Flavour Physics 2
Theory Tools and Phenomenology

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Theory Challenges for LHC Physics

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What is the Problem?

- Weak interaction: Transitions between quarks
- Observations: Transitions between Hadrons
- We have to deal with nonperturbative QCD
- Extraction of fundamental parameters (CKM Elements, CP Phases) requires precise predictions, including error estimates
- $\rightarrow$ Simple models are out ...
- Effective Field Theory methods
- QCD Sum Rules
- Lattice QCD

Contents

Effective Field Theories

- Weak decays:
  Very different mass scales are involved:
  - $\Lambda_{\text{QCD}} \sim 200$ MeV: Scale of strong interactions
  - $m_c \sim 1.5$ GeV: Charm Quark Mass
  - $m_b \sim 4.5$ GeV: Bottom Quark Mass
  - $m_t \sim 175$ GeV and $M_W \sim 81$ GeV:
    Top Quark Mass and Weak Boson Mass
  - Λ_{NP} Scale of "new physics"
- At low scales the high mass particles / high energy degrees of freedom are irrelevant.
- Construct an "effective field theory" where the massive / energetic degrees of freedom are removed ("integrated out")

Integrating out heavy degrees of freedom

- ϕ: light fields, Φ: heavy fields with mass Λ
- Generating functional as a functional integral
  Integration over the heavy degrees of freedom

$$Z[j] = \int [d\phi][d\Phi] \exp\left(\int d^4x \, [\mathcal{L}(\phi, \Phi) + j\phi]\right)$$

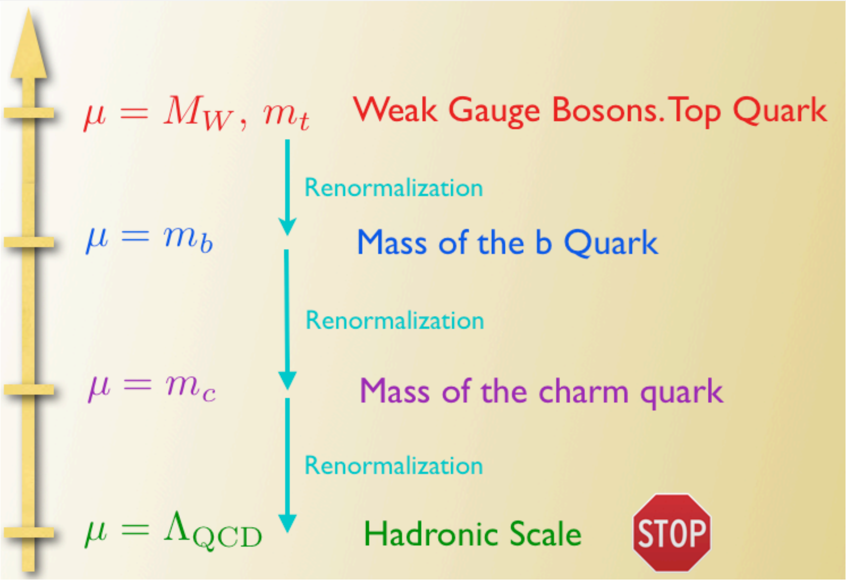
$$= \int [d\phi] \exp\left(\int d^4x \, [\mathcal{L}_{\text{eff}}(\phi) + j\phi]\right) \quad \text{with}$$

$$\exp\left(\int d^4x \, \mathcal{L}_{\text{eff}}(\phi)\right) = \int [d\Phi] \exp\left(\int d^4x \, \mathcal{L}(\phi, \Phi)\right)$$

- For length scales $x \gg 1/\Lambda$: local effective Lagrangian
- Technically: (Operator Product) Expansion in inverse powers of Λ

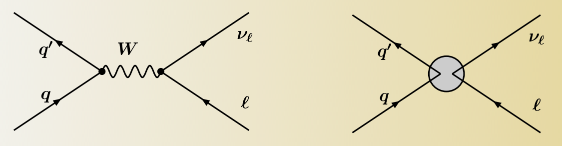
$$\mathcal{L}_{\rm eff}(\phi) = \mathcal{L}_{\rm eff}^{(4)}(\phi) + \frac{1}{\Lambda}\mathcal{L}_{\rm eff}^{(5)}(\phi) + \frac{1}{\Lambda^2}\mathcal{L}_{\rm eff}^{(6)}(\phi) + \cdots$$

- $\mathcal{L}_{\rm eff}$ is in general non-renormalizable, but ...
- $\mathcal{L}_{\rm eff}^{(4)}$ is the renormalizable piece
- For a fixed order in $1/\Lambda$: Only a finite number of insertions of $\mathcal{L}_{\rm eff}^{(4)}$ is needed!
- $\to$ can be renormalized
- Renormalizability is not an issue here


$\mu = M_W, m_t$
Weak Gauge Bosons. Top Quark
Renormalization
$\mu = m_b$
Mass of the b Quark
Renormalization
$\mu = m_c$
Mass of the charm quark
Renormalization
$\mu = \Lambda_{QCD}$
Hadronic Scale
STOP

Effective Weak Hamiltonian

- Start out from the Standard Model
- $W^\pm$, Z^0, top: much heavier than any hadron mass
- "integrate out" these particles at the scale $\mu \sim M_{\rm Hadron}$



- W has zero range in this limit:

$$\langle 0 | T[W^*_\mu(x) W_\nu(y)] | 0 \rangle \to g_{\mu\nu} \frac{1}{M_W^2} \delta^4(x-y)$$

- Effective Interaction (Fermi Coupling)

$$H_{\rm eff} = \frac{g^2}{\sqrt{2} M_W^2} V_{q'q} [\bar{q}' \gamma_\mu (1-\gamma_5) q][\bar{\nu}_\ell \gamma_\mu (1-\gamma_5) \ell] = \frac{4 G_F}{\sqrt{2}} V_{q'q} \, j_{\mu,{\rm had}} \, j^\mu_{\rm lep}$$

Decays of Hadrons

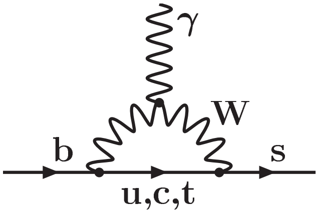
- Leptonic and semi-leptonic decays

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{q'q} [\bar{q}' \gamma_\mu \frac{(1-\gamma_5)}{2} q][\bar{\nu}_\ell \gamma_\mu \frac{(1-\gamma_5)}{2} \ell]$$

- Hadronic decays

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{q'q} V^*_{QQ'} [\bar{q}' \gamma_\mu \frac{(1-\gamma_5)}{2} q][\bar{Q} \gamma_\mu \frac{(1-\gamma_5)}{2} Q']$$

- Rare (FCNC) Decays: Loop Corrections (QCD and electroweak)



- Example: $b \to s\gamma$

$$\mathcal{A}(b \to s\gamma) = V_{ub} V_{us}^* f(m_u) + V_{cb} V_{cs}^* f(m_c) + V_{tb} V_{ts}^* f(m_t)$$

