Alternative renormalization group for the Standard Model

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- Why?
- How?
  - Generalized Gell-Mann—Low equations
  - Generalized nonabelian gauge theories
- Example of ϕ^4
- Prospects for the SM

Evolution of the scalar mass: ϕ^4

Mass squared anomalous dimension: $\gamma_\Phi = \frac{g^2}{12(16\pi^2)^2}$

- Naive (quadratic divergence in the self-energy, Susskind, 1979): $m^2(Q^2)/Q^2 = m^2_{ph}/Q^2 + \gamma_\Phi$
- Smart (MS-scheme, Macfarlane & Collins 1974): $m^2(Q^2)/Q^2 = (m^2_{ph}/Q^2)^{1-\gamma_\Phi}$

MS-scheme ignores quadratic divergences. For scalar fields, choice of scheme may not be a technical issue.

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Impications for the naturalness problem

$$\frac{\delta m^2(Q^2)}{m^2(Q^2)}) \equiv r(\frac{m_{ph}^2}{Q^2}, \gamma_\Phi)\frac{\delta m_{ph}^2}{m_{ph}^2}$$

- Naive: $r = \frac{m_{ph}^2}{Q^2}/(\frac{m_{ph}^2}{Q^2} + \gamma_\Phi)$. Accuracy grows infinitely with Q^2. Expectation of new physics at 1 TeV.
- Smart: $r = 1 - \gamma_\Phi$. No naturalness problem. Expectation of the "great desert"

At least for naturalness, the use of "naive" and "smart" RG evolution does matter

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History

From Gell-Mann—Low to 't Hooft—Weinberg

From conceptually sound nonlinear equations for QED Green functions to technically convenient linear equations of 't Hooft and Weinberg (present-day standard)

Scalar fields

Weinberg—scalar field is a singular case, needs extra consideration (1973)
Macfarlane & Collins—scalar fields are OK in MS-scheme (1974)
Veltman—MS-scheme ignores poles at dimensions $(4 - 2/(\#\text{ of loops})) \equiv$ ignors quadratic divergences (1981)

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To take into account ssb and quadratic divergences in renormalization group evolution of the Sandard Model

Problem 1:
Definition of the Gell-Mann–Low scheme for the most general theory

Problem 2:
Proliferation of terms in the Lagrangian after shifting the scalar field to the vacuum value (what are the evolving parameters?)

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Plan with more details

Generalization of Gell-Mann–Low (Problem 1)

- Specify extraction of parameters from Green's functions
- Specify expression of Green functions in terms of these parameters

Generalization of gauge theories (Problem 2)

- Ignore unitarity
- Keep locality, Lorentz invariance, and renormalizability
- Keep $g_4 = g_3^2$

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Generalized Gell-Mann–Low scheme

Sequence of steps in Gell-Mann–Low scheme

1. Extraction of parameters from Green's functions depending on energy scale
2. Expression of Green's functions in terms of extracted parameters
3. RG equations for the parameters require them to depend on the energy scale in such a way, that the resulting Green's functions do not depend on the energy scale of the extraction procedure

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Extraction of Parameters

Generating functional for connected Green's functions $W(J) \rightarrow$ Generating functional for amplitudes $A_{Q^2}(\Phi)$ (partial amputation depending on the energy scale) $\rightarrow$ Extraction of local part of the two-, three-, and four-point amplitudes with an operation $\Delta_{Q^2} \rightarrow$ Inaction functional $I_{Q^2}(\Phi) \equiv \Delta_{Q^2} A_{Q^2}(\Phi)$

- Inaction is a finite, Lorentz invariant, and local polynomial in fields of not more than fourth degree in the fields
- Inaction depends on the extraction scale Q^2
- Inaction is a container for parameters of the theory. It resembles the classical action of the model

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Expression of Green's functions in terms of $I_{Q^2}(\Phi)$

condition

If the bare action of the model $S_B(\Phi)$ satisfies the condition $\Delta_{Q^2} S_B(\Phi) = S_B(\Phi)$, the connected Green's functions of this model can be expressed in terms of $I_{Q^2}(\Phi)$

The recipe

$W(J) = R_{Q^2} \log T_{I_{Q^2,F}} \exp(I_{Q^2,3}(\Phi) + I_{Q^2,4}(\Phi) + iJ)$
Here R_{Q^2} is a specially tuned renormalization operation.

For details see V.T. Kim & GP, 2010—2015

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Comments

Peculiarity of R_{Q^2}

R_{Q^2} subtracts not only divergences. It also subtracts some finite parts originating from connected subgraphs (not only 1pi-subgraphs generate counter terms)

Role of combinatorics

No struggle with divergences: as soon as $\Delta_{Q^2} S_B(\Phi) = S_B(\Phi)$, R_{Q^2} can be combinatorially constructed from Δ_{Q^2}.

