S-matrix approach to the Z line shape

or: The Z resonance without weak loops

or: The SMATASY/ZFITTER approach to the Z resonance

Tord Riemann, DESY **Thanks to: M. Grünewald + S. Riemann**

talk held at workshop CALC - "Calculations for Modern and Future Colliders"

July 23-30 2015, JINR, Dubna, Russia

https://indico.cern.ch/event/368497/overview

Why ?

Present interest in precision approaches to the Z boson

- $\sqrt{s} \sim 10$ GeV $\rightarrow$ **Belle-II** will measure 10^9 $\mu^+\mu^-$ events,
  $\rightarrow$ T. Ferber, Belle-II, DESY, talk at DPG meeting 2015.
- $\sqrt{s} \sim M_Z$ $\rightarrow$ **Fcc-ee** expects 10^{13} events at the Z resonance,
  $\rightarrow$ Need **complete electroweak 2-loop calculation**; see e.g. A. Freitas, talk at Pisa meeting 2015.
  Model-independent alternative: How to do?
  $\rightarrow$ Request by the Fcc-ee physics study group
  $\rightarrow$ **S-matrix approach** a la SMATASY/ZFITTER; see e.g. T. Riemann, talk at Pisa meeting 2015.

2-loop weak corrections to the Z resonance

At the Z peak, one needs essentially vertex corrections.
Much has been done by Hollik et al., Czakon et al., Freitas et al., see: [1] and many refs. therein.

AMBRE/MB/MBnumerics: 2-loop weak vertex corrections

We (J. Gluza, I. Dubovyk, J. Usovitsch, T.R.) are developing a tool for the (semi-)automatized calculation of 2-loop massive vertex (and other) integrals.
Under development.

Outline

S-matrix approach to the Z line shape

- Developed as a model-independent analysis tool of $e^+e^- \to (\gamma, Z) \to f^+f^-$ **around the Z boson resonance**
- Aim: determinations of M_Z and Γ_Z
- $\to \sigma_T$: Leike/TR/Rose 1991 [2]
  $\to A_{FB,LR,pol}$: TR 1992 [3],
  $\to$ `SMATASy` code: Kirsch/TR 1994 [4]
- First application: LEP/L3 1993 [5], also: Tristan/TOPAZ,VENUS, LEP/OPAL, ...

Introduction

The reaction

$$e^+e^- \to (\gamma, Z) \to f^+f^- \; + \; (n\,\gamma) \qquad (1)$$

allows to study the Z boson, its mass M_Z, its width Γ_Z, its couplings, and potentially deviations from the Standard Model.

Need correct “model”

See experiences with *constant* and *s-dependent* Z width:

$$\frac{1}{[s - M_Z^2 + iM_Z\,\Gamma_Z(s)]} \quad \textbf{versus} \quad \frac{1}{[s - M_Z^2 + iM_Z\,\Gamma_Z]} \qquad (2)$$

To a very good accuracy, it holds: $\Gamma_Z(s) \approx s/M_Z^2 \times \Gamma_Z$

see next slide, $\to$ Bardin/Leike/Riemann/Sachwitz 1988 [6]; also: Berends/Burgers/Hollik/v.Neerven 1988 [7]

Need correct unfolding ..

.. of *Realistic Observables* in order to get *Pseudo Observables*. $\to$ e.g.: Borrelli/Consoli/Maiani/Sisto 1990 [8], Later: Bardin/Passarino 1999 [9], Bardin/Grünewald/Passarino 1999 [10], Passarino 2003 [11], Passarino 2013 [12] and refs. therein.

Lesson: The model influences numerical results

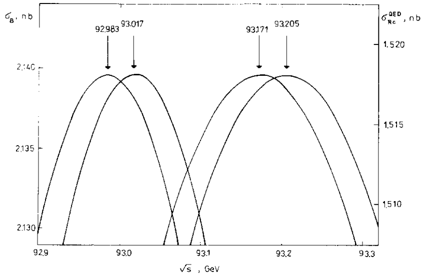

Fig. 1. Total cross sections $\sigma_{\rm B}$, $\sigma_{\rm RC}^{\rm QED}$ for the reactions $e^+e^- \to \mu^+\mu^-(\gamma)$ in Born approximation (left scale) and including $O(\alpha)$ QED corrections right scale). Peaks of σ with energy-dependent width $\Gamma_Z(s)$ are shifted by 34 MeV to the left.

Total cross section for $e^+e^- \to \mu^+\mu^-$ production at LEP …

…without (left) and with (right) QED corrections. Both sample data produced with an energy-dependent Z width.
The assumptions on the Z-propagator in the fit formulas influence the location of the peak, but not the "experimental errors".
Fig.: from [6], license Number: 3557090997554.

	Born		Born + QED	
from fit: $\to$	M_Z	Γ_Z	M_Z	Γ_Z
$\Gamma_Z(s)$	93.000 $\pm$ 0.013	2.498 $\pm$ 0.009	93.000 $\pm$ 0.016	2.498 $\pm$ 0.011
Γ_Z	92.966 $\pm$ 0.013	2.498 $\pm$ 0.009	92.966 $\pm$ 0.016	2.498 $\pm$ 0.011

Introduction

The Standard Model analysis tool for the Z resonance: ZFITTER (D. Bardin et al.)

