On the running α_s (and m_b) in the SM: two-loop electroweak corrections

A. Bednyakov

CALC2015, JINR,Dubna

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Outline

Motivation (α_s)

- Three-loop RGE for all the SM Lagrangian parameters were calculated recently in the $\overline{MS}$ scheme [MSS12, BPV13, CZ13].
- Boundary values at the electroweak (EW) scale are required for a RGE analysis of the model
  - Matching predictions in terms of parameters with "observables" or "pseudo"-observables - in perturbation theory at two loops.
- In a vacuum stability analysis of the SM the uncertainty of the instability scale (or critical values of the SM parameters at the EW scale) is dominated by those of y_t, λ and α_s [BKKS12, DDVEM$^+$12]
  - When one determines $\alpha_s(\mu)$ in the SM (from that of $n_f = 5$ flavour QCD) usually only strong interactions are taken into account.
  - However, the electroweak corrections can be potentially enhanced by top Yukawa coupling.

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  - Contary to the top-quark the ambiguity $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ significantly limits relative precision of M_b $\Rightarrow$new definitions of the b-quark mass parameter (short-distance).
  - Moreover, the relation between M_b and the corresponding running parameter (either m_b or y_b) at the matching scale involves logs of very different scale (m_b and EW scale), which can not be simultaneously made small via a suitable choice of the renormalization scale$\Rightarrow$call for effective theories

QCD embedded in the SM

$$\mathcal{L}_{\rm SM} = \mathcal{L}^{\rm gauge}_{\rm QCD} + \mathcal{L}^{\rm gauge}_{\rm SU(2)\times U(1)} + \mathcal{L}_{\rm Yukawa} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm g.f.} + \mathcal{L}_{\rm ghosts}$$

- In the QCD embedded in the SM, quark mass terms are generated via Yukawa interactions with the Higgs vacuum expectation value v:

$$m_q = \frac{y_q v}{\sqrt{2}}$$

- Due to spontaneous symmetry breaking (SSB) all other SM masses are also proportional to v

$$M_W^2 = \frac{g_2^2 v^2}{4}, \qquad M_Z^2 = \frac{g_1^2 + g_2^2}{4} v^2, \qquad M_h^2 = 2\lambda v^2$$

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- Introducing fine-structure constant α and Weinberg angle θ_W

$$(4\pi)\alpha = \frac{g_1^2 g_2^2}{g_1^2 + g_2^2} = g_2^2 \sin^2\theta_W = g_1^2 \cos^2\theta_W$$

- Parametrization used in this work

$$y_q^2 = \frac{4\pi\alpha}{\sin^2\theta_W}\frac{m_q^2}{M_W^2}, \qquad \lambda = \frac{4\pi\alpha}{8\sin^2\theta_W}\frac{M_h^2}{M_W^2}$$

  - All the parameters here are bare (or $\overline{MS}$ renormalizied) ones.
  - NB: In the formal limit $v \to \infty$ the mass ratios are finite.

Parameter values and the choice of renormlization scheme

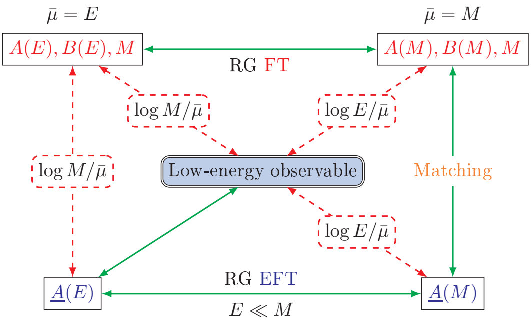
- The values of the SM parameter are not predicted by the theory but should be extracted from an experiment via matching procedure.

- In the QCD sector, due to confinement, one usually adopts $\overline{MS}$ scheme to define the running $\alpha_s(\mu)$.
- In order to determine the corresponding value, an observable $\mathcal{O}$ is matched to the corresponding theoretical prediction

$$\mathcal{O} = \alpha_s^k(\mu)\left[c_0(\mu) + c_1(\mu)\alpha_s(\mu) + c_2(\mu)\alpha_s^2(\mu) + ...\right],$$

  so that $\alpha_s(\mu_0)$ at some matching μ_0 is extracted.
- To avoid large logarithms the scale μ_0 is usually chosen around the typical scale involved in the measurement of $\mathcal{O}$ (e.g. momentum transfer Q^2).
- However, in $\overline{MS}$ additional effort is required if a theory involves different mass scales (apparent violation of the Appelquist & Carazzone decoupling theorem[AC75])

Re-summation and effective theories



Matching can be used to find $A(\bar{\mu})$ given $\bar{A}(\bar{\mu})$, $B(\bar{\mu})$ and M.

