



Search for BFKL-evolution effects at high energies

Victor T. Kim

**St. Petersburg Nuclear Physics Institute NRC KI, Gatchina
St. Petersburg State Polytechnical University**

Outline:

- **Motivation: high energy asymptotics**
- **BFKLP: NLA BFKL within generalized BLM**
- **$\gamma^*\gamma^*$ - collisions**
- **Dijets from QCD dynamics: GLAPD vs. BFKL**
- **Forward dijets at LHC: dijet “K-factor” vs $|\eta|$**
- **Forward dijets at LHC: azimuthal decorrelations vs $|\eta|$**
- **Summary**

High energy asymptotics

- Large-angle scattering:

QCD in Bjorken limit

- **GLAPD: V. Gribov & L. Lipatov (71-72); L. Lipatov (74);
G. Altarelli & G. Parisi (77); Yu. Dokshitzer (77)**

- Small-angle scattering:

QED in Gribov-Regge limit

- **V. Gribov, V. Gorshkov, L. Lipatov & G. Frolov (67-70)
H. Cheng & T. Wu (66-70)**

QCD in Gribov-Regge limit

- **BFKL: V. Fadin, E. Kuraev & L. Lipatov (75-78)
I. Balitsky & L. Lipatov (78)**

High-energy QCD asymptotics: GLAPD and BFKL

$$s=(p_1+p_2)^2$$

$$t=(p_1-p_3)^2$$

$$Q^2=-t$$

Scattering in the Standard Model (QCD) at high energies:

Large logarithms: as $\log(s)$, as $\log(Q^2)$

Bjorken limit (large-angle scattering):

$$s \sim Q^2 \gg m^2$$

$$Q^2/s = x \sim 1$$

Gribov-Lipatov-Altarelli-Parisi-Dokshitzer (GLAPD):

(as $\log(Q^2)$)ⁿ resummation

Inclusive cross section $\sim 1/Q^4$

Gribov-Regge limit (small-angle scattering):

$$s \gg Q^2 \gg m^2$$

$$Q^2/s = x \Rightarrow 0$$

Balitsky-Fadin-Kuraev-Lipatov (BFKL):

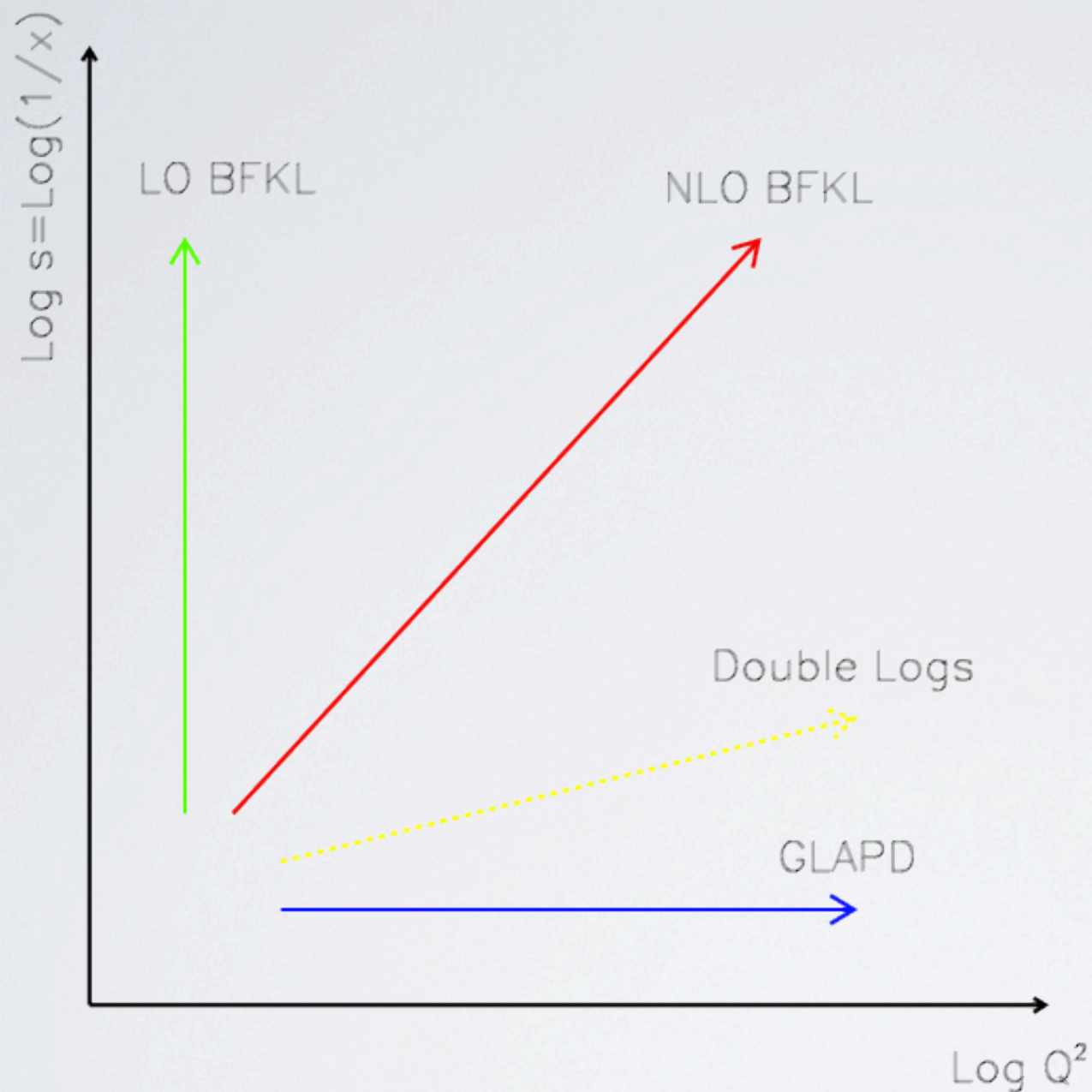
(as $\log(s)$)ⁿ resummation

Total cross section $\sim s^{(a_P-1)}$

a_P – Pomeron intercept

soft scattering data: $a_P = 1.1$

x-section asymptotics



Bjorken limit (GLAPD):

$$s \sim Q^2 \gg m^2$$

$$Q^2/s = x \sim 1$$

Large-angle (large-x) scattering

Gribov-Regge limit (BFKL):

$$s \gg Q^2 \gg m^2$$

$$Q^2/s = x \rightarrow 0$$

Small-angle (small-x) scattering

Leading Log (LL) BFKL: problems

LL BFKL: designed for infinite collision energies

LL BFKL problems (at finite energies):

- **fixed (non-running) coupling α_s**
- **energy-momentum conservation**
- **transverse momentum conservation**

Cross section in LL BFKL:

$$\sigma = \sigma_0 (S/S_0)^{(\alpha_P - 1)}$$

$$\alpha_P = 1 + C \alpha_s \approx 1.5 - 1.6$$

Data: $\alpha_P \approx 1.2 - 1.3$

BFKL: next-to-leading logs (NLL)

V.S. Fadin & L.N. Lipatov (89-98)

C. Camici & M. Ciafaloni (96-98)

next-to-leading log approximation (NLL) BFKL
MSbar-renormalization scheme: large corrections

S.J. Brodsky, V.S. Fadin, V.K., L.N. Lipatov, G.B. Pivovarov (98-99) BFKLP

BFKLP: NLL BFKL + resummation of running coupling α_s

(Brodsky, Lepage & Mackenzie - 83) BLM approach

NLL BFKL: BLM?

S. Brodsky, P. Lepage & P. Mackenzie (83) BLM approach for NLO

- **QCD – asymptotically conformal**
- **non-conformal corrections (running coupling corrections) are resummed into optimal scale**

**BLM in high orders: S. Mikhailov & A. Kataev (2015),
PMC - S. Brodsky et al. (2012-15)**

Naïve BLM application at NLO does not work (!):

- **NLL BFKL in $\overline{\text{MS}}$ scheme**
- **Upsilon \rightarrow ggg decay in NLO in $\overline{\text{MS}}$ scheme**

$\overline{\text{MS}}$ -renormalization scheme: nonphysical RG scheme (!)