- Complete electroweak radiative corrections;
- QED corrections by convolution:
  with some $\sigma_0(s')$
  beware: for initial-final state interferences with some $\sigma_0(s,s')$;
- **semi-analytical QED** integrations;
- free choice of $\sigma_0(s')$ by **user interfaces**;
- Standard Model interfaces: four weak form factors $\rho, \kappa_e, \kappa_f, \kappa_{ef}$.

ZFITTER is **well-tested**, **flexible**, **accurate** and **fast**.

References

- ZFITTER has been published in CPC in 1990 [13], 2001 [14], 2006 [15].
- The actual Fortran package is v.6.44; version 6.42 is public in CPC program library, with CPC-licence.
- Beware: **Gfitter/GSM (2007-2011) is an illegal clone of ZFITTER**, available at
  http://zfitter-gfitter.desy.de/ and
  http://fh.desy.de/projekte/gfitter01/Gfitter01.htm.
  See also: http://zfitter.education, http://zfitter.com.

Introduction

Stuart 1991 [16], S-Matrix ansatz for $e^+e^- \longrightarrow Z \longrightarrow f^+f^-$

$$M = \frac{R}{s - s_0} + F(s), \qquad s_0 = M_Z^2 - iM_Z\Gamma_Z \tag{3}$$

Allows to study:

- Mass M_Z and width Γ_Z $\rightarrow$ Leike/Riemann/Rose 1991 [2]
- How many independent degrees of freedom? $\rightarrow$ Leike/Riemann/Rose 1991 [2], Kirsch/S.Riemann, L3 [17, 5]
- But also: How to define mass and width of the Z boson at higher orders of perturbation theory? $\rightarrow$ Denner 2014 [18], Freitas 2014 [1] and Fcc-ee [19], Degrassi FCC-ee [20] and refs. therein

Total cross sections

There are immediate questions, from an experimental point:

- What about the photon exchange?
- What about QED corrections, e.g. the $2 \to 3$ part of the cross sections?
- What about asymmetries, besides σ_{tot}?

We have to describe

$$e^+e^- \longrightarrow (\gamma, Z) \longrightarrow f^+f^-(\gamma) \tag{4}$$

Ansatz in the complex energy plane, for **four helicity matrix elements**:

$$\mathcal{M}^i(s) = \frac{R^i_\gamma}{s} + \frac{R^i_Z}{s - s_Z} + F^i(s), \quad i = 1, \ldots 4. \tag{5}$$

Beware: Eqn. (5) is mathematically not consistent → Böhm/Sato 2004 [21]

The poles of $\mathcal{M}$ have complex residua R_Z and R_γ, the latter corresponding to the photon, and the background $F(s)$ is an analytic function without poles:

$$F^i(s) = \sum_{n=0}^{\infty} F^i_n (s - s_0)^n \tag{6}$$

Comment on the photon term (3 Feb 2015)

$$
\begin{aligned}
\frac{R^i_\gamma(s)}{s} &= \frac{\sum_{n=0}^\infty R^i_n (s-s_0)^n}{s} \\
&= \frac{\sum_{n=0}^\infty R^i_n (s-s_0)^n}{s_0-(s_0-s)} \\
&= \sum_{n=0}^\infty R^i_n (s-s_0)^n \, \frac{1}{s_0} \, \frac{1}{1-\frac{s_0-s}{s_0}} \\
&= \sum_{n=0}^\infty R^i_n (s-s_0)^n \, \frac{1}{s_0} \left[1 + \frac{s_0-s}{s_0} + \left(\frac{s_0-s}{s_0}\right)^2 \cdots \right]
\end{aligned}
\tag{7}
$$

The term $R^i_\gamma(s)/s$ is part of the the background term $F(s)$.

- It is useful to sum up a selected part of self-energy insertions in the propagators in order to derive the Breit-Wigner resonance form,
- It is useful to sum up a selected part of the photonic background of the Z resonance in order to take explicit notice of physically known pieces of the input expressions.

Ansatz for realistic applications

The analysis of the Z line shape will be based here on the cross section

$$\sigma(s) = \sum_{i=1}^{4} \sigma^i(s) = \frac{1}{4}\sum_{i=1}^{4} s|\mathcal{M}^i(s)|^2, \quad (8)$$

where the sum must be performed over four helicity amplitudes with different residua R_Z^i and functions $F^i(s)$. The result is, with QED corrections folded in:

$$\sigma_T(s) = \frac{4}{3}\pi\alpha^2 \int \frac{ds'}{s}\left[\frac{r^\gamma}{s} + \frac{sR + (s-M_Z^2)J}{(s-M_Z{}^2)^2 + M_Z^2\Gamma_Z{}^2} + \cdots\right]\rho_{ini}\left(\frac{s'}{s}\right). \quad (9)$$

The radiation connected with initial-final state interferences can be taken into account by an analogue formula to (9) with a slightly more complicated structure [22, 23]:

$$\sigma_{\rm int}(s) = \int ds' \sigma(s,s')\rho_{\rm int}(s'/s). \quad (10)$$

Some details

The correct ansatz for the S-matrix based cross section is:

$$\sigma(s,s') = \frac{1}{8} s' \sum_i \left[\mathcal{M}_i(s)\mathcal{M}_i^*(s') + \mathcal{M}_i^*(s)\mathcal{M}_i(s') \right] . \quad (11)$$

