New developments in the MCSANC tool

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MCSANC overview

MCSANC is a Monte Carlo tool for fixed order calculations of *pp*-collisions:

- the processes are: Drell–Yan (inclusive); associated Higgs and gauge boson production; single-top quark production in s- and t-channel
- based on the SANC modules
- complete NLO QCD and EW corrections
- new! photon induced corrections and certain two-loop corrections
- the tool supports different electroweak schemes, running/fixed scales, fully differential cross-sections on the output
- Parallel calculation on multicore machines thanks to Cuba library (http://www.feynarts.de/cuba/)

Supported processes

pid	$f\bar{f} \to$	SANC ref.
001:003	$l^+l^-(l = e, \mu, \tau)$	arXiv:0711.0625,0901.2785
004	$Z^0 + H$	arXiv:hep-ph/0506120,0812.4207
$\pm$ 101:103	$l^\pm + \nu_l$	arXiv:hep-ph/0506110,
$\pm$ 104	$W^\pm + H$	-
105	$t + \bar{b}$ (s-channel)	arXiv:1110.3622,1207.4400
106	$t + q$ (t-channel)	-//-
-105	$\bar{t} + b$ (s-channel)	-//-
-106	$\bar{t} + q$ (t-channel)	-//-

The process id notation: first digit is a sign of EW-current, last two digits — final particle choice:

$0xx$ - neutral current, $xx = 01(e), 02(\mu), 03(\tau), 04(HZ)$

$\pm 1xx$ - charged current, $xx = 01(e), 02(\mu), 03(\tau), 04(HW)$, $05, 06$(top-production, s- and t-channels)

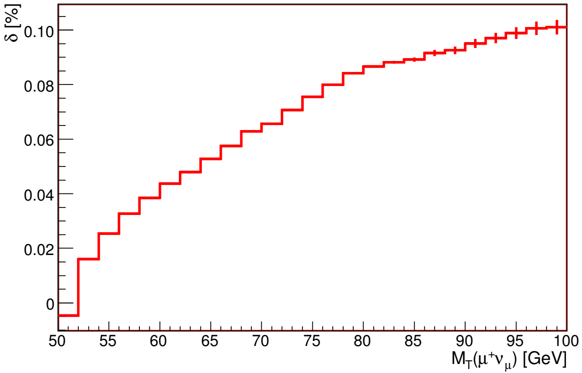
Motivation: Drell–Yan processes

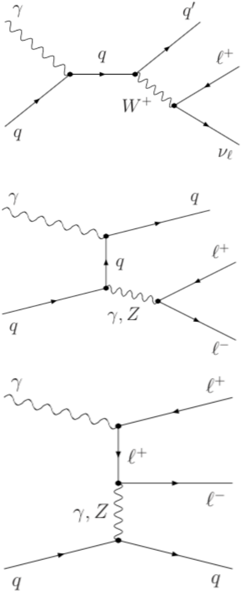
- The Drell–Yan processes have leptonic final states with clean signatures (see M.Managano's lectures)
- Measurements are used for PDF constraints and SM verification
- Current fixed order precision is NNLO QCD + NLO EW (FEWZ, DYNNLO, HORACE, MCSANC)
- LHC gives access to energies where additional corrections have to be included:
  - photon-induced contributions
  - higher order radiative corrections

Photon-induced contributions

Processes with photonic initial states:

- $q\gamma \to q'\ell^{\pm}\nu_{\ell}$ (for CC DY),
- $q\gamma \to q\ell^{-}\ell^{+}$ (for NC DY),
- $\gamma\gamma \to \ell^{+}\ell^{-}$ (for NC DY);





Photon-induced contributions

Introduced into SANC environment [J.Exp.Theor.Phys. 106 (2008)]

$$\sigma^{pp}(s) = \sum_{q_i} \int_0^1 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 q_i(x_1, M^2) \gamma(x_2, M^2) \hat{\sigma}^{q_i\gamma}(\hat{s})$$

The quark masses are finite, therefore σ^{pp} depends on the masses as $\ln(M^2/m_q^2)$. Such terms have to be subtracted to avoid double-counting due to their foregoing inclusion into PDFs.

$$\delta_1 = \sum_{q_i} \int_0^1 \int_0^1 \mathrm{d}x_1 \, \mathrm{d}x_2 \, \gamma(x_1, M^2) q_i(x_2, M^2) \times \int_0^1 \mathrm{d}x_3 D_{q'\gamma}(x_3, M, m_{q'}) \tilde{\sigma}^{q_i q'_i \to l_1 \bar{l}_2}(\tilde{s})$$

is the convolution of two subprocesses: photon-to-quark pair conversion, and DY $q\bar{q} \to \ell\bar{\ell}$.