S. Brodsky, Rathmann et al (1997)

BFKLP: NLL BFKL within generalized BLM

Naïve BLM application does not work (!):

- **NLL BFKL in \overline{MS} scheme**
- **Upsilon \rightarrow ggg decay in NLO in \overline{MS} scheme**

\overline{MS} -scheme: nonphysical RG scheme (!)

numerically close to V-scheme (heavy quark potential) – Abelian in LO

physical RG scheme: MOM scheme (gauge dependent)

- **NLL BFKL in non-Abelian in LO**
- **Upsilon \rightarrow ggg decay in non-Abelian in LO**

one can use MOM-scheme based on ggg-vertex non-Abelian in LO

BLM generalized on non-Abelian case:

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov(98-99) BFKLP

BFKLP: NLL BFKL + resummation of running coupling as

BLM resummation depends on non-Abelian structure in LO

BFKLP: NLL BFKL within generalized BLM

$$\omega_{\overline{MS}}(Q_1^2, \nu) = \int d^2 Q_2 K_{\overline{MS}}(\mathbf{Q}_1, \mathbf{Q}_2) \left(\frac{Q_2^2}{Q_1^2} \right)^{-\frac{1}{2} + i\nu} \quad \sigma \sim S^{\alpha_{IP} - 1} = S^{\omega^{\max}}$$

$$= N_C \chi_L(\nu) \frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi} \left[1 + r_{\overline{MS}}(\nu) \frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi} \right],$$

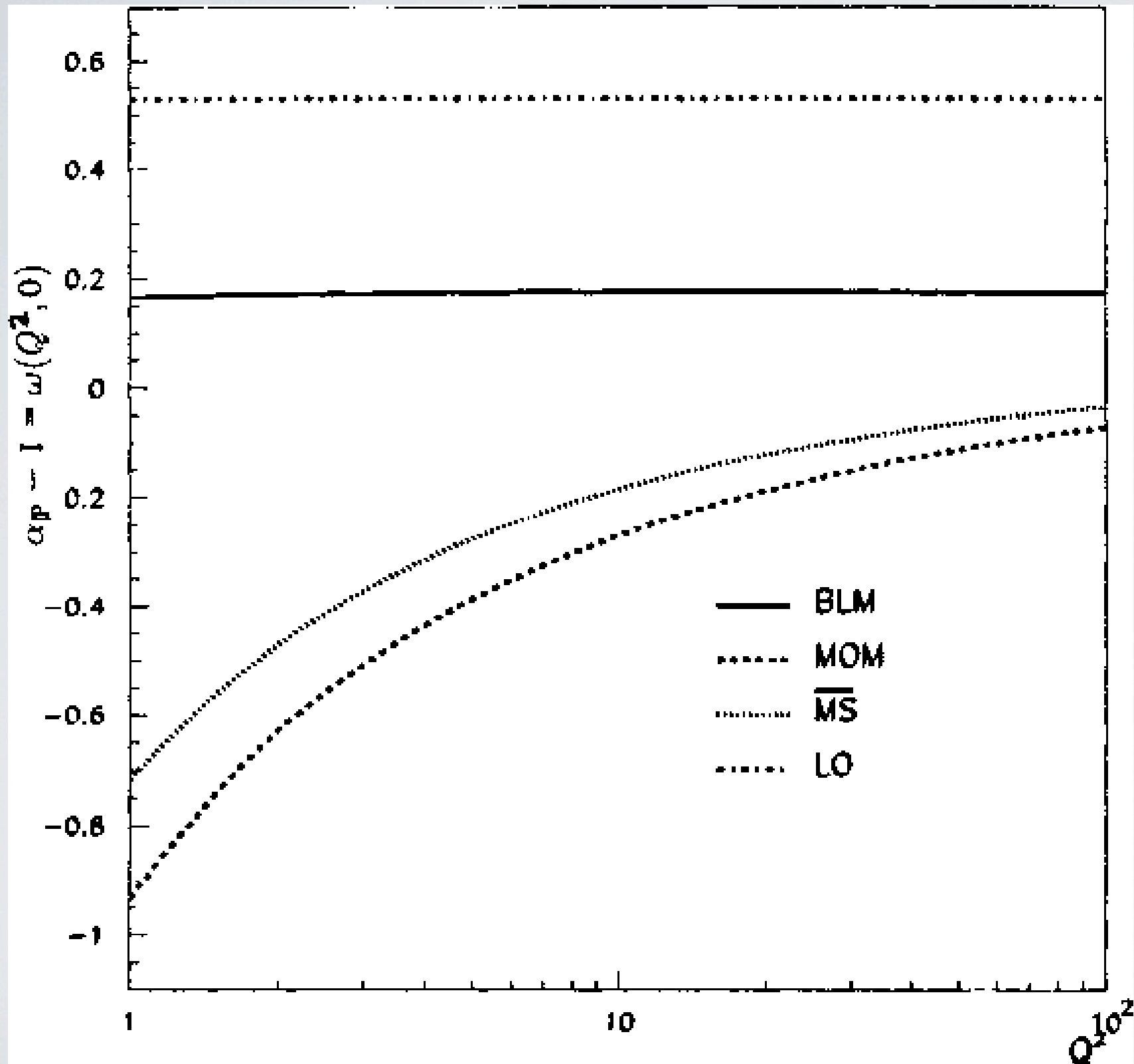
$$\chi_L(\nu) = 2\psi(1) - \psi(1/2 + i\nu) - \psi(1/2 - i\nu)$$

$$r_{\overline{MS}}(\nu) = r_{\overline{MS}}^{\beta}(\nu) + r_{\overline{MS}}^{\text{conf}}(\nu)$$

$$r_{\overline{MS}}^{\beta}(\nu) = -\frac{\beta_0}{4} \left[\frac{1}{2} \chi_L(\nu) - \frac{5}{3} \right]$$

$$r_{\overline{MS}}^{\text{conf}}(\nu) = -\frac{N_C}{4\chi_L(\nu)} \left[\frac{\pi^2 \sinh(\pi\nu)}{2\nu \cosh^2(\pi\nu)} \left(3 + \left(1 + \frac{N_F}{N_C^3} \right) \frac{11 + 12\nu^2}{16(1 + \nu^2)} \right) - \chi_L''(\nu) + \frac{\pi^2 - 4}{3} \chi_L(\nu) - \frac{\pi^3}{\cosh(\pi\nu)} - 6\zeta(3) + 4\varphi(\nu) \right]$$

BFKLP: NLL BFKL within generalized BLM



$$\sigma \sim S^{\alpha_{IP} - 1} = S^{\omega^{\max}}$$

BFKLP: NLL BFKL within generalized BLM

V.S. Fadin & L.N. Lipatov (89-98)