From Leike/Riemann/Rose:

$$\sigma(s) = \sum_A \sigma_A(s), \qquad A = Z, \gamma, F, \gamma Z, ZF, F\gamma, \quad (12)$$

with the contributions:

$$\begin{aligned}
\sigma_Z(s) &= \frac{s r_Z}{|s-s_Z|^2}, & r_Z &= \tfrac{1}{4}\sum |R_Z^i|^2, \\
\sigma_\gamma(s) &= \frac{r_\gamma}{s}, & r_\gamma &= |R_\gamma|^2, \\
\sigma_F(s) &= s r_F(s), & r_F(s) &= \tfrac{1}{4}\sum |F^i(s)|^2, \\
\sigma_{\gamma Z}(s) &= 2\mathrm{Re}\frac{C_\gamma^* C_Z}{s-s_Z}, & C_\gamma &= R_\gamma, \qquad C_Z = \tfrac{1}{4}\sum R_Z^i, \\
\sigma_{ZF}(s) &= 2\mathrm{Re}\frac{s C_{ZF}(s)}{s-s_Z}, & C_{ZF}(s) &= \tfrac{1}{4}\sum R_Z^i F^{i*}(s), \\
\sigma_{F\gamma}(s) &= 2\mathrm{Re}\left[C_\gamma^* C_F(s)\right], & C_F(s) &= \tfrac{1}{4}\sum F^i(s).
\end{aligned}$$

Some details

After making denominators real one remains with the following formula for the effective Born formula for the Z line shape:

$$\sigma(s) = \frac{R + (s - M_Z^2)I}{|s - s_Z|^2} + \frac{r_\gamma}{s} + r_0 + (s - M_Z^2) r_1 + \ldots \quad (13)$$

Besides M_Z, Γ_Z, the real constants R, I, r_0 and $r_1 \cdots$ are introduced:

$$\begin{aligned} R &= M_Z^2 \left[r_Z + 2(\Gamma_Z/M_Z) \left(\Im m C_R + M_Z \Gamma_Z \Re e(C_R') \right) \right], \\ I &= r_Z + 2 \Re e C_R, \\ C_R(s) &= C_\gamma^* C_Z + s_Z C_{ZF}(s), \\ r_0 &= M_Z^2 \left[r_F - M_Z \Gamma_Z \Im m(r_F') \right] + \Re e C_r - M_Z \Gamma_Z \Im m C_r', \\ r_1 &= r_F + M_Z^2 \left[\Re e(r_F') - (\Gamma_Z/M_Z) \Im m(r_F') \right] + \Re e C_r', \\ C_r(s) &= C_\gamma^* C_F(s) + C_{ZF}(s). \end{aligned} \quad (14)$$

The energy-dependent functions C_{ZF}, C_F, r_F, and their (primed) derivatives with respect to s have to be taken at $s = s_Z$. As may be seen, the cross section may be described by only six real parameters as long as one takes into account only the first two terms in the expansion of the functions $F^i(s)$ around $s = s_Z$ and at most terms of the order $(s - M_Z^2)^n, n = 0, 1$ in the cross section parametrization.

Born Asymmetries

On a Sunday in Summer 1992, I had a discussion with Luciano Maiani in the CERN library.
He had doubt that an analogue to the model-independent ansatz for σ_{tot} might be usefully formulated, especially in view of the QED corrections.

I believed one can do that, and I followed the rule “The proof of the pudding is in the eating” [3]. The result:

$$A^{Born}(s) = A_0 + A_1\left(\frac{s}{M_Z^2}-1\right) + A_2\left(\frac{s}{M_Z^2}-1\right)^2 + \ldots \tag{15}$$

$$A_{FB} = \frac{\sigma_{FB}}{\sigma_T}, \qquad A_{pol} = \frac{\sigma_{pol}}{\sigma_T}. \tag{16}$$

The A_{FB} and A_{pol} are helicity combinations as also σ_T is, i.e. have also S-matrix ansatzes.

The parameters in $A^{Born}(s)$ are in QED-Born approximation:

$$A_0 = \frac{R_A}{R_T}, \tag{17}$$

and

$$A_1 = \left[\frac{J_A}{R_A} - \frac{J_T}{R_T}\right] A_0. \tag{18}$$

Some details

$$
\begin{aligned}
R_T &= \kappa^2(a_e^2+v_e^2)(a_f^2+v_f^2) + 2\kappa|Q_eQ_f|v_ev_f\frac{\Gamma_Z}{M_Z}\Im m\frac{\alpha(s)}{\alpha}, \\
R_{FB} &= 3\kappa^2 a_ev_ea_fv_f + \frac{3}{2}\kappa|Q_eQ_f|a_ea_f\frac{\Gamma_Z}{M_Z}\Im m\frac{\alpha(s)}{\alpha}, \\
R_{pol} &= -2\kappa^2(a_e^2+v_e^2)a_\tau v_\tau - 2\kappa|Q_eQ_f|v_ea_\tau\frac{\Gamma_Z}{M_Z}\Im m\frac{\alpha(s)}{\alpha}, \qquad (19)\\
J_A &= 2|Q_eQ_f|\Re e\frac{\alpha(s)}{\alpha}\kappa\kappa_A, \\
\kappa_T &= v_ev_f, \qquad (20)\\
\kappa_{FB} &= \frac{3}{4}a_ea_f, \qquad (21)\\
\kappa_{pol} &= -v_ea_\tau, \qquad (22)
\end{aligned}
$$