- In case of degenarate masses up-type masses:

$$\mathcal{A}(b \to s\gamma) = f(m) \left[V_{ub} V_{us}^* + V_{cb} V_{cs}^* + V_{tb} V_{ts}^* \right] = 0$$

Renormalization Group Running

- $H_{\rm eff}$ is defined at the scale Λ, where we integrated out the particles with mass Λ: General Structure

$$H_{\rm eff} = \frac{4G_F}{\sqrt{2}} \lambda_{\rm CKM} \sum_k \hat{C}_k(\Lambda) \mathcal{O}_k(\Lambda)$$

- $\mathcal{O}_k(\Lambda)$: The matrix elements of $\mathcal{O}_k$ have to be evaluated ("normalized") at the scale Λ.
- $\hat{C}_k(\Lambda)$: Short distance contribution, contains the information about scales $\mu > \Lambda$
- Matrixelements of $\mathcal{O}_k(\Lambda)$: Long Distance Contribution, contains the information about scales $\mu < \Lambda$

- We could as well imagine a situation with a different definition of “long” and “short” distances, defined by a scale μ, in which case

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \lambda_{\text{CKM}} \sum_k C_k(\Lambda/\mu) \mathcal{O}_k(\mu)$$

- Key Observation: The matrix elements of H_{eff} are physical Quantities, thus cannot depend on the arbitrary choice of μ

$$0 = \mu \frac{d}{d\mu} H_{\text{eff}}$$

- compute this ....

$$0 = \sum_i \left(\mu \frac{d}{d\mu} C_i(\Lambda/\mu) \right) \mathcal{O}_i(\mu) + C_i(\Lambda/\mu) \left(\mu \frac{d}{d\mu} \mathcal{O}_i(\mu) \right)$$

- Operator Mixing: Change in scale can turns the operator $\mathcal{O}_i$ into a linear combination of operators (of the same dimension)

$$\mu \frac{d}{d\mu} \mathcal{O}_i(\mu) = \sum_j \gamma_{ij}(\mu) \mathcal{O}_j(\mu)$$

and so

$$\sum_i \sum_j \left(\left[\delta_{ij} \mu \frac{d}{d\mu} + \gamma_{ij}(\mu) \right] C_i(\Lambda/\mu) \right) \mathcal{O}_j(\mu) = 0$$

- Assume: The operators $\mathcal{O}_j$ from a basis, then

$$\sum_i \left[\delta_{ij}\mu\frac{d}{d\mu} + \gamma_{ij}^T(\mu)\right] C_j(\Lambda/\mu) = 0$$

- QCD: Coupling constant α_s depends on μ: β-function

$$\mu\frac{d}{d\mu}\alpha_s(\mu) = \beta(\alpha_s(\mu))$$

- C_j depend also on α_s

$$\mu\frac{d}{d\mu} = \left(\mu\frac{\partial}{\partial\mu} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s}\right)$$

- In an appropriate scheme γ_{ij} depend on μ only trough α_s: $\gamma_{ij}(\mu) = \gamma_{ij}(\alpha_s(\mu))$

- Renormalization Group Equation (RGE) for the coefficients

$$\sum_i \left[\delta_{ij} \left(\mu \frac{\partial}{\partial \mu} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) + \gamma_{ij}^T(\alpha_s) \right] C_j(\Lambda/\mu, \alpha_s) = 0$$

- This is a system of linear differential equations:
  $\rightarrow$ Once the initial conditions are known, the solution is in general unique
- RGE Running: Use the RGE to relate the coefficients at different scales

- The coefficients are at $\mu = \Lambda$ (at the "matching scale")

$$C_i(\Lambda/\mu = 1, \alpha_s) = \sum_n a_i^{(n)} \left(\frac{\alpha_s}{4\pi}\right)^n \qquad \text{perturbative calculation}$$

- Perturbative calculation of the RG functions β and γ_{ij}

$$\beta(\alpha_s) = \alpha_s \sum_{n=0} \beta^{(n)} \left(\frac{\alpha_s}{4\pi}\right)^{n+1} \qquad \gamma_{ij}(\alpha_s) = \sum_{n=0} \gamma_{ij}^{(n)} \left(\frac{\alpha_s}{4\pi}\right)^{n+1}$$

- RG functions can be calculated from loop diagrams:

$$\beta^{(0)} = -\frac{2}{3}(33 - 2n_\mathrm{f}) \qquad \gamma_{ij} \text{ depends on the set of } \mathcal{O}_i$$

- Structure of the perturbative expansion of the coefficient at some other scale

$$c_i(\Lambda/\mu, \alpha_s) =$$

$$b_i^{00}$$

$$+ \quad b_i^{11}\left(\frac{\alpha_s}{4\pi}\right)\ln\frac{\Lambda}{\mu} + b_i^{10}\left(\frac{\alpha_s}{4\pi}\right)$$

$$+ \quad b_i^{22}\left(\frac{\alpha_s}{4\pi}\right)^2\ln^2\frac{\Lambda}{\mu} + b_i^{21}\left(\frac{\alpha_s}{4\pi}\right)^2\ln\frac{\Lambda}{\mu} + b_i^{20}\left(\frac{\alpha_s}{4\pi}\right)^2$$

$$+ \quad b_i^{33}\left(\frac{\alpha_s}{4\pi}\right)^3\ln^3\frac{\Lambda}{\mu} + b_i^{32}\left(\frac{\alpha_s}{4\pi}\right)^3\ln^2\frac{\Lambda}{\mu} + b_i^{31}\left(\frac{\alpha_s}{4\pi}\right)^3\ln\frac{\Lambda}{\mu} + \cdots \quad ,$$

- LLA (Leading Log Approximation):
  Resummation of the b_i^{nn} terms

$$C_i(\Lambda/\mu, \alpha_s) = \sum_{n=0}^{\infty} b_i^{nn} \left(\frac{\alpha_s}{4\pi}\right)^n \ln^n \frac{\Lambda}{\mu}$$

  $\rightarrow$ leading terms in the expansion of the RG functions

- NLLA (Next-to-leading log approximation):

$$C_i(\Lambda/\mu, \alpha_s) = \sum_{n=0}^{\infty} \left[b_i^{nn} + b_i^{n+1,n} \left(\frac{\alpha_s}{4\pi}\right)\right] \left(\frac{\alpha_s}{4\pi}\right)^n \ln^n \frac{\Lambda}{\mu}$$

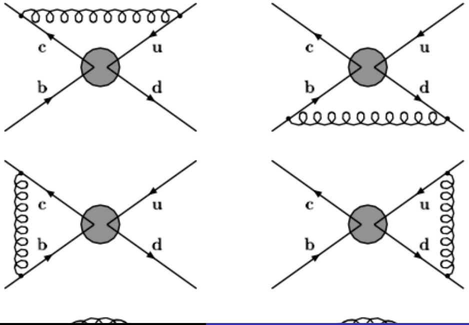
  $\rightarrow$ next-to-leading terms of the RG functions

- Typical Proceedure:
  - "Matching" at the scale $\mu = M_W$
  - "Running" to a scale of the order $\mu = m_b$
  - $\rightarrow$ includes operator mixing
- Resummation of the large logs $\ln(M_W^2/m_b^2)$
  - "Matching" at the scale $\mu = m_b$
  - "Running" to the scale m_c
- Resummation of the "large" logs $\ln(m_b^2/m_c^2)$
- ...
- Untli $\alpha_s(\mu)$ becomes too large ...