C. Camici & M. Ciafaloni (96-98)

next-to-leading log approximation (NLL) BFKL

MSbar-renormalization scheme: large corrections

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov (98-99) BFKLP

D. Colferai, M. Ciafaloni & G. Salam (99) ...

BFKLP: NLL BFKL + resummation of running coupling α_s

BFKLP: Conformal BFKL kernel in NLL \rightarrow SUSY N=4

Pomeron intercept: $a_P = 1.2 - 1.3$

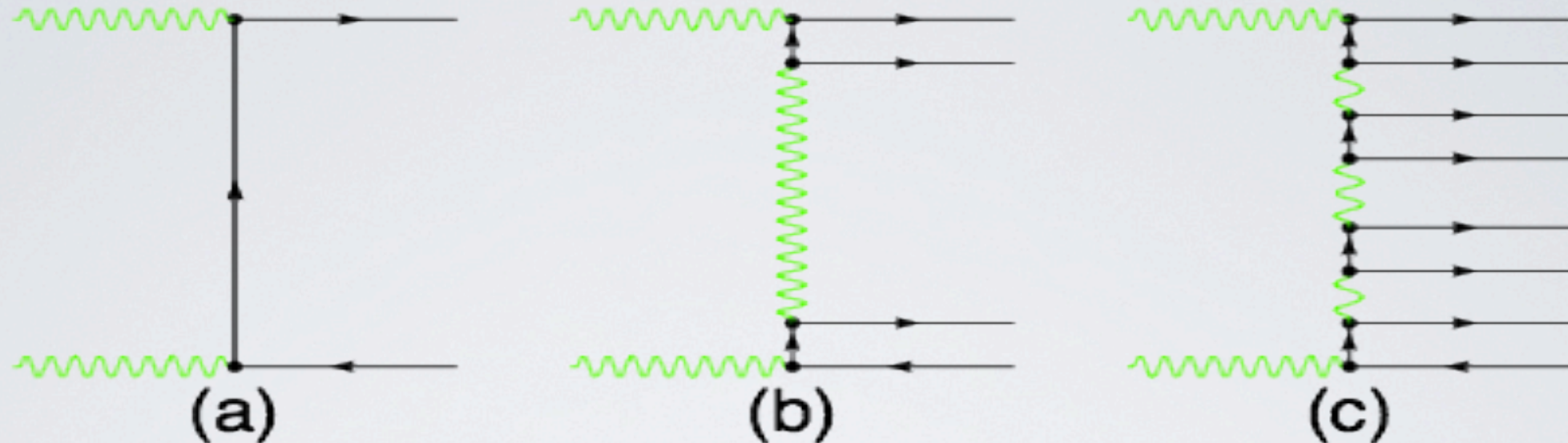
Cross section: $\sigma_0 (S/S_0)^{(a_P-1)}$ $a_P = 1 + C \alpha_s$

L.N. Lipatov, A.V. Kotikov et al. (2000-06)

SUSY N=4 BFKL-Pomeron

Anomalous dimensions: test of AdS/CFT-conjecture

Asymptotics of QED cross sections



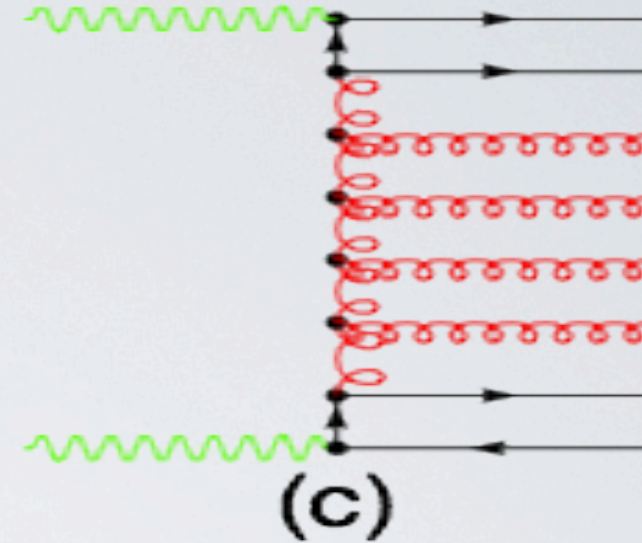
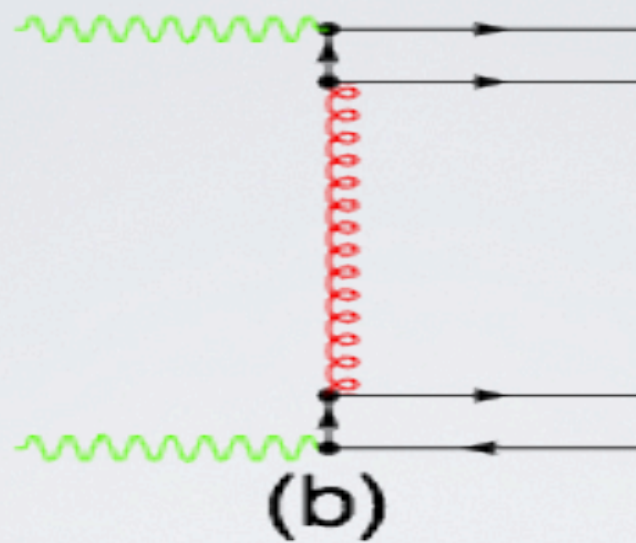
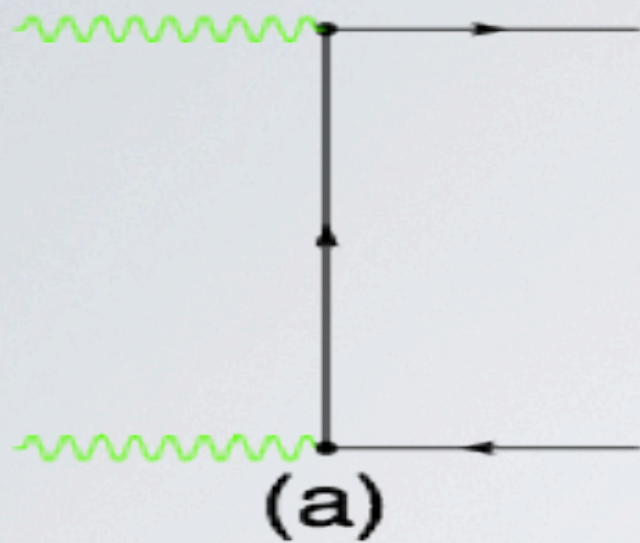
$$\sigma \sim (a_{\text{QED}})^2 \log(s)/s \quad \sigma \sim (a_{\text{QED}})^4 \text{const}(s)$$

All orders: V.N. Gribov, L.N. Lipatov, G.V. Frolov & V.G. Gorshkov (69-71)
H. Cheng & T.T. Wu (69-70)

Cross section at $s \rightarrow \infty$: $\sim (a_{\text{QED}})^4 (S/S_0)^{(a_P-1)}$

$$a_P = 1 + C (a_{\text{QED}})^2 \approx 1.002$$

Asymptotics of QCD cross sections: $\gamma\gamma$



$$\sigma \sim (a_{QED})^2 \log(s)/s$$

$$\sigma \sim (a_{QED})^2 (a_s)^2 \text{const}(s)$$

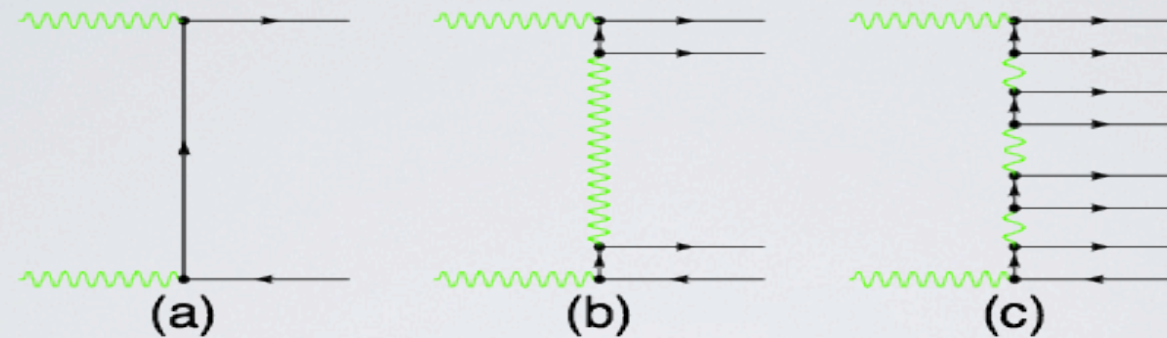
All orders: LL BFKL

Cross section at $s \rightarrow \infty$: $\sim (a_{QED})^2 (a_s)^2 (S/S_0)^{a_P-1}$

$a_P = 1 + C(a_s) \approx 1.5$ LL BFKL S. Brodsky & F. Hautmann (96)

