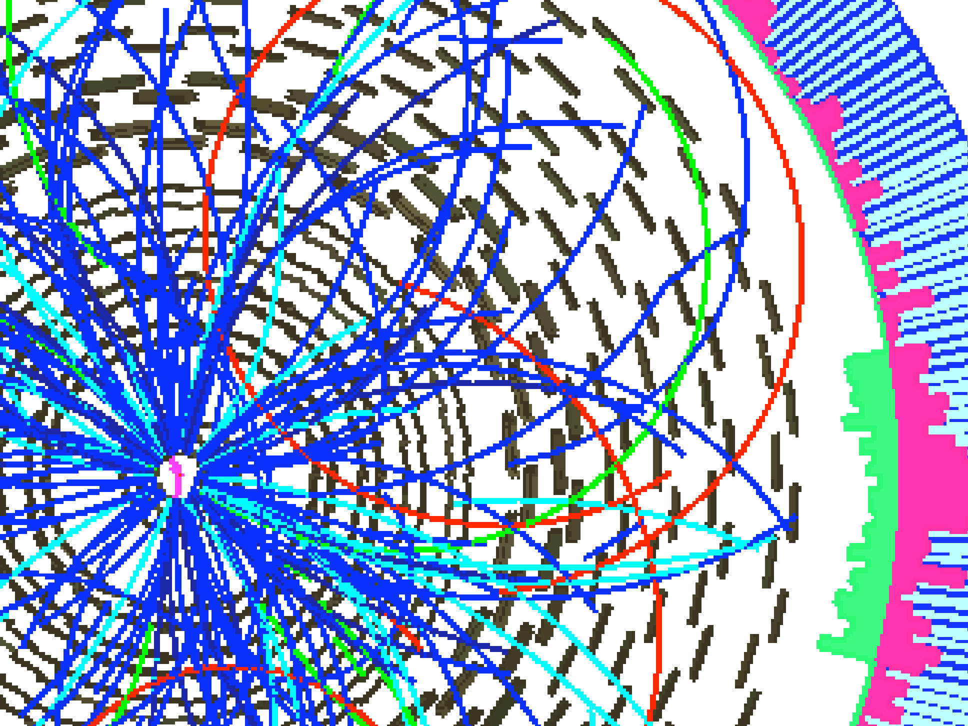
Why Monte Carlos?

We want to understand

$$\mathscr{L}_{\text{int}} \longleftrightarrow \text{Final states}\ .$$

Can you spot the Higgs?





Why Monte Carlos?

LHC experiments require
sound understanding of signals and *backgrounds*.

$\uparrow$

Full detector simulation.

$\uparrow$

Fully exclusive hadronic final state.

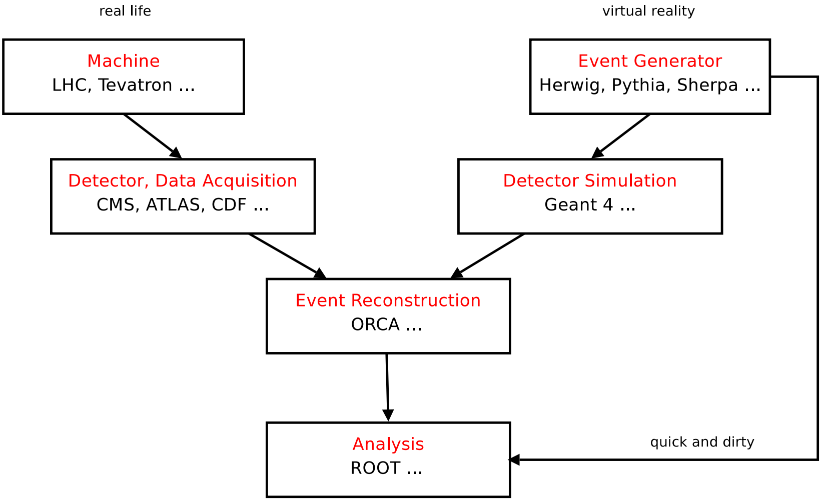
$\uparrow$

Monte Carlo event generator with
parton shower, hadronization model, decays of unstable particles.

$\uparrow$

Parton level computations.

Experiment and Simulation



Monte Carlo Event Generators

- Complex final states in full detail (jets).
- Arbitrary observables and cuts from final states.
- Studies of new physics models.

- Rates and topologies of final states.
- Background studies.
- Detector Design.
- Detector Performance Studies (Acceptance).

- *Obvious* for calculation of observables on the quantum level

$$|A|^2 \longrightarrow \text{Probability.}$$

Plan for this school

I Parton Showers

II Hadronization and Hadronic Decays

III Matching and Merging with Higher Orders
(if time permits)

pp Event Generator

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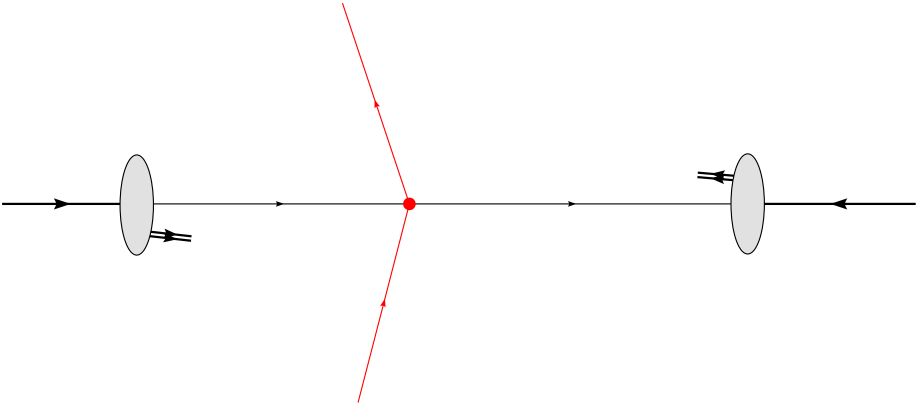

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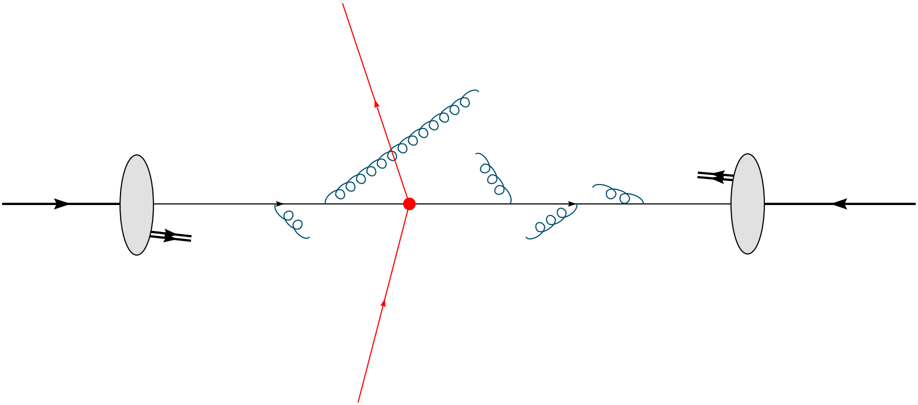

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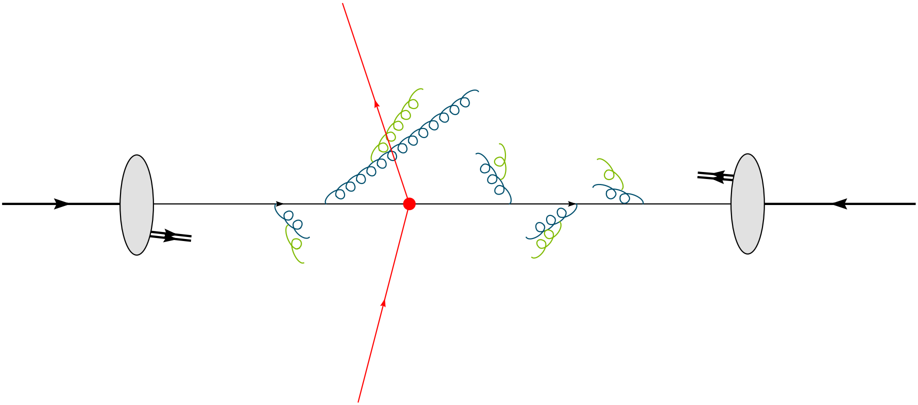

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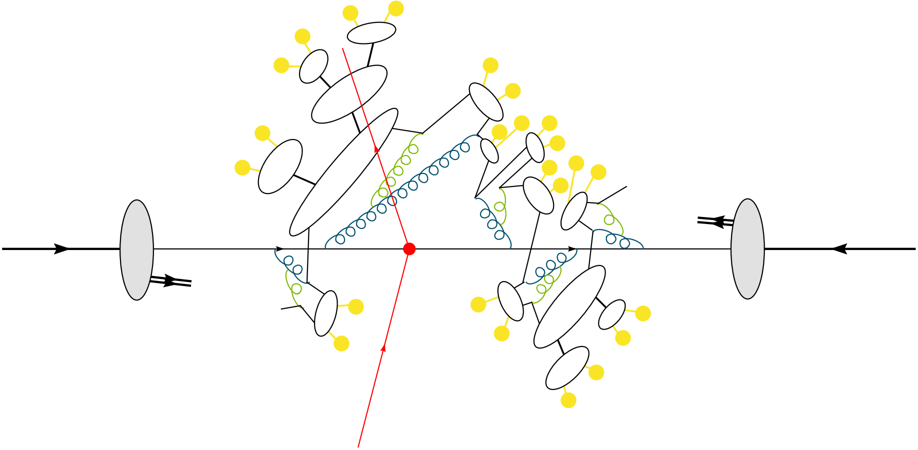

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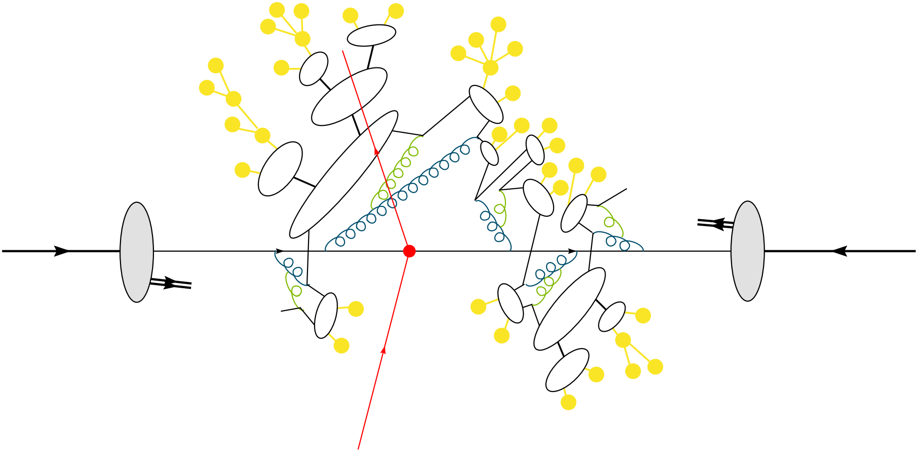

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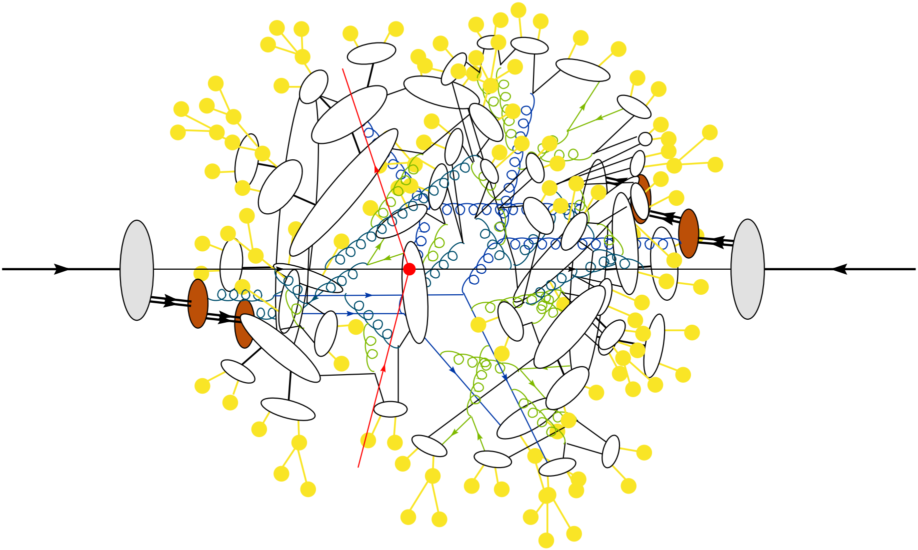


Divide and conquer

Partonic cross section from Feynman diagrams

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\text{hard}}\mathrm{d}P(\text{partons} \to \text{hadrons})$$

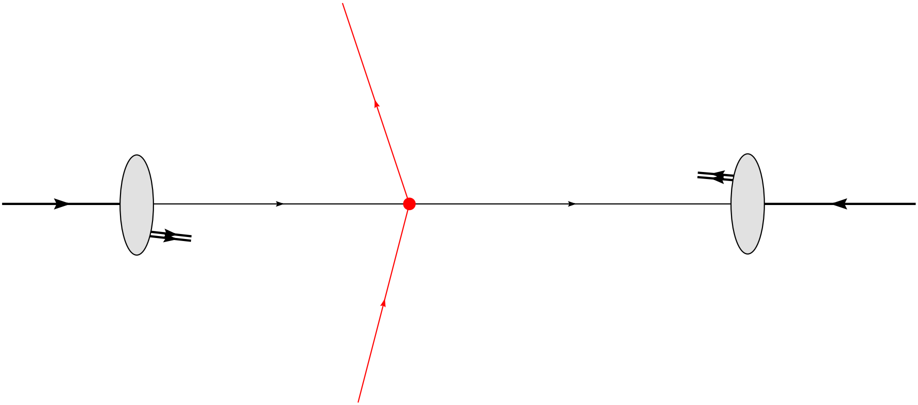
$$\begin{aligned}
\mathrm{d}P(\text{partons} \to \text{hadrons}) = \;& \mathrm{d}P(\text{resonance decays}) && [\Gamma > Q_0] \\
& \times \mathrm{d}P(\text{parton shower}) && [\text{TeV} \to Q_0] \\
& \times \mathrm{d}P(\text{hadronisation}) && [\sim Q_0] \\
& \times \mathrm{d}P(\text{hadronic decays}) && [O(\text{MeV})]
\end{aligned}$$

Underlying event from multiple partonic interactions

$$\mathrm{d}\sigma \longleftarrow \mathrm{d}\sigma(\text{QCD } 2 \to 2)$$

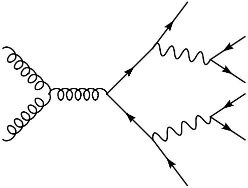
Hard scattering

Hard scattering



Matrix elements

- Perturbation theory/Feynman diagrams give us (fairly accurate) final states for a few number of legs ($O(1)$).



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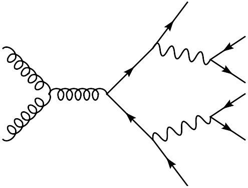

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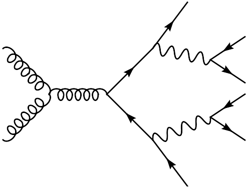

- OK for very inclusive observables.
- Starting point for further simulation.
- Want exclusive final state at the LHC ($O(100)$).
- Want arbitrary cuts.
- $\rightarrow$ use Monte Carlo methods.

Matrix elements

Where do we get (LO) $|M|^2$ from?

- Most/important simple processes (SM) are 'built in'.
- Calculate yourself (≤ 3 particles in final state).
- Matrix element generators:
  - MadGraph/MadEvent.
  - Comix/AMEGIC (part of Sherpa).
  - HELAC/PHEGAS.
  - Whizard.
  - CalcHEP/CompHEP.

  generate code or event files that can be further processed.
- $\rightarrow$ FeynRules interface to ME generators.