$$
\kappa = \frac{G_\mu}{\sqrt{2}}\frac{M_Z^2}{8\pi\alpha} = 0.3724\left(\frac{M_Z}{91}\right)^2. \qquad (23)
$$

Scetch of derivation of the expression for $A_{FB}(s)$

$$
\begin{aligned}
A_{FB}(s) &= \frac{\sigma_{FB}(s)}{\sigma_T(s)} \\
&= \frac{\frac{sR_{FB}}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} + \frac{(s-M_Z^2)J_{FB}}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} + \cdots}{\frac{sR_T}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} + \frac{(s-M_Z^2)J_T}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} + \cdots} \\
&= \frac{R_{FB} + \frac{s-M_Z^2}{s} J_{FB} + \cdots}{R_T + \frac{s-M_Z^2}{s} J_T + \cdots} \\
&= \frac{R_{FB}}{R_T} \times \frac{1 + \frac{s-M_Z^2}{s}\frac{J_{FB}}{R_{FB}} + \cdots}{1 + \frac{s-M_Z^2}{s}\frac{J_T}{R_T} + \cdots} \quad = \frac{R_{FB}}{R_T}\left(1 + \frac{s-M_Z^2}{s}\frac{J_{FB}}{R_{FB}} + \cdots\right)\left(1 - \frac{s-M_Z^2}{s}\frac{J_T}{R_T} + \cdots\right) \\
&= \frac{R_{FB}}{R_T} + \left(1 - \frac{M_Z^2}{s}\right)\left(\frac{J_{FB}}{R_{FB}} - \frac{J_T}{R_T}\right)\frac{R_{FB}}{R_T} + \cdots
\end{aligned}
$$

$$
A_{FB}(s) = A_{0,FB} + \left(1 - M_Z^2/s\right) A_{1,FB} + \cdots
$$

$$
A_{0,FB} = \frac{R_{FB}}{R_T}, \quad A_{1,FB} = \left(\frac{J_{FB}}{R_{FB}} - \frac{J_T}{R_T}\right) A_{0,FB} \tag{24}
$$

QED corrections for asymmetries

QED corrections to asymmetries lead to few simple correction factors [3, 4]:

$$A_{LR}^{QED}(s) = A_{0,LR}^{Born} + c_{1,T}(s)\, A_{1,LR}^{Born}\left(\frac{s}{M_Z^2}-1\right) + \cdots \tag{25}$$

$$A_{FB}^{QED}(s) = c_{0,FB}(s)\, A_{0,FB}^{Born} + c_{1,FB}(s)\, A_{1,FB}^{Born}\left(\frac{s}{M_Z^2}-1\right) + \cdots \tag{26}$$

The A_0 and A_1 are constant, and the same as in Born approximation.
The QED corrections are contained in the model-independent factor $C(s)$.

$$c_{0,FB}(s) = \frac{C_{FB}^R}{C_T^R}, \qquad c_{0,T}(s) = 1 \tag{27}$$

$$c_{1,A}(s) = c_{0,A}\,\frac{C_T^J}{C_T^R} \tag{28}$$

Sample QED factor

$$C_{T,FB}^R(s) = \int dk\ \rho_{T,FB}(s'/s)\,\frac{s'R}{sR}\,\frac{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}{(s'-M_Z^2)^2+M_Z^2\Gamma_Z^2} \tag{29}$$

Scetch of derivation of the expression for $A_{FB}(s)$ with QED corr's

The QED corrected cross-sections $\sigma_T, \sigma_{FB}, \sigma_{LR}, \sigma_{pol}\dots$ can be represented (with $k = s'/s$) as:

$$\sigma_A^{QED} = \int dk\ \rho_A(k)\ \sigma_A^{Born}, \quad A = T, FB, LR, \cdots \tag{30}$$

Beware that e.g. $\rho_T(k) = \rho_{LR}(k)$, but definitely [23]:

$$\rho_{FB}(k) \neq \rho_T(k) \tag{31}$$

Now with QED corrections, again try the Born-like asymmetry ansatz, and then re-write the new constants $\bar{R}, \bar{J}$ etc. accordingly. The QED corrected total cross section is:

$$\bar{\sigma}(s) = \frac{4}{3}\pi\alpha^2 \left[\frac{\bar{r}_\gamma}{s} + \frac{s\bar{R} + (s - M_Z^2)\bar{J}}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2} + \cdots \right]. \tag{32}$$

The QED corrected asymmetries are also searched as Born-like expressions:

$$\bar{A}(s) = \bar{A}_0 + \bar{A}_1 \left(\frac{s}{M_Z^2} - 1 \right) + + \cdots, \quad A = T, FB, LR, pol, \cdots \tag{33}$$