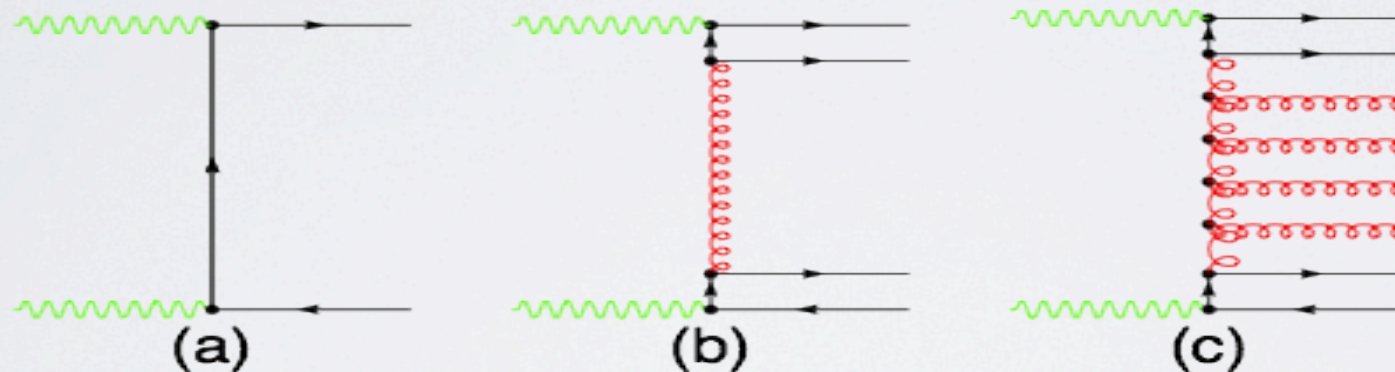
**$a_P = 1 + C(a_s) \approx 1.2$ NLL BFKL S. Brodsky, V Fadin, VK,
L. Lipatov, G. Pivovarov (2001-02)**

Asymptotics of QED cross sections



V.N. Gribov, L.N. Lipatov, G.V. Frolov & V.G. Gorshkov (69-71)
Cheng & T.T. Wu (69-71)

Asymptotics of QCD cross sections



LL BFKL

J. Bartels et al (96), S.J. Brodsky & Hautmann (97)

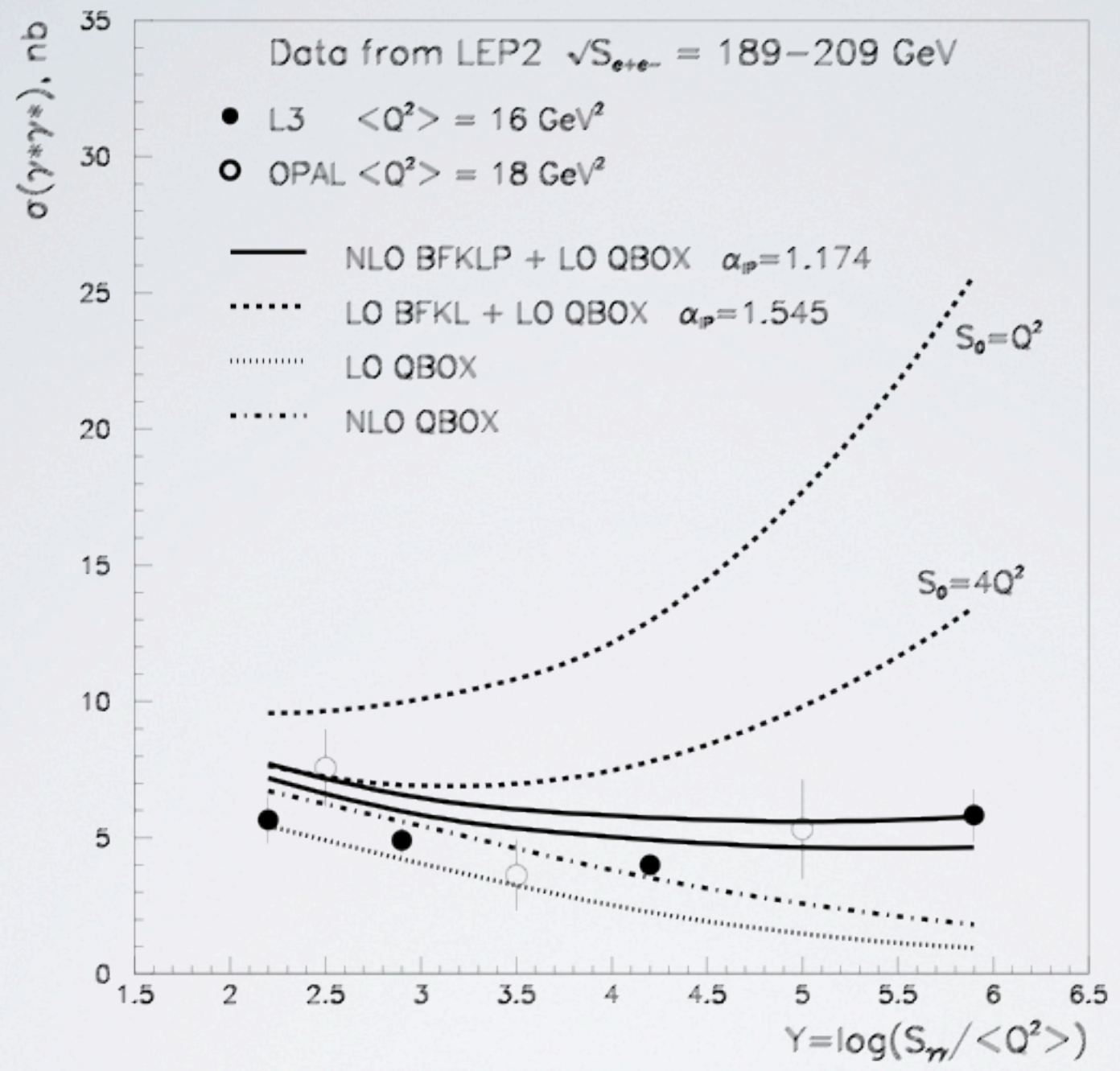
NLL BFKL (with LO impact factors)

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov & G.B. Pivovarov (2001-02)

NLO impact factors and full NLL BFKL:

I. Balitsky, J.Chirilli, J. Bartels et al.

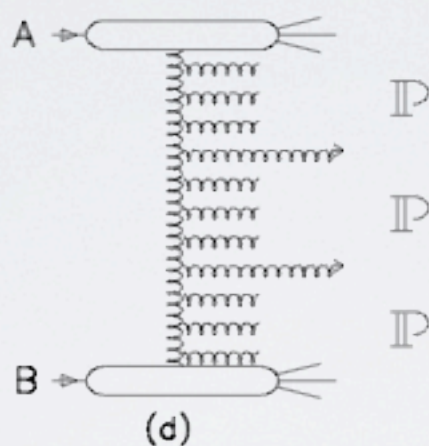
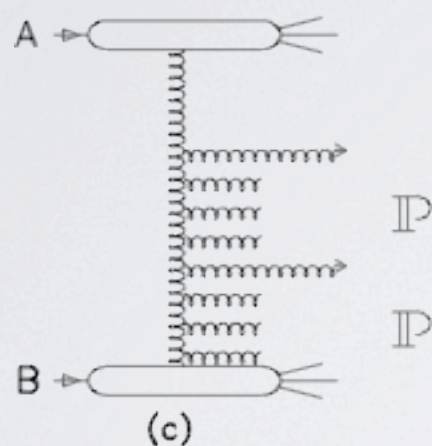
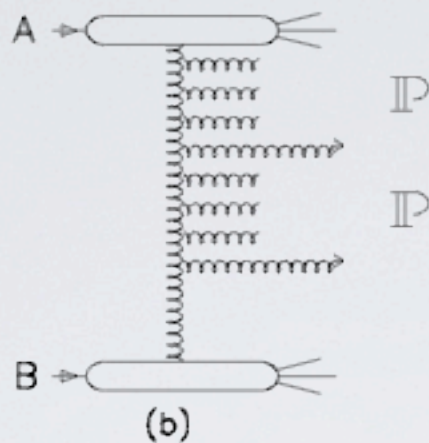
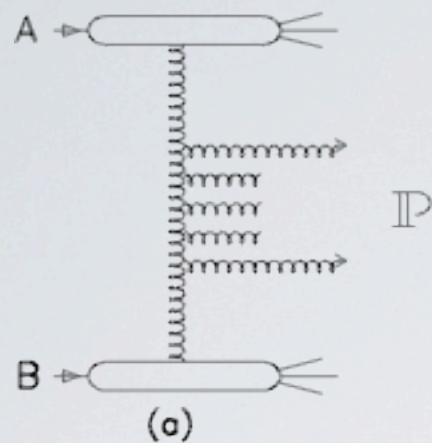
Highly virtual photon scattering at LEP-2