Cross section formula

From Matrix element, we calculate

$$\sigma = \int f_i(x_1,\mu^2) f_j(x_2,\mu^2) \frac{1}{F} \overline{\sum} |M|^2 \qquad \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}\Phi_n \ ,$$

Cross section formula

From Matrix element, we calculate

$$\sigma = \int f_i(x_1,\mu^2) f_j(x_2,\mu^2) \frac{1}{F} \overline{\sum} |M|^2 \, \Theta(\text{cuts}) \, \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}\Phi_n \ ,$$

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now,

$$\frac{1}{F} \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}\Phi_n = J(\vec{x}) \prod_{i=1}^{3n-2} \mathrm{d}x_i \qquad \left(\mathrm{d}\Phi_n = (2\pi)^4 \delta^{(4)}(\ldots) \prod_{i=1}^{n} \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3 2E_i} \right)$$

such that

$$\sigma = \int g(\vec{x}) \, \mathrm{d}^{3n-2}\vec{x} \ , \qquad \left(g(\vec{x}) = J(\vec{x}) f_i f_j \overline{\sum} |M|^2 \Theta(\text{cuts}) \right)$$

$$= \frac{1}{N} \sum_{i=1}^{N} \frac{g(\vec{x}_i)}{p(\vec{x}_i)} = \frac{1}{N} \sum_{i=1}^{N} w_i \ .$$

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We generate events $\vec{x}_i$ with weights w_i.

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$$P_i = \frac{w_i}{w_{\max}} \ ,$$

where $w_{\max}$ has to be chosen sensibly.
$\rightarrow$ reweighting, when $\max(w_i) = \bar{w}_{\max} > w_{\max}$, as

$$P_i = \frac{w_i}{\bar{w}_{\max}} = \frac{w_i}{w_{\max}} \cdot \frac{w_{\max}}{\bar{w}_{\max}} \ ,$$

i.e. reject events with probability $(w_{\max}/\bar{w}_{\max})$ afterwards.
(can be ignored when #(events with $w_i > \bar{w}_{\max}$) small.)

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Generate events with same frequency as in nature!

Some comments:

- Use common Monte Carlo techniques to generate events efficiently. Goal: small variance in w_i distribution!

Matrix elements

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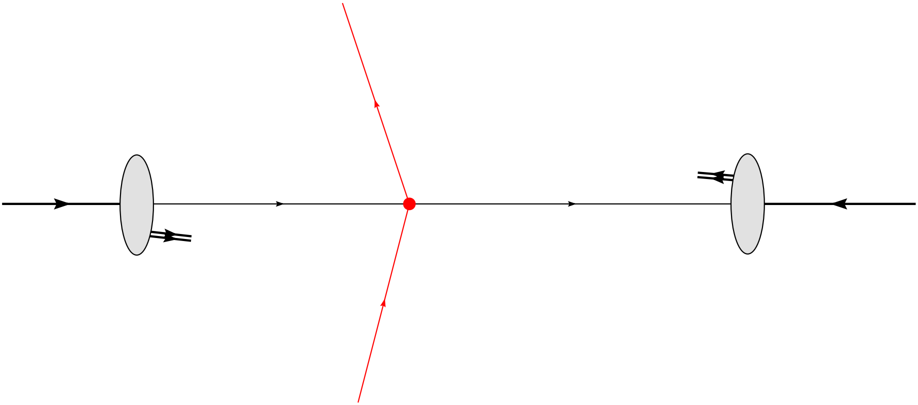
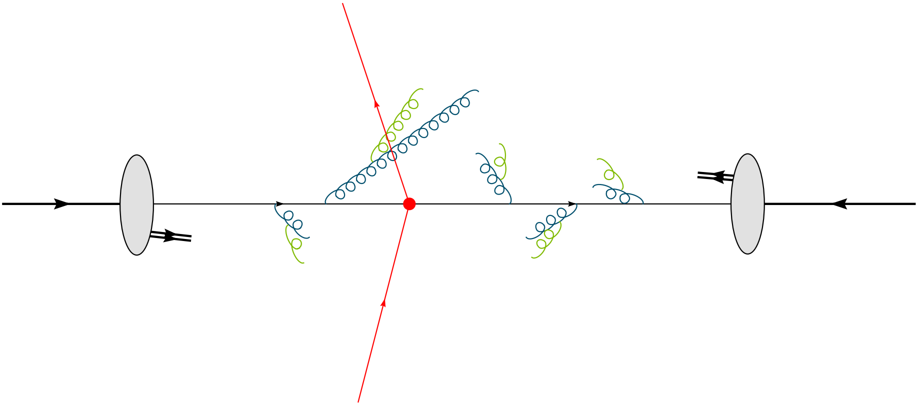
- Use common Monte Carlo techniques to generate events efficiently. Goal: small variance in w_i distribution!
- Efficient generation closely tied to knowledge of $f(\vec{x}_i)$, *i.e.* the matrix element's propagator structure.
  $\rightarrow$ build phase space generator already while generating ME's automatically.

Hard matrix element



Hard matrix element → parton showers



Parton showers

Quarks and gluons in final state, pointlike.

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Dominated by large logs, terms

$$\alpha_S^n \log^{2n} \frac{Q}{Q_0} \sim 1 \, .$$

Generated from emissions *ordered* in Q.

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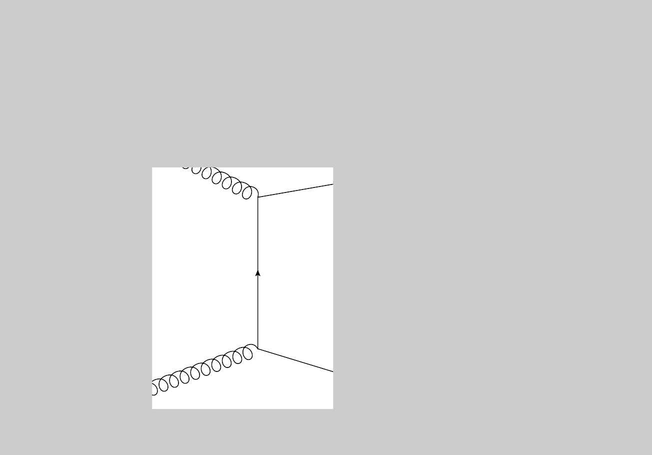
$$\alpha_S^n \log^{2n} \frac{Q}{Q_0} \sim 1 \, .$$

Generated from emissions *ordered* in Q.
Soft and/or collinear emissions.

ME approximated by parton cascade

Evolution in scale, typically $Q \sim 1\,\mathrm{TeV}$ down to $Q \sim 1\,\mathrm{GeV}$.



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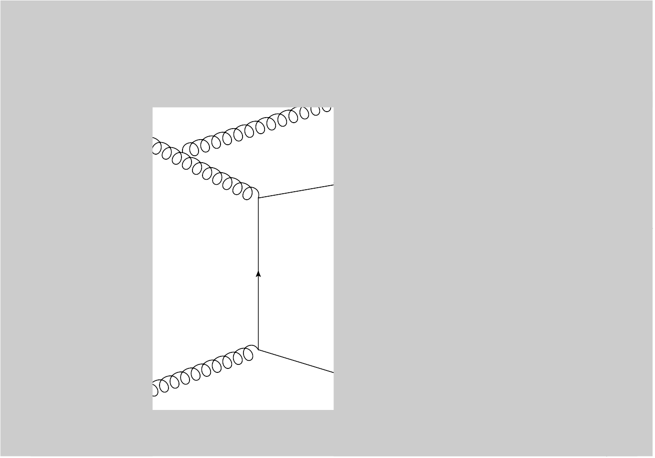

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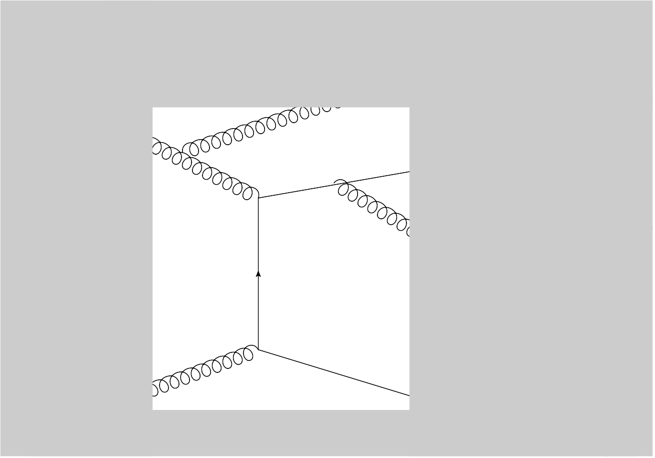

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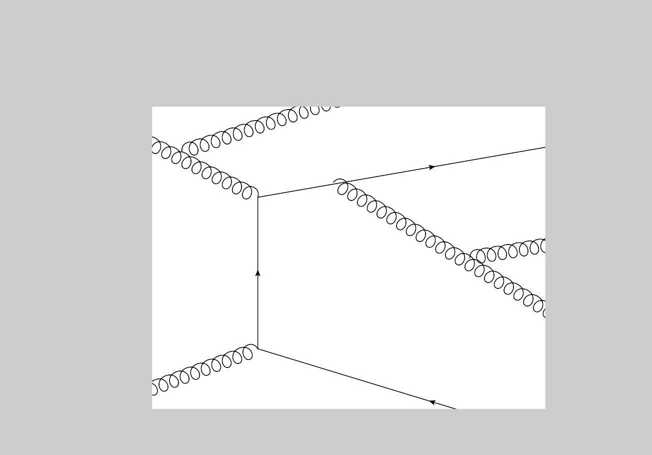

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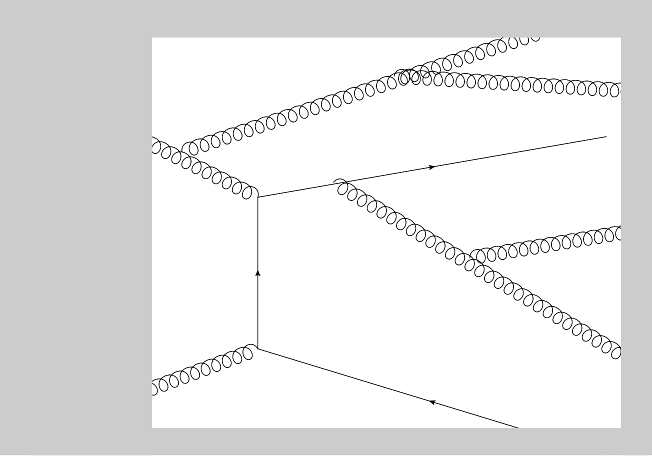


e^+e^- annihilation

Good starting point: $e^+e^- \to q\bar{q}g$:

Final state momenta in one plane (orientation usually averaged).
Write momenta in terms of

$$x_i = \frac{2p_i \cdot q}{Q^2} \quad (i = 1,2,3) \; ,$$

$$0 \le x_i \le 1 \; , x_1 + x_2 + x_3 = 2 \; ,$$

$$q = (Q,0,0,0) \; ,$$

$$Q \equiv E_{cm} \; .$$

$(x_1,x_2) = (x_q, x_{\bar{q}})$ –plane:

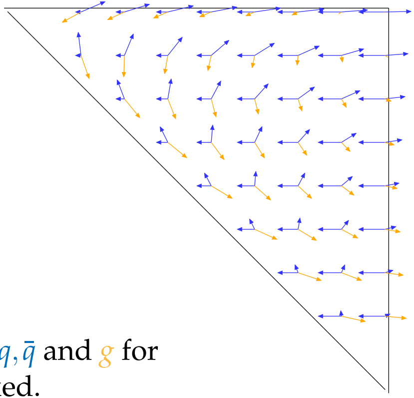


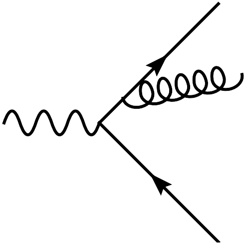
Fig: momentum configuration of $q, \bar{q}$ and g for given point (x_1,x_2), $\bar{q}$ direction fixed.

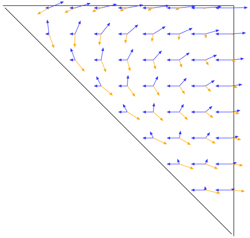
e^+e^- annihilation

Differential cross section:

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 \frac{C_F \alpha_S}{2\pi} \frac{x_1 + x_2}{(1-x_1)(1-x_2)}$$

Collinear singularities: $x_1 \to 1$ or $x_2 \to 1$. Soft singularity: $x_1, x_2 \to 1$.





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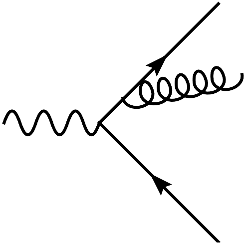

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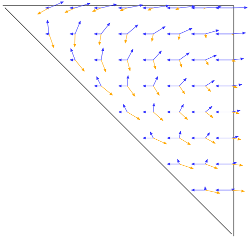
Rewrite in terms of x_3 and $\theta = \angle(q,g)$:

$$\frac{d\sigma}{d\cos\theta\, dx_3} = \sigma_0 \frac{C_F \alpha_S}{2\pi} \left[\frac{2}{\sin^2\theta} \frac{1+(1-x_3)^2}{x_3} - x_3 \right]$$



Singular as $\theta \to 0$ and $x_3 \to 0$.

e^+e^- annihilation

Can separate into two jets as

$$\begin{aligned}\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} &= \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\theta}{1+\cos\theta} \\ &= \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\bar{\theta}}{1-\cos\bar{\theta}} \\ &\approx \frac{\mathrm{d}\theta^2}{\theta^2} + \frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2}\end{aligned}$$

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So, we rewrite $\mathrm{d}\sigma$ in collinear limit as

$$\mathrm{d}\sigma = \sigma_0 \sum_{\text{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} C_F \frac{1+(1-z)^2}{z^2} \mathrm{d}z$$

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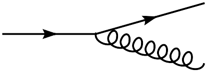
$$= \sigma_0 \sum_{\text{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} P(z) \mathrm{d}z$$

with DGLAP splitting function $P(z)$.

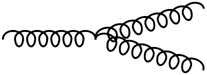
Collinear limit

Universal DGLAP splitting kernels for collinear limit:

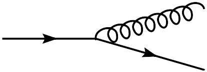
$$\mathrm{d}\sigma = \sigma_0 \sum_{\text{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} P(z) \mathrm{d}z$$



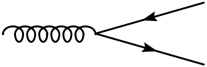
$$P_{q\to qg}(z) = C_F \frac{1+z^2}{1-z}$$



$$P_{g\to gg}(z) = C_A \frac{(1-z(1-z))^2}{z(1-z)}$$



$$P_{q\to gq}(z) = C_F \frac{1+(1-z)^2}{z}$$



$$P_{g\to qq}(z) = T_R(1-2z(1-z))$$

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Note: Other variables may equally well characterize the colline-ar limit:

$$\frac{\mathrm{d}\theta^2}{\theta^2} \sim \frac{\mathrm{d}Q^2}{Q^2} \sim \frac{\mathrm{d}p_\perp^2}{p_\perp^2} \sim \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \sim \frac{\mathrm{d}t}{t}$$

whenever $Q^2, p_\perp^2, t \to 0$ means "collinear".

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$$\mathrm{d}\sigma = \sigma_0 \sum_{\mathrm{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} P(z)\mathrm{d}z$$

Note: Other variables may equally well characterize the collinear limit:

$$\frac{\mathrm{d}\theta^2}{\theta^2} \sim \frac{\mathrm{d}Q^2}{Q^2} \sim \frac{\mathrm{d}p_\perp^2}{p_\perp^2} \sim \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \sim \frac{\mathrm{d}t}{t}$$

whenever $Q^2, p_\perp^2, t \to 0$ means "collinear".

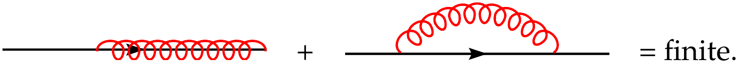
- θ: HERWIG
- Q^2: PYTHIA ≤ 6.3, SHERPA.
- $p_\perp$: PYTHIA ≥ 6.4, ARIADNE, Catani–Seymour showers.
- $\tilde{q}$: Herwig++.