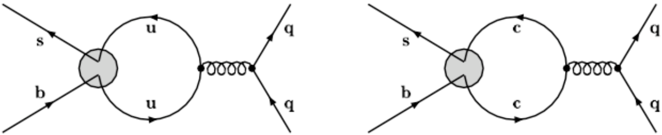
$H_{\rm eff}$ for b decays at low scales

- Effective interaction: $H_{\rm eff} = \frac{4G_F}{\sqrt{2}} \lambda_{\rm CKM} \sum_k \hat{C}_k(\Lambda) \mathcal{O}_k(\Lambda)$
- "Tree" Operators"

$$\begin{aligned} \mathcal{O}_1 &= (\bar{c}_{L,i}\gamma_\mu s_{L,j})\,(\bar{d}_{L,j}\gamma_\mu u_{L,i}) \quad , \\ \mathcal{O}_2 &= (\bar{c}_{L,i}\gamma_\mu s_{L,i})\,(\bar{d}_{L,j}\gamma_\mu u_{L,j}) \quad . \end{aligned}$$

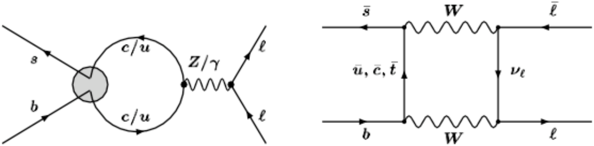


- If two flavours are equal: QCD Penguin Operators



$$
\begin{aligned}
\mathcal{O}_3 &= (\bar{s}_{\mathrm{L},i}\gamma_\mu b_{\mathrm{L},i}) \sum_{q=u,d,s,c,b} (\bar{q}_{\mathrm{L},j}\gamma^\mu q_{\mathrm{L},j}) \,, \\
\mathcal{O}_4 &= (\bar{s}_{\mathrm{L},i}\gamma_\mu b_{\mathrm{L},j}) \sum_{q=u,d,s,c,b} (\bar{q}_{\mathrm{L},j}\gamma^\mu q_{\mathrm{L},i}) \,, \\
\mathcal{O}_5 &= (\bar{s}_{\mathrm{L},i}\gamma_\mu b_{\mathrm{L},i}) \sum_{q=u,d,s,c,b} (\bar{q}_{\mathrm{R},j}\gamma^\mu q_{\mathrm{R},j}) \,, \\
\mathcal{O}_6 &= (\bar{s}_{\mathrm{L},i}\gamma_\mu b_{\mathrm{L},j}) \sum_{q=u,d,s,c,b} (\bar{q}_{\mathrm{R},j}\gamma^\mu q_{\mathrm{R},i}) \,.
\end{aligned}
$$

- Electroweak Penguins:
  Replace the Gluon by a Z_0 or Photon: $\mathcal{P}_7 \cdots \mathcal{P}_{10}$
- Rare (FCNC) Processes:



$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}_{\mathrm{L},\alpha} \sigma_{\mu\nu} b_{\mathrm{R},\alpha}) F^{\mu\nu}$$

$$\mathcal{O}_8 = \frac{g}{16\pi^2} m_b (\bar{s}_{\mathrm{L},\alpha} T^a_{\alpha\beta} \sigma_{\mu\nu} b_{\mathrm{R},\alpha}) G^{a\mu\nu}$$

$$\mathcal{O}_9 = \frac{1}{2} (\bar{s}_{\mathrm{L}} \gamma_\mu b_{\mathrm{L}})(\bar{\ell} \gamma^\mu \ell)$$

$$\mathcal{O}_{10} = \frac{1}{2} (\bar{s}_{\mathrm{L}} \gamma_\mu b_{\mathrm{L}})(\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

- Coefficients of the Operators (One Loop)

$C_i(\mu)$	$\mu = 10.0\,\mathrm{GeV}$	$\mu = 5.0\,\mathrm{GeV}$	$\mu = 2.5\,\mathrm{GeV}$
C_1	0.182	0.275	0.40
C_2	−1.074	−1.121	−1.193
C_3	−0.008	−0.013	−0.019
C_4	0.019	0.028	0.040
C_5	−0.006	−0.008	−0.011
C_6	0.022	0.035	0.055

$C_i(\mu)$	$\mu = 2.5\ \mathrm{GeV}$	$\mu = 5\ \mathrm{GeV}$	$\mu = 10\ \mathrm{GeV}$
C_7^{eff}	−0.334	−0.299	−0.268
C_8^{eff}	−0.157	−0.143	−0.131
$\frac{2\pi}{\alpha} C_9$	1.933	1.788	1.494

Heavy Quark Limit

Isgur, Wise, Voloshin, Shifman, Georgi, Grinstein, ...

- $1/m_Q$ Expansion: Substantial Theoretical Progress!
- Static Limit: $m_b,\ m_c \to \infty$ with fixed (four)velocity

$$v_Q = \frac{p_Q}{m_Q}, \qquad Q = b, c$$

- In this limit we have

$$\left.\begin{array}{r} m_{Hadron} = m_Q \\ p_{Hadron} = p_Q \end{array}\right\} v_{Hadron} = v_Q$$

- For $m_Q \to \infty$ the heavy quark does not feel any recoil from the light quarks and gluons (Cannon Ball)
- This is like the H-atom in Quantum Mechanics !!

Heavy Quark Symmetries

- The interaction of gluons is identical for all quarks
- Flavour enters QCD only through the mass terms
  - $m \to 0$: (Chiral) Flavour Symmetry (Isospin)
  - $m \to \infty$ Heavy Flavour Symmetry
  - Consider b and c heavy: Heavy Flavour SU(2)
- Coupling of the heavy quark spin to gluons:

$$H_{int} = \frac{g}{2m_Q} \bar{Q}(\vec{\sigma} \cdot \vec{B}) Q \quad \stackrel{m_Q \to \infty}{\longrightarrow} \quad 0$$

  - Spin Rotations become a symmetry
  - Heavy Quark Spin Symmetry: SU(2) Rotations
- Spin Flavour Symmetry Multiplets

Mesonic Ground States

Bottom:

$$|(b\bar{u})_{J=0}\rangle = |B^-\rangle \qquad |(b\bar{u})_{J=1}\rangle = |B^{*-}\rangle$$

$$|(b\bar{d})_{J=0}\rangle = |\overline{B}^0\rangle \qquad |(b\bar{d})_{J=1}\rangle = |\overline{B}^{*0}\rangle$$

$$|(b\bar{s})_{J=0}\rangle = |\overline{B}_s\rangle \qquad |(b\bar{s})_{J=1}\rangle = |\overline{B}_s^*\rangle$$

