Introduction to hadron collider physics

Theory challenges for LHC Physics

Dubna International Advanced School of Theoretical Physics

Helmholtz International Summer School

July 20-30 2015

Lecture 1

Michelangelo L. Mangano

TH Unit, Physics Department, CERN

michelangelo.mangano@cern.ch

Outline

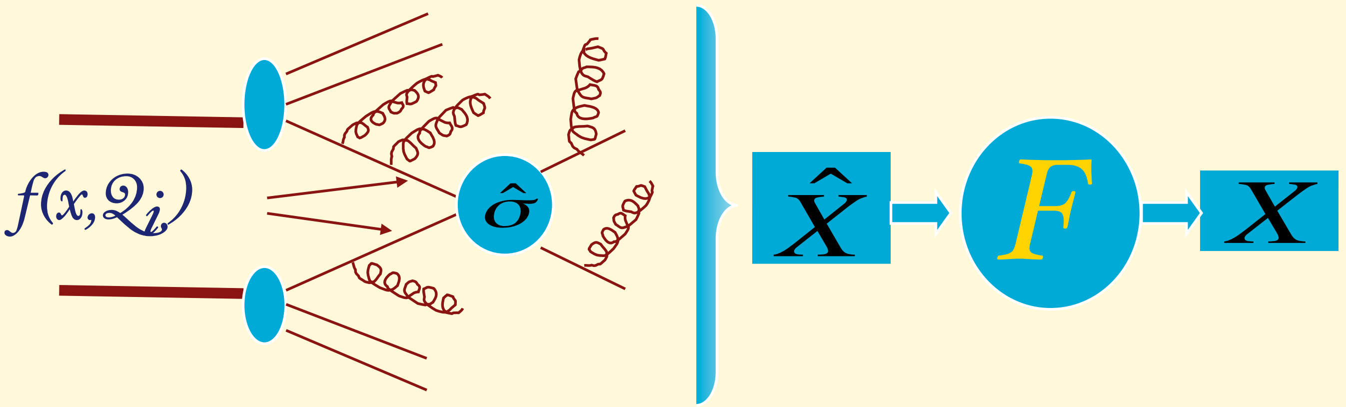
1. Introduction to the theoretical principles of hadron collisions:
   1.1. Factorization, initial state evolution and PDFs
2. Phenomenological applications, review and interpretation of LHC data:
   2.1. Drell-Yan processes
   2.2. Jet production
   2.3. Top quark physics

.. things I'll give for granted you know ..

- quarks and gluons
- mesons and baryons
- asymptotic freedom
- Feynman diagrams and Feynman rules
- basic knowledge of what high-energy hadronic collisions are about:
  - production and study of jets, heavy quarks (bottom, top)
  - production and study of W/Z bosons,
  - production and study of Higgs bosons,
  - search for phenomena beyond the Standard Model (supersymmetry, dark matter, new gauge forces, etc.)

Factorization Theorem

$$\frac{d\sigma}{dX} = \sum_{j,k} \int_{\hat{X}} f_j(x_1,Q_i)\, f_k(x_2,Q_i)\, \frac{d\hat{\sigma}_{jk}(Q_i,Q_f)}{d\hat{X}}\, F(\hat{X} \to X; Q_i, Q_f)$$



$f_j(x,Q)$ Parton distribution functions (PDF)

- sum over all initial state histories leading, at the scale Q, to:

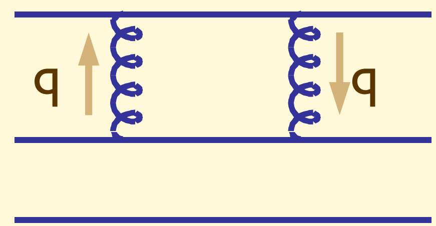
$$\vec{p}_j = x\,\vec{P}_{proton}$$

$F(\hat{X} \to X; Q_i, Q_f)$

- transition from partonic final state to the hadronic observable (hadronization, fragm. function, jet definition, etc)
- Sum over all histories with X in them