S.J Brodsky, VK, L.N. Lipatov, V.S. Fadin & G.B. Pivovarov (2002)
BFKLP: NLL BFKL + generalized BLM

LL BFKL: ruled out

BFKL: dijet processes



Jet production

**GLAPD: ordering on κT
y – no ordering**

**BFKL: ordering on y
 κT – no ordering**

A. Mueller & H. Navelet, Nucl. Phys. (87)

Most forward/backward (Mueller-Navelet) dijets: x-section $\sim \exp(|\Delta|y)$

V.T. Kim & G.B. Pivovarov, Phys. Rev. (96)

Inclusive dijets

J.C. Collins, R.K. Ellis (91), S. Catani et al (91)

E.M.Levin, M.G.Ryskin, Yu.M.Shabelsky, A.G.Shuvaev (91)

kT-factorization

Dijet K-factor

K-factor = x-section / Born x-section

GLAPD: x-section $\rightarrow C_1 \alpha_s^2 + C_2 \alpha_s^3 + \dots$
Born x-section $\rightarrow C_1 \alpha_s^2$

K-factor = $(1 + C_2 / C_1 \alpha_s + C_3 / C_1 \alpha_s^2 + \dots)$

Mueller-Navelet (87):

BFKL \rightarrow enhanced $(\alpha_s \Delta y)$ -terms
x-section $\rightarrow B_1 \alpha_s^2 \Delta y + B_2 \alpha_s^3 \Delta y^2 + \dots$
Born x-section $\rightarrow B_1 \alpha_s^2 \Delta y$

K-factor_MN $\rightarrow \exp(\alpha_s \Delta y)$

$\Delta y = |y_1 - y_2|$

Dijet K-factor: not measurable

K-factor = x-section / Born x-section

Born x-section: no real and no virtual corrections

only a theoretical quantity - > not measurable (!)

**Experiment: one cannot forbid virtual corrections
by kinematical conditions**

**Exclusive dijet x-section: always contains virtual
corrections**

VK & G. Pivovarov:

**Using dijets with extra jet veto
instead of Born dijets**

Dijet observables:

“K-factor” = inclusive dijet / “exclusive” dijet

“K-factor” = MN dijet / “exclusive” dijet

as a function of rapidity separation between jets

**Inclusive dijet: $N_{\text{jets}} \geq 2$ $p_T \geq p_{T\text{min}}$
all jet pairs**

**Mueller-Navelet dijet: $N_{\text{jets}} \geq 2$ $p_T \geq p_{T\text{min}}$
most forward & most backward jets**

“exclusive” dijet (2-jet events) with extra jet veto:

**$N_{\text{jets}} = 2, p_T \geq p_{T\text{min}}$
veto for extra jets $p_T \geq p_{T\text{veto}}$
 $p_{T\text{veto}} \leq p_{T\text{min}}$**

Forward dijets at Tevatron and LHC

Tevatron : D0 -> $|\Delta y| < 6$ $p_{Tmin} = 20$ GeV
- azimuthal decorr. (1997)
- 1800/630 GeV x-section ratio (2001)

LHC: ATLAS -> $|\Delta y| < 6$ 70 GeV $< p_T < 90$ GeV
- (inverse) “K-factor” (2011)

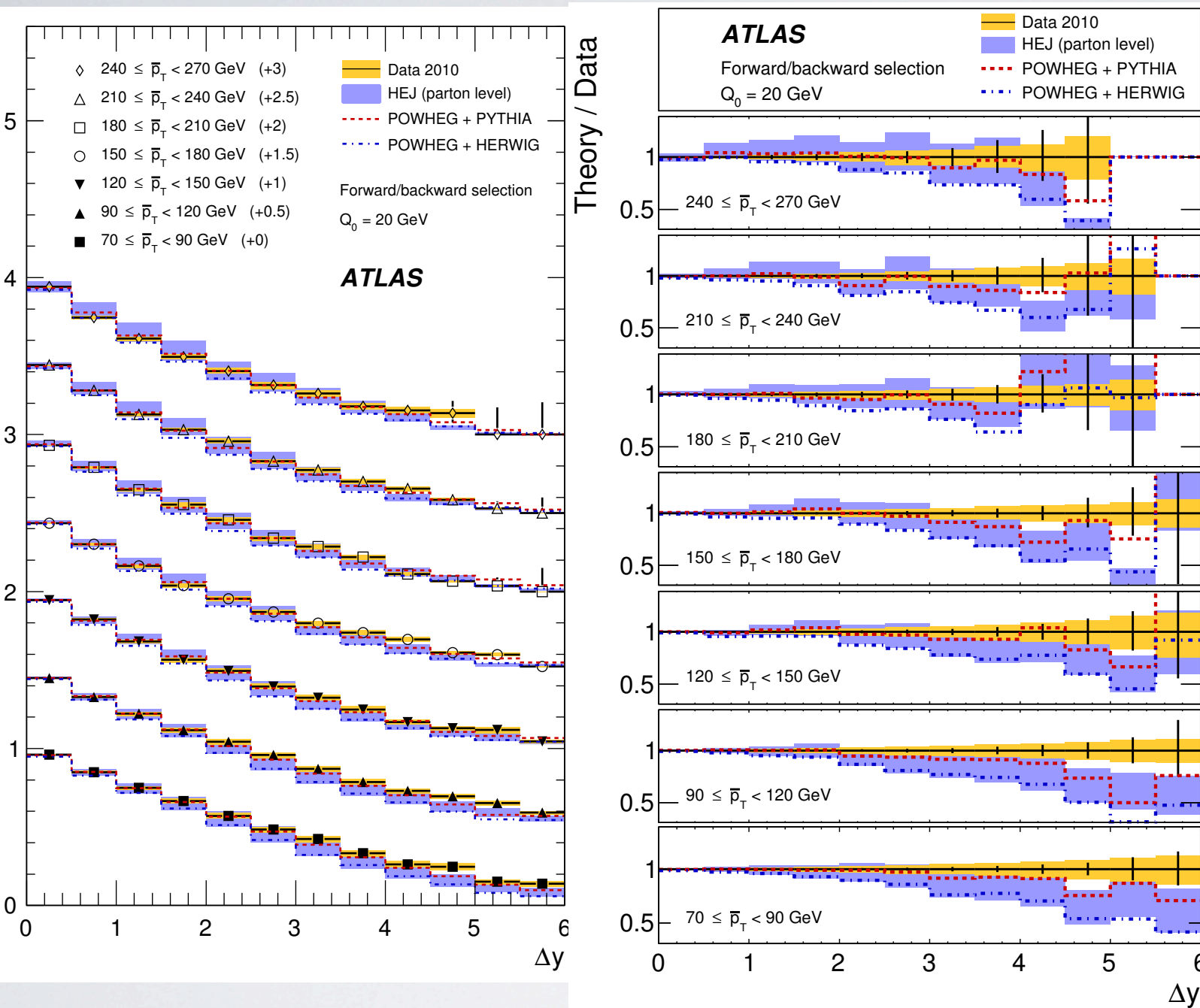
LHC: CMS -> $|\Delta y| < 9.4$ $p_{Tmin} = 35$ GeV
- “K-factor” (2012)
- azimuthal angle decorr. (prel. 2013)