Charm:

$$|(c\bar{u})_{J=0}\rangle = |D^0\rangle \qquad |(c\bar{u})_{J=1}\rangle = |D^{*0}\rangle$$

$$|(c\bar{d})_{J=0}\rangle = |D^+\rangle \qquad |(c\bar{d})_{J=1}\rangle = |D^{*+}\rangle$$

$$|(c\bar{s})_{J=0}\rangle = |D_s\rangle \qquad |(c\bar{s})_{J=1}\rangle = |D_s^*\rangle$$

Baryonic Ground States

$$\left|[(ud)_0 Q]_{1/2}\right\rangle = |\Lambda_Q\rangle$$

$$\left|[(uu)_1 Q]_{1/2}\right\rangle, \left|[(ud)_1 Q]_{1/2}\right\rangle, \left|[(dd)_1 Q]_{1/2}\right\rangle = |\Sigma_Q\rangle$$

$$\left|[(uu)_1 Q]_{3/2}\right\rangle, \left|[(ud)_1 Q]_{3/2}\right\rangle, \left|[(dd)_1 Q]_{3/2}\right\rangle = |\Sigma_Q^*\rangle$$

$$\left|[(us)_0 Q]_{1/2}\right\rangle, \left|[(ds)_0 Q]_{1/2}\right\rangle = |\Xi_Q\rangle$$

$$\left|[(us)_1 Q]_{1/2}\right\rangle, \left|[(ds)_1 Q]_{1/2}\right\rangle = |\Xi'_Q\rangle$$

$$\left|[(us)_1 Q]_{3/2}\right\rangle, \left|[(ds)_1 Q]_{3/2}\right\rangle = |\Xi_Q^*\rangle$$

$$\left|[(ss)_1 Q]_{1/2}\right\rangle = |\Omega_Q\rangle \qquad \left|[(ss)_1 Q]_{3/2}\right\rangle = |\Omega_Q^*\rangle$$

Wigner Eckart Theorem for HQS

- HQS imply a “Wigner Eckart Theorem”

$$\left\langle H^{(*)}(v) \right| Q_v \Gamma Q_{v'} \left| H^{(*)}(v') \right\rangle = C_\Gamma(v, v') \xi(v \cdot v')$$

with $H^{(*)}(v) = D^{(*)}(v)$ or $B^{(*)}(v)$

  - $C_\Gamma(v, v')$: Computable Clebsh Gordan Coefficient
  - $\xi(v \cdot v')$: Reduced Matrix Element
  - $\xi(v \cdot v')$: universal non-perturbative Form Faktor: Isgur Wise Funktion
  - Normalization of ξ at $v = v'$:

$$\xi(v \cdot v' = 1) = 1$$

Heavy Quark Effective Theory

- The heavy mass limit can be formulated as an effective field theory
- Expansion in inverse powers of m_Q
- Define the static field h_v for the velocity v

$$h_v(x) = e^{im_Q v\cdot x}\frac{1}{2}(1+\not{v})b(x) \qquad p_Q = m_Q v + k$$

- HQET Lagrangian

$$\mathcal{L} = \bar{h}_v(iv\cdot D)h_v + \frac{1}{2m_Q}\bar{h}_v(i\not{D})^2 h_v + \cdots$$

- Dim-4 Term: Feynman rules, loops, renormalization...

Application: Determination of V_{cb} from $B \to D^{(*)}\ell\bar{\nu}_\ell$

- Kinematic variable for a heavy quark: Four Velovity v
- Differential Rates

$$\frac{d\Gamma}{d\omega}(B \to D^* \ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 m_{D^*}^3 (\omega^2 - 1)^{1/2} P(\omega)(\mathcal{F}(\omega))^2$$

$$\frac{d\Gamma}{d\omega}(B \to D \ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^{3/2} (\mathcal{G}(\omega))^2$$

- with $\omega = vv'$ and
- $P(\omega)$: Calculable Phase space factor
- $\mathcal{F}$ and $\mathcal{G}$: Form Factors

Heavy Quark Symmetries

- Normalization of the Form Factors is known at $vv' = 1$: (both initial and final meson at rest)
- Corrections can be calculated / estimated

$$
\begin{aligned}
\mathcal{F}(\omega) &= \eta_{\text{QED}}\eta_A \left[1 + \delta_{1/\mu^2} + \cdots\right] (\omega - 1)\rho^2 + \mathcal{O}((\omega - 1)^2) \\
\mathcal{G}(1) &= \eta_{\text{QED}}\eta_V \left[1 + \mathcal{O}\left(\frac{m_B - m_D}{m_B + m_D}\right)\right]
\end{aligned}
$$

- Parameter of HQS breaking: $\dfrac{1}{\mu} = \dfrac{1}{m_c} - \dfrac{1}{m_b}$
- $\eta_A = 0.960 \pm 0.007$, $\eta_V = 1.022 \pm 0.004$, $\delta_{1/\mu^2} = -(8 \pm 4)\%$, $\eta_{\text{QED}} = 1.007$

$B \to D^{(*)}$ Form Factors

- Lattice Calculations of the deviation from unity

$$\boxed{\mathcal{F}(1) = 0.903 \pm 0.13}$$

$$\boxed{\mathcal{G}(1) = 1.033 \pm 0.018 \pm 0.0095}$$

A. Kronfeld et al.

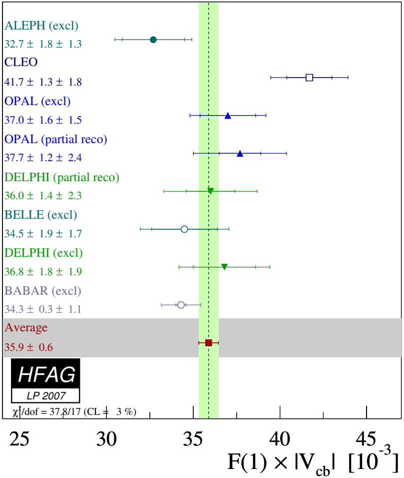
- Zero Recoil Sum Rules

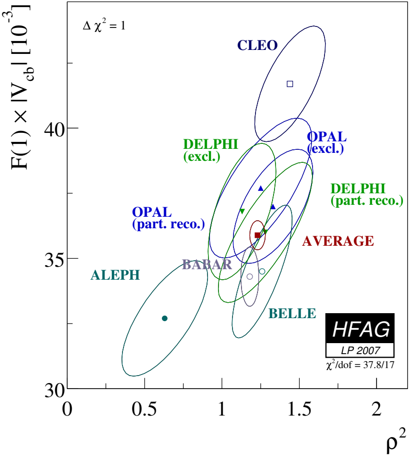
$$\boxed{\mathcal{F}(1) = 0.86 \pm 0.04}$$

$$\boxed{\mathcal{G}(1) = 1.04 \pm 0.02}$$

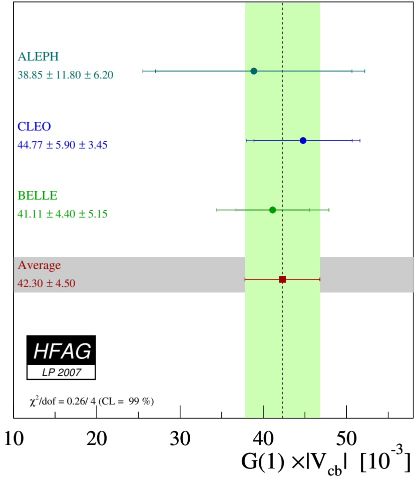
P. Gambino et al.