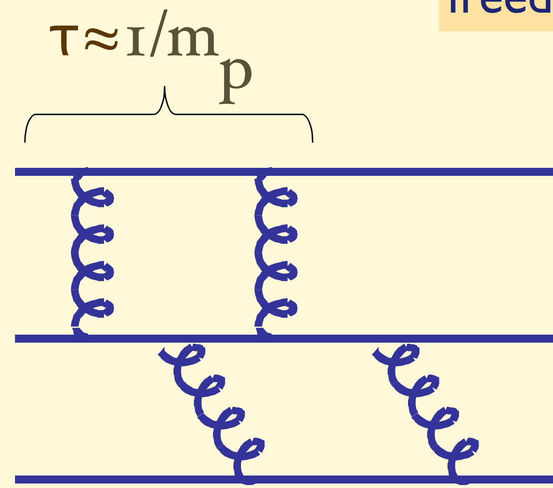
Universality of parton densities and factorization, an intuitive picture

1) Exchange of **hard gluons** among quarks inside the proton is suppressed by powers of $(m_p/Q)^2$



$$\overset{q>Q}{\sim} \int_Q^\infty \frac{d^4q}{q^6} \sim \frac{1}{Q^2}$$

Assuming asymptotic freedom!

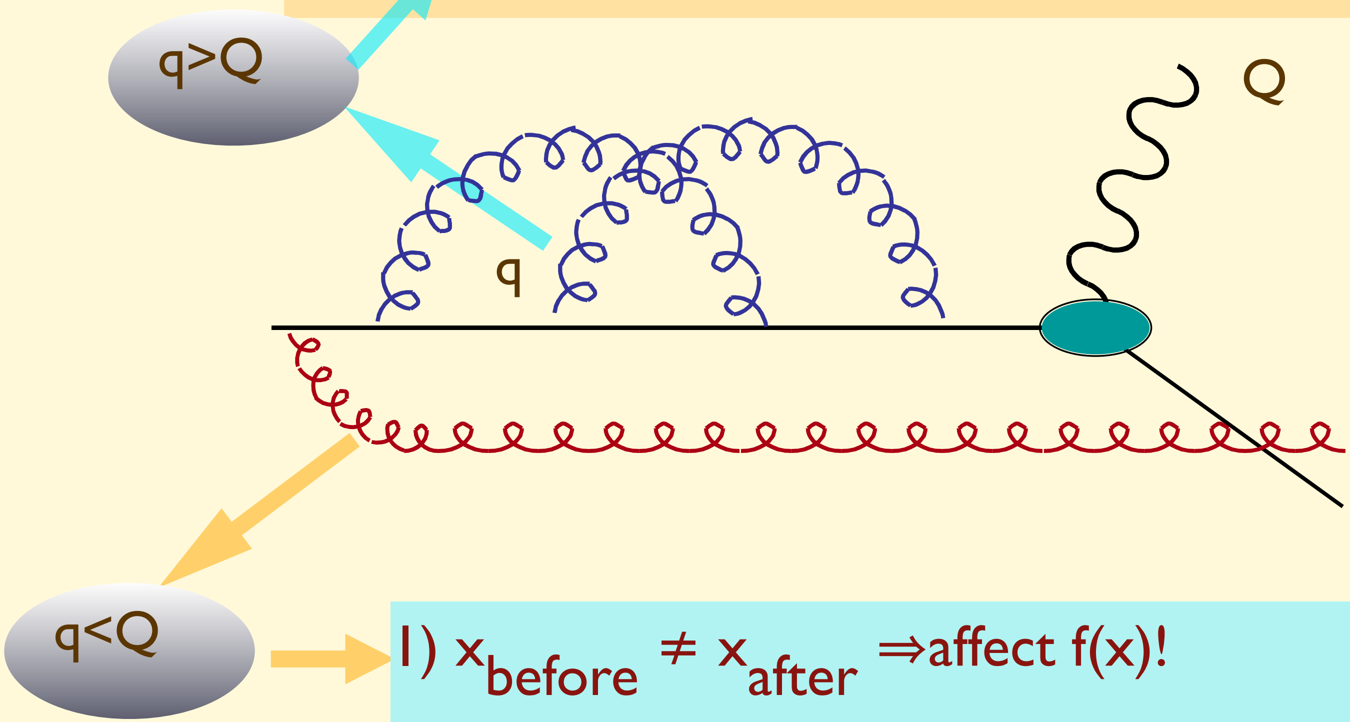


2) **Typical time-scale of interactions binding the proton** is therefore of $O(1/m_p)$ (in a frame in which the proton has energy E, $\tau=\gamma/m_p = E/m_p^2$)

3) If a hard probe ($Q>>m_p$) hits the proton, on a time scale $=1/Q$, there is no time for quarks to negotiate a coherent response. The struck quark receives no feedback from its pals, and acts as a free particle