Resolution

Need to introduce resolution t_0, e.g. a cutoff in $p_\perp$. Prevent us from the singularity at $\theta \to 0$.

Emissions below t_0 are unresolvable.

Finite result due to virtual corrections:



unresolvable + virtual emissions are included in Sudakov form factor via unitarity (see below!).

Towards multiple emissions

Starting point: factorisation in collinear limit, single emission.

$$\sigma_{2+1}(t_0) = \sigma_2(t_0) \int_{t_0}^{t} \frac{\mathrm{d}t'}{t'} \int_{z_-}^{z_+} \mathrm{d}z \, \frac{\alpha_S}{2\pi} \hat{P}(z) \; = \sigma_2(t_0) \int_{t_0}^{t} \mathrm{d}t \, W(t) \; .$$

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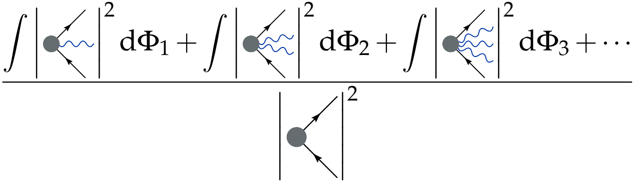
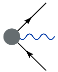
Simple example:
Multiple photon emissions, strongly ordered in t.
We want

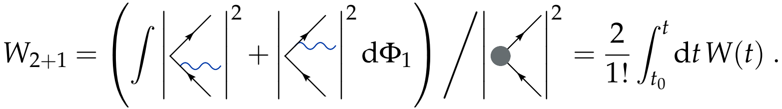
$$W_{\mathrm{sum}} = \sum_{n=1} W_{2+n} =$$



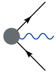
for any number of emissions.

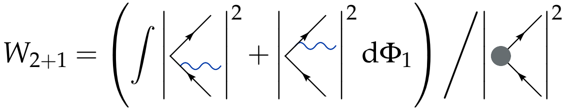
Towards multiple emissions

$(n = 1)$ 

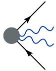


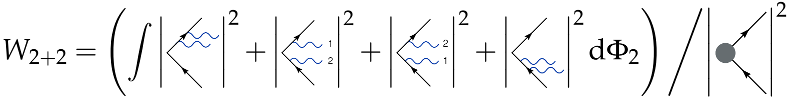
Towards multiple emissions

$(n = 1)$ 



$$= \frac{2}{1!}\int_{t_0}^{t} \mathrm{d}t\, W(t)\ .$$

$(n = 2)$ 

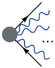


$$= 2^2 \int_{t_0}^{t} \mathrm{d}t' \int_{t_0}^{t'} \mathrm{d}t''\, W(t')W(t'') = \frac{2^2}{2!}\left(\int_{t_0}^{t} \mathrm{d}t\, W(t)\right)^2\ .$$

We used

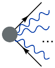
$$\int_{t_0}^{t} \mathrm{d}t_1 \ldots \int_{t_0}^{t_{n-1}} \mathrm{d}t_n\ W(t_1)\ldots W(t_n) = \frac{1}{n!}\left(\int_{t_0}^{t} \mathrm{d}t\, W(t)\right)^n\ .$$

Towards multiple emissions

Easily generalized to n emissions  by induction. *i.e.*

$$W_{2+n} = \frac{2^n}{n!}\left(\int_{t_0}^{t} \mathrm{d}t\, W(t)\right)^n$$

Towards multiple emissions

Easily generalized to n emissions  by induction. *i.e.*

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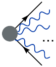
So, in total we get

$$\sigma_{>2}(t_0) = \sigma_2(t_0) \sum_{k=1}^{\infty} \frac{2^k}{k!} \left(\int_{t_0}^{t} \mathrm{d}t\, W(t) \right)^k = \sigma_2(t_0) \left(\mathrm{e}^{2 \int_{t_0}^{t} \mathrm{d}t\, W(t)} - 1 \right)$$

Towards multiple emissions

Easily generalized to n emissions  by induction. *i.e.*

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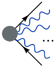
$$= \sigma_2(t_0) \left(\frac{1}{\Delta^2(t_0, t)} - 1 \right)$$

Sudakov Form Factor

$$\Delta(t_0, t) = \exp\left[-\int_{t_0}^{t} \mathrm{d}t\, W(t) \right]$$

Towards multiple emissions

Easily generalized to n emissions  by induction. *i.e.*

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$$= \sigma_2(t_0) \left(\frac{1}{\Delta^2(t_0, t)} - 1 \right)$$

Sudakov Form Factor in QCD

$$\Delta(t_0, t) = \exp\left[-\int_{t_0}^{t} \mathrm{d}t\, W(t) \right] = \exp\left[-\int_{t_0}^{t} \frac{\mathrm{d}t}{t} \int_{z_-}^{z_+} \frac{\alpha_S(z,t)}{2\pi} \hat{P}(z,t) \mathrm{d}z \right]$$

Sudakov form factor

Note that

$$\sigma_{\text{all}} = \sigma_2 + \sigma_{>2} = \sigma_2 + \sigma_2 \left(\frac{1}{\Delta^2(t_0, t)} - 1 \right) ,$$

$$\Rightarrow \Delta^2(t_0, t) = \frac{\sigma_2}{\sigma_{\text{all}}} .$$

$$\text{Two jet rate} = \Delta^2 = P^2(\text{No emission in the range } t \to t_0) .$$

$$\text{Sudakov form factor} = \text{No emission probability} .$$

Often $\Delta(t_0, t) \equiv \Delta(t)$.

- Hard scale t, typically CM energy or $p_\perp$ of hard process.
- Resolution t_0, two partons are resolved as two entities if inv mass or relative $p_\perp$ above t_0.
- P^2 (not P), as we have two legs that evolve independently.

Sudakov form factor from Markov property

Unitarity

$$P(\text{"some emission"}) + P(\text{"no emission"}) = P(0 < t \leq T) + \bar{P}(0 < t \leq T) = 1 \; .$$

Multiplication law (no memory)

$$\bar{P}(0 < t \leq T) = \bar{P}(0 < t \leq t_1)\bar{P}(t_1 < t \leq T)$$

Sudakov form factor from Markov property

Unitarity

$$P(\text{“some emission”}) + P(\text{“no emission”}) = P(0 < t \le T) + \bar{P}(0 < t \le T) = 1 \; .$$

Multiplication law (no memory)

$$\bar{P}(0 < t \le T) = \bar{P}(0 < t \le t_1)\bar{P}(t_1 < t \le T)$$

Then subdivide into n pieces: $t_i = \frac{i}{n}T, 0 \le i \le n$.

$$\bar{P}(0 < t \le T) = \lim_{n\to\infty}\prod_{i=0}^{n-1}\bar{P}(t_i < t \le t_{i+1}) = \lim_{n\to\infty}\prod_{i=0}^{n-1}\big(1 - P(t_i < t \le t_{i+1})\big)$$

$$= \exp\left(-\lim_{n\to\infty}\sum_{i=0}^{n-1}P(t_i < t \le t_{i+1})\right) = \exp\left(-\int_0^T \frac{\mathrm{d}P(t)}{\mathrm{d}t}\mathrm{d}t\right) \; .$$

Sudakov form factor

Again, no–emission probability!

$$\bar{P}(0 < t \leq T) = \exp\left(-\int_0^T \frac{\mathrm{d}P(t)}{\mathrm{d}t}\mathrm{d}t\right)$$

So,

$$\begin{aligned}\mathrm{d}P(\text{first emission at } T) &= \mathrm{d}P(T)\bar{P}(0 < t \leq T) \\ &= \mathrm{d}P(T)\exp\left(-\int_0^T \frac{\mathrm{d}P(t)}{\mathrm{d}t}\mathrm{d}t\right)\end{aligned}$$

That's what we need for our parton shower! Probability density for next emission at t:

$$\mathrm{d}P(\text{next emission at } t) =$$
$$\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\ \exp\left[-\int_{t_0}^{t}\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\right]$$

Parton shower Monte Carlo

Probability density:

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$$\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z \, \exp\left[-\int_{t_0}^{t}\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\right]$$

Conveniently, the probability distribution is $\Delta(t)$ itself.

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Conveniently, the probability distribution is $\Delta(t)$ itself.
Hence, parton shower very roughly from (HERWIG):

1. Choose flat random number $0 \leq \rho \leq 1$.
2. If $\rho < \Delta(t_{\max})$: no resolbable emission, stop this branch.
3. Else solve $\rho = \Delta(t_{\max})/\Delta(t)$
   (= no emission between $t_{\max}$ and t) for t.
   Reset $t_{\max} = t$ and goto 1.

Determine z essentially according to integrand in front of exp.

Parton shower Monte Carlo

Probability density:

$$\mathrm{d}P(\text{next emission at } t) = \frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\ \exp\left[-\int_{t_0}^{t}\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\right]$$

Conveniently, the probability distribution is $\Delta(t)$ itself.

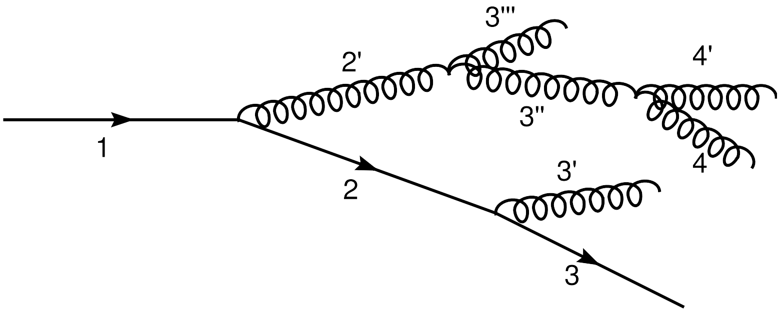
- That was old HERWIG variant. Relies on (numerical) integration/tabulation for $\Delta(t)$.
- Pythia, now also Herwig++, use the Veto Algorithm.
- Method to sample x from distribution of the type

$$\mathrm{d}P = F(x)\exp\left[-\int^{x}\mathrm{d}x'\,F(x')\right]\mathrm{d}x\ .$$

Simpler, more flexible, but slightly slower.

Parton cascade

Get tree structure, ordered in evolution variable t:



Here: $t_1 > t_2 > t_3$; $t_2 > t_{3'}$ etc.
Construct four momenta from (t_i, z_i) and (random) azimuth ϕ.

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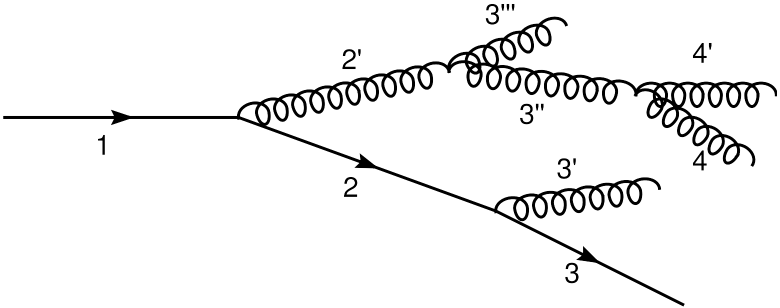
Get tree structure, ordered in evolution variable t:

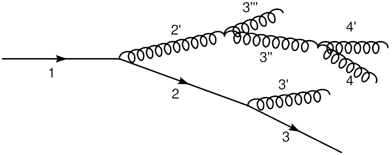


Here: $t_1 > t_2 > t_3$; $t_2 > t_{3'}$ etc.
Construct four momenta from (t_i, z_i) and (random) azimuth ϕ.

Not at all unique!
Many (more or less clever) choices still to be made.

Parton cascade

Get tree structure, ordered in evolution variable t:



- t can be θ, Q^2, $p_\perp$, ...
- Choice of hard scale t_{max} not fixed. "Some hard scale".
- z can be light cone momentum fraction, energy fraction, ...
- Available parton shower phase space.
- Integration limits.
- Regularisation of soft singularities.
- ...

Good choices needed here to describe wealth of data!

Soft emissions

- Only *collinear* emissions so far.
- Including *collinear+soft*.
- *Large angle+soft* also important.

Soft emissions

- Only *collinear* emissions so far.
- Including *collinear+soft*.
- *Large angle+soft* also important.

Soft emission: consider *eikonal factors*,
here for $q(p+q) \to q(p)g(q)$, soft g:

$$u(p) \not{\varepsilon} \frac{\not{p} + \not{q} + m}{(p+q)^2 - m^2} \longrightarrow u(p) \frac{p \cdot \varepsilon}{p \cdot q}$$

soft factorisation. Universal, *i.e.* independent of emitter.
In general:

$$d\sigma_{n+1} = d\sigma_n \frac{d\omega}{\omega} \frac{d\Omega}{2\pi} \frac{\alpha_S}{2\pi} \sum_{ij} C_{ij} W_{ij} \quad \text{(“QCD–Antenna”)}$$

with

$$W_{ij} = \frac{1 - \cos\theta_{ij}}{(1 - \cos\theta_{iq})(1 - \cos\theta_{qj})} \; .$$

Soft emissions

We define

$$W_{ij} = \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{iq})(1-\cos\theta_{qj})} \equiv W_{ij}^{(i)} + W_{ij}^{(j)}$$

with

$$W_{ij}^{(i)} = \frac{1}{2}\left(W_{ij} + \frac{1}{1-\cos\theta_{iq}} - \frac{1}{1-\cos\theta_{qj}}\right) .$$

$W_{ij}^{(i)}$ is only collinear divergent if $q \| i$ etc .

Soft emissions

We define

$$W_{ij} = \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{iq})(1-\cos\theta_{qj})} \equiv W_{ij}^{(i)} + W_{ij}^{(j)}$$

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$W_{ij}^{(i)}$ is only collinear divergent if $q \| i$ etc .
After integrating out the azimuthal angles, we find

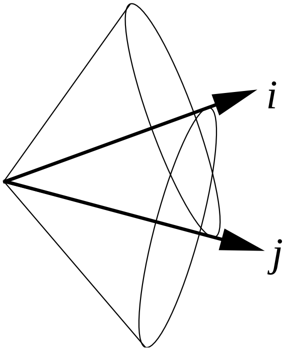
$$\int \frac{d\phi_{iq}}{2\pi} W_{ij}^{(i)} = \begin{cases} \dfrac{1}{1-\cos\theta_{iq}} & (\theta_{iq} < \theta_{ij}) \\ 0 & \text{otherwise} \end{cases}$$

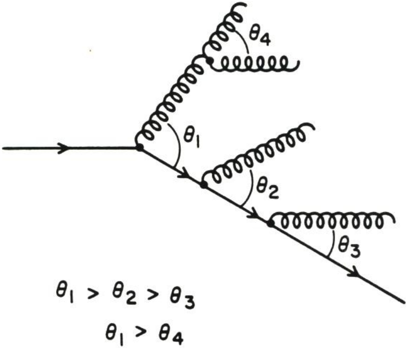
That's angular ordering.

Angular ordering

Radiation from parton *i* is bound to a cone, given by the colour partner parton *j*.

Results in angular ordered parton shower and suppresses soft gluons viz. hadrons in a jet.





Colour coherence from CDF

Events with 2 hard ($> 100\,\text{GeV}$) jets and a soft 3rd jet ($\sim 10\,\text{GeV}$)

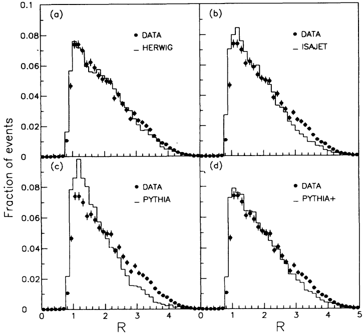


FIG. 14. Observed R distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.