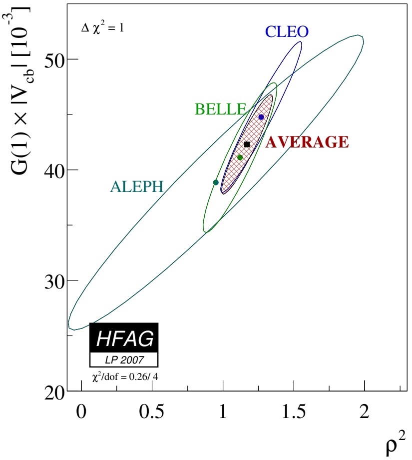
$B \to D^* \ell \bar{\nu}_\ell$





$B \to D\ell\bar{\nu}_\ell$





Inclusive Decays: Heavy Quark Expansion

Operator Product Expansion = Heavy Quark Expansion

(Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar. Wise, Neubert, M,...)

$$\Gamma \propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{eff} | B(v) \rangle|^2$$

$$= \int d^4x \, \langle B(v) | \mathcal{H}_{eff}(x) \mathcal{H}^\dagger_{eff}(0) | B(v) \rangle$$

$$= 2 \, \mathrm{Im} \int d^4x \, \langle B(v) | T\{ \mathcal{H}_{eff}(x) \mathcal{H}^\dagger_{eff}(0) \} | B(v) \rangle$$

$$= 2 \, \mathrm{Im} \int d^4x \, e^{-im_b v\cdot x} \langle B(v) | T\{ \widetilde{\mathcal{H}}_{eff}(x) \widetilde{\mathcal{H}}^\dagger_{eff}(0) \} | B(v) \rangle$$

- Last step: $p_b = m_b v + k$,
  Expansion in the residual momentum k

- Perform an OPE: m_b is much larger than any scale appearing in the matrix element

$$\int d^4x e^{-im_b vx} T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}_{eff}^\dagger(0)\}$$
$$= \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n C_{n+3}(\mu)\mathcal{O}_{n+3}$$

$\rightarrow$ The rate for $B \to X_c \ell \bar{\nu}_\ell$ can be written as

$$\Gamma = \Gamma_0 + \frac{1}{m_Q}\Gamma_1 + \frac{1}{m_Q^2}\Gamma_2 + \frac{1}{m_Q^3}\Gamma_3 + \cdots$$

- The Γ_i are power series in $\alpha_s(m_Q)$:
$\rightarrow$ Perturbation theory!

- Γ_0 is the decay of a free quark ("Parton Model")
- Γ_1 vanishes due to Heavy Quark Symmetries
- Γ_2 is expressed in terms of two parameters

$$\begin{aligned} 2M_H\mu_\pi^2 &= -\langle H(v)|\bar{Q}_v(iD)^2 Q_v|H(v)\rangle \\ 2M_H\mu_G^2 &= \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(iD^\nu)Q_v|H(v)\rangle \end{aligned}$$

μ_π: Kinetic energy and μ_G: Chromomagnetic moment

- Γ_3 two more parameters

$$\begin{aligned} 2M_H\rho_D^3 &= -\langle H(v)|\bar{Q}_v(iD_\mu)(ivD)(iD^\mu)Q_v|H(v)\rangle \\ 2M_H\rho_{LS}^3 &= \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(ivD)(iD^\nu)Q_v|H(v)\rangle \end{aligned}$$

ρ_D: Darwin Term and ρ_{LS}: Chromomagnetic moment

New: $1/m_b^4$ Contribution Γ_4 (Dassinger, Turczyk, M.)

- Five new parameters:

$\langle \vec{E}^2 \rangle$: Chromoelectric Field squared

$\langle \vec{B}^2 \rangle$: Chromomagnetic Field squared

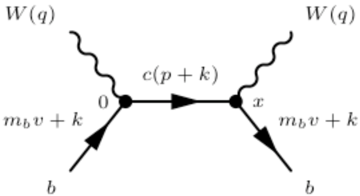
$\langle (\vec{p}^2)^2 \rangle$: Fourth power of the residual b quark momentum

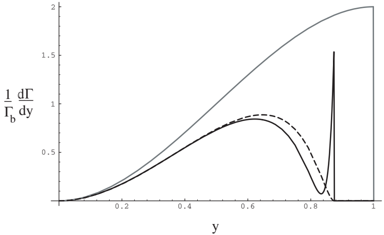
$\langle (\vec{p}^2)(\vec{\sigma} \cdot \vec{B}) \rangle$: Mixed Chromomag. Mom. and res. Mom. sq.

$\langle (\vec{p} \cdot \vec{B})(\vec{\sigma} \cdot \vec{p}) \rangle$: Mixed Chromomag. field and res. helicity

- Some of these can be estimated in naive factorization

Spectra of Inclusive Decays





- Endpoint region: $\rho = m_c^2/m_b^2$, $y = 2E_\ell/m_b$

$$\frac{d\Gamma}{dy} \sim \Theta(1-y-\rho)\left[2 + \frac{\lambda_1}{(m_b(1-y))^2}\left(\frac{\rho}{1-y}\right)^2\left\{3 - 4\frac{\rho}{1-y}\right\}\right]$$

- Reliable calculation in HQE possible for the moments of the spectrum

Application: V_{cb} from $b \to c\ell\bar{\nu}$ inclusive

- Tree level terms up to and including $1/m_b^5$ known
- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the partonic rate known
- $\mathcal{O}(\alpha_s)$ for the μ_π^2/m_b^2 is known
- QCD insprired modelling for the HQE matrix elements
- New: Complete α_s/m_b^2, including the μ_G terms
  Alberti, Gambino, Nandi (arXiv:1311.7381)
  ThM, Pivovarov, Rosenthal (arXiv:1405.5072, arXiv:1506.08167)
- This was the remaining parametrically largest uncertainty

- Alberti et al.: **Phys.Rev.Lett. 114 (2015) 6, 061802** and **JHEP 1401 (2014) 147**
  - Calculation of the differential rate including the charm mass
  - partially numerical calculation
- ThM, Pivovarov, Rosenthal: **Phys.Lett. B741 (2015) 290-294**
  - Fully analytic calculation
  - limit $m_c \to 0$
  - Possibility to include m_c in a Taylor series
- Results do agree, surprisingly steep m_c dependence