This is how $\alpha_s^{(6)}(\bar{\mu})$ is found from the quoted value of $\alpha_s^{(5)}(M_Z)$!

- Effective theory (ET) describes the interactions of light fields at low energies $E \ll M$ and parametrized by running $\overline{A}(\bar{\mu})$ coupling .
- The latter can be expessed via matching in terms of (running) parameters of the "full" theory (FT) - $A(\bar{\mu})$,$B(\bar{\mu})$ and heavy masses M.
- Large $\log E/M$ are re-summed by solving renormalization group (RG) equations in the effective theory with initial conditions at $\bar{\mu} = M$.

QED x QCD as an effective low-energy theory

- As a "low-energy" effective theory for the SM we consider a (toy) QCD x QED theory describing strong and electromagnetic interactions of five massless quarks (u, d, c, s, b) and leptons.

$$\mathcal{L}_{SM}\left(\alpha_s^{SM}, g_1, g_2, y_t, \lambda, ...\right) \Rightarrow \mathcal{L}^{(n_f=5)}_{QCD\times QED}\left(\alpha_s^{(5)}, \alpha_{EM}\right)$$

- Similar to the QCD case we "integrate out" top quark, electroweak gauge bosons and Higgs fields. We also neglect Fermi-like non-renormalizable interactions "$G_F\bar{\psi}\psi\bar{\psi}\psi$" with $G_F \propto \frac{g_s^2}{M_W^2}$.

- Formally, we consider the limit $v \to \infty$, which is different from that $y_t, g_2, \lambda \to \infty, v = \textit{fixed}$ usually implied in the discussions of "non-decoupling" feature of the models with SSB (see [Pic98]).

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- From the phenomelogical point of view we miss a lot of electroweak physics, governed at low energies by the Fermi constant G_F!

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- Nevertheless, our task is to study the running of $\alpha_s^{SM}(\mu)$ in $\overline{MS}$ extracted from $\alpha_s^{(5)}(\mu)$ at some matching scale $\mu_0 \simeq 100 - 200$ GeV
- Due to the *chosen $\overline{MS}$ scheme*, the result is also valid in the effective QED×QCD×Fermi theory!

Matching bare parameters

$$\alpha_{s,0}^{(5)} = \zeta_{\alpha_s,0}[\alpha_{s,0}, \alpha_0, M_0] \times \alpha_{s,0}$$

Due to SU(3) gauge invariance, the bare decoupling constant $\xi_{\alpha_{s,0}}$ can be found in a number of ways:

$$\zeta_{\alpha_s,0} = \zeta_{cGc,0}^2 \zeta_{c,0}^{-2} \zeta_{G,0}^{-1} = \zeta_{qGq,0}^2 \zeta_{q,0}^{-2} \zeta_{G,0}^{-1} = \ldots$$

in which different ζs are found by considering three- and two-point 1PI Green functions in the SM so that

- $\zeta_{cGc,0}$and $\zeta_{qGq,0}$ correspond to the leading terms in Taylor expansion of the integrand of the ghost-gluon and (light)-quark-gluon vertices, respectively.
- $\zeta_{c,0}, \zeta_{G,0}, \zeta_{q,0}$ involve only $\ln M/\mu$ terms coming from ghost, gluon and quark propagators.

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Taylor expansion can produce spurious IR-divergent $\frac{1}{(q^2)^2}$ terms, which, upon integration, lead to additional IR poles in $\epsilon = (4-d)/2$ in bare ζs.

Matching bare parameters

$$\alpha_s^{(5)}(\mu) = \frac{Z_{\alpha_s}[\alpha_s, \alpha, M]}{Z_{\alpha_{s(5)}}\left[\alpha_s^{(5)}\right]} \zeta_{\alpha_s,0}\left[Z_{\alpha_s}\alpha_s, Z_\alpha \alpha, Z_M M\right] \times \alpha_s(\mu)$$

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But the spurious IR poles are canceled in the matching relation for the running couplings *after* renormalization.

Matching via "pseudo"-observables

$$
\begin{aligned}
M_b &= m_b(\mu)\left(1 + \sum_{i+j=1}^{2} \alpha_s^i \cdot \alpha^j \cdot \sigma_{ij}(M_b, M, \mu) + \cdots\right)_{\mathrm{SM}} \\
&= \underline{m}_b(\mu)\left(1 + \sum_{i+j=1}^{2} \underline{\alpha}_s^i \cdot \underline{\alpha}^j \cdot \underline{\sigma}_{ij}(M_b, \mu) + \cdots\right)_{\mathrm{QEDxQCD}} + \mathcal{O}\left(\frac{m_b}{M}\right)
\end{aligned}
$$

Matching via pseudo-observable - the pole mass M_b - calculated either in the full SM or in $n_f = 5$ QCDxQED with $\underline{m}_b(\mu) \equiv m_b^{(5)}(\mu)$, etc.