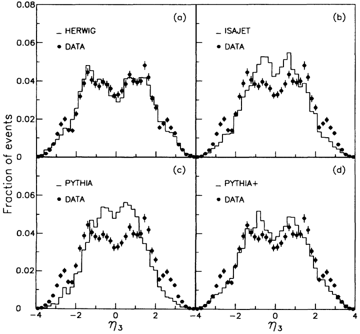


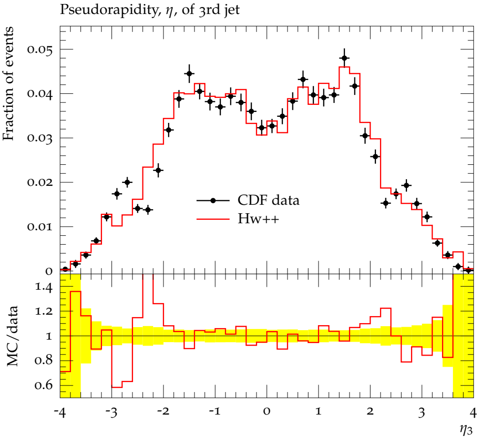
FIG. 13. Observed η_3 distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.

F. Abe *et al.* [CDF Collaboration], Phys. Rev. D **50** (1994) 5562.

Best description with angular ordering.

Colour coherence from CDF

Events with 2 hard (> 100 GeV) jets and a soft 3rd jet (∼ 10 GeV)



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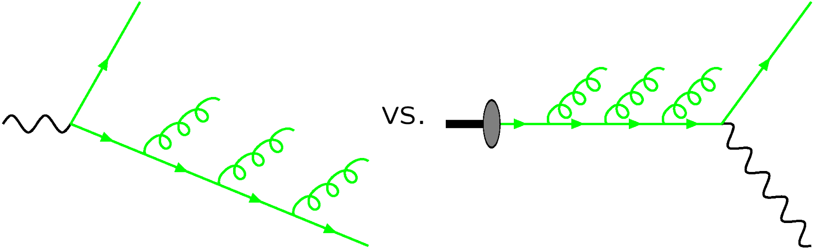
Best description with angular ordering.

Initial state radiation



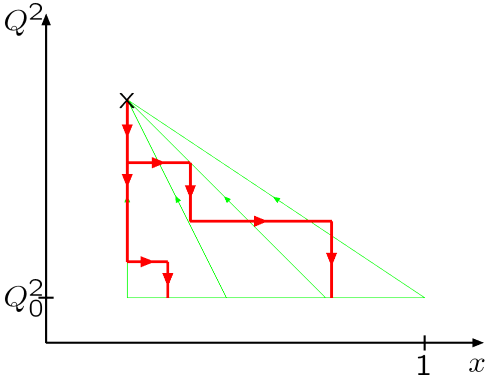
Similar to final state radiation. Sudakov form factor ($x' = x/z$)

$$\Delta(t, t_{\max}) = \exp\left[-\sum_b \int_t^{t_{\max}} \frac{\mathrm{d}t}{t} \int_{z_-}^{z_+} \mathrm{d}z \frac{\alpha_S(z,t)}{2\pi} \frac{x' f_b(x',t)}{x f_a(x,t)} \hat{P}_{ba}(z,t) \right]$$

Have to divide out the pdfs.

Initial state radiation

Evolve backwards from hard scale Q^2 *down* towards cutoff scale Q_0^2. Thereby increase x.

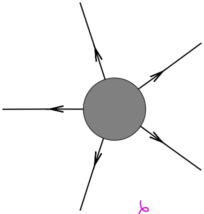


With parton shower we *undo* the DGLAP evolution of the pdfs.

Reconstruction of Kinematics

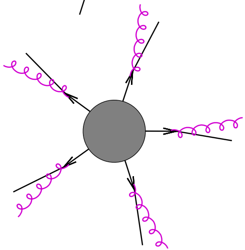
After shower: original partons acquire virtualities q_i^2
$\rightarrow$ boost/rescale jets:
Started with

$$\sqrt{s} = \sum_{i=1}^{n} \sqrt{m_i^2 + \vec{p}_i^2}$$



we *rescale* momenta with common factor k,

$$\sqrt{s} = \sum_{i=1}^{n} \sqrt{q_i^2 + k\vec{p}_i^2}$$

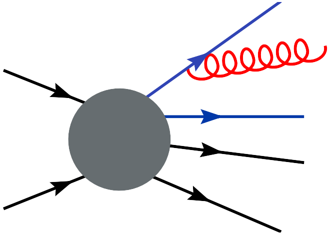


to preserve overall energy/momentum.
$\rightarrow$ resulting jets are boosted accordingly.

Dipoles

Exact kinematics when recoil is taken by spectator(s).

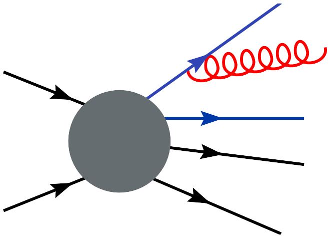
- Dipole showers.
- Ariadne.
- Recoils in Pythia.



Dipoles

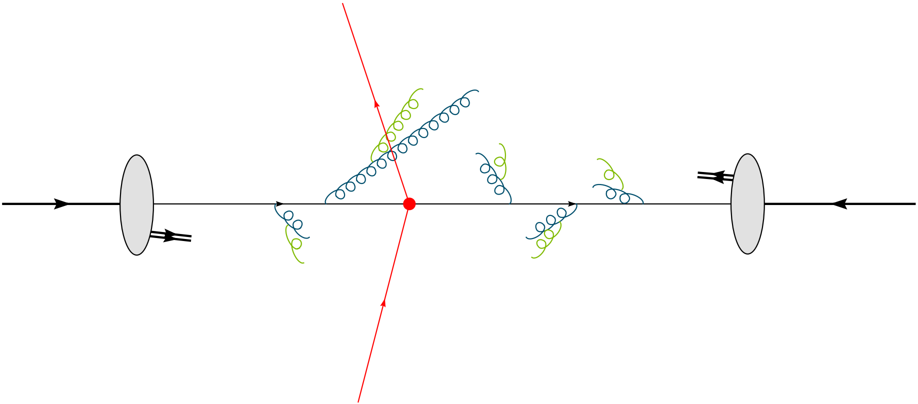
Exact kinematics when recoil is taken by spectator(s).

- Dipole showers.
- Ariadne.
- Recoils in Pythia.
- New dipole showers, based on
  - Catani Seymour dipoles.
  - QCD Antennae.
  - Goal: matching with NLO.
- Generalized to IS–IS, IS–FS.

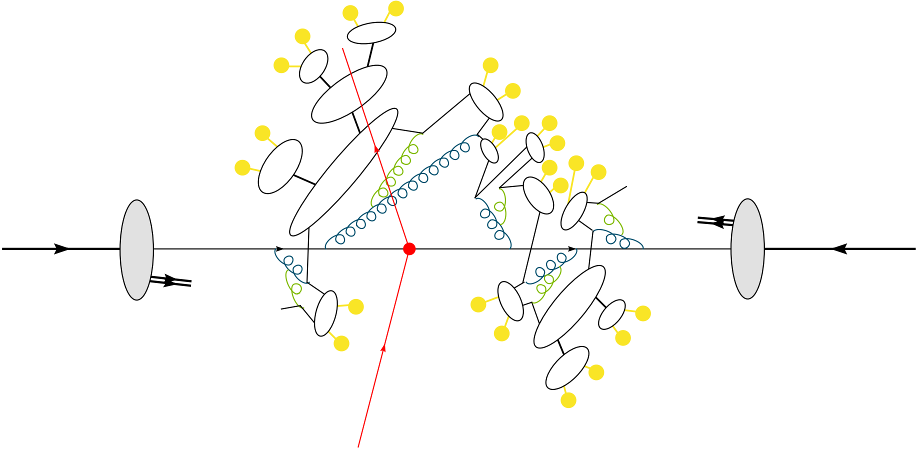


Hadronization

Parton shower



Parton shower ⟶ hadrons

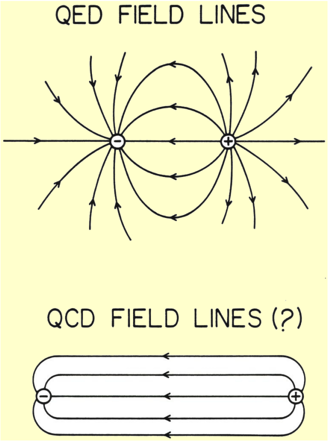


Parton shower $\longrightarrow$ hadrons

- Parton shower terminated at t_0 = lower end of PT.
- Can't measure quarks and gluons.
- Degrees of freedom in the detector are hadrons.
- Need a description of confinement.

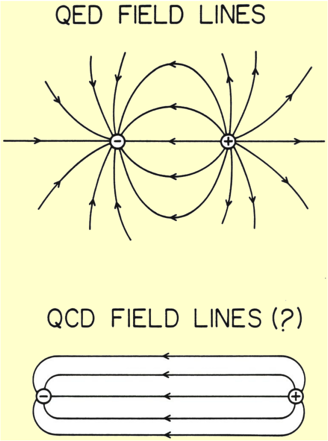
Physical input

Self coupling of gluons
$\leftrightarrow$ "attractive field lines"

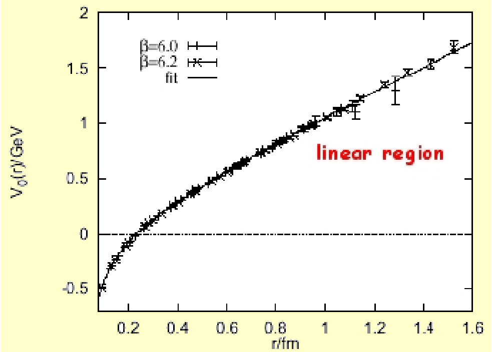


Physical input

Self coupling of gluons
$\leftrightarrow$ "attractive field lines"



Linear static potential $V(r) \approx \kappa r$.



Supported by lattice QCD,
hadron spectroscopy.

Hadronization models

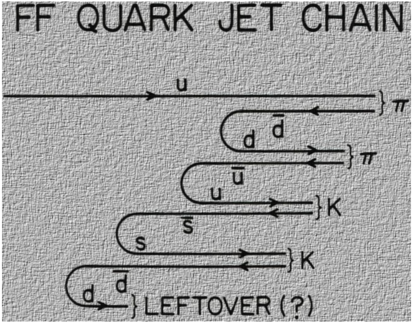
Older models:

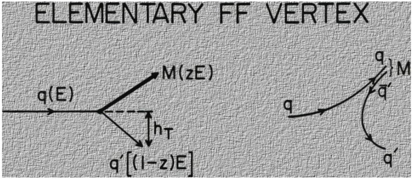
- Flux tube model.
- Independent fragmentation.

Today's models.

- Lund string model (Pythia).
- Cluster model (Herwig).

Independent fragmentation

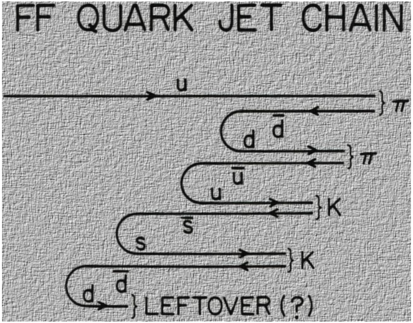


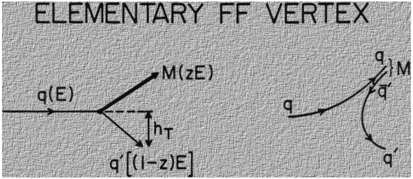


Feynman–Field fragmentation ('78).

- $q\bar{q}$ pairs created from vacuum to dress bare quarks.
- Fragmentation function $f_{q\to h}(z)$ = density of momentum fraction z carried away by hadron h from quark q.
- Gaussian $p_\perp$ distribution.

Independent fragmentation



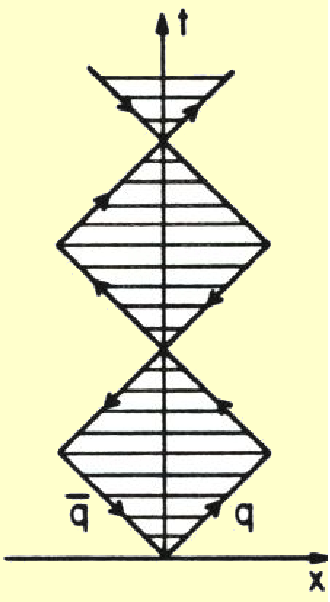


Feynman–Field fragmentation ('78).

- $q\bar{q}$ pairs created from vacuum to dress bare quarks.
- Fragmentation function $f_{q\to h}(z)$ = density of momentum fraction z carried away by hadron h from quark q.
- Gaussian $p_\perp$ distribution.
- Problems:
  - "last quark".
  - not Lorentz invariant.
  - infrared safety.
  - …
- Good at that time.
- Still usefull for inclusive descriptions.

Lund string model

String model of mesons.
$L = 0$ mesons move in yoyo modes.
Area law: $m^2 \sim$ area.



Lund string model

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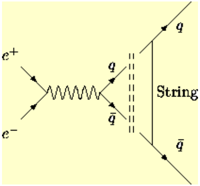
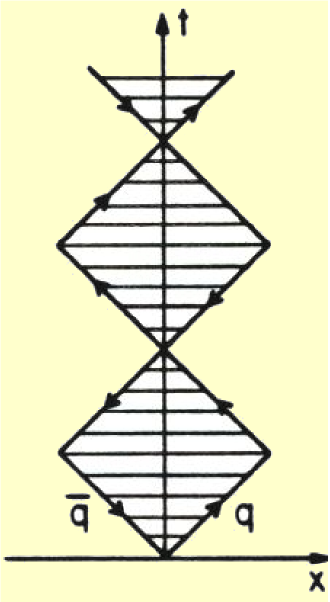
$L = 0$ mesons move in yoyo modes.
Area law: $m^2 \sim$ area.
Simple model for particle production in e^+e^- annihilation:



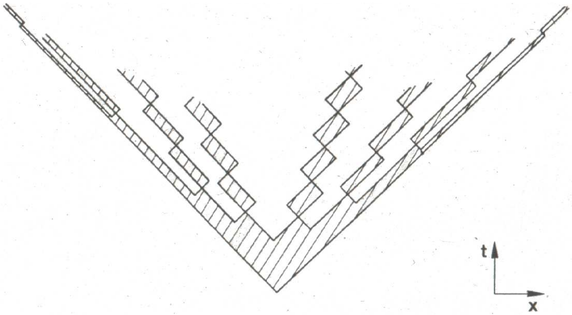
$q\bar{q}$ pair as pointlike source of string.



Lund string model

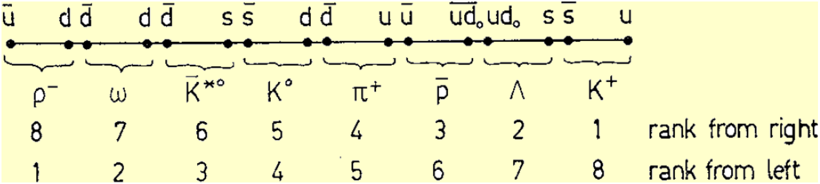
String energy $\sim$ intense chromomagnetic field.
$\longrightarrow$ Additional $q\bar{q}$ pairs created by QM tunneling.

$$\frac{\mathrm{dProb}}{\mathrm{d}x\mathrm{d}t} \sim \exp\left(-\pi m_q^2/\kappa\right) \qquad \kappa \sim 1\,\mathrm{GeV}\,.$$



String breaking expected long before yoyo point.

Ajacent breaks form hadrons.



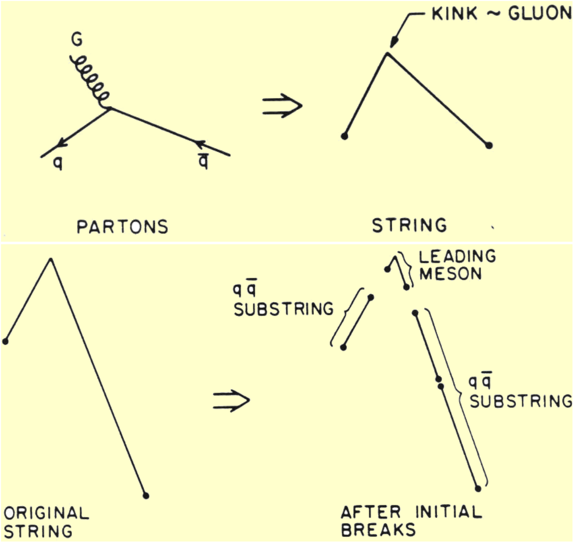
Works in both directions (symmetry).
Lund symmetric fragmentation function

$$f(z, p_\perp) \sim \frac{1}{z}(1-z)^a \exp\left(-\frac{b(m_h^2 + p_\perp^2)}{z}\right)$$

a, b, m_h^2 main adjustable parameters.
Note: diquarks $\rightarrow$ baryons.