Result for V_{cb}

- Inclusive Decay (HQE / OPE)

$$\boxed{V_{cb} = 42.21 \pm 0.78}$$ (Gambino et al. 2015)

- Exclusive decay (Lattice FF)

$$\boxed{V_{cb} = 39.36 \pm 0.78}$$ (Fermilab/Milc 2015)

- Exclusive decay (Zero Recoil Sum rule)

$$\boxed{V_{cb} = 41.4 \pm 0.9}$$ (Gambino et al. 2015)

Soft Collinear Effective Theory

- Problem: How to deal with "energetic" light degrees of freedom = Endpoint regions of the spectra ?
- More than two scales involved!
- Inclusive Rates in the Endpoint become (Korchemski, Sterman)

$$d\Gamma = H * J * S$$

with * = Convolution

- H: Hard Coefficient Function, Scales $\mathcal{O}(m_b)$
- J: Jet Function, Scales $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
- S: Shape function, Scales $\mathcal{O}(\Lambda_{\rm QCD})$

Basics of Soft Collinear Effective Theory

- Heavy-to-light decays:
  Kinematic Situations with energetic light quarks
  hadronizing into jets or energetic light mesons
  p_{fin}: Momentum of a light final state meson

$$p_{\rm fin}^2 \sim \mathcal{O}(\Lambda_{\rm QCD} m_b) \quad v \cdot p_{\rm fin} \sim \mathcal{O}(m_b)$$

- Use light-cone vectors $n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$:

$$p_{\rm fin} = \frac{1}{2}(n \cdot p_{\rm fin})\bar{n} \quad \text{and} \quad v = \frac{1}{2}(n + \bar{n})$$

- Momentum of a light quark in such a meson:

$$p_{\rm light} = \frac{1}{2}[(n \cdot p_{\rm light})\bar{n} + (\bar{n} \cdot p_{\rm light})n] + p_{\rm light}^{\perp}$$

SCET Power Counting

- Define the parameter $\lambda = \sqrt{\Lambda_{\rm QCD}/m_b}$
- The light quark invariant mass (or virtuality) is assumed to be

$$p_{\rm light}^2 = (n \cdot p_{\rm light})(\bar{n} \cdot p_{\rm light}) + (p_{\rm light}^\perp)^2 \sim \lambda^2 m_b^2$$

- The components of the quark momentum have to scale as

$$(n \cdot p_{\rm light}) \sim m_b \quad (\bar{n} \cdot p_{\rm light}) \sim \lambda^2 m_b \qquad p_{\rm light}^\perp \sim \lambda m_b$$

A brief look at SCET (Bauer, Stewart, Pirjol, Beneke, Feldmann ...)

- QCD quark field q is split into a collinear component ξ and a soft one with $\xi = \frac{1}{4}\not{n}_-\not{n}_+ q$
- The Lagrangian $\mathcal{L}_{\mathrm{QCD}} = \bar{q}(i\not{D})q$ is rewritten in terms of the collinear field

$$\mathcal{L} = \frac{1}{2}\bar{\xi}\not{n}_+(in_-D)\xi - \bar{\xi}i\not{D}_\perp \frac{1}{in_+D + i\epsilon}\frac{\not{n}_+}{2}i\not{D}_\perp\xi$$

- Expansion according to the above power couning:

$$in_+D = in_+\partial + gn_+A_{\mathrm{c}} + gn_+A_{\mathrm{us}} = in_+D_{\mathrm{c}} + gn_+A_{\mathrm{us}}$$

- Leading $\mathcal{L}$ becomes non-local: Wilson lines

Practical Consequences of SCET

- Similar to HQS: Relations between for factors at large momentum transfer

$$\langle B(v)|\bar{b}\Gamma q|\pi(p)\rangle \propto \zeta(vp),\ \zeta_{//}(vp),\ \zeta_{\perp}(vp)$$

For energetic pion only three independent form factors (Charles et al.)

- Correction can be calculated as in HQET

Basic Idea (Shifman, Vainshtein, Zakharaov, 1978)

- Start from a suitably chosen correlation function, e.g.

$$T(q^2) = \int d^4x\, e^{-iqx} \langle 0 | T[j(x) j^\dagger(0) | 0 \rangle$$

- This can be calculated perturbatively as $q^2 \to -\infty$.
- On the other hand, it has a dispersion relation

$$T(q^2) = \int \frac{ds}{2\pi} \frac{\rho(s)}{s - q^2 + i\epsilon} + \text{possible subtractions}$$

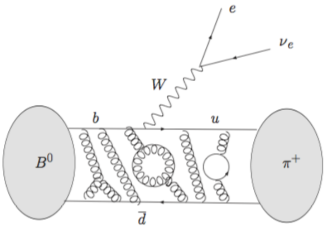
with $\rho(s) \sim \langle 0 | j(x) j^\dagger(0) 0 \rangle = \sum_n \langle 0 | j(x) | n \rangle \langle n | j^\dagger(0) 0 \rangle$

- Estimates for $\langle 0 | j(x) | n \rangle$ from e.g. positivity statements

Application: Determination of V_{ub} from exclusive $b \to u\ell\bar{\nu}$

- $B \to \pi\ell\nu_\ell$, determination of $|V_{ub}|$

  - decay amplitude parametrized by hadronic form factors



$$\langle\pi^+(p)|\bar{u}\gamma_\mu b|\bar{B}^0(p+q)\rangle = f^+_{B\pi}(q^2)\left[...\right]_\mu + f^0_{B\pi}(q^2)\left[...\right]_\mu$$

  - $|V_{ub}|$ determination [BaBar,Belle]

$$\left(\frac{1}{\tau_B}\right)\frac{dBR(\bar{B}^0 \to \pi^+ l^- \nu)}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3}p_\pi^3|f^+_{B\pi}(q^2)|^2 + O(m_l^2)$$

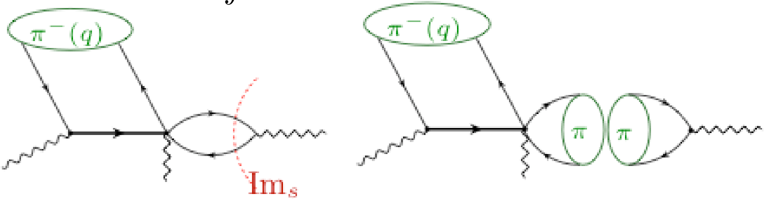
$0 < q^2 < (m_B - m_\pi)^2 \sim 26\ \text{GeV}^2$,

  - form factors accessible in lattice QCD at $q^2 \gtrsim 16\ \text{GeV}^2$

$f_+(q^2)$ from QCD Sum Rules (Ball, Zwicky, Khodjamirian, ...)