Dijet “K-factor” at 7 TeV

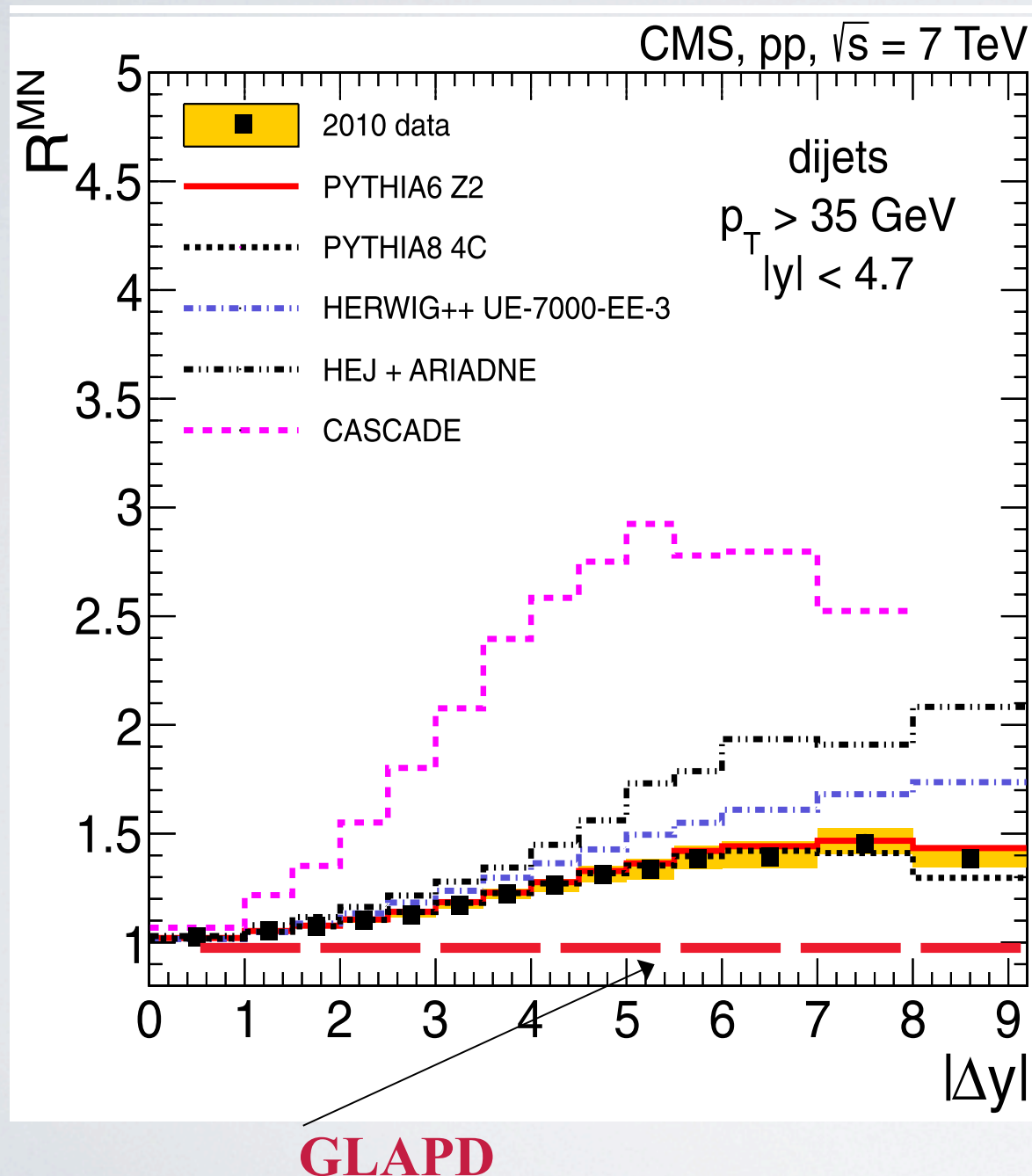
**1/ (MN dijet K-factor) =
“exclusive” dijet/ MN dijet**

**ATLAS, JHEP (2011)
arXiv: 1107.1641
7 TeV**

**70 < p_T < 90 GeV
|Δy| < 6**



CMS: dijet “K-factor”



Eur. Phys. J. C (2012) 72:2216
DOI 10.1140/epjc/s10052-012-2216-6

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article - Experimental Physics

Ratios of dijet production cross sections as a function of the absolute difference in rapidity between jets in proton–proton collisions at $\sqrt{s} = 7$ TeV

The CMS Collaboration*
CERN, Geneva, Switzerland

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Abstract A study of dijet production in proton–proton collisions was performed at $\sqrt{s} = 7$ TeV for jets with $p_T > 35$ GeV and $|y| < 4.7$ using data collected with the CMS detector at the LHC in 2010. Events with at least one pair of jets are denoted as “inclusive”. Events with exactly one pair of jets are called “exclusive”. The ratio of the cross sec-

jets are well separated in rapidity, the description of the data becomes worse [2].

When the collision energy \sqrt{s} is considerably larger than the hard scattering scale given by the jet transverse momentum, p_T , the average number of produced jets grows rapidly, along with the phase space available in rapidity.

Forward dijets at LHC:

Color coherence and AO effects

GLAPD: strong kT-ordering & no rapidity ordering

BFKL: strong rapidity ordering & no kT-ordering

Color coherence effects => rapidity ordering

Polar angle ordering (AO):

jet cone veto for larger cone angles => rapidity ordering

Pythia 6 and 8: GLAPD + AO (AO cannot be fully switched off!)

Herwig++: GLAPD + color coherence (CC cannot be switched off)

**No pure GLAPD MC generators (!) available
at present: Pythia and Herwig generators contain $|\Delta y|$ -effects**

**small CC and AO $|\Delta y|$ -effects in GLAPD-regime
can be large in BFKL-regime at large $|\Delta y|$**

Forward dijets at LHC

GLAPD generators Pythia 6 and 8 (with AO) are consistent with CMS dijet “K-factor” data rather well:

- 1) no sizeable BFKL effects?**
- 2) or BFKL effects cancels out in dijet ratio**

**in the latter case the “K-factor” with extra jet veto
can be more sensitive BFKL effects**

2-jet “exclusive” events: impose an extra jet veto $p_{T\text{veto}} < p_{T\text{min}}$

Forward dijets: azimuthal angle decorrelations

Cosines

V. Del Duca & C. Schmidt (94)

J. Stirling (94)

Cosine ratios → more sensitive to BFKL (!)