Lund string model

gluon = kink on string = motion pushed into the $q\bar{q}$ system.



Lund string model

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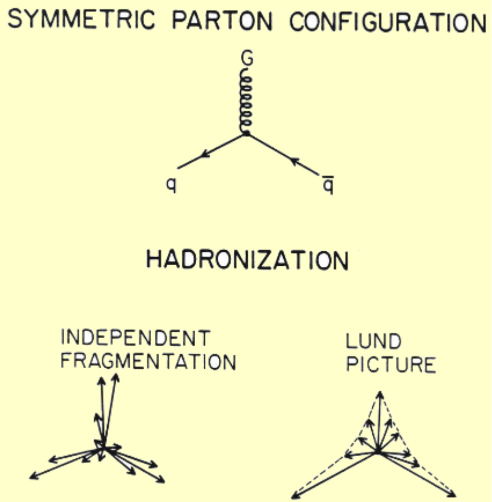

Lund string model

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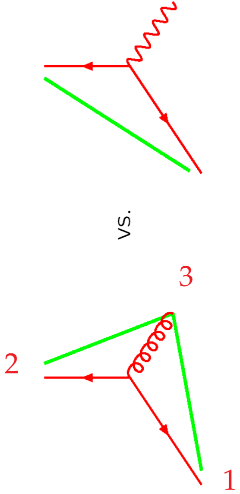
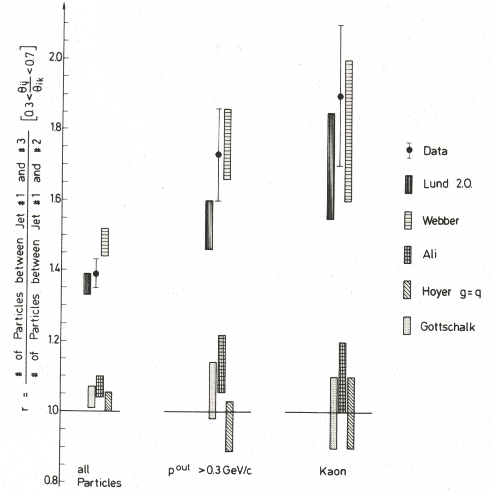


“String effect”



Lund string model

Some remarks:

- Originally invented without parton showers in mind.

Lund string model

Some remarks:

- Originally invented without parton showers in mind.
- Stong physical motivation.
- Very successful desription of data.
- Universal desription of data
  (fit at e^+e^-, transfer to hadron-hadron).
- Many parameters, ~ 1 per hadron.
- Too easy to hide errors in perturbative description?

Lund string model

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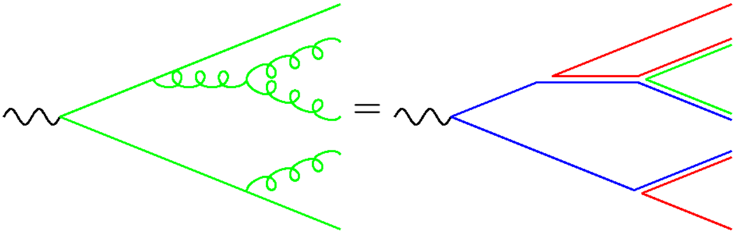
- Universal desription of data
  (fit at e^+e^-, transfer to hadron-hadron).
- Many parameters, ~ 1 per hadron.
- Too easy to hide errors in perturbative description?

$\longrightarrow$ try to use more QCD information/intuition.

Colour preconfinement

Large N_C limit $\longrightarrow$ planar graphs dominate.
Gluon = colour — anticolourpair



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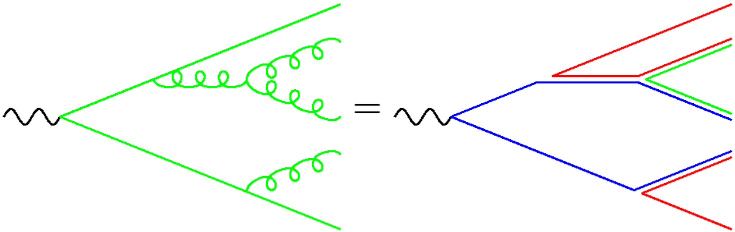
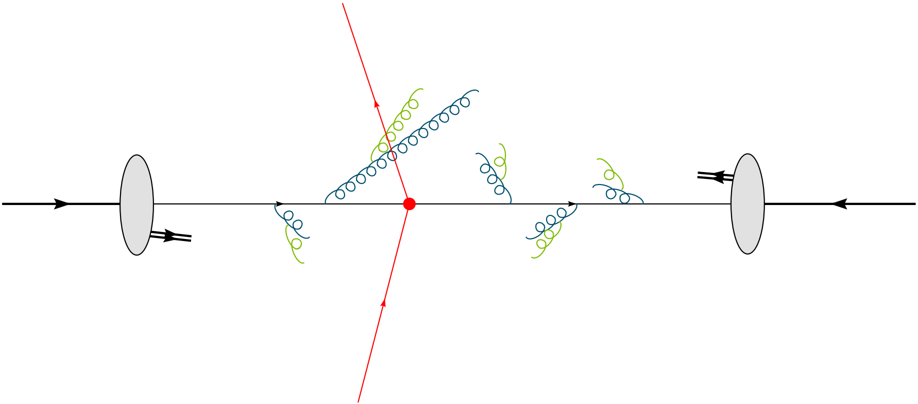


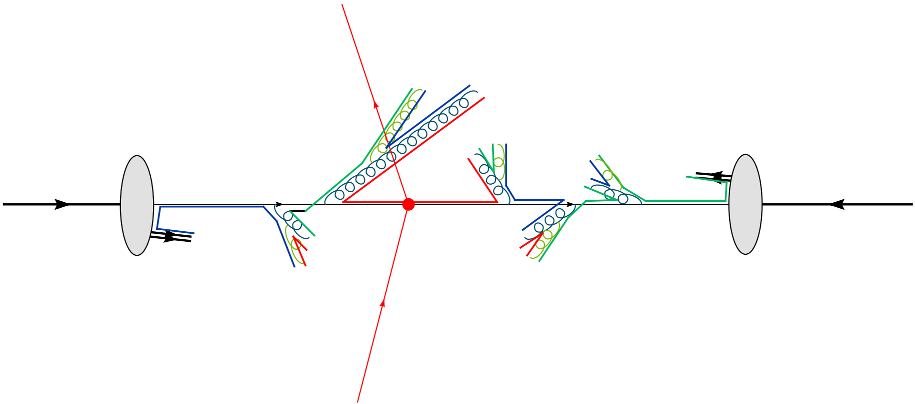
Parton shower organises partons in colour space. Colour partners (=colour singlet pairs) end up close in phase space.

$\longrightarrow$ Cluster hadronization model

Cluster hadronization



Cluster hadronization



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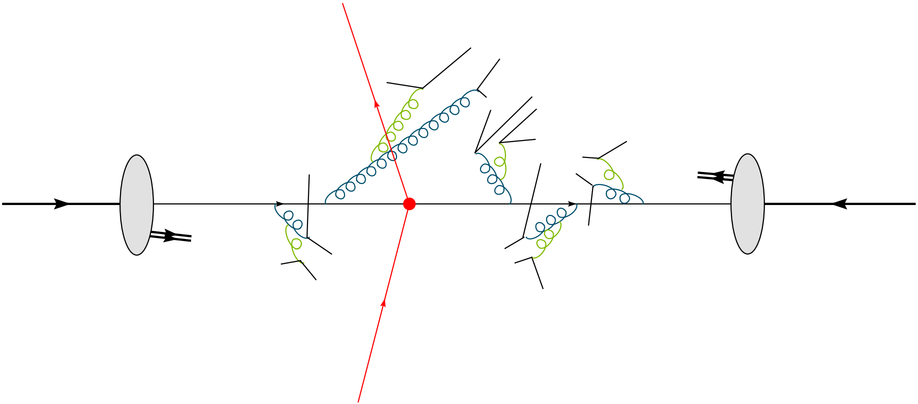
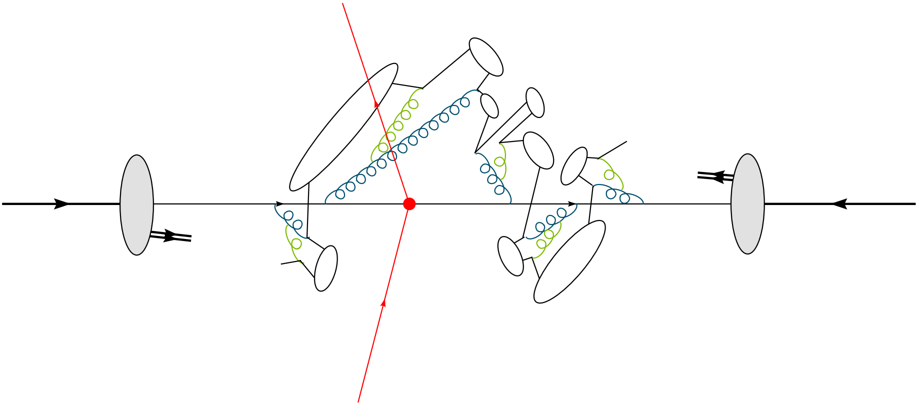
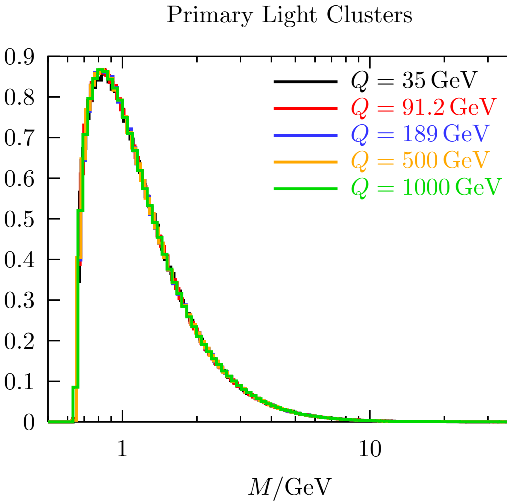


Cluster hadronization



Cluster hadronization

Primary cluster mass spectrum independent of production mechanism. Peaked at some low mass.



Cluster hadronization

Primary cluster mass spectrum independent of production mechanism. Peaked at some low mass.

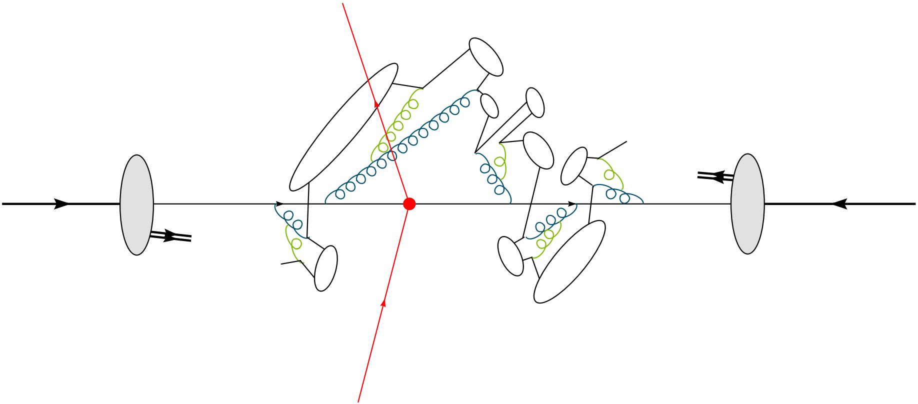
Cluster = continuum of high mass resonances.
Decay into well–known lighter mass resonances
= discrete spectrum of hadrons.

No spin information carried over, i.e. only phase space.

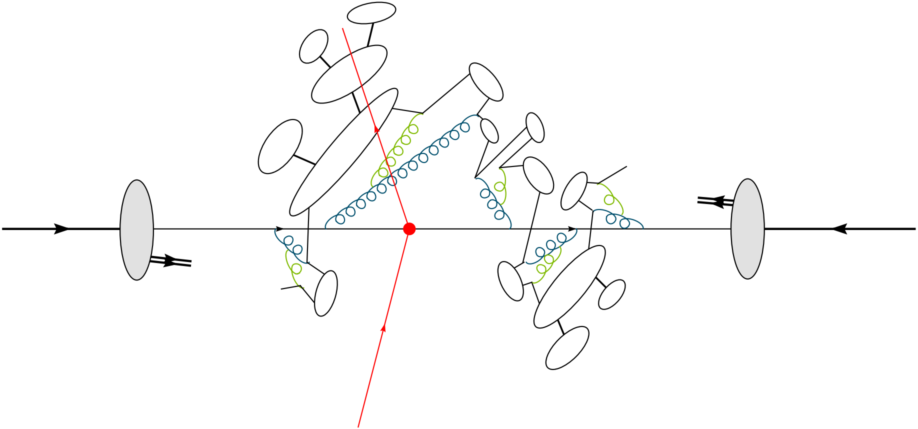
Suppression of heavier particles
(particularly baryons, can be problematic).

Cluster spectrum determined entirely by parton shower, i.e. perturbation theory. Hence, t_0 crucial parameter.

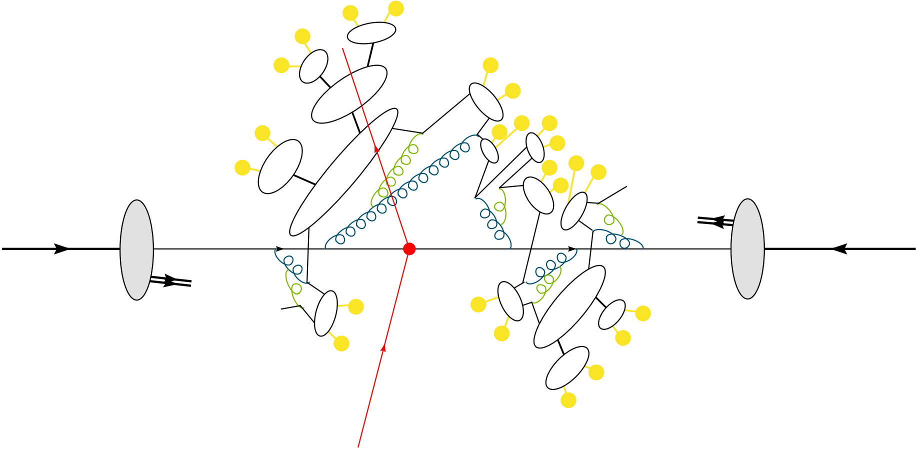
Cluster hadronization



Cluster hadronization



Cluster hadronization



Cluster hadronization in a nutshell

- Nonperturbative $g \to q\bar{q}$ splitting ($q = uds$) isotropically. Here, $m_g \approx 750\,\mathrm{MeV} > 2m_q$.
- Cluster formation, universal spectrum (see below)
- Cluster fission, until

$$M^p < M^p_{\max} + (m_1 + m_2)^p$$

where masses are chosen from

$$M_i = \left[\left(M^P - (m_i + m_3)^P\right) r_i + (m_i + m_3)^P\right]^{1/P},$$

with additional phase space contraints. Constituents keep moving in their original direction.