- Dispersion Relation and Light Cone Expansion
- Study a Correlation Function

$$F_\lambda(p,q) = i\int d^4x\ e^{ipx}\langle\pi^+(q)|T\{\bar{u}\gamma_\lambda b(x)\ m_b\bar{b}i\gamma_5 d(0)\}|0\rangle$$



- Yields an estimate for $f_B f_+(q^2)$
- Limited to small q^2

Results from LCSR

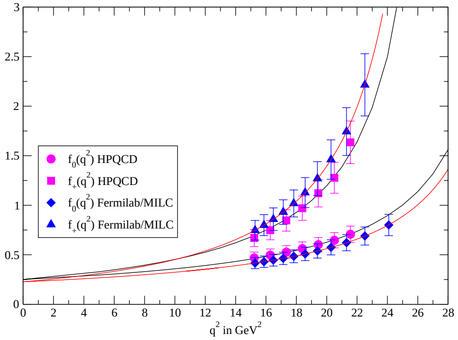
- Uncertainties from
    - Higher Twists (≥ 4)
    - b quark mass and renormalization scale
    - Values of the condensates
    - Threshold and Borel parameters
    - Pion Distribution amplitude

$$f_+(0) = 0.27 \times \left[1 \pm (5\%)_{tw>4} \pm (3\%)_{m_b,\mu} \pm (3\%)_{\langle \bar{q}q \rangle} \pm (3\%)_{s_0^B,M} \pm (8\%)_{a^\pi_{2,4}}\right]$$

- Extrapolation to $q^2 \neq 0$ by a pole model

Lattice QCD for Heavy to Light Form Factors

- Results reliable for large q^2
- Unquenched results are available
- Extrapolation to small q^2 by a pole model Becirevic, Kaidalov



Rate for $q^2 \geq 16\,\mathrm{GeV}^2$

$$|V_{ub}|^2 \times (1.31 \pm 0.33)\,\mathrm{ps}^{-1}$$
$$|V_{ub}|^2 \times (1.80 \pm 0.48)\,\mathrm{ps}^{-1}$$

(HPQCD / Fermilab MILC)

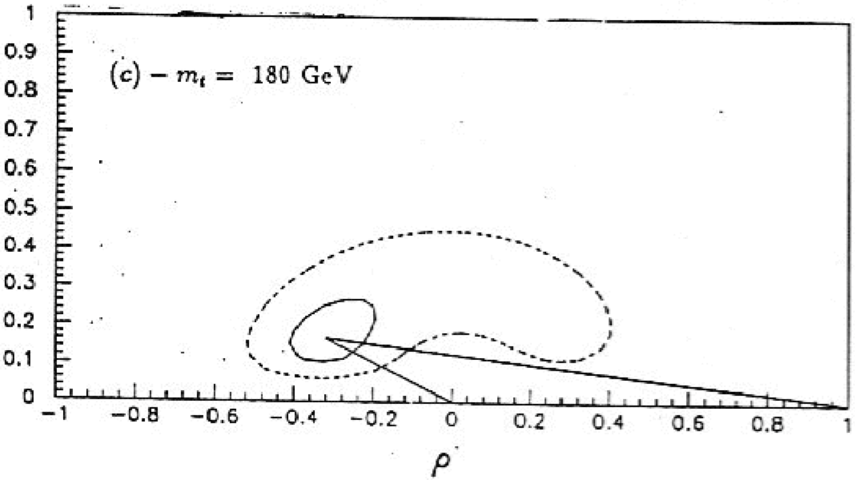
Status of V_{ub}

- From QCD LCSR: $V_{ub} = (3.32 \pm 0.26) \times 10^{-3}$
- PDG 2104:
  - Inclusive (LC-OPE): $V_{ub} = (4.41 \pm 0.25) \times 10^{-3}$
  - Exclusive (Combined): $V_{ub} = (3.28 \pm 0.29) \times 10^{-3}$
- This is the famous tension between the V_{ub}s

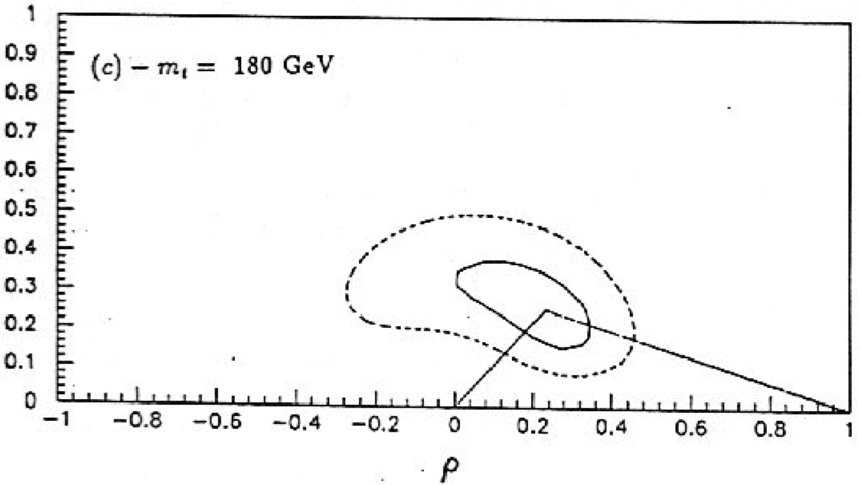
The history of the UT since $\sim$ 1993

- Situation in 1993:
  - HQET was still young ($\sim$ 3 years)
  - Hadronic Matrix elements for $\Delta m_d \sim f_B^2$ were quite uncertain
  - V_{ub}/V_{cb} was known at the level of $\sim$ 20%
  - The top quark mass was still $m_t \sim (140 \pm 40)$ GeV
  - No CP violation has been observed except ϵ_K
- The UT still could have been “flat”

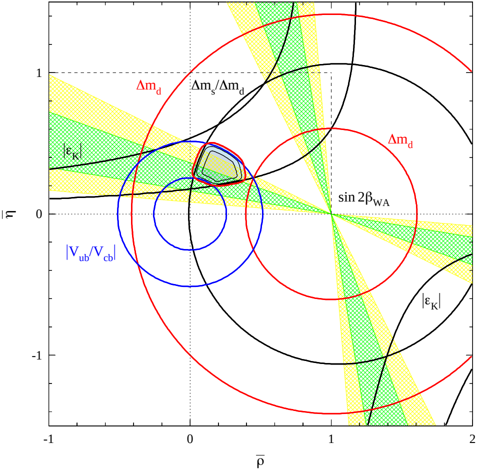
Unitarity triangle 1993: $f_B = 135 \pm 25$ MeV