- Both σ_{ij} and $\underline{\sigma}_{ij}$ are extracted from the b-quark self-energies. Only photon and gluon exchange contribute to $\underline{\sigma}_{ij}$, while σ_{ij} also involve exchange of heavy virtual particles with mass $M = \{M_t, M_W, M_Z, M_H\}$

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$\underline{m}_b(\mu)$, $\underline{\alpha}(\mu)$, $\underline{\alpha}_s(\mu)$ are related to their counterparts in the SM by means of decoupling constants ζs.

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But what is $m_b(\mu)$ in the SM formula for M_b?

Running m_b in the SM

- Running mass is not a fundamental SM Lagrangian parameter and is expressed in terms of y_b and the Higgs field v.e.v

$$m_b(\mu) = y_b(\mu) v/\sqrt{2}$$

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- $\frac{d}{d\ln\mu^2} m_b = \gamma_b m_b, \qquad \frac{d}{d\ln\mu^2} y_b = \beta_{y_b} y_b$

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  - $v^2 = \frac{1}{\sqrt{2} G_F}$ - gauge-independent vev from the tree-level matching to the Fermi-theory, $\gamma_{m_b} = \beta_{y_b}$.

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Running m_b in the SM

- Reorganize series for M_b in the SM, use [HK95, KV14, KPV15]

$$m_b(\mu) \to m_{Y,b}(\mu) = y_b(\mu) 2^{-3/4} G_f^{-1/2}$$

  - via matching to Fermi theory - $v^2(\mu) = \frac{m^2(\mu)}{\lambda(\mu)} = \frac{1}{\sqrt{2} G_f (1 + \bar{\delta} r(\mu))}$ with $\delta r(\mu)$ being an appropriate variant of Sirlin's Δr parameter
- Advantage - the relation between M_b and $m_{Y,b}(\mu)$ does not involve numerically "dangerous" tadpole terms, scaling as $M_t^4 / M_W^2 M_h^2$.

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$$\underline{m}_b(\mu) = m_{Y,b}(\mu) \cdot \zeta_{m_b}(\alpha_s, \alpha, M, \cancel{M_b}, \mu) = m_{Y,b}(\mu) \cdot (1 + \delta\zeta_{m_b})$$

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- The relation is used to find the boundary value of the Yukawa coupling from the $n_f = 5$ QCDxQED b-quark running mass at $\mu = M_t$

$$y_b(\mu) = 2^{3/4} G_f \cdot \underline{m}_b(\mu) \cdot \zeta_{m_b}^{-1}(\mu)$$

Results

- The results of matching after proper re-expansion can be casted, e.g., into the following form

$$\alpha_s = {\alpha_s}^{(5)}\left(1 + \frac{{\alpha_s}^{(5)}}{4\pi}\delta\zeta^{(1)}_{\alpha_s'} + \frac{\left({\alpha_s}^{(5)}\right)^2}{(4\pi)^2}\delta\zeta^{(2)}_{\alpha_s'} + \frac{{\alpha_s}^{(5)}\alpha_F}{(4\pi)^2}\delta\zeta^{(2)}_{\alpha_s'\alpha} + \cdots\right.$$

$$m_{Y,b} = \underline{m}_b\left(1 + \frac{\alpha_F}{4\pi}\delta\zeta^{(1)}_{\alpha_F} + \frac{\left({\alpha_s}^{(5)}\right)^2}{(4\pi)^2}\delta\zeta^{(2)}_{\alpha_s'}\right.$$
$$\left. + \frac{{\alpha_s}^{(5)}\alpha_F}{(4\pi)^2}\delta\zeta^{(2)}_{\alpha_s'\alpha} + \frac{(\alpha_F)^2}{(4\pi)^2}\delta\zeta^{(2)}_{\alpha}\cdots\right)$$

- $\alpha_F \equiv \frac{\sqrt{2}G_F M_W^2}{\pi}\left(1 - \frac{M_W^2}{M_Z^2}\right) = 132.233$ (for PDG'14)
- The required expressions for the strong coupling is available from [Bed14]and the results for the b-quark mass are preliminary and extracted from full two-loop electroweak corrections considered in the recent paper by Kniehl, Pikelner and Veretin' 2015.