Photon-induced contributions

The structure function $D_{q'\gamma}(x_3, M, m_{q'})$ describes the probability to find quark q' with energy fraction x_3 in the photon. For the $\overline{\mathrm{MS}}$ scheme at NLO this function reads

$$D^{\overline{\mathrm{MS}}}_{q'\gamma}(x_3, M, m_{q'}) = \frac{\alpha}{2\pi} Q^2_{q'} \ln \frac{M^2}{m^2_{q'}} [x_3^2 + (1 - x_3)^2],$$

Same structure function in DIS scheme:

$$D^{\mathrm{DIS}}_{q'_i\gamma}(x_3, M, m_{q'}) = -\ln \frac{M^2(1 - x_3)}{x_3 m_q^2} (x_3^2 + (1 - x_3)^2) - 2 + 16x_3 - 16x_3^2$$

Photon-induced contributions

Another sub-process with photons in the initial state is $\gamma\gamma \to \ell^+\ell^-$ (NC DY only). The cross section in massive case:

$$\sigma(s, \cos\vartheta) = \frac{\alpha^2\pi}{s}\left\{\frac{1}{1-\beta_l\cos\vartheta}\left[1+\beta_l\cos\vartheta+\frac{4m_f^2}{s}\left(1-\frac{1+\beta_l^2}{1-\beta_l\cos\vartheta}\right)\right]\right.$$
$$\left.+\frac{1}{1+\beta_l\cos\vartheta}\left[1-\beta_l\cos\vartheta+\frac{4m_f^2}{s}\left(1-\frac{1+\beta_l^2}{1+\beta_l\cos\vartheta}\right)\right]\right\},$$

where $\beta_l = \beta(s, m_l^2, m_l^2) = \sqrt{1-4\frac{m_l^2}{s}}$ and θ is the angle between the photon and the outgoing lepton momenta in the center-of-mass system.

PI: numerical results

Inclusive LO cross section and photon-induced contribution $\delta_{q/\bar{q}\gamma} = \frac{\sigma_{q/\bar{q}\gamma}}{\sigma_0}$ for lepton pair transverse mass M_T and the transverse momentum p_T

M_T/GeV	σ_0/pb	$\delta_{q/\bar{q}\gamma}$/%
50-∞	4495.8(1) 4495.7(2)	0.047(3) 0.052(1)
100-∞	27.590(1) 27.589(2)	0.11(1) 0.12(1)
200-∞	1.7907(1) 1.7906(1)	0.24(1) 0.25(1)
500-∞	0.084696(1) 0.084697(4)	0.36(1) 0.37(1)
1000-∞	0.0065221(1) 0.0065222(4)	0.38(1) 0.39(1)
2000-∞	0.00027322(1) 0.00027322(1)	0.35(1) 0.36(1)

p_T/GeV	σ_0/pb	$\delta_{q/\bar{q}\gamma}$/%
25-∞	4495.8(1) 4495.7(2)	0.059(3) 0.065(1)
50-∞	27.590(1) 27.589(2)	4.6(1) 4.7(1)
100-∞	1.7907(1) 1.7906(1)	11.9(1) 12.3(1)
200-∞	0.18129(1) 0.18128(1)	16.6(1) 17.1(1)
500-∞	0.0065221(1) 0.0065222(4)	16.2(1) 16.7(1)
1000-∞	0.00027322(1) 0.00027322(1)	13.1(1) 13.5(1)

First line is MCSANC, second line is [Phys.Rev. D77 (2008) 073006]