- Cluster Decay

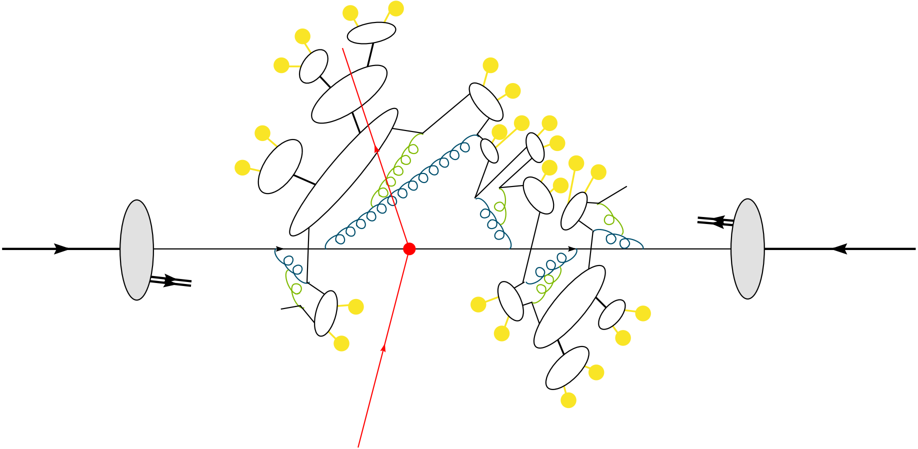
$$P(a_{i,q}, b_{q,j}|i,j) = \frac{W(a_{i,q}, b_{q,j}|i,j)}{\sum_{M/B} W(c_{i,q'}, d_{q',j}|i,j)}.$$

Hadronization

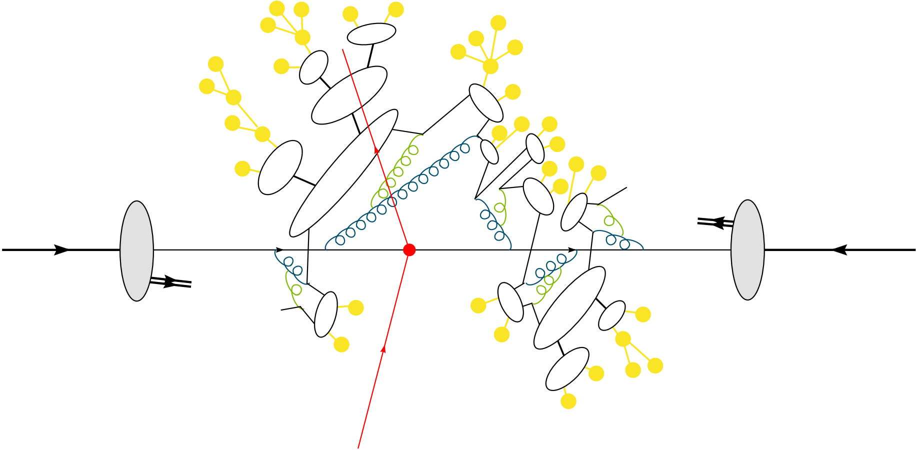
- Only string and cluster models used in recent MC programs.
  Independent fragmentation only for inclusive observables.
- Strings started non–perturbatively,
  improved by parton shower.
- Cluster model started mostly on perturbative side,
  improved by string like cluster fission.

Hadronic Decays

Hadronic decays



Hadronic decays



Hadronic decays

Many aspects:

$$
\begin{aligned}
B^{*0} &\to \gamma B^0 \\
&\qquad \hookrightarrow \bar{B}^0 \\
&\qquad\quad \hookrightarrow e^- \bar{\nu}_e D^{*+} \\
&\qquad\qquad\qquad \hookrightarrow \pi^+ D^0 \\
&\qquad\qquad\qquad\qquad \hookrightarrow K^- \rho^+ \\
&\qquad\qquad\qquad\qquad\qquad \hookrightarrow \pi^+ \pi^0 \\
&\qquad\qquad\qquad\qquad\qquad\qquad \hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Hadronic decays

Many aspects:

$$B^{*0} \to \gamma B^0$$

$$\hookrightarrow \bar{B}^0$$

$$\hookrightarrow e^- \bar{\nu}_e D^{*+}$$

$$\hookrightarrow \pi^+ D^0$$

$$\hookrightarrow K^- \rho^+$$

$$\hookrightarrow \pi^+ \pi^0$$

$$\hookrightarrow e^+ e^- \gamma$$

EM decay.

Hadronic decays

Many aspects:

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$$\hookrightarrow \bar{B}^0$$
$$\hookrightarrow e^- \bar{\nu}_e D^{*+}$$
$$\hookrightarrow \pi^+ D^0$$
$$\hookrightarrow K^- \rho^+$$
$$\hookrightarrow \pi^+ \pi^0$$
$$\hookrightarrow e^+ e^- \gamma$$

Weak mixing.

Hadronic decays

Many aspects:

$$
\begin{aligned}
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$$

Weak decay.

Hadronic decays

Many aspects:

$$B^{*0} \to \gamma B^0$$
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Strong decay.

Hadronic decays

Many aspects:

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& \qquad\qquad \hookrightarrow \pi^+ \pi^0 \\
& \qquad\qquad\quad \hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Weak decay, ρ^+ mass smeared.

Hadronic decays

Many aspects:

$$
\begin{aligned}
B^{*0} \to \gamma B^0 & \\
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\end{aligned}
$$

ρ^+ polarized, angular correlations.

Hadronic decays

Many aspects:

$$
\begin{aligned}
B^{*0} \to \gamma B^0 & \\
& \hookrightarrow \bar{B}^0 \\
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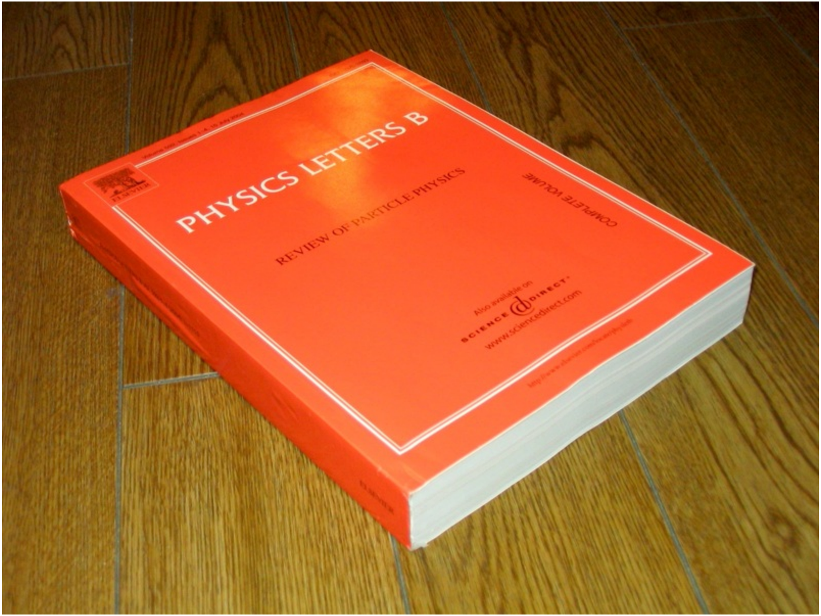
& \qquad\qquad\qquad\qquad \hookrightarrow \pi^+ \pi^0 \\
& \qquad\qquad\qquad\qquad\qquad \hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Dalitz decay, m_{ee} peaked.

Hadronic decays

Tedious.
100s of different particles, 1000s of decay modes, phenomenological matrix elements with parametrized form factors…

Hadronic decays



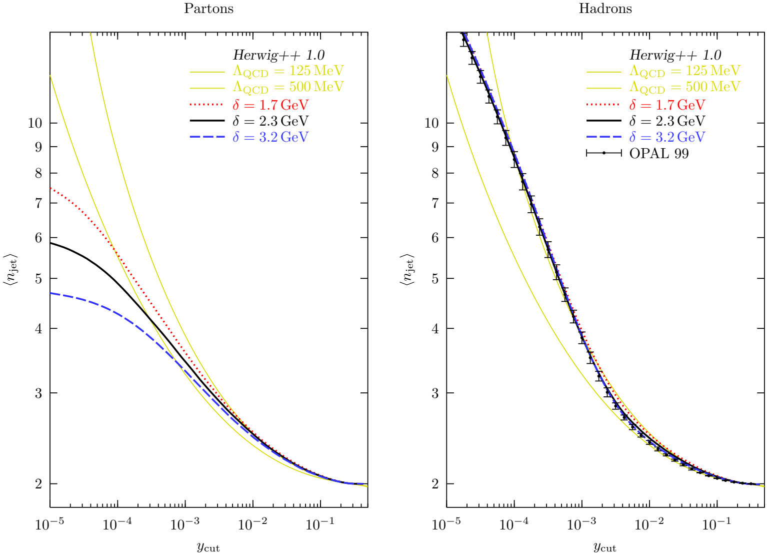
A few plots

How well does it work?

- $e^+e^- \to$ hadrons, mostly at LEP.
- Jet shapes, jet rates, event shapes, identified particles…
- 'Tuning' of parameters.
- Want to get *everything* right with *one* parameter set.
- Compare to literally 100s of plots.

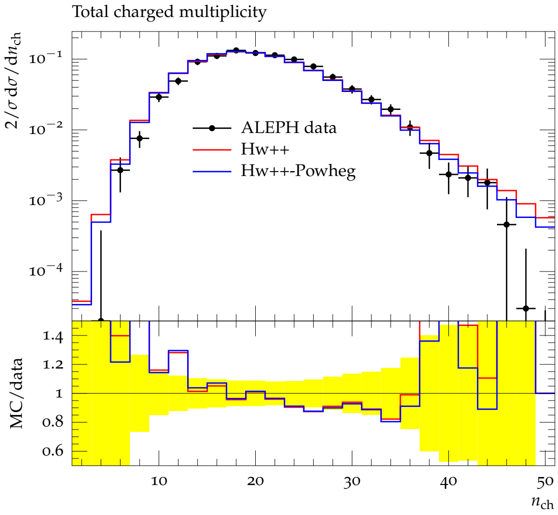
How well does it work?

Smooth interplay between shower and hadronization.



How well does it work?

$N_{\rm ch}$ at LEP. Crucial for t_0 (Herwig++ 2.5.2)

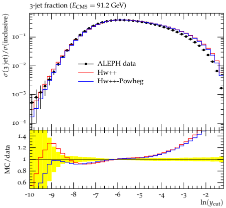


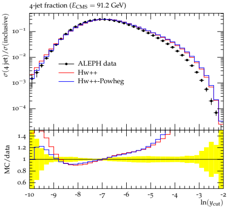
How well does it work?

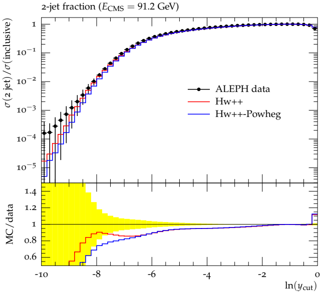
Jet rates at LEP.

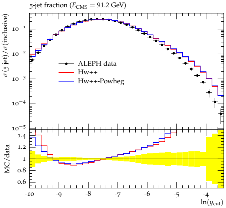
$$R_n = \sigma(n\text{–jets})/\sigma(\text{jets})$$
$$R_6 = \sigma(> 5\text{–jets})/\sigma(\text{jets})$$

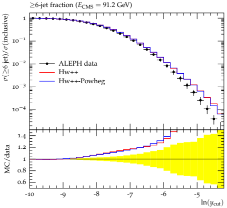
(Herwig++ 2.5.2)





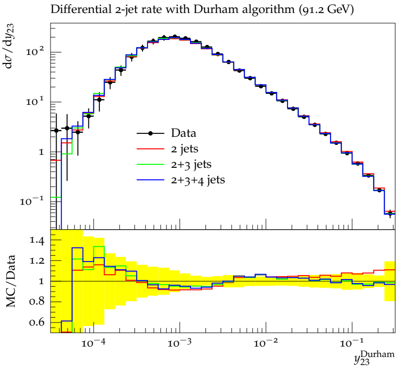


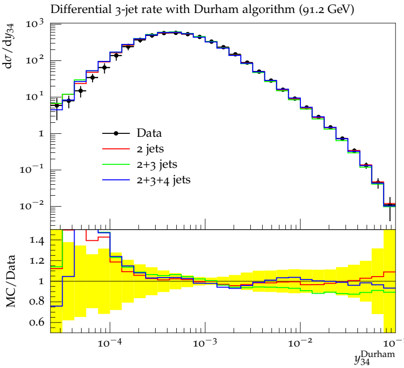




How well does it work?

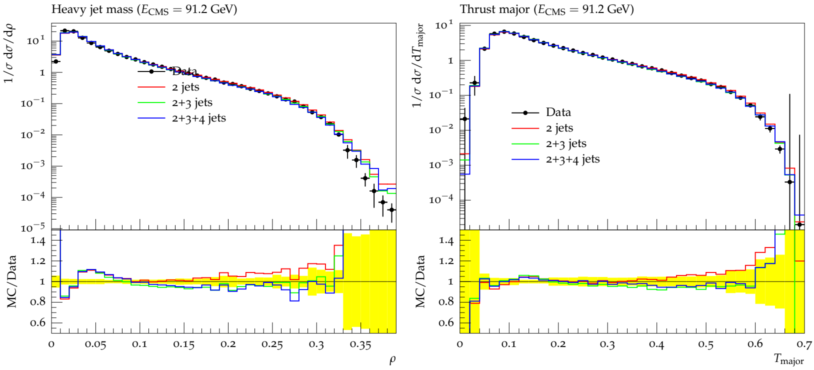
Differential Jet Rates at LEP (Herwig++ pre-3.0).
Dipole shower + some merging





How well does it work?

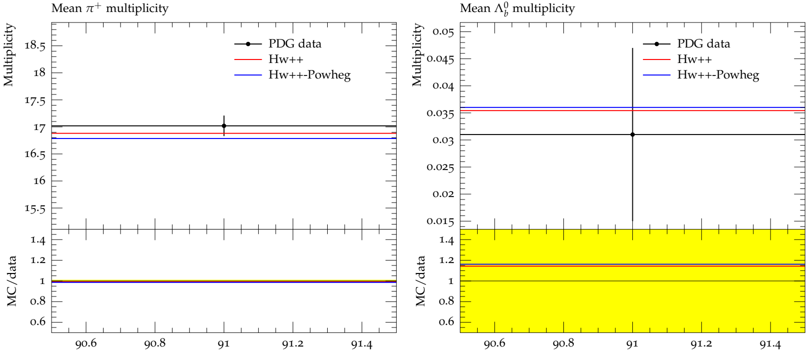
Event Shapes at LEP (Herwig++ pre-3.0). Dipole shower + some merging



Parton showers do very well, today!

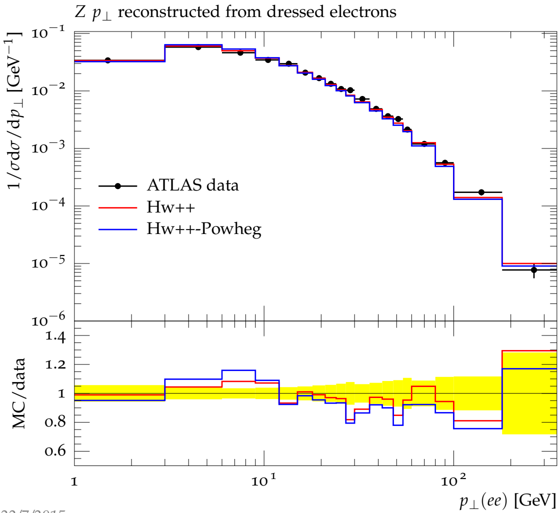
How well does it work?

Hadron Multiplicities at LEP (e.g. π^+, Λ_b^0).

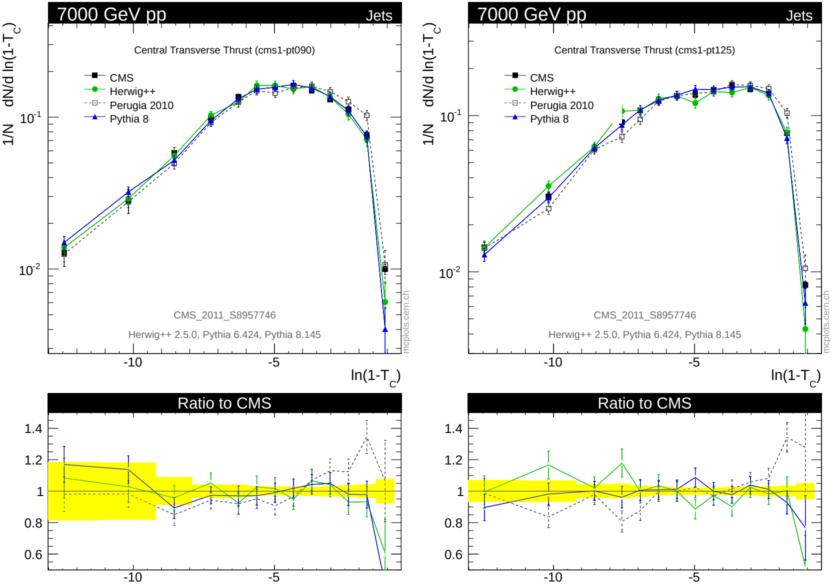


How well does it work?