Rewrite now as follows:

$$\bar{\sigma}(s) \quad \sim \quad \int dk\ \rho(k) \left[\frac{s'R + (s' - M_Z^2)J}{(s' - M_Z^2)^2 + M_Z^2\Gamma_Z^2} + \cdots \right]$$

Scetch of derivation of the expression for $A_{FB}(s)$ with QED corr's II

Look now only at the term R:

$$\sim \quad \int dk\ \rho(k) \frac{s'R}{sR} \frac{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}{(s'-M_Z^2)^2+M_Z^2\Gamma_Z^2} \times \frac{sR}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}$$

$$= \quad \int dk\ \rho(k) \frac{s'R}{sR} \frac{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}{(s'-M_Z^2)^2+M_Z^2\Gamma_Z^2} \times \frac{sR}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} \tag{34}$$

$$= \quad C^R(s) \times \frac{sR}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} \tag{35}$$

So we got:

$$\bar{R} = C^R(s) \times R, \tag{36}$$

$$\bar{J} = C^J(s) \times J, \quad \textbf{with} \quad C^R(s) \neq C^J(s) \tag{37}$$

and

$$\bar{A}_{0,FB} = \frac{C^R_{FB}(s)}{C^R_T(s)} A_{0,FB}, \quad \textbf{with} \quad C^R_{FB}(s) \neq C^R_T(s) \tag{38}$$

$$\bar{A}_{1,FB} = \frac{C^J_T(s)}{C^R_T(s)} \left[\frac{J_{FB}}{R_{FB}} - \frac{J_T}{R_T} \right] \bar{A}_{0,FB} \tag{39}$$

QED corrections for asymmetries

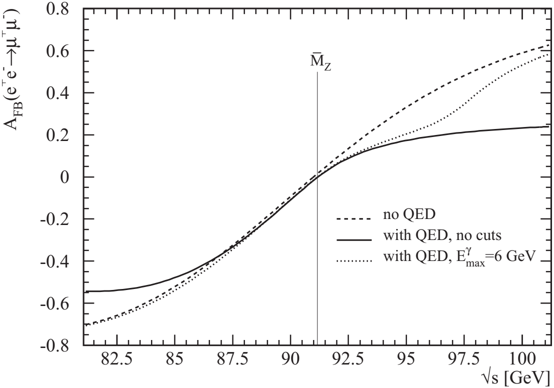

Figure 1 : The forward-backward asymmetry for the process $e^+e^- \to \mu^+\mu^-$ near the Z boson peak. From Kirsch/Riemann 1994 [4], license Number: 3557090997554.

Applications

In Leike/S.Riemann/Riemann 1992 [24] correlations are discussed.

For the Z **peak position** s_{peak}, one may derive the relation:

$$\Delta\sqrt{s_{peak}} = \Delta M_Z + \frac{1}{4}\frac{\Gamma_Z^2}{M_Z}\,\Delta\left(\frac{J_T}{R_T}\right) + \ldots \qquad (40)$$

between an uncertainty in M_Z and an uncertainty in the γZ interference.
The latter also influences A_1.

Similarly, for a **hypothetical heavy gauge boson** Z', the effects from its virtual exchange transform after a partial fraction decomposition into simple shifts of the γZ interferences [24]:

$$\Delta\left(\frac{J_T}{R_T}\right) = -2\frac{g'^2}{g^2}\frac{M_{Z'}^2}{M_{Z'}^2 - M_Z^2}\frac{(a_e a'_e + v_e v'_e)(a_f a'_f + v_f v'_f)}{(a_e^2 + v_e^2)(a_f^2 + v_f^2)}, \qquad (41)$$

Correlations

From LEPEWWG et al., Phys. Rept. 2006, section 2 [25]:

The extra free parameter j^{tot}_{had} is strongly anti-correlated with m_Z, resulting in errors on m_Z enlarged by a factor of almost three, as is observed in the existing S-matrix analyses of LEP-I data [77].
The dependence of m_Z on j^{tot}_{had} is given by:

$$j^{tot}_{had} = \frac{G m_Z^2}{\sqrt{2}\pi\alpha(m_Z^2)} Q_e g_{V_e} \cdot 3 \sum_{q \neq t} Q_q g_{V_q} \tag{42}$$

$$\frac{\partial m_Z}{\partial j^{tot}_{had}} = -1.6\mathrm{MeV}/0.1 \tag{43}$$

...
Improved experimental constraints on the hadronic interference term are obtained by including measurements of the hadronic total cross-section at centre-of-mass energies further away from the Z pole than just the off-peak energies at LEP-I. Including the measurements of the **TRISTAN** *collaborations at KEK,* **TOPAZ** *[78] and* **VENUS** *[79], at $q(s) = 58$ GeV,* **the error on j^{tot}_{had} is about** ± 0.1*, while its central value is in good agreement with the SM expectation.*

Correlations

From LEPEWWG et al., Phys. Rept. 2006, section 2 [25], continued:

Measurements at centre-of-mass energies above the Z resonance at **LEP-II** *[80-83] also provide constraints on j^{tot}_{had}, and in addition test modifications to the interference terms arising from the possible existence of a heavy Z' boson."*