As a result, to study inclusive processes at large Q it is sufficient to consider the interactions between the external probe and a single parton:

1) calculable in perturbative QCD (pQCD)
2) do not affect f(x): $x_{\text{before}} = x_{\text{after}}$



This gluon cannot be reabsorbed because the quark is gone

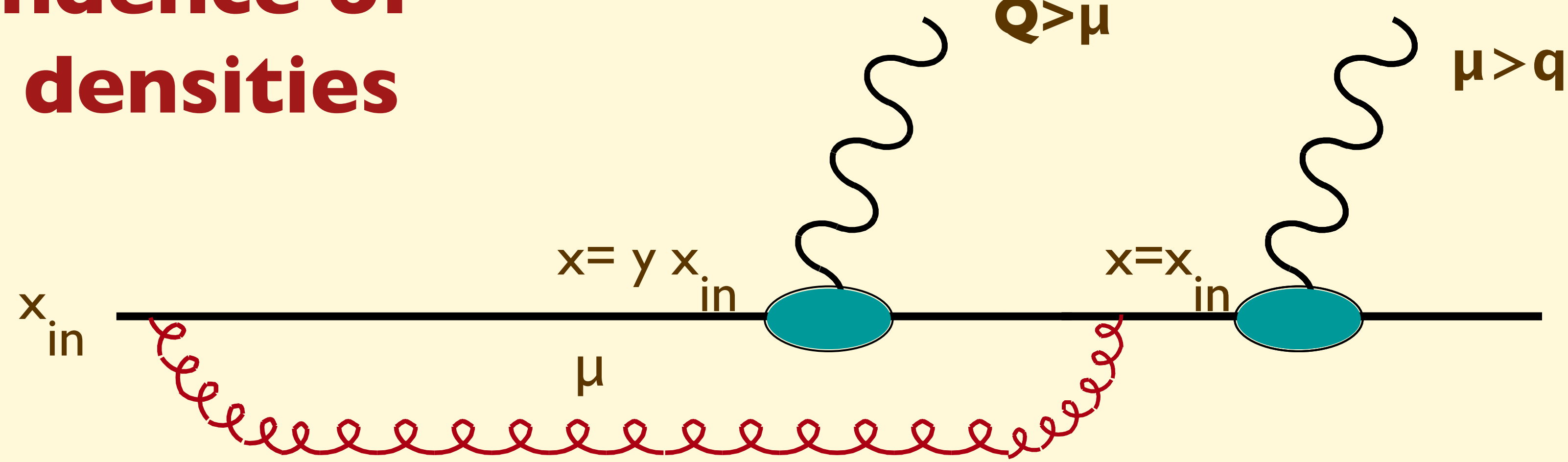
1) $x_{\text{before}} \neq x_{\text{after}}$ ⇒affect f(x)!

2) for q≈1 GeV **not** calculable in pQCD

However, since τ(q≈1GeV)>>1/Q, the emission of low-virtuality gluons will take place long before the hard collision, and therefore cannot depend on the detailed nature of the hard probe. While it is not calculable in pQCD, f(q<<Q) can be measured using a reference probe, and used elsewhere

➡ Universality of f(x)

Q dependence of parton densities



The larger is Q, the more gluons will **not** have time to be reabsorbed

PDF's depend on Q!

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in}\, f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy\, P(y,q^2)\, \delta(x - yx_{in})$$

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in} f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy P(y,q^2)\,\delta(x - y x_{in})$$

f(x,Q) should be independent of the intermediate scale μ considered:

$$\frac{df(x,Q)}{d\mu^2} = 0 \quad \Rightarrow \boxed{\frac{df(x,\mu)}{d\mu^2} = \int_x^1 \frac{dy}{y} f(y,\mu)\, P(x/y,\mu^2)}$$

One can prove that:

calculable in pQCD

$$P(x,Q^2) = \frac{\alpha_s}{2\pi}\frac{1}{Q^2}P(x)$$

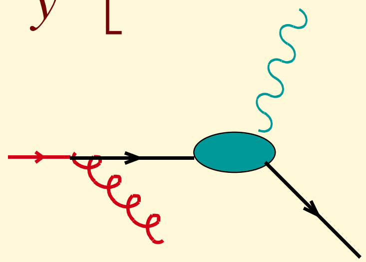
and finally (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi DGLAP equation):