$p_\perp(Z^0) \to$ intrinsic $k_\perp$ (LHC 7 TeV). See also in context of matching/marging.

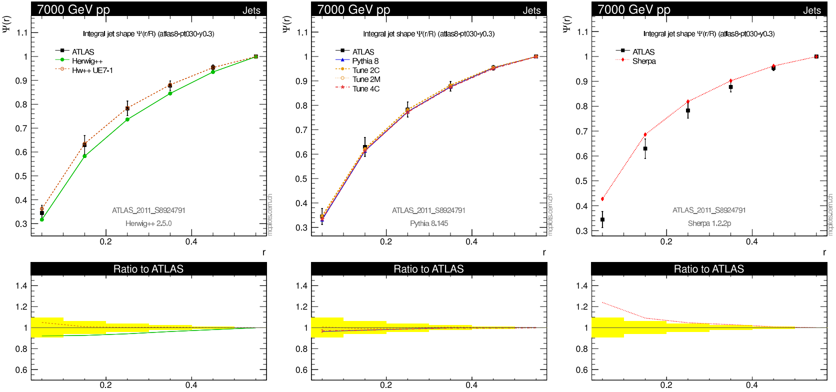


Transverse thrust



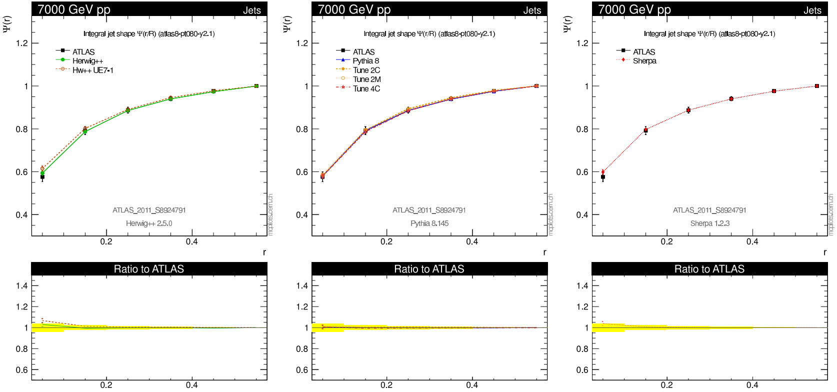
Integral jet shapes

not too hard, central ($30 < p_T/\text{GeV} < 40; 0 < |y| < 0.3$)



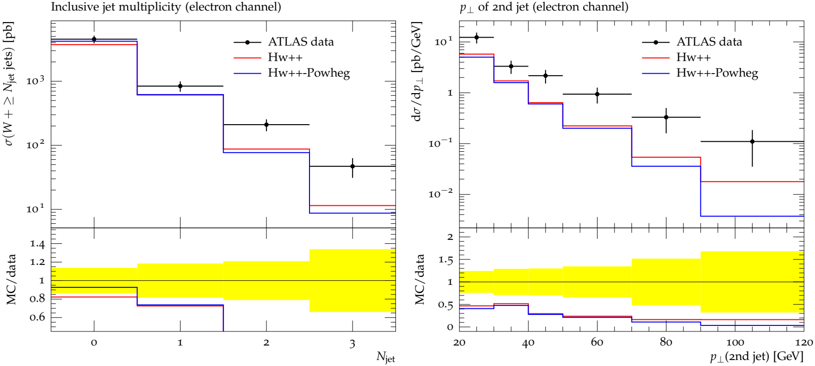
Integral jet shapes

harder, more forward ($80 < p_T/\mathrm{GeV} < 110; 1.2 < |y| < 2.1$)



Limits of parton shower

$W + $ jets, LHC 7 TeV.



Higher jets not covered by parton shower only → matching.

Matching and Merging

Matching NLO computations and parton showers

The problem:
Consider n and $n+1$ body ME

$$|M_n^{(0)}|^2 \qquad 2\mathrm{Re}M_n^{(0)}M_n^{(1)} \qquad |M_{n+1}^{(0)}|^2 \ .$$

- Both present in NLO as Born+Virtual and Real ME.
- Parton shower adds $n+1$ st emission as well (accurate to leading log accuracy).

⇒ Potential double counting!

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$\Rightarrow$ Potential double counting!

Two popular approaches:

- MC@NLO
- POWHEG

NLO with subtraction method

Toy model: NLO calculation with subtraction method,
$x =$ real emission phase space, *B*orn, *O*bservable, *R*eal, *V*irtual.

$$\langle O\rangle_{\mathrm{NLO}} = BO(0) + VO(0) + \int_0^1 dx\, \frac{O(x)R(x)}{x},$$

NLO with subtraction method

Toy model: NLO calculation with subtraction method,
x = real emission phase space, *B*orn, *O*bservable, *R*eal, *V*irtual.

$$\langle O\rangle_{\mathrm{NLO}} = BO(0) + VO(0) + \int_0^1 dx\,\frac{O(x)R(x)}{x},$$

Add/subtract soft/collinear piece $A(x)$ ($\lim_{x\to 0} A(x) = R(x)$):

$$\langle O\rangle_{\mathrm{NLO}} = BO(0) + \bar{V}O(0) + \int_0^1 dx\,\frac{O(x)R(x) - O(0)A(x)}{x},$$

where

$$\bar{V} = V + \int_0^1 dx\,\frac{A(x)}{x} = \text{IR finite}\,.$$

Calculate parton shower contribution with Sudakov FF,

$$\Delta = \exp\left\{ -\int_\mu \frac{dx}{x} P(x) \right\} \qquad .$$

From Born $\otimes$ zero/one parton shower emission:

$$\langle O \rangle_{\mathrm{PS}} = \int dx\, O(x) \left[B\Delta\delta(x) + B\frac{P(x)}{x}\Delta\Theta(x-\mu) \right]$$

PS@NLO

Calculate parton shower contribution with Sudakov FF,

$$\Delta = \exp\left\{-\int_\mu \frac{dx}{x}P(x)\right\} \approx 1 - \int dx\,\frac{P(x)}{x}\ .$$

From Born $\otimes$ zero/one parton shower emission:

$$\begin{aligned}\langle O\rangle_{\rm PS} &= \int dx\, O(x)\left[B\Delta\delta(x) + B\frac{P(x)}{x}\Delta\Theta(x-\mu)\right]\\ &= BO(0)\left[1 - \int_\mu \frac{dx}{x}P(x)\right] + \int_\mu dx\,O(x)B\frac{P(x)}{x}\ .\end{aligned}$$

PS@NLO

Calculate parton shower contribution with Sudakov FF,

$$\Delta = \exp\left\{-\int_\mu \frac{dx}{x} P(x)\right\} \approx 1 - \int dx\, \frac{P(x)}{x} \,.$$

From Born $\otimes$ zero/one parton shower emission:

$$\begin{aligned}\langle O\rangle_{\mathrm{PS}} &= \int dx\, O(x) \left[B\Delta\delta(x) + B\frac{P(x)}{x}\Delta\Theta(x-\mu)\right] \\ &= BO(0)\left[1 - \int_\mu \frac{dx}{x} P(x)\right] + \int_\mu dx\, O(x) B\frac{P(x)}{x} \,.\end{aligned}$$

Terms that contribute at $O(\alpha_S)$/NLO $\Rightarrow$ double counting.

Matching MC and NLO

Solution: subtract doubly counted terms.

$$\langle O\rangle_{\mathrm{NLO}} = BO(0) + \bar{V}O(0) + \int_0^1 dx\, \frac{O(x)R(x) - O(0)A(x)}{x}$$

$$\langle O\rangle_{\mathrm{PS}} = BO(0)\left[1 - \int_\mu \frac{dx}{x}P(x)\right] + \int_\mu dx\, O(x)B\frac{P(x)}{x}$$

Matching MC and NLO

Solution: subtract doubly counted terms.

$$\langle O\rangle_{\mathrm{NLO}}{}' = BO(0) + \bar{V}O(0) + \int_0^1 dx\, \frac{O(x)R(x) - O(0)A(x)}{x}$$
$$+ \int_\mu \frac{dx}{x} P(x) - \int_\mu dx\, O(x) B \frac{P(x)}{x}$$

Matching MC and NLO

Solution: subtract doubly counted terms.

$$\langle O\rangle_{\mathrm{NLO}}{}' = BO(0) + \bar{V}O(0) + \int_0^1 dx\, \frac{O(x)R(x) - O(0)A(x)}{x}$$
$$+ \int_\mu \frac{dx}{x} P(x) - \int_\mu dx\, O(x) B \frac{P(x)}{x}$$

Result ("MC@NLO master formula")

$$\langle O\rangle_{\mathrm{MC@NLO}} = O(0)\left[B + \bar{V} + \int_0^1 dx\, \frac{BP(x) - A(x)}{x}\right]$$
$$+ \int dx\, O(x) \frac{R(x) - BP(x)}{x} \; .$$

Note: $(O(0)B \otimes$ parton shower) adds back subtracted terms
$\Rightarrow$ NLO result is exactly reproduced after parton shower.

Matching MC and NLO

$$\langle O\rangle_{\mathrm{MC@NLO}} = O(0)\left[B+\bar{V}+\int_0^1 dx\,\frac{BP(x)-A(x)}{x}\right] + \int dx\,O(x)\frac{R(x)-BP(x)}{x}\;.$$

Observations/remarks:

- Events with n and $n+1$ legs are seperately finite. No cancellation of large weights.
- NLO result can be recovered strictly upon expansion in powers of α (with parton shower emission).
- Interface to MC program very well defined.
- Dropping $\mu \to 0$ is only a power correction.

Matching MC and NLO

$$
\begin{aligned}
\langle O \rangle_{\mathrm{MC@NLO}} = & O(0) \left[B + \bar{V} + \int_0^1 dx \, \frac{BP(x) - A(x)}{x} \right] \\
& + \int dx \, O(x) \frac{R(x) - BP(x)}{x} \; .
\end{aligned}
$$

Three types of matching

1. MC@NLO (classic, Frixione and Webber).
2. Simpler: parton shower with $P(x) = A(x)/B$.
3. Or, also simpler, $P(x) = R(x)/B$.

Matching MC and NLO

$$\langle O\rangle_{\mathrm{MC@NLO}} = O(0)\left[B+\bar{V}+\int_0^1 dx\,\frac{BP(x)-A(x)}{x}\right] + \int dx\,O(x)\,\frac{R(x)-BP(x)}{x}\,.$$

1. Classic MC@NLO (Frixione and Webber)
   - $A(x)$ = FKS subtraction terms
   - $P(x)$ and phase space specific for HERWIG.
   - Generic, calculate once and for all.
   - New for every process.

Matching MC and NLO

$$\langle O\rangle_{\mathrm{MC@NLO}} = O(0)\left[B+\bar{V}+\int_0^1 dx\,\frac{BP(x)-A(x)}{x}\right] + \int dx\,O(x)\,\frac{R(x)-BP(x)}{x}\,.$$

2. 'Custom' parton shower

e.g. with Catani–Seymour subtraction kernels

- CS subtraction already used in many NLO calculations.
- $P(x) = A(x)/B$, so terms vanish.
- $R(x) - A(x)$ already in NLO parton level program.

⇒ (almost) no need to modify NLO calculation!

Matching MC and NLO

$$\langle O \rangle_{\text{MC@NLO}} = O(0) \left[B + \bar{V} + \int_0^1 dx \, \frac{BP(x) - A(x)}{x} \right] + \int dx \, O(x) \frac{R(x) - BP(x)}{x} \, .$$

3. Simpler in a different way, $P(x) = R(x)/B$
   - $R(x) - A(x)$ now only needed as integral available in NLO parton level program.
   - No $n+1$ body events.
   - ≥ 1 PS emission from $R(x)/B$ as splitting kernel $\rightarrow$POWHEG.
   - Positive weights (terms $\neq 0$ are $\sigma_{\text{NLO}}^{\text{incl}}$).
   - Further emissions from (truncated) standard PS.

MC@NLO

- Introduced 2002 Frixione, Webber, JHEP 0206:029,2002 [hep-ph/0204244].
- Extended to heavy quarks

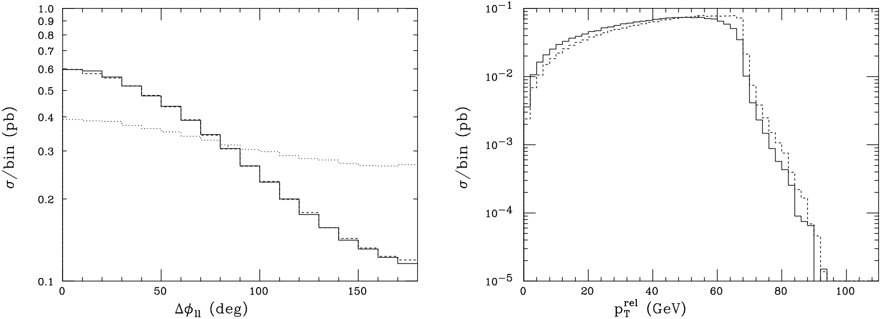
  Frixione, Nason, Webber, JHEP 0308:007,2003 [hep-ph/0305252].
- further extensions to many processes (single top etc.)
- MC@NLO customised to use with HERWIG.
- Some processes in Herwig++ as well
  $e^+e^- \to$ jets, DY, W', h^0 decay

  Latunde–Dada 0708.4390, 0903.4135, Latunde–Dada, Papaefstatiou, 0901.3685.
- MC@NLO package adopted to Herwig++ as well.

  S. Frixione, F. Stoeckli P. Torrielli and B.R. Webber, 1010.0568.

MC@NLO

Examples with Herwig++ (solid) Herwig6 (dash)



$h^0 \to WW \to l\nu l\nu$, (no spin corr dotted)

$t\bar{t}$, $p_t(b)$ rel to t (right).

S. Frixione, F. Stoeckli P. Torrielli and B.R. Webber, 1010.0568.

POWHEG

- Alternative proposed by P. Nason.
- Modified Sudakov FF for first emission.
- Angular ordered Parton Shower tricky (see below).
- *Truncated Shower* adds in missing radiation afterwards.
- Finally evolution with 'ordinary' Parton Shower.

[Nason, hep-ph/0409146; Nason, Ridolfi hep-ph/0606275]

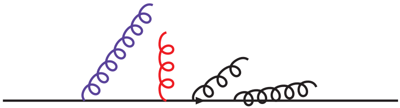
Recently systematically extended.

- `POWHEG` formulation independent of the event generator implementation.
- Worked out for different subtraction schemes.

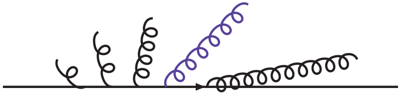
[Frixione, Nason, Ridolfi, 0707.3081, 0707.3088; Frixione, Nason, Oleari, 0709.2092]

POWHEG

Angular ordered showers and POWHEG



$p_\perp$ ordered shower. Angular ordering from additional vetos.



Angular ordered shower. Some softer emissions before hardest one.

Need truncated showers.