A. Sabio Vera et al (2011)

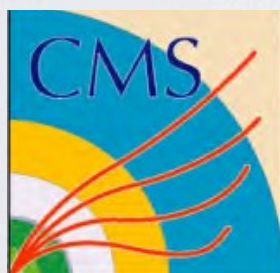
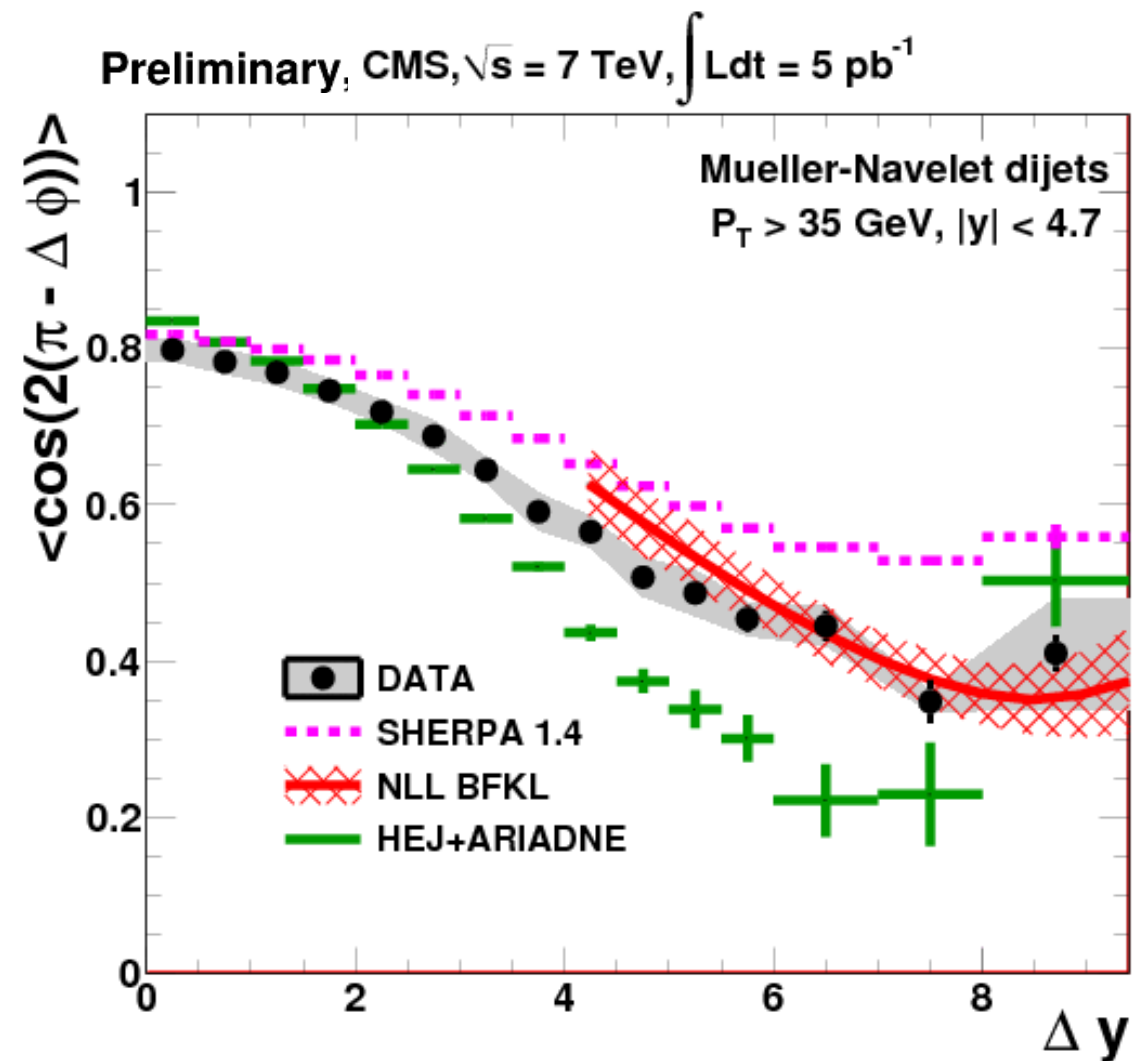
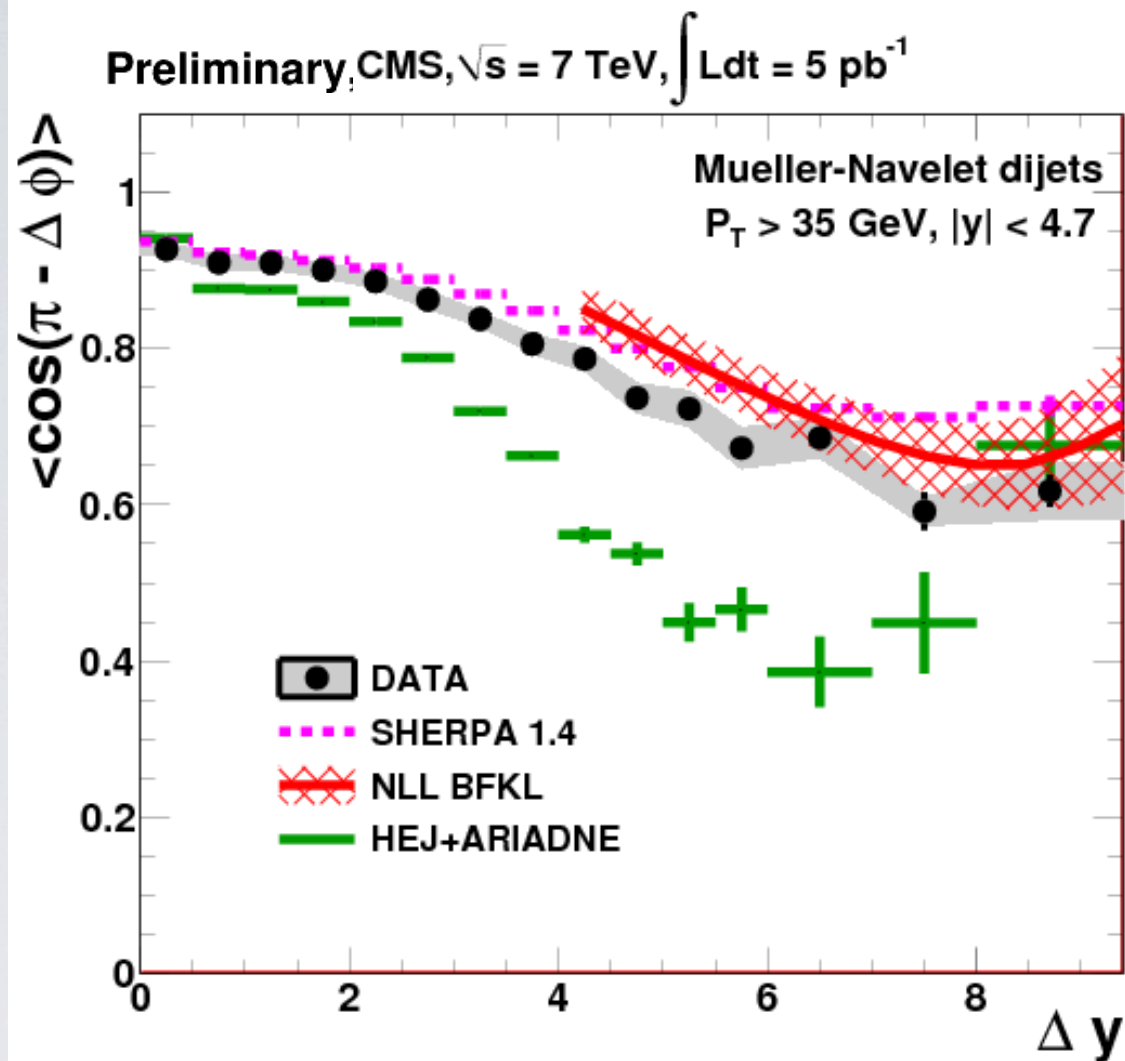
Forward dijets: azimuthal decorrelations

$$\frac{1}{\sigma} \frac{d\sigma}{d(\Delta\phi)}(\Delta y, p_{T\min}) = \frac{1}{2\pi} \left[1 + 2 \sum_{n=1}^{\infty} C_n(\Delta y, p_{T\min}) \cdot \cos(n(\pi - \Delta\phi)) \right]$$

$$C_n(\Delta y, p_{T\min}) = \langle \cos(n(\pi - \Delta\phi)) \rangle, \text{ where } \Delta\phi = \phi_1 - \phi_2$$