The loop

$S_B(\Phi) \to W(J) \to I_{Q^2}(\Phi) \to S_B(\Phi)$ The inaction is the classical action with parameters extracted from Green functions (Dominicis-Englert duality for the last step)

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Abstraction against complexity

Proliferation of local field monomials in the action after ssb.
Consider most general case keeping the desired features.

- No need to restrict consideration to unitary theories, if the aim is to study the renormalization group
- renormalizability + locality + Lorentz invariance
- Example: $\mathrm{g} f_{\mathrm{abc}} \mathrm{A}^{\mathrm{a}}_{\mu} \mathrm{A}^{\mathrm{b}}_{\nu} (\partial^{\mu} \mathrm{A}^{\nu\mathrm{c}} - \partial^{\nu} \mathrm{A}^{\mu\mathrm{c}}) \to \mathrm{g}^{(3)}_{\mathrm{abc}} \mathrm{A}^{\mathrm{a}}_{\mu} \mathrm{A}^{\mathrm{b}}_{\nu} \partial^{\mu} \mathrm{A}^{\nu\mathrm{c}}$
- Couplings are tensors with respect to gauge group and flavour indexes, like $\mathrm{g}^{(3)}_{\mathrm{abc}}$
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Peculiarity of non-abelian theories

Quartic couplings are functions of cubic couplings

- $g^{(4)} = f(g^{(3)}) = O((g^{(3)})^2)$
- Tensorial functions: $g^{(4)}_{abcd} = c_1 g^{(3)}_{abn} (Z^{-1})^{nm} g^{(3)}_{mcd} + \ldots$
- Even after ssb, all quartic couplings are functions of cubic ones
- In terms of inaction: $I^{(4)}_{Q^2} = F(I^{(3)}_{Q^2})$. Only quadratic and cubic couplings evolve independently.

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Consistency condition

- Functions expressing four-point couplings in terms of three point couplings are far from arbitrary
- They should satisfy a consistency condition making the theory renormalizable
- Discrete set of solutions to the consistency condition. Non-abelian gauge theories give particular solutions of the consistency condition.

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Comments

- COUNTER TERMS TO the FOUR-POINT FUNCTIONS ARE CONSTRUCTED AS NONLINEAR COMBINATIONS OF THE COUNTER TERMS TO THREE-POINT FUNCTIONS, TWO-POINT FUNCTIONS AND TADPOLES
- The possibility of this construction is a general phenomenon. The non-abelian gauge theories are particular instances of this phenomenon
- The standard approach (renormalization as "always" and derivation of relations between renormalization constants) allows one to ignore this phenomenon
- Deriving relations between renormalization constants replaces a basic feature of the theory with a technical problem, and complicates life. Hence, the need to replace Gell-Mann–Low with 't Hooft-Weinberg

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Example: scalar field with ssb

Potential

$V = \frac{\lambda}{24}(\phi^2 - v^2)^2$

SSB

$\phi \to \phi + \tilde{v}, \langle\phi\rangle = 0$

Potential after ssb

$V(\phi) = \Lambda^3\phi + \frac{m^2}{2}\phi^2 + \frac{\mu}{6}\phi^3 + \frac{\lambda}{24}\phi^4$

quartic couping

$\lambda = \frac{\mu^2}{3m^2(1+\sqrt{1-\frac{\Lambda^3\mu}{3m^4}})} \approx \frac{\mu^2}{6m^2} + \mathcal{O}(\mu^4)$

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Multicomponent generalization

Tensorial structure

The only admissible structure:

$\lambda_{\mathrm{abcd}} \approx \omega \mu_{\mathrm{abe}} (\mathrm{m}^{-2})^{\mathrm{ee}'} \mu_{\mathrm{e}'\mathrm{cd}}$

Uniqueness

$\omega = \frac{1}{6}$ from renormalizability

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Mass evolution in ϕ^4 revisited

The naive and the smart are superseded by the correct (2010, gp):

$$\frac{m^2(Q^2)}{Q^2} = \frac{\gamma_\Phi}{1-4\gamma_\Phi \log \frac{Q^2}{m^2_{ph}}}$$

There is a Landau pole in the scalar mass squared for ϕ^4, and no naturalness problem

What if...

in the Standard Model, the Landau pole in the scalar mass squared is replaced with asymptotic freedom?

$$\frac{m^2(Q^2)}{Q^2} = \frac{B(g)}{1+A(g)\log\frac{Q^2}{m_{ph}^2}}$$

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- Generalizations of non-abelian gauge theories are defined
- Ready for computing Gell-Mann—Law evolution for the Standard Model parameters
- If there will be asymptotic freedom for scalar mass squared in the standard model (instead of the Landau pole of Φ^4), the naturalness problem will appear in the standard model. But the growth of the accuracy will be logarithmically slow. In this case the scale of new physics will be much larger than 1 TeV.

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