POWHEG in Herwig++

- First implementation of method for e^+e^- annihilation

  [O. Latunde–Dada, SG, B. Webber, hep-ph/0612281]

- Many more processes now available with release:
  DY $(\gamma^*/Z^0/W^\pm), h^0, h^0Z^0, h^0W^\pm, W^+W^-, W^\pm Z^0, Z^0Z^0$

  [K. Hamilton, P. Richardson and J. Tully, 0806.0290, 0903.4345, Hamilton, JHEP 1101:009]

- and with contributed code:
  $e^+e^- \to \text{jets}, t\bar{t}, t - \text{decay}, W', h^0 - \text{decay}$

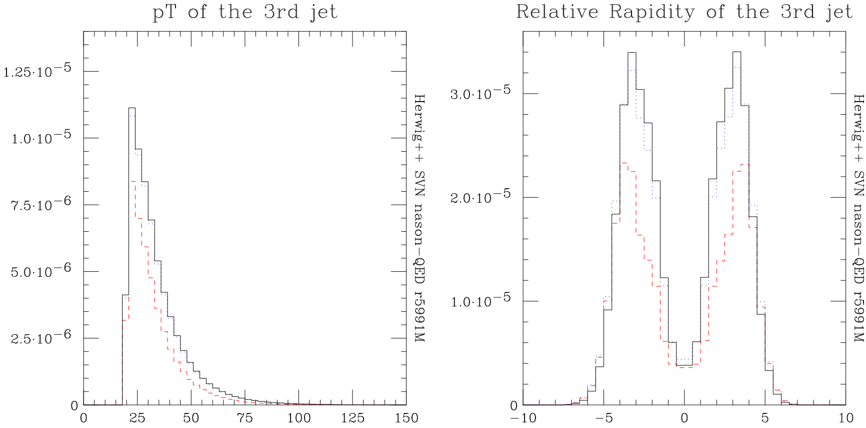
  [O. Latunde-Dada, 0812.3297, Eur. Phys. J. C **58**, 543 (2008)]

  [A. Papaefstathiou and O. Latunde-Dada, JHEP **0907**, 044]

- includes full truncated showers.
- Interface to PowhegBox straightforward.
- More processes underway ($\gamma\gamma$, VBF, SUSY pair prod...).

POWHEG in Herwig++

Higgs production in VBF. (POWHEG, MEC, LO+PS)



[L. D'Errico, P. Richardson in preparation]

Matchbox in Herwig++

- Upcoming Herwig++ 3.0 with Matchbox working horse.
  $\rightarrow$ NLO as default.
- Interfaces to various programs.
- Formalism and code to generate matched/merged events.

What's in Matchbox?

- Matching/merging formalism completely genereric.
- Two showers
  - Angular ordered shower.
  - Catani–Seymour dipoles.
- Two matching formalisms
  - MC@NLO like.
  - POWHEG like.
- Many interfaces to (automatic) NLO programs.
- Automatic CS subtraction terms.
- Improved phase space.

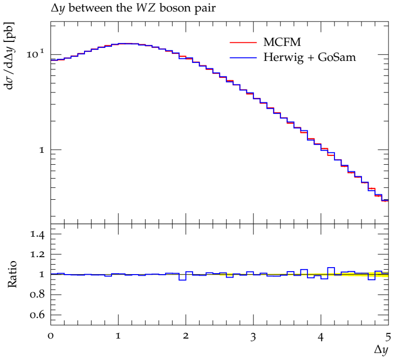
Interfaces to Matchbox

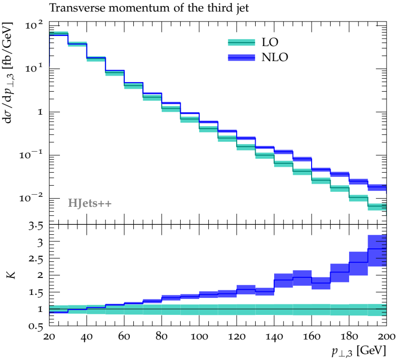
- Amplitude level
  - Hand-coded MEs
  - Hjet++ [F. Campanario, T. Figy, S. Plätzer, M. Sjödahl]
  - MadGraph5 [MadGraph, SG, S. Plätzer, J. Bellm]
  - Colour correlations with ColourFull [S. Plätzer, M. Sjödahl]
- Squared amplitude level
  - GoSam [GoSam & J. Bellm, SG, S. Plätzer, C. Reuschle]
  - OpenLoops [OpenLoops & J. Bellm, SG, S. Plätzer]
  - NJet [NJet & S. Plätzer]
  - VBFNLO [VBFNLO & J. Bellm, SG, S. Plätzer]

Many details validated, see e.g. below.

Processes at the parton level

E.g. WZ production, $H + 2$ jets (EW) as more complicated example. Many processes tested.



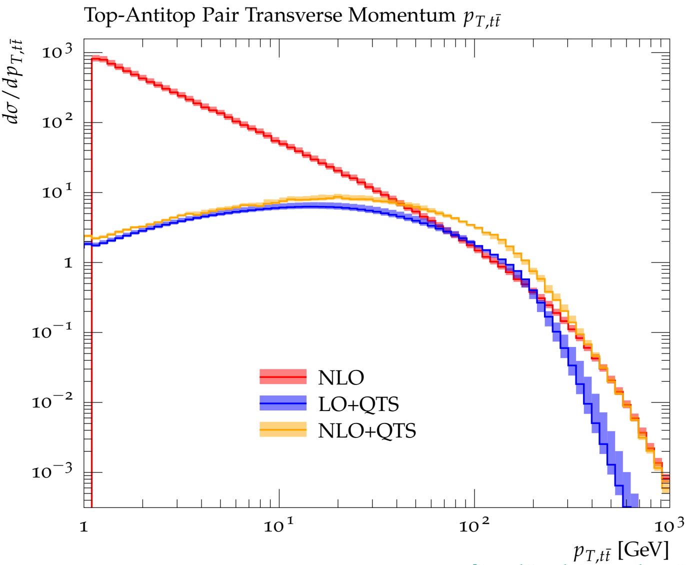


[N. Fischer, KIT 2013]

[F. Campanario, T. Figy, S. Plätzer, M. Sjödahl, PRL 111 (2013) 211802]

All SM $2 \to 2$ processes validated in detail. Plus more.

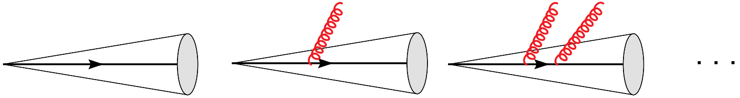
$t\bar{t}$ Matched with parton shower



[Daniel Rauch, Master thesis KIT 2014]

Matching tree level ME and parton showers

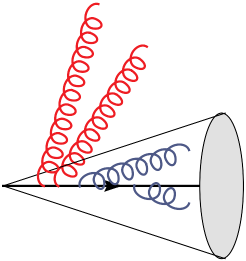
- Problem: have multiple tree level MEs for $X + 0, 1, \ldots n$ jets.



- Jets well separated and *inclusive*.
- Merge this into one exclusive multijet sample.
- Idea: use Sudakov form factors to disallow "+ anything softer" (which is normally inside an inclusive ME).
- That's done in the CKKW(-L) approach. [Catani, Krauss, Kuhn, Webber, JHEP 0111:063,2001, Krauss JHEP 0208:015,2002, L. Lönnblad, JHEP 0205:046,2002, Gleisberg, Höche, Winter, Schälicke, Schumann.]
- Alternative: MLM matching. [M.L. Mangano]
- Systematic study and comparison of implementations. [J. Alwall, S. Höche, F. Krauss, N. Lavesson, L. Lönnblad, F. Maltoni, M.L. Mangano, M. Moretti, C.G. Papadopoulos, F. Piccinini, S. Schumann, M. Treccani, J. Winter, M. Worek, EPJC53:473-500,2008.]

Matching tree level ME and parton showers

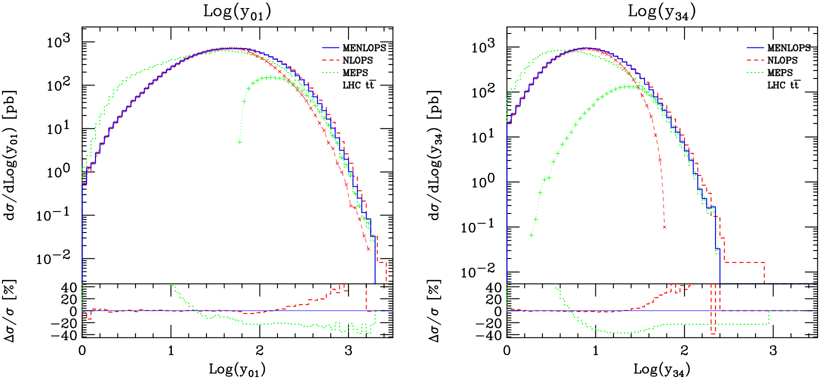
- Separates ME and parton shower at intermediate scale Q_{ini}.
- Parton shower fills region below Q_{ini}.
- All emissions resolvable above Q_0.



Merges ME and parton shower at scale Q_{ini}.

MENLOPS

ME+PS merging with lowest multiplicity at NLO.

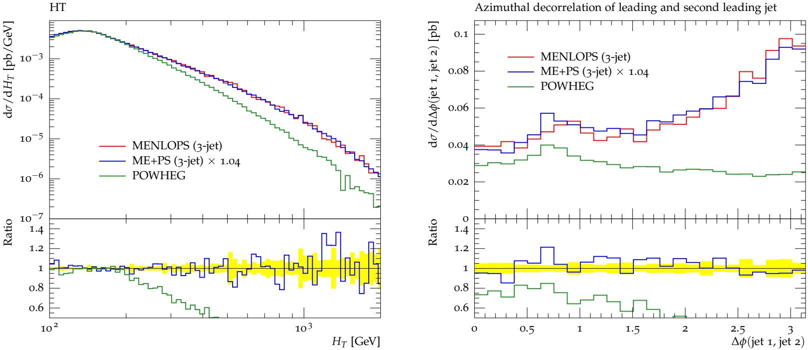


Test generic method with Pythia. y_{nm} in $t\bar{t}$+jets

[Hamilton, Nason, JHEP 1006:039]

MENLOPS

ME+PS merging with lowest multiplicity at NLO.



WW+jets implementation in Sherpa.

[Hoeche, Krauss, Schönherr, Siegert, 1009.1127]

Outlook: unitarized Matching/Merging

New approach in Herwig++/Matchbox. [S. Plätzer, 1211.5467]

Idea: Approximation of Sudakov "$\Delta \approx 1 - \int BP$" violates parton shower unitarity. Replace BP by full LO matrix element also in reweighting of events.

Leads to unified NLO matching and (LO/NLO)-merging prescription. [J. Bellm, SG, S. Plätzer]

Outlook: unitarized Matching/Merging

Consider parton shower acting on Born ME,

$$PS[B_0] = \Delta_\mu^0 B_0 + PS[P_1\,\Delta_0^1\,B_0]\ ,$$

iterate once,

$$PS[B_0] = \Delta_\mu^0 B_0 + \Delta_\mu^1\,P_1\,\Delta_1^0 B_0 + PS[P_2\,\Delta_2^1\,P_1\,\Delta_1^0\,B_0]\ ,$$

replace

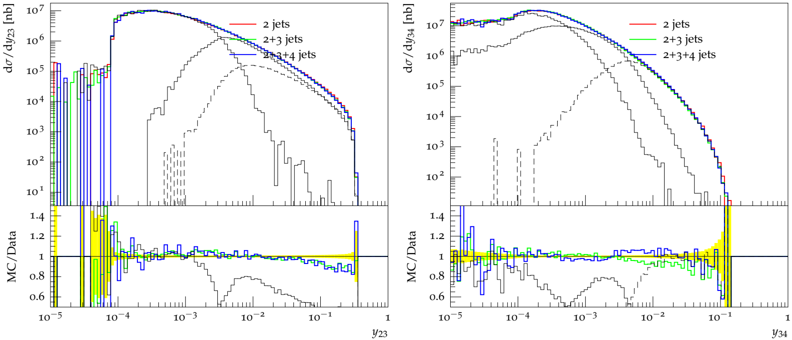
$$P_1\,B_0 \to \frac{\alpha_S(q_1)}{\alpha_S(q_0)} B_1\ ,$$

etc., but induces unitarity violation in Sudakov weights, so

$$\Delta_\mu^1 \approx 1 - P_1 B_0 \to 1 - \frac{\alpha_S(q_1)}{\alpha_S(q_0)} B_1\ .$$

Outlook: unitarized Matching/Merging

Preliminary example: LEP with merging contributions

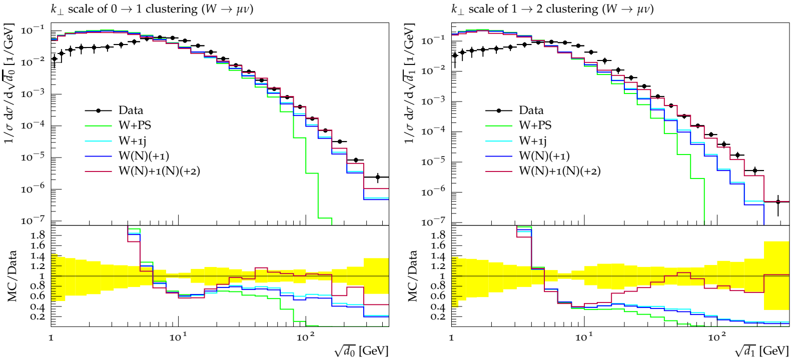


[J. Bellm, KIT]

Note: no hadronization in small y_{ij} region.

Outlook: unitarized Matching/Merging

W+jets. Note residual hadronization dependance.

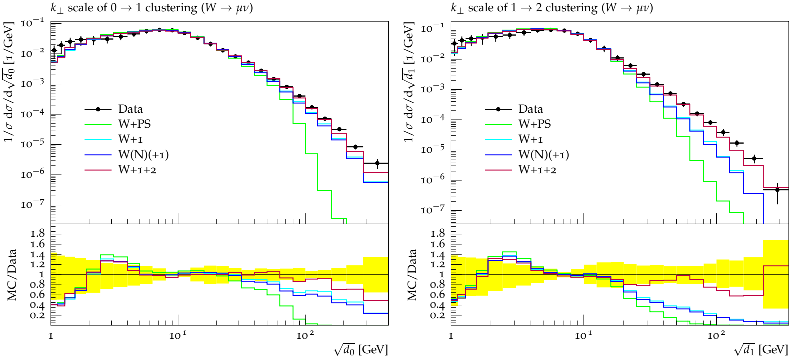


[J. Bellm, KIT]

MPI/Hadronization off.
$W+1, W+1+2$: LO merging with 1(2) jets.
$W(N)+1$: 0j NLO with 0j+1j LO (“matching through merging”).

Outlook: unitarized Matching/Merging

W+jets. Note residual MPI/hadronization dependance.

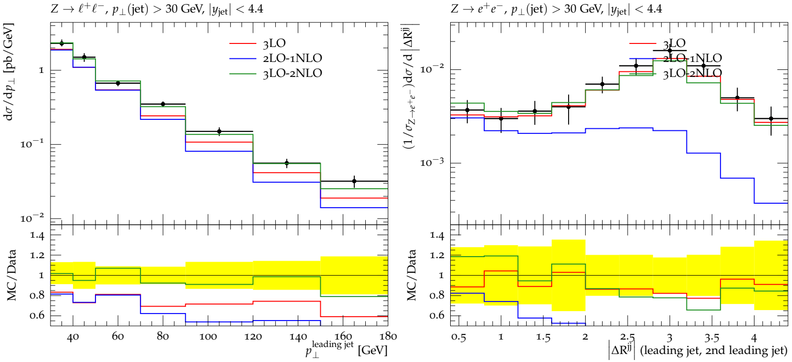


[J. Bellm, KIT]

MPI/Hadronization on.

Outlook: unitarized Matching/Merging

Preliminary example: Z production, jet-jet correlation.



[J. Bellm, KIT]

3LO-2NLO = Z+0, 1, 2 (tree) and Z+0,1 NLO (virtual).

Brief summary

I Parton Showers

Factorization in collinear and soft limits
$\to$ multiple parton final states at the GeV scale.

II Hadronization and Hadronic Decays

Modeling our physics picture.

III Matching and Merging with Higher Orders

Get rid of double counting $\to$ showered results are still NLO at the inclusive level. Beware of what's LO, what's NLO etc.

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