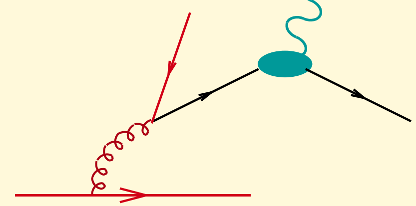
$$\boxed{\frac{df(x,\mu)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} f(y,\mu)\, P(x/y)}$$

More in general, one should consider additional processes which lead to the evolution of partons at high Q ($t=\log Q^2$):

$$[g(x)]_+ : \quad \int_0^1 dx\, f(x)\, g(x)_+ \equiv \int_0^1 [f(x) - f(1)]\, g(x)\, dx$$

$$\frac{dq(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[q(y,Q)\, P_{qq}(\frac{x}{y}) + g(y,Q)\, P_{qg}(\frac{x}{y}) \right]$$

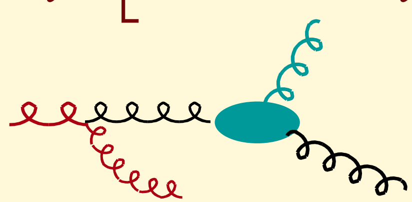


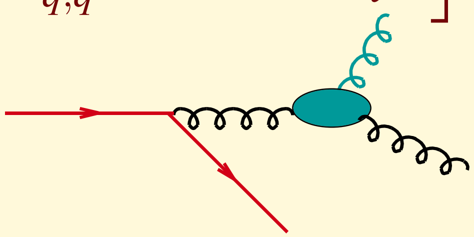


$$P_{qq}(x) = C_F \left(\frac{1+x^2}{1-x} \right)_+$$

$$P_{qg}(x) = \frac{1}{2} \left[x^2 + (1-x)^2 \right]$$

$$\frac{dg(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[g(y,Q)\, P_{gg}(\frac{x}{y}) + \sum_{q,q} q(y,Q)\, P_{gq}(\frac{x}{y}) \right]$$

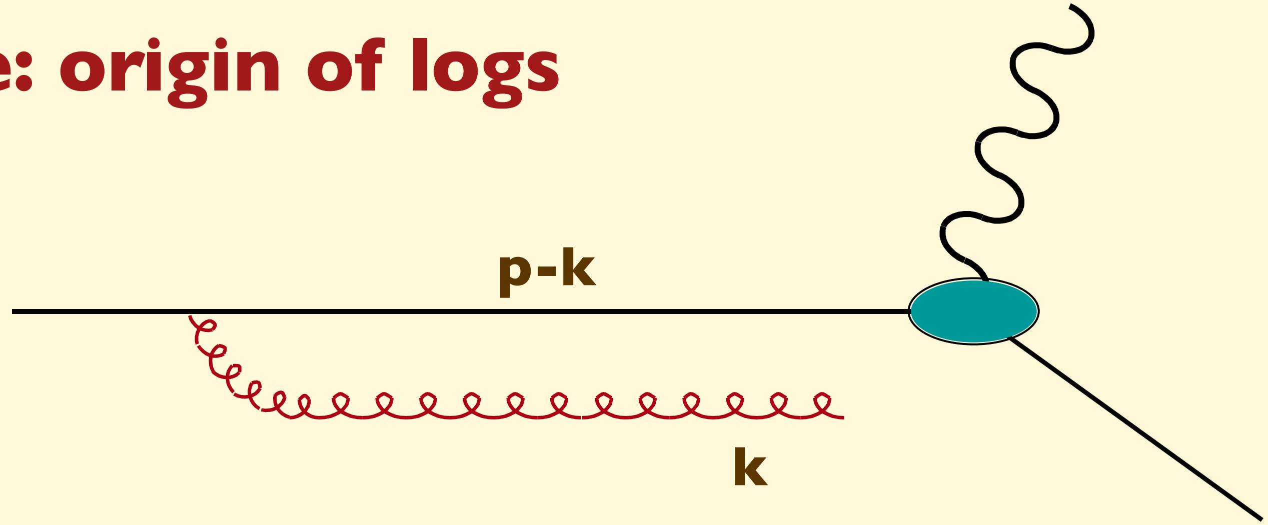




$$P_{gq}(x) = C_F \left(\frac{1+(1-x)^2}{x} \right)$$

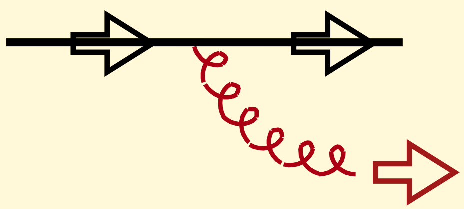
$$P_{gg}(x) = 2N_c \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \delta(1-x) \left(\frac{11N_c - 2n_f}{6} \right)$$