Leading two-loop electroweak corrections

In MCSANC v1.20 we follow the recipe introduced in [Phys.Lett. B319 (1993) 249-256]
The ρ parameter is defined as the ratio of the neutral current to charged current amplitudes at zero momentum transfer:

$$\rho = \frac{G_{NC}(0)}{G_{CC}(0)} = \frac{1}{1 - \Delta\rho} \,,$$

where $G_{CC}(0) = G_\mu$ is the Fermi constant defined from the μ-decay width, perturbatively

$$\Delta\rho = \Delta\rho^{(1)} + \Delta\rho^{(2)} + \dots$$

We try to estimate the effect due to the two-loop EW corrections $\Delta\rho^{(2)}$ by basic replacements in our form factors (FF). Expanding ρ to quadratic terms $\Delta\rho^2$, we have

$$\rho = 1 + \Delta\rho + \Delta\rho^2 \,.$$

Leading two-loop electroweak corrections

The leading in $G_\mu m_t^2$ NLO EW contribution to $\Delta\rho$ is explicitly given by

$$\Delta\rho^{(1)}\Big|^{G_\mu} = 3x_t = \frac{3\sqrt{2}G_\mu m_t^2}{16\pi^2}\,.$$

At the two-loop level, quantity $\Delta\rho$ contains two contributions:

$$\Delta\rho = 3x_t\,\left[1+\rho^{(2)}\,\left(m_H^2/m_t^2\right)\,x_t\right]\,\left[1-\frac{2\alpha_s(m_Z^2)}{9\pi}(\pi^2+3)\right].$$

They consist of the following:

- two-loop EW part at $\mathcal{O}(G_\mu^2)$
- mixed two-loop EW$\otimes$QCD at $\mathcal{O}(G_\mu\alpha_s)$

Leading two-loop electroweak corrections

Using intermediate vector boson propagators $\sim 1/(Q^2 + M_V^2)$, we derive:

$$\rho = \frac{m_W^2}{\bar{c}_W^2 m_Z^2} \, ,$$

where we introduced a new parameter $\bar{c}_W^2$ to distinguish from the usual c_W^2 for which we maintain the meaning $c_W^2 = m_W^2/m_Z^2$ to be valid to all perturbative orders. At the lowest order (LO)

$$\rho^{(0)} = \frac{m_W^2}{c_W^2 m_Z^2} = 1 \, .$$

Then:

$$\bar{c}_W^2 = \frac{m_W^2}{\rho m_Z^2} = (1 - \Delta\rho) \, c_W^2 \, .$$

Leading two-loop electroweak corrections

The leading in $G_\mu m_t^2$ universal higher order (h.o.) corrections may be taken into account via the replacements, [JHEP 1001 (2010) 060]:

$$
\begin{aligned}
\alpha_{G_\mu} &\to \alpha_{G_\mu} \frac{\bar{s}_W^2}{s_W^2}\,, \\
s_W^2 &\to \bar{s}_W^2 \equiv s_W^2 + \Delta\rho\, c_W^2\,, \\
c_W^2 &\to \bar{c}_W^2 \equiv 1 - \bar{s}_W^2 = (1-\Delta\rho)\, c_W^2
\end{aligned}
$$

in the LO expression for NC DY cross section.
This approach correctly reproduces terms up to $\mathcal{O}(\Delta\rho^2)$.

Given these replacements, we get the contributions of h.o. corrections to the scalar form factors of the invariant amplitude

Leading two-loop electroweak corrections

In the LQ basis the Z exchange amplitude has the following Born-like structure in terms of four (LL, QL, LQ and QQ) form factors:

$$\mathcal{A}_Z^{\mathrm{IBA}} = -\,i\,\frac{4\pi\alpha}{s}\left[\frac{1}{4s_W^2 c_W^2}\frac{s}{s - M_Z^2 + i\frac{\Gamma_Z}{M_Z}s}\right]$$

$$\Big\{\gamma_\mu\gamma_+ \otimes \gamma_\mu\gamma_+ I_e^{(3)} I_t^{(3)} F_{LL}(s,t),$$

$$+\,\gamma_\mu \otimes \gamma_\mu\gamma_+(-2Q_e s_W^2) I_t^{(3)} F_{QL}(s,t)$$

$$+\,\gamma_\mu\gamma_+ \otimes \gamma_\mu I_e^{(3)}(-2Q_f s_W^2) F_{LQ}(s,t)$$

$$+\,\gamma_\mu \otimes \gamma_\mu(-2Q_e s_W^2)(-2Q_f s_W^2) F_{QQ}(s,t)\Big\},$$

