Dynamical Gluon Mass and Linear Confinement

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work in collaboration with
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Introduction

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- Asymptotic Freedom $\rightarrow$ strong interaction at short distances (high energy) of constituents of hadronic matter in DIS.
- Confinement $\rightarrow$ strong interaction at large distances (low energy) where the hadron spectrum is already known. (lattice QCD, phenomenological potential models, dispersive extensions of the QCD beta function, DSE, etc...).

The quark-antiquark potential (Cornell)

The relation between quark-antiquark potential in QCD is via the Cornell potential

$$V^C(r) = -\frac{a}{r} + b\ r,$$

which reproduces the experimental heavy meson spectrum.
Potential can be obtained from the running coupling with Fourier transform (Richardson)

$$V(r) = -\frac{8}{3\pi}\int_0^\infty \alpha_s(q^2)\frac{\sin(qr)}{qr}dq.$$

Cornell Potential

The limits of the Cornell potential imply in the momentum space the following restrictions on the running coupling

$$\alpha_s(q^2 \mapsto \infty) \sim \mathit{const} \rightarrow \text{asymptotic freedom}$$

$$\alpha_s(q^2 \mapsto 0) \sim 1/q^2 \rightarrow \text{confinement}$$

Motivated in these constraints, Richardson proposed

$$\alpha_s^{(C)}(q^2) = \frac{4\pi}{\beta_0 \ln(1 + q^2/\Lambda_C^2)},$$

where $\beta_0 = 11 - 2/3\ n_f$ is the first β-function coefficient for QCD and n_f is the number of active quarks [J.L.Richardson, Phys.Lett.B82,272(1979)].

Cornell potential and running coupling

Therefore, any model that satisfied the aforementioned restrictions on asymptotic freedom and linear confinement (pole at the origin of the running coupling which leads in Richardson's approach to a Gribov singularity) is a good candidate to reproduce experimental data.
For example

$$\alpha_s(q^2) = \frac{A}{1 - e^{-Bq^2}} \ . \tag{1}$$

Cornell potential and running coupling

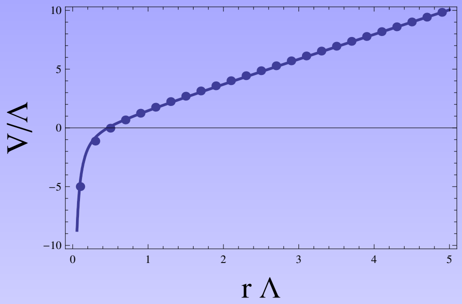

Figure: The dots represent the Cornell potential with parameter values $a = 0.52, \sqrt{b} = 427.3\ MeV$. With these values the correct masses of charmed mesons are reproduced within errors. The full line shows the toy model potential for $A = 0.41, \sqrt{1/B} = 107.8 MeV$. We choose $\Lambda = 300 MeV$, close to the value of the QCD scale constant, as our dimensional scale. The two potentials reproduce the spectrum within experimental errors.

Massive Approach to Confinement

While the Richardson's running coupling reproduces confinement, at high energies we cannot ensure that it coincide at all with the perturbative counterpart. In fact, it deviates from the usual perturbative (pQCD) coupling by

$$\alpha_s^{(C)}(q^2) - \alpha_s^{(\mathrm{pQCD})}(q^2) \sim \frac{\Lambda^2}{q^2 \ln(q^2/\Lambda^2)} \qquad |q^2| > \Lambda^2 \ ,$$

where $\alpha_s^{(\mathrm{pQCD})} = 4\pi/(\beta_0 \ln(q^2/\Lambda^2))$.
This is what we don't want.

A coupling with the correct behavior can be defined by introducing an effective mass $m(q^2)$ in the following way

$$\alpha_s^{(\mathrm{m})}(q^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{q^2+m^2(q^2)}{\Lambda_m^2}\right)} \,, \qquad (2)$$

such that

$$m^2(q^2) = \begin{cases} \Lambda_m^2, & q^2 \mapsto 0 \\ 0, & q^2 \mapsto \infty \end{cases}$$

It is evident that the deviation from the perturbative case disappears if the mass function goes down quickly to zero with q^2.

Gluon mass parametrization from DSE

To connect the q^2-dependence of the effective gluon mass with QCD we use,

$$m^2(q^2) = \frac{m_0^2}{1 + (q^2/\mathcal{M}^2)^{1+p}}, \tag{3}$$

a function obtained in the Dyson-Schwinger context [A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys.Rev.D89(2014)8,085032].
Substituting this mass formula in Eq.2, we proceed to obtain the parameters $m_0^2 = \Lambda_m^2$, $\mathcal{M}$ and p by comparing with the Cornell potential .