Note: origin of logs

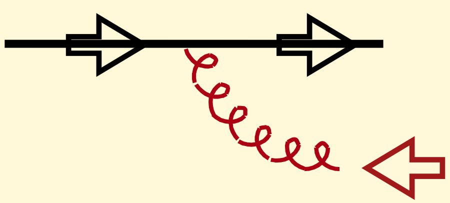


$$(p-k)^2 = -2\,p^0 k^0 (1-\cos\theta_{pk})$$

1/2 → 3/2



1/2 → -1/2



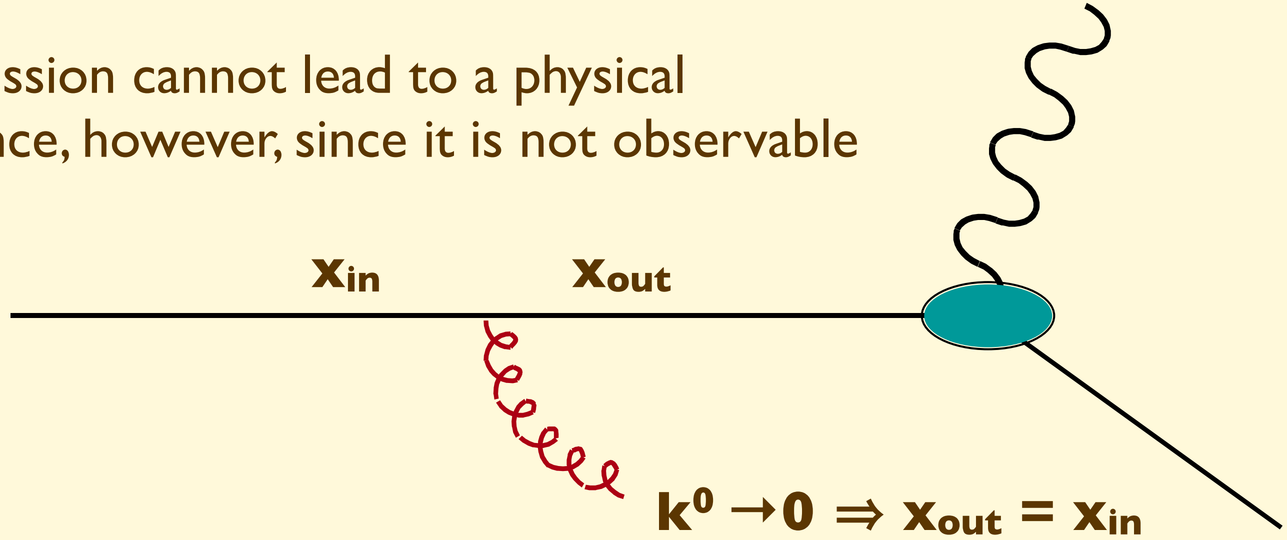
Helicity conservation $\sim p \cdot k$

$$|M|^2 \sim \left[\frac{1}{(p-k)^2}\right]^2 \times (p \cdot k) \quad \rightarrow \quad \frac{1}{p^0}\frac{dk^0}{k^0}\frac{d\theta}{\theta}$$