Leading two-loop electroweak corrections

$$\mathcal{A}_Z^{\mathrm{IBA}}(with\ h.o.) = i\frac{4\pi\bar{\alpha}_{G_\mu}}{s}\left[\frac{1}{4}\frac{1}{s_W^2}\left[\frac{s_W^2}{\bar{\mathbf{s}}_W^2}\right]\frac{s}{s - M_Z^2 + i\frac{\Gamma_Z}{M_Z}s}\right]$$

$$\left\{\gamma_\mu\gamma_+ \otimes \gamma_\mu\gamma_+ I_e^{(3)} I_t^{(3)}\left[\frac{1}{\bar{\mathbf{c}}_W^2}\right] F_{LL}(s,t),\right.$$

$$+\gamma_\mu \otimes \gamma_\mu\gamma_+(-2Q_e)s_W^2\left[\frac{\bar{\mathbf{s}}_W^2}{s_W^2}\frac{1}{\bar{\mathbf{c}}_W^2}\right] I_t^{(3)} F_{QL}(s,t)$$

$$+\gamma_\mu\gamma_+ \otimes \gamma_\mu I_e^{(3)}(-2Q_f)s_W^2\left[\frac{\bar{\mathbf{s}}_W^2}{s_W^2}\frac{1}{\bar{\mathbf{c}}_W^2}\right] F_{LQ}(s,t)$$

$$\left.+\gamma_\mu \otimes \gamma_\mu(-2Q_e)(-2Q_f)s_W^4\left[\frac{\bar{\mathbf{s}}_W^4}{s_W^4}\frac{1}{\bar{\mathbf{c}}_W^2}\right] F_{QQ}(s,t)\right\},$$

where for convenience, we multiply and divide $\bar{\mathbf{s}}_W^2$ on the expression s_W^2/s_W^2.

Leading two-loop electroweak corrections

In respect with replacements on slide 13: object $\frac{1}{\bar{\mathbf{c}}_W^2}$ will go into:

$$\frac{1}{\bar{\mathbf{c}}_W^2} = \frac{1}{(1-\Delta\rho)c_W^2} = (1+\Delta\rho+\Delta\rho^2)\frac{1}{c_W^2},$$

and the object $\frac{\bar{\mathbf{s}}_W^2}{s_W^2}$ will go into:

$$\frac{\bar{\mathbf{s}}_W^2}{s_W^2} = (1+\frac{c_W^2}{s_W^2}\Delta\rho).$$

Leading two-loop electroweak corrections

Therefore, amplitude in NNLO order is:

$$\mathcal{A}_Z^{\mathrm{IBA}}(with\ h.o.) =$$

$$- i\frac{4\pi\alpha_{G_\mu}}{s}\left[\frac{1}{4s_W^2c_W^2}\frac{s}{s - M_Z^2 + i\frac{\Gamma_Z}{M_Z}s}\right]$$

$$\Bigg\{\gamma_\mu\gamma_+ \otimes \gamma_\mu\gamma_+ I_e^{(3)} I_t^{(3)}\left[1 + \Delta\rho + \Delta\rho^2\right] F_{LL}(s,t)$$

$$+ \gamma_\mu \otimes \gamma_\mu\gamma_+(-2Q_e)s_W^2\left[(1 + \Delta\rho + \Delta\rho^2)(1 + \frac{c_W^2}{s_W^2}\Delta\rho)\right] I_t^{(3)} F_{QL}(s,t)$$

$$+ \gamma_\mu\gamma_+ \otimes \gamma_\mu I_e^{(3)}(-2Q_f)s_W^2\left[(1 + \Delta\rho + \Delta\rho^2)(1 + \frac{c_W^2}{s_W^2}\Delta\rho)\right] F_{LQ}(s,t)$$

$$+ \gamma_\mu \otimes \gamma_\mu(-2Q_e)(-2Q_f)s_W^4\left[(1 + \Delta\rho + \Delta\rho^2)(1 + \frac{c_W^2}{s_W^2}\Delta\rho)^2\right] F_{QQ}(s,t)\Bigg\}$$

Leading two-loop electroweak corrections

To avoid a double counting one should remove the leading NLO EW contribution from the linear in $\Delta\rho$ terms: $\Delta\rho \longrightarrow \left(\Delta\rho - \Delta\rho^{(1)}\Big|^{G_\mu}\right)$.