[77] = L3, OPAL 1993 . . . 2003 [26, 27, 28] (see also K. Sachs, L3 [29, 30])
[78] = TOPAZ 1994 [31]
[79] = VENUS 1999 [32]
[80] => correct ref: ALEPH 1996 [33]
[81] = DELPHI 1999 [34]
[82] = L3 1993 . . . 2000 [5, 35, 30]
[83] = OPAL 1997 [36]

Correlations

From LEPEWWG et al., Phys. Rept. 2013, App. A [37]:

In the LEP-I combination the measured value of **the Z boson mass**

$m_Z = 91.1929 \pm 0.0059$ **GeV**

agrees well with the results of the **standard nine parameter fit,**

$[m_Z =]\ 91.1876 \pm 0.0021$ **GeV***,*

albeit with a significantly larger error, resulting from the correlation with the large uncertainty on j_{had}^{tot}*.*
This uncertainty is the dominant source of uncertainty on m_Z *in the S-Matrix fits.*
The measured value of

$j_{had}^{tot} = -0.10 \pm 0.33$

also agrees with the prediction of the SM,

$[j_{had}^{tot} =]\ 0.2201^{+0.0032}_{-0.0137}$.

Correlations

An analysis of the LEP-2 data in terms of j_{had}^{tot} is lacking.
But see Holt 2001 [38] and Sachs 2003 [29].

Including more measurements from LEP*II* solves this problem, reducing the correlation. The final result of $M_Z = 91\ 186.9 \pm 2.3$ MeV [8] is in very good agreement with the result of the standard lineshape fit $M_Z = 91\ 187.6 \pm 2.1$ MeV [9] with only slightly increased error.

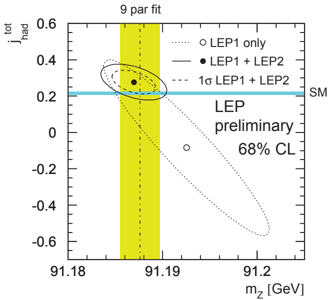

Figure 2: Correlation between the mass of the Z and $j_{\rm had}^{\rm tot}$. Results are shown for LEP*I* data only and for a combined fit to LEP*I* and LEP*II* data. The yellow band indicates the 1 σ error from the 9 parameter fit.

Figure 2 : K. Sachs, "Standard model at LEP II", talk held at Moriond 2001, fig. 2 [29]

Correlations

LEP experiments use cross-section and forward-backward asymmetry results from $\sqrt{s} \sim M_Z$ and LEP II. OPAL and L3 have reported preliminary results which are given in Table 1, and are compared to the value obtained by VENUS [9] using data at $\sqrt{s} \sim 60$ GeV and preliminary LEP I S-Matrix results. The results are consistent with each other, and with the $\mathcal{SM}$ prediction $j_{had}^{tot} = 0.22$.

Expt	Data	j_{had}^{tot}
L3:	LEP I + LEP II	0.30 ± 0.10
OPAL:	LEP I + LEP II	0.21 ± 0.12
VENUS:	VENUS + LEP I	0.20 ± 0.08

Table 1: Measurements of j_{had}^{tot}

Figure 3 : P. Holt, "Fermion pair production above the Z^0 resonance", talk held at HEP 2001, table 1 [38]

Fortran programs: ZPOLE and SMATASY/ZFITTER

ZPOLE – The stand-alone Fortran test package (Leike/Riemann, v.0.5, July 1991) is available on request.
It was used for the numerics of [2].

ZUSMAT . . . the S-Matrix interface of older ZFITTER versions.
ZUSMAT was used for analysing the total cross sections, but could not treat asymmetries.

SMATASY/ZFITTER – With interface package **SMATASY** one has the full functionality of **ZFITTER** corrections [14, 15, 39].

The actual Fortran program for the S-matrix Z line shape approach:

M. Grünewald, S. Kirsch, T. Riemann 1994 [4]
SMATASY v.6.42.01 = SMATA642 (2 June 2005)
available at `https://gruenew.web.cern.ch/gruenew/smatasy.html`

Summary

- The S-matrix approach is absolutely independent of the Standard Model approach.
- The degrees of freedom for σ_{tot} are, at minimum:
  M_Z
  Γ_Z
  R – the residue of the Z resonance, *per scattering channel*
  J – the value of the γZ interference, *per scattering channel*
- So we have at least four degrees of freedom.
  This deserves at least five data points as a function of s.
- Asymmetries may be described as well as σ_{tot}.
- For a exact numerical analysis of data, an accurate description of QED corrections is mandatory.
  This has been realised by combining SMATASY with ZFITTER.
- With so much more statistics at the Fcc-ee compared to LEP-1 and LEP-2:

  The S-matrix approach might gain at the Fcc-ee even more interest as an alternative to the Standard Model approach.

References I

[1] A. Freitas, Higher-order electroweak corrections to the partial widths and branching ratios of the Z boson, JHEP 1404 (2014) 070.
arXiv:1401.2447, doi:10.1007/JHEP04(2014)070.