V. del Duca & C. Schmidt (94-95) Strling (94)
V. Kim & G. Pivovarov (96-98)
A. Sabio Vera et al (2007-11)

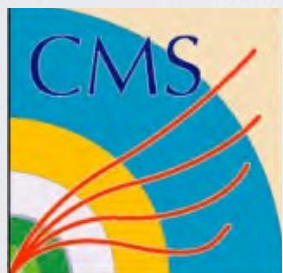
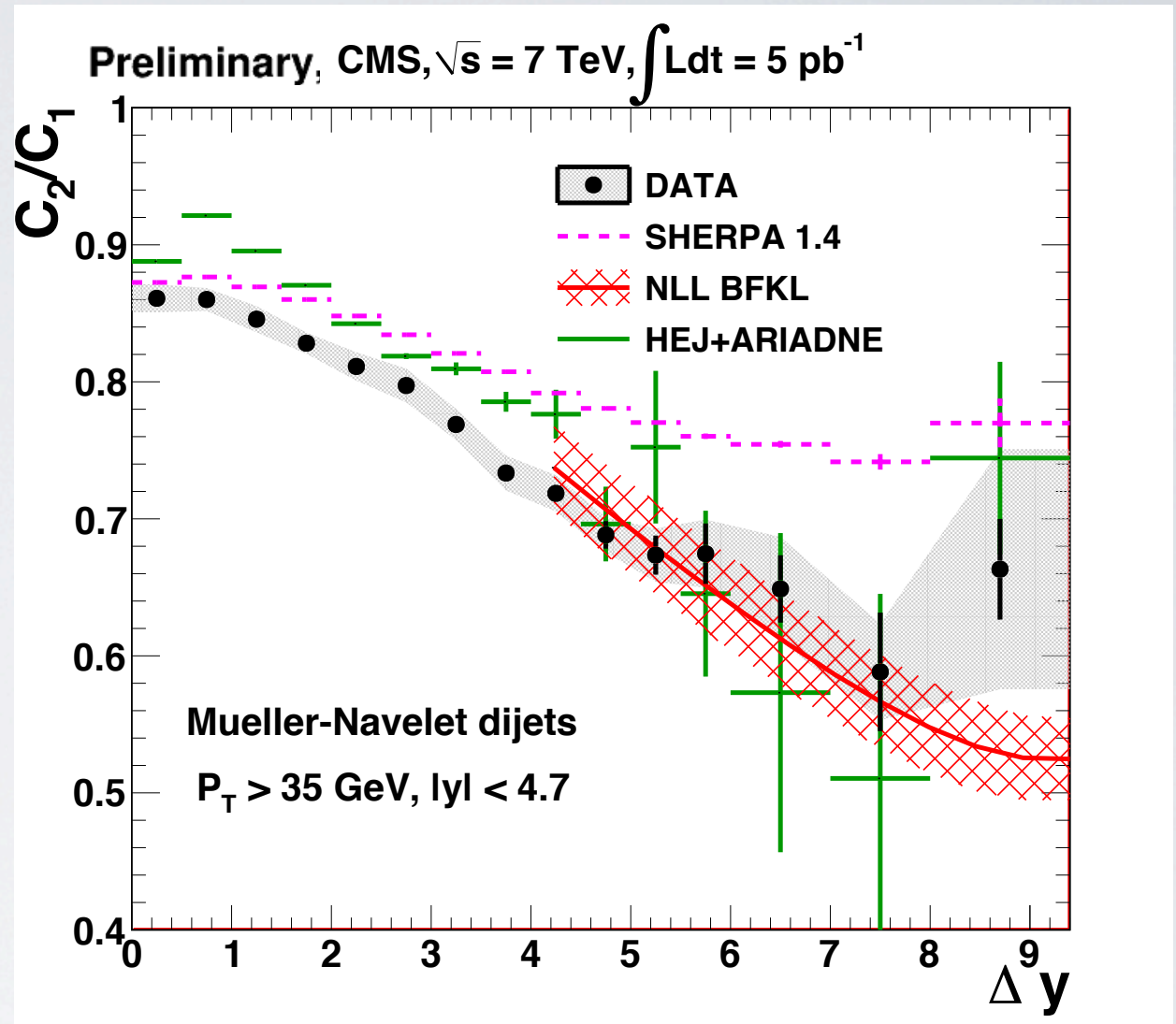
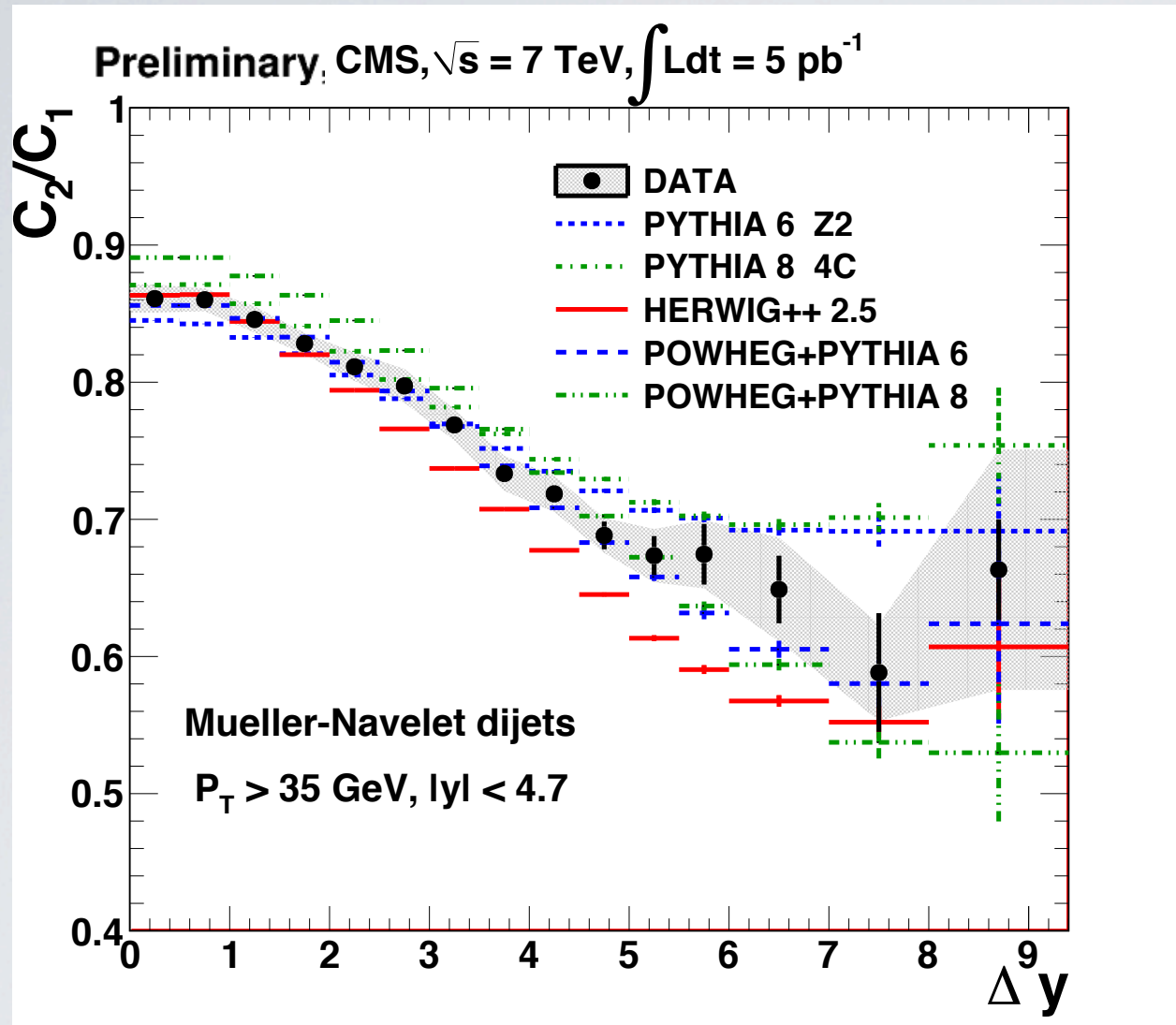
Dijets: $\langle \cos \rangle$ vs NLL BFKL+BFKLP



CMS PAS-FSQ-12-002
7 TeV, $p_{T_min} = 35 \text{ GeV}$
 $\Delta y = | | < 9.4$

NLL BFKL + BFKLP (2014)
B. Ducloue, L. Szymanowski & S. Wallon

Dijets: $\langle \cos^2 \rangle / \langle \cos \rangle$ vs NLL BFKL + BFKLP



CMS PAS-FSQ-12-002
7 TeV, $p_{T_min} = 35 \text{ GeV}$
 $\Delta y < 9.4$

NLL BFKL + BFKLP (2014)
B. Ducloue, L. Szymanowski & S. Wallon

Summary – 1

$\gamma^*\gamma^*$ - collisions at LEP2

NLL BFKL improved by BFKLP (generalized BLM) (2001-02):

Indication on BFKL evolution

Outlooks: Future linear colliders

Summary - 2:

- **Forward dijet “K-factor” by CMS at 7 TeV :**
moderate rise with increasing $|\Delta y|$
 - **Pythia describes the rise, Herwig overshoots the rise**
 - **however: pure GLAPD \rightarrow const ?**
 - **Azimuthal angle decorrelations (AAD) of CMS dijets:**
 - **agreement with NLL BFKL improved by BFKLP (generalized BLM)**
 - **GLAPD generators (Pythia, Herwig) describes AAD differently because different color coherence (CC) implementations**
- \rightarrow the first indication on BFKL at LHC ?**
No pure LL GLAPD predictions
(now only LL GLAPD with color coherence, angle ordering, ...)

Other observables:

- **K-factor with extra jet veto, number of extra jets, ... ?**
LHC Run 2 at 13 TeV ?!