We showed analytically that the results obtained in this way agree with the corresponding expressions from paper [JHEP 1001 (2010) 060]

HE EW: numerical results

Table shows inclusive LO cross section σ_0 for $pp \to e^+e^-X$ and higher order corrections

$$\delta_{h.o.weak} = \frac{\sigma_{h.o.weak}}{\sigma_0}$$

First line is MCSANC, second line is [Phys.Rev. D77 (2008) 073006].

We see an excellent agreement between these two calculations.

M_{ll}/GeV	σ_0/pb	$\delta_{h.o.weak}$/%
50-∞	738.813(5) 738.773(6)	0.030(1) 0.030
100-∞	32.7293(2) 32.7268(3)	0.013(1) 0.012
200-∞	1.48488(1) 1.48492(1)	-0.23(1) -0.23
500-∞	0.080942(3) 0.0809489(6)	-0.29(1) -0.29
1000-∞	0.0067998(1) 0.00680008(3)	-0.31(1) -0.31
2000-∞	0.00030375(1) 0.000303767(1)	-0.31(1) -0.32

Forward-backward asymmetry

The forward-backward asymmetry A_{FB}^{ff} is usually defined as

$$A_{FB} = \frac{F - B}{F + B}$$

where

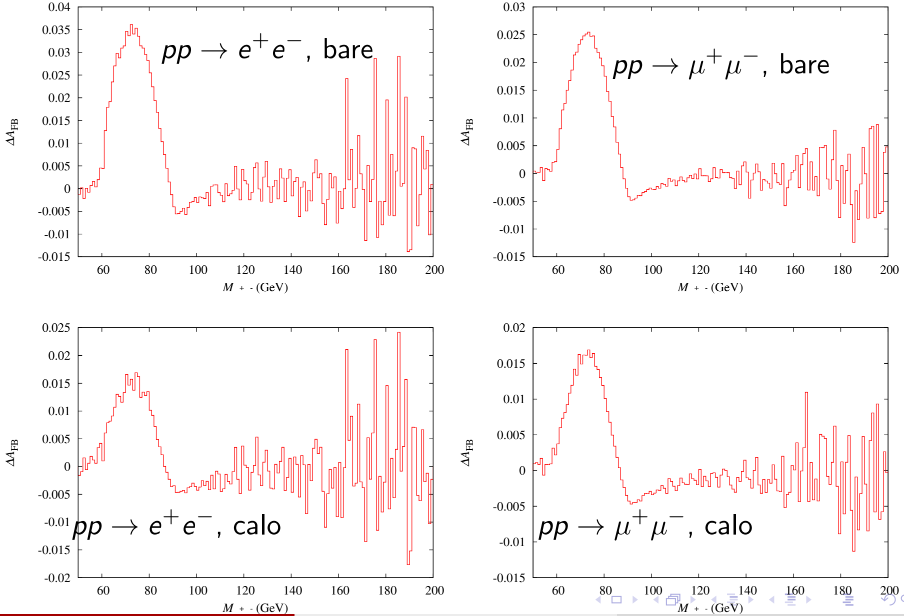
$$F = \int_0^1 \frac{d\sigma}{d\cos\theta^*} d\cos\theta^*, \qquad B = \int_{-1}^0 \frac{d\sigma}{d\cos\theta^*} d\cos\theta^*.$$

The cosine of the angle between the lepton and quark in the $\ell^+\ell^-$ rest frame is then approximated by $p^\pm = \frac{1}{\sqrt{2}}(E \pm p_z)$:

$$\cos\theta^* = \frac{|p_z|}{p_z} \cdot \frac{2(p^+ p^- - p^- p^+)}{m\sqrt{m^2 + p_T^2}}$$

where E is the energy, and p_z is the longitudinal component of the momentum vector.

A_{FB}



$pp \to e^+e^-$, bare
$pp \to \mu^+\mu^-$, bare
$pp \to e^+e^-$, calo
$pp \to \mu^+\mu^-$, calo
ΔA_{FB}
M_{+-} (GeV)

Summary

Several improvements were made in the MCSANC program v1.20:

- Important contribution of photon induced processe are added for for Drell–Yan processes
- Higher order corrections are included for DY neutral current
- Calculation of forward-backward asymmetry is implemented into standard output
- New option for up-to 3-dimensional histograms for corresponding experimental measurements