[2] A. Leike, T. Riemann, J. Rose, S matrix approach to the Z line shape, Phys. Lett. B273 (1991) 513–518.
arXiv:hep-ph/9508390, doi:10.1016/0370-2693(91)90307-C.

[3] T. Riemann, Cross-section asymmetries around the Z peak, Phys. Lett. B293 (1992) 451–456.
arXiv:hep-ph/9506382, doi:10.1016/0370-2693(92)90911-M.

[4] S. Kirsch, T. Riemann, SMATASY: A program for the model independent description of the Z resonance, Comput. Phys. Commun. 88 (1995) 89–108.
arXiv:hep-ph/9408365, doi:10.1016/0010-4655(95)00016-9.

[5] L3 collab., O. Adriani, et al., An S matrix analysis of the Z resonance, Phys. Lett. B315 (1993) 494–502.
doi:10.1016/0370-2693(93)91646-5.

[6] D. Y. Bardin, A. Leike, T. Riemann, M. Sachwitz, Energy Dependent Width Effects in e^+e^- Annihilation Near the Z Boson Pole, Phys. Lett. B206 (1988) 539–542.
doi:10.1016/0370-2693(88)91625-5.

[7] F. A. Berends, G. Burgers, W. Hollik, W. van Neerven, The Standard Z Peak, Phys. Lett. B203 (1988) 177.
doi:10.1016/0370-2693(88)91593-6.

[8] A. Borrelli, M. Consoli, L. Maiani, R. Sisto, Model Independent Analysis of the Z Line Shape in e^+e^- Annihilation, Nucl.Phys. B333 (1990) 357.
doi:10.1016/0550-3213(90)90042-C.

[9] D. Y. Bardin, G. Passarino, The standard model in the making: Precision study of the electroweak interactions, International series of monographs on physics, 104 (Oxford University Press, 1999).
http://www.amazon.de/Standard-Model-Making-Interactions-International/dp/019850280X/ref=sr_1_1?ie=UTF8&qid=1422902184&sr=8-1&keywords=bardin+passarino.

[10] D. Bardin, M. Grünewald, G. Passarino, Precision calculation project report.
arXiv:hep-ph/9902452.

[11] G. Passarino, Pseudo versus realistic observables: All that theories can tell us is how the world could be, talk at 'Workshop on Electroweak Precision Data and the Higgs Mass', DESY, Zeuthen, Feb. 28 - March 1, 2003 137–146.
http://www--library.desy.de/preparch/desy/proc//proc03--01/14.ps.gz.

References II

[12] G. Passarino, Higgs CAT, Eur. Phys. J. C74 (2014) 2866.
arXiv:1312.2397, doi:10.1140/epjc/s10052-014-2866-7.

[13] D. Y. Bardin, M. S. Bilenky, T. Riemann, M. Sachwitz, H. Vogt, P. C. Christova, DIZET: A program package for the calculation of electroweak one loop corrections for the process $e^+e^- \to f^+f^-$ around the Z^0 peak, Comput. Phys. Commun. 59 (1990) 303–312, doi:10.1016/0010-4655(90)90179-5.
doi:10.1016/0010-4655(90)90179-5.

[14] D. Bardin, M. Bilenky, P. Christova, M. Jack, L. Kalinovskaya, A. Olchevski, S. Riemann, T. Riemann, ZFITTER v.6.21: A semi-analytical program for fermion pair production in e^+e^- annihilation, Comput. Phys. Commun. 133 (2001) 229–395.
arXiv:hep-ph/9908433, doi:10.1016/S0010-4655(00)00152-1.

[15] A. Arbuzov, M. Awramik, M. Czakon, A. Freitas, M. Grünewald, K. Mönig, S. Riemann, T. Riemann, ZFITTER: A Semi-analytical program for fermion pair production in e^+e^- annihilation, from version 6.21 to version 6.42, Comput. Phys. Commun. 174 (2006) 728–758.
arXiv:hep-ph/0507146, doi:10.1016/j.cpc.2005.12.009.

[16] R. G. Stuart, Gauge invariance, analyticity and physical observables at the Z0 resonance, Phys. Lett. B262 (1991) 113–119.
doi:10.1016/0370-2693(91)90653-8.

[17] S. Kirsch, S. Riemann, A Combined Fit to the L3 Data Using the S-Matrix Approach (First Resultats) , L3 note 1233, 1992.
http://l3.web.cern.ch/l3/note/notes1992.html.

[18] A. Denner, J.-N. Lang, The Complex-Mass Scheme and Unitarity in perturbative Quantum Field Theory.
arXiv:1406.6280.

[19] A. Freitas, About projected theory uncertainties, Talk at the 2015 Pisa Fcc-ee meeting,
https://agenda.infn.it/conferenceOtherViews.py?view=standard&confId=8830.

[20] G. Degrassi, Precision observables in the Standard Model: a reexamination, Talk at the 2015 Pisa Fcc-ee meeting,
https://agenda.infn.it/conferenceOtherViews.py?view=standard&confId=8830.

[21] A. R. Böhm, Y. Sato, Relativistic resonances: Their masses, widths, lifetimes, superposition, and causal evolution, Phys. Rev. D71 (2005) 085018.
arXiv:hep-ph/0412106, doi:10.1103/PhysRevD.71.085018.