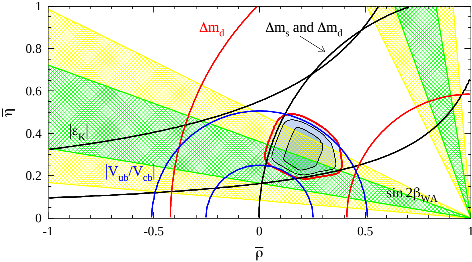


Unitarity triangle 1993: $f_B = 200 \pm 30$ MeV

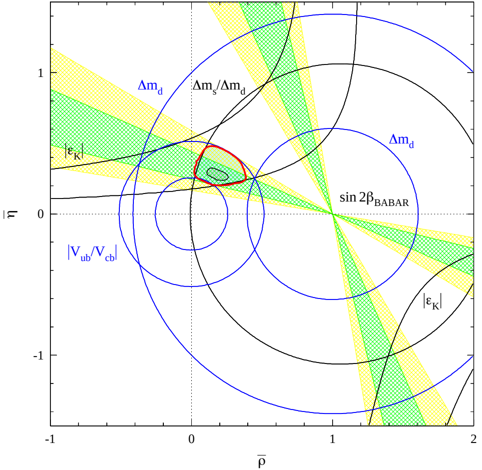


2001: First observation of “Non-Kaon CPV”

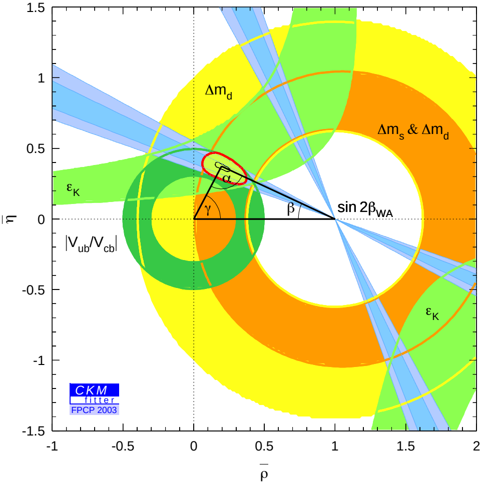




Unitarity Triangle 2001

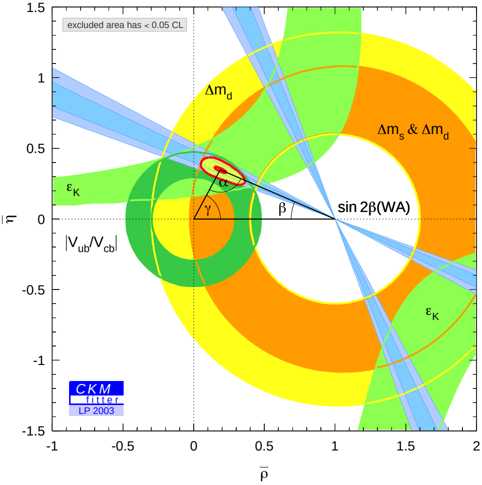


Unitarity Triangle 2002



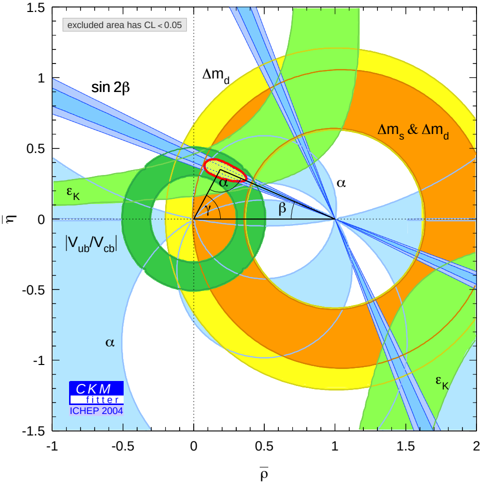
- Some improvement of V_{ub}/V_{cb} through the Heavy Quark Expansion
- More data on $\mathcal{A}_{\rm CP}(B \to J/\Psi K_s)$

Unitarity Triangle 2003



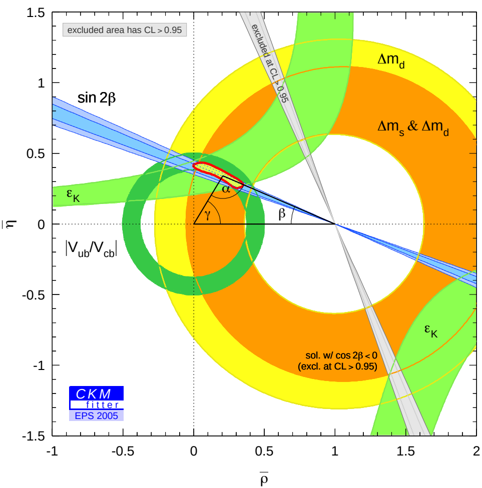
- Slight improvement of $f_B^2 B_B$ from lattice calculations
- Still more data on $\mathcal{A}_{\rm CP}(B \to J/\Psi K_s)$
- Central value of V_{ub}/V_{cb} slightly moved

Unitarity Triangle 2004



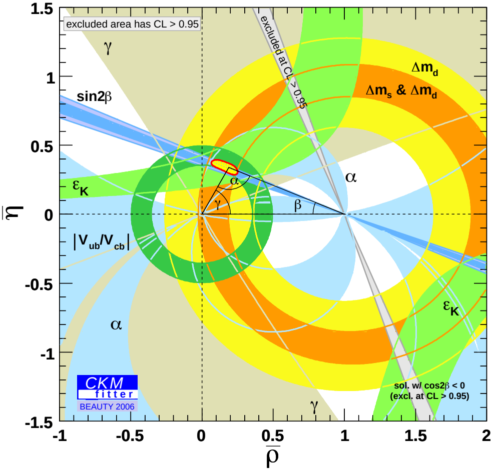
- More improvement of $f_B^2 B_B$ from lattice calculations
- Still more data on $\mathcal{A}_{\rm CP}(B \to J/\Psi K_s)$
- First constraints on the angle α from $B \to \rho\rho$

Unitarity Triangle 2005



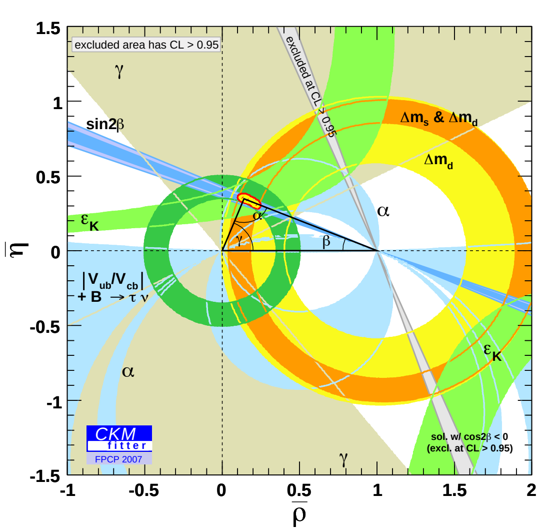
- Still more data on $\mathcal{A}_{\mathrm{CP}}(B \to J/\Psi K_s)$
- Exclusion of the "wrong branch" of β
- Dramatic Improvement of V_{ub} from the HQE

Unitarity Triangle 2006



- TEVATRON measurement of Δm_s
- Tighter constraints on α
- First constraints on γ from CPV in $B \to K\pi$

Unitarity traingle 2007



Unitarity Triangle Now