Numerical analysis of the $\mathcal{O}(\alpha_s\alpha)$ correction to α_s

- In order to analyze the calculated correction we take the matching scale is $\mu = M_Z$ and use PDG'14 values of the pole masses.
- The quoted world average $\alpha_s^{(5)}(M_Z) = 0.1185$ is assumed to be fitted within the effective theory.
- At Z - boson mass scale (three-loop contribution $\mathcal{O}(\alpha_s^3)$ is also shown):

$$\alpha_s(M_Z) = 0.1185 \cdot \left[1 - \underbrace{0.008067}_{\alpha_s} - \underbrace{0.000965}_{\alpha_s^2} + \underbrace{0.000143}_{\alpha_s\alpha} + \underbrace{0.000018}_{\alpha_s^3} \right],$$

- In principle, final result for the running $\alpha_s^{SM}(\mu \gg M_Z)$ should not depend on the matching scale. However, due to truncation of the series, there is a residual dependence on μ
- As a consequence, the matching scale is usually chosen of the order of electroweak scale so that no large logs appear in the relation (effectively re-sum logarithms $\ln M_Z/\mu$).

Preliminary numerical analysis of EW corrections to m_b

- At Z - boson mass scale (with RunDec):

$$y_b(M_Z) = 2^{3/4} G_f \cdot \underline{m}_b(M_Z) \cdot \left[1 - \underbrace{0.00838}_{\alpha} - \underbrace{0.00074}_{\alpha_s^2} - \underbrace{0.00023}_{\alpha_s^3} + \underbrace{0.00068}_{\alpha_s \alpha} + \underbrace{0.00005}_{\alpha^2} + \cdots \right]$$

in comparision with

$$y_b(M_Z) = 2^{3/4} G_f \cdot M_b \left[1 - \underbrace{0.010}_{\alpha} - \underbrace{0.0270}_{\alpha_s} - \underbrace{0.0784}_{\alpha_s^2} + \underbrace{0.0032}_{\alpha_s \alpha} + \underbrace{0.0003}_{\alpha^2} + \cdots \right]$$

Scale dependence of the decoupling corrections to α_s

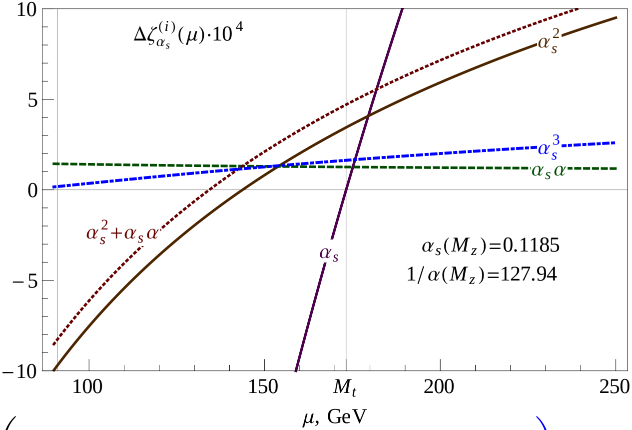
The scale dependence of different matching corrections:

α_s in terms of ${\alpha_s}^{(5)}$

$$\Delta\zeta_{\alpha_s}^{(\alpha_s)} \equiv \frac{{\alpha_s}^{(5)}}{(4\pi)}\delta\zeta^{(1)}_{{\alpha_s}^{(5)}},$$

etc

Four-loop running up to the matching scale via RunDec [CKS00] package.
[CKS06, SS06, KKOV06] - α_s^4



$$\alpha_s(M_t) = 0.10800\cdot\left(1+\underbrace{0.00034}_{\alpha_s^2}+\underbrace{0.00013}_{\alpha_s\alpha}+\underbrace{0.00016}_{\alpha_s^3}+\underbrace{0.00006}_{\alpha_s^4}\right)$$

Conclusions

- Electroweak corrections to the matching relations between α_s of the SM and effective $\alpha_s{}^{(5)}$ are found and expressed either in terms of particle pole masses or $\overline{MS}$ running masses in an explicit gauge-invariant way.
- The corrections for α_s,when evaluated at the electroweak scale, are found to be comparable with pure three(four)-loop QCD contribution usually taken into account in three-loop RGE analysis of the SM.
- Two-loop electroweak decoupling corrections for $y_b(\mu)$ are found by considering the b-quark poles mass in the SM and effective QCDxQED. The obtained result is gauge-indepdent and is free from soft scales, thus, allowing to use RGE for resummation of log m_b/M_t.
- Still a comprehensive numerical analysis is lacking for m_b.

Thank you for your attention!

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