References III

[22] D. Y. Bardin, M. S. Bilenky, A. Chizhov, A. Sazonov, O. Fedorenko, T. Riemann, M. Sachwitz, Analytic approach to the complete set of QED corrections to fermion pair production in e+ e- annihilation, Nucl.Phys. B351 (1991) 1–48.
arXiv:hep-ph/9801208, doi:10.1016/0550-3213(91)90080-H.

[23] D. Y. Bardin, M. S. Bilenky, A. Chizhov, A. Sazonov, Y. Sedykh, T. Riemann, M. Sachwitz, The convolution integral for the forward - backward asymmetry in e^+e^- annihilation, Phys. Lett. B229 (1989) 405.
doi:10.1016/0370-2693(89)90428-0.

[24] A. Leike, S. Riemann, T. Riemann, Z Z-prime mixing and radiative corrections at LEP-1, Phys.Lett. B291 (1992) 187–194.
arXiv:hep-ph/9507436, doi:10.1016/0370-2693(92)90142-Q.

[25] ALEPH collab., DELPHI collab., L3 collab., OPAL collab., SLD Collaboration, LEP Electroweak Working Group, SLD Electroweak Group, SLD Heavy Flavour Group, S. Schael, et al., Precision electroweak measurements on the Z resonance, Phys. Rept. 427 (2006) 257–454.
arXiv:hep-ex/0509008, doi:10.1016/j.physrep.2005.12.006.

[26] L3 collab., O. Adriani, et al., Results from the L3 experiment at LEP, Phys. Rept. 236 (1993) 1–146.
doi:10.1016/0370-1573(93)90027-B.

[27] L3 collab., M. Acciarri, et al., Measurements of cross-sections and forward backward asymmetries at the Z resonance and determination of electroweak parameters, Eur. Phys. J. C16 (2000) 1–40.
arXiv:hep-ex/0002046, doi:10.1007/s100520050001.

[28] OPAL collab., G. Abbiendi, et al., Precise determination of the Z resonance parameters at LEP: 'Zedometry', Eur. Phys. J. C19 (2001) 587–651.
arXiv:hep-ex/0012018, doi:10.1007/s100520100627.

[29] K. Sachs, Standard model at LEP II, Proc. of XXXVIII Rencontres de Moriond: Electroweak Interactions and Unified Theories, Les Arcs, March 15-22, 2003.
arXiv:hep-ex/0307009.

[30] L3 collab., M. Acciarri, et al., Determination of γ/Z interference in e^+e^- annihilation at LEP, Phys. Lett. B489 (2000) 93–101.
arXiv:hep-ex/0007006, doi:10.1016/S0370-2693(00)00889-3.

[31] TOPAZ collab., K. Miyabayashi, et al., Measurement of the total hadronic cross-section and determination of $\gamma - Z$ interference in e^+e^- annihilation, Phys. Lett. B347 (1995) 171–178.
doi:10.1016/0370-2693(95)00038-M.

References IV

[32] VENUS collab., K. Yusa, et al., Precise measurement of the total hadronic cross-section in e^+e^- annihilation at $\sqrt{s} = 57.77$ GeV, Phys. Lett. B447 (1999) 167–177.
doi:10.1016/S0370-2693(98)01560-3.

[33] ALEPH collab., D. Buskulic, et al., Measurement of hadron and lepton pair production from e^+e^- annihilation at center-of-mass energies of 130 GeV and 136 GeV, Phys. Lett. B378 (1996) 373–384.
doi:10.1016/0370-2693(96)00504-7.

[34] DELPHI collab., P. Abreu, et al., Measurement and interpretation of fermion pair production at LEP energies from 130 GeV to 172 GeV, Eur. Phys. J. C11 (1999) 383–407.
doi:10.1007/s100520050643.

[35] L3 collab., M. Acciarri, et al., Measurement of hadron and lepton pair production at 130 GeV < $\sqrt{S}$ < 189 GeV at LEP, Phys. Lett. B479 (2000) 101–117.
arXiv:hep-ex/0002034, doi:10.1016/S0370-2693(00)00280-X.

[36] OPAL collab., K. Ackerstaff, et al., Tests of the standard model and constraints on new physics from measurements of fermion pair production at 130 GeV to 172 GeV at LEP, Eur. Phys. J. C2 (1998) 441–472.
arXiv:hep-ex/9708024, doi:10.1007/s100520050152.

[37] ALEPH collab., DELPHI collab., L3 collab., OPAL collab., LEP Electroweak Working Group, S. Schael, et al., Electroweak Measurements in Electron-Positron Collisions at W-Boson-Pair Energies at LEP, Phys. Rept. 532 (2013) 119–244.
arXiv:1302.3415, doi:10.1016/j.physrep.2013.07.004.

[38] P. Holt, Fermion pair production above the Z^0 resonance, PoS HEP2001 (2001) 115.
http://pos.sissa.it/archive/conferences/007/115/hep2001_115.pdf.

[39] A. Akhundov, A. Arbuzov, S. Riemann, T. Riemann, The ZFITTER project, Phys. Part. Nucl. 45 (3) (2014) 529–549.
arXiv:1302.1395, doi:10.1134/S1063779614030022.