Gluon mass parametrization from DSE

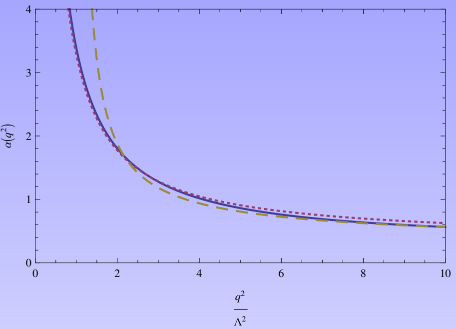

Figure: The full line represents the Exponential coupling, the dotted line the Gluon Mass coupling and the dashed line the LO perturbative QCD coupling. The parameters of the first two have been fitted to reproduce the Cornell potential. It is apparent the identical behavior at the Gribov singularity of the first two and the rising of the Landau pole in the latter. Asymptotically the Gluon Mass term contains the perturbative logarithm which the Exponential potential or the Cornell potential do not have.

Gluon mass parametrization from DSE

The two potentials, massive and Cornell, are very close for large r and differ inappreciably at small r. This small change does not affect the fitting of the heavy spectrum and is associated with the perturbative logarithm dependence of the gluon mass potential at large q^2.

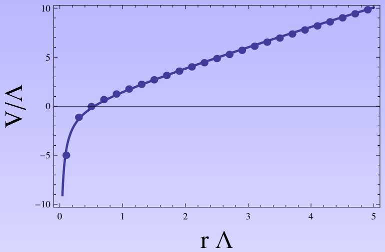

Figure: The dots represent the Cornell potential for mentioned parameter values a and b. The full line shows the gluon DS-mass model potential for $m_0 = 339 MeV, \mathcal{M} = 436 MeV, p = 0.15$. The values of $\mathcal{M}$ and p were taken from DSE, and m_0 was fitted to the Cornell potential. We choose $\Lambda = 300\ MeV$ as our dimensional scale. The gluon mass potential reproduces the data within errors.

Gluon mass parametrization from DSE

DYSON SCHWINGER			GLUON MASS		
$m_0(MeV)$	$\mathcal{M}(MeV)$	p	$m_0(MeV)$	$\mathcal{M}(MeV)]$	p
425	436	0.15	339	436	0.15

Table: Parameter sets of the Dyson-Schwinger mass formula [A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys.Rev.D89(2014)8,085032] and of the present analysis.

Gluon mass parametrization from DSE

If we relax the above condition (now: $m_0 > \Lambda_m \Rightarrow \alpha_s^{(\mathrm{m})}$ IR finite) and keep exactly the Dyson-Schwinger form for the running mass, we are led to a soft low energy behavior represented by a Yukawa potential. This mechanism reproduces the behavior of the quark thresholds for unquenched lattice QCD [G. S. Bali, Phys.Rept.343,1(2001)].

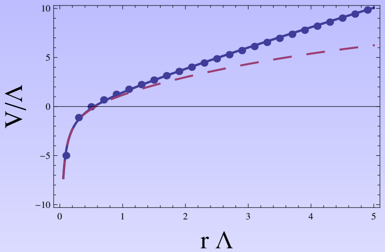

Figure: The dots represent the Cornell potential for parameters mentioned parameters a and b. The full line shows the gluon mass model potential for $m_0 = 339 MeV, \mathcal{M} = 436 MeV, p = 0.15$; the dashed line the DS-mass potential with $m_0 = 339 MeV, \mathcal{M} = 436 MeV, p = 0.15$and $\Lambda_m = 330 MeV < m_0$. We use $\Lambda = 300\ MeV$ as our dimensional scale.

Conclusions

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Conclusions

1. We have shown that in Richardson prescription it is necessary to take a massless gluon propagator and a nonperturbative coupling constant to reproduce the Cornell potential.
2. Taking the gluon mass function from Dyson-Schwinger analysis, we have obtained an excellent description of the Cornell potential when we impose the correct singularity structure, i.e., $m_0 = \Lambda_m$.
3. If we shift the initial mass parameter m_0, $m_0 > \Lambda_m$, we reproduce the behavior of the potential for unquenched lattice QCD.

THANK YOU!!