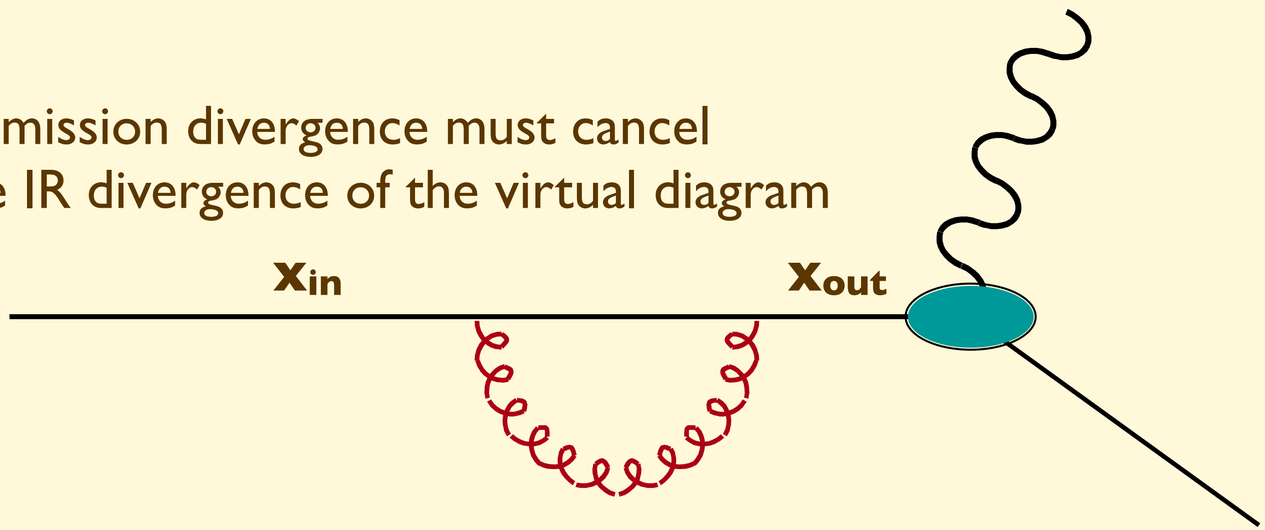
Soft divergence

Collinear divergence

Soft emission cannot lead to a physical divergence, however, since it is not observable

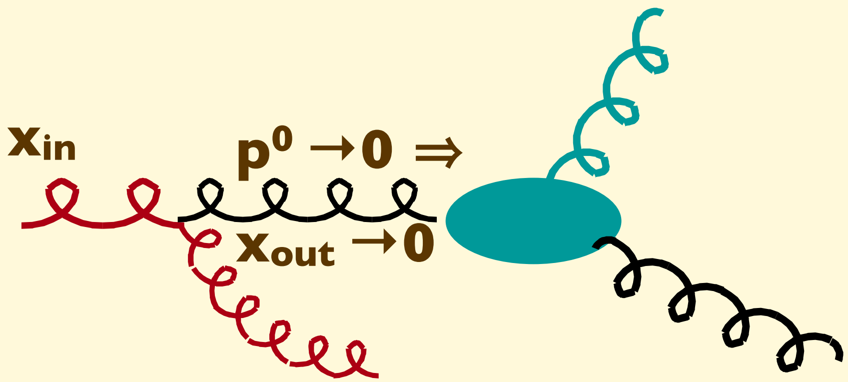


The soft-emission divergence must cancel against the IR divergence of the virtual diagram



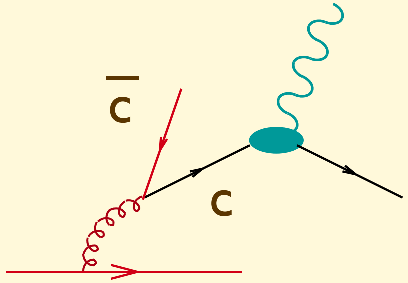
The cancellation cannot take place in the case of collinear divergence, since $x_{out} \neq x_{in}$, so virtual and real configurations are not equivalent

Things are different if $\mathbf{p^0 \to 0}$. In this case, again, $\mathbf{x_{out} \neq x_{in}}$, no virtual-real cancellation takes place, and an extra singularity due to the $\mathbf{1/p^0}$ pole appears



These are called **small-x** logarithms. They give rise to the double-log growth of the number of gluons at small **x** and large **Q**

Example: charm in the proton



$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q)\, P_{qg}(\frac{x}{y})$$

Assuming a typical behaviour of the gluon density: $g(x,Q) \sim A/x$

and using $P_{qg}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$ we get:

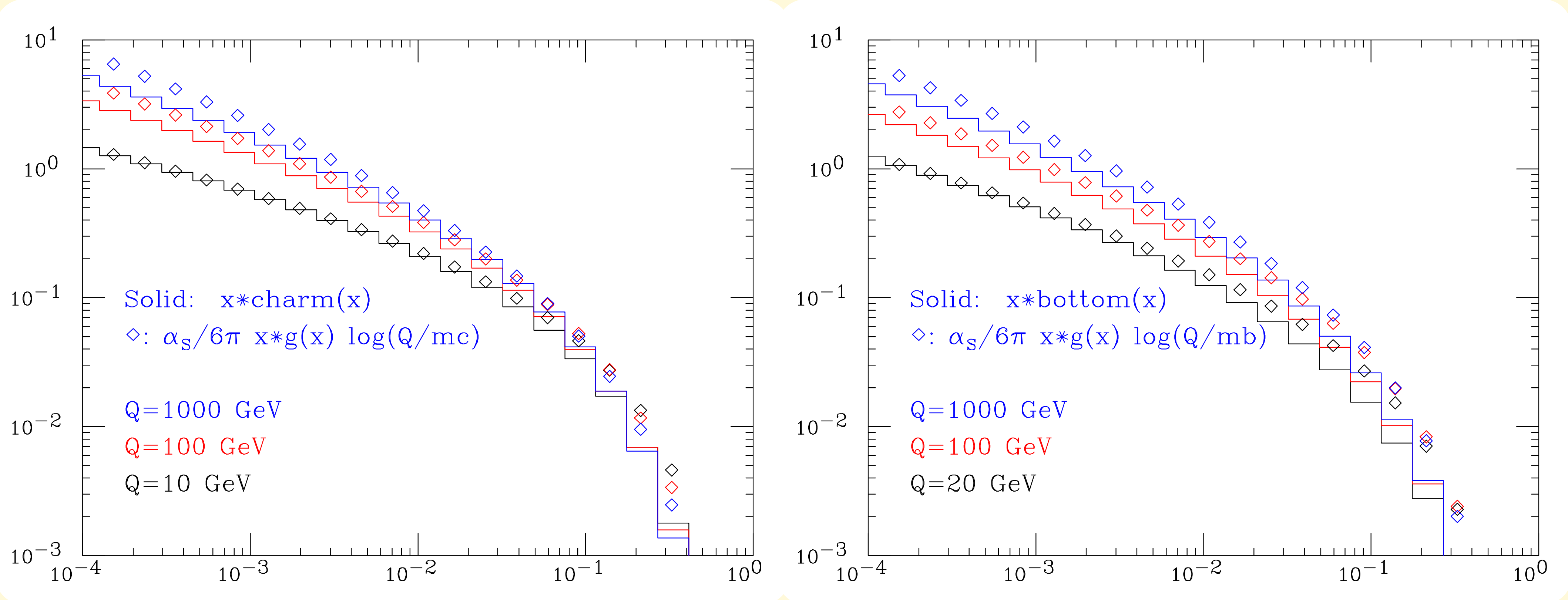
$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(x/y,Q)\, P_{qg}(y) = \frac{\alpha_s}{2\pi}\int_x^1 dy \frac{A}{x}\frac{1}{2}\left[y^2 + (1-y)^2\right] = \frac{\alpha_s}{6\pi}\frac{A}{x}$$

and therefore:

$$c(x,Q) \sim \frac{\alpha_s}{6\pi}\log(\frac{Q^2}{m_c^2})\, g(x,Q)$$

Corrections to this simple formula will arise due to the Q dependence of g(x) and of α_s

Numerical example



Excellent agreement, given the simplicity of the approximation!

Can be improved by tuning the argument of the log (threshold onset), including a better parameterization of g(x), etc....

General properties of the PDF evolution

Definition of n-th moment: $$g_n = \int_0^1 \frac{dx}{x}\, x^n\, g(x)$$

In moment space, the evolution eqs become coupled linear differential equations

$$\frac{df_i^{(n)}}{dt} = \frac{\alpha_s}{2\pi}[P_{qq}^{(n)} f_i^{(n)} + P_{qg}^{(n)} f_g^{(n)}]$$

$$\frac{df_g^{(n)}}{dt} = \frac{\alpha_s}{2\pi}[P_{gg}^{(n)}\, f_g + P_{gq}^{(n)} f_i^{(n)}]$$

exercise!

or, equivalently:

$$\frac{df_g^{(n)}}{dt} = \frac{\alpha_s}{2\pi}\, \left[P_{gq}^{(n)}\, \Sigma^{(n)} + P_{gg}^{(n)}\, f_g^{(n)}\right]$$

$$\frac{d\Sigma^{(n)}}{dt} = \frac{\alpha_s}{2\pi}\, \left[P_{qq}^{(n)}\, \Sigma^{(n)} + 2n_f\, P_{qg}^{(n)}\, f_g^{(n)}\right]$$

$$\frac{dV^{(n)}}{dt} = \frac{\alpha_s}{2\pi}\, P_{qq}^{(n)}\, V^{(n)}$$

where we define “singlet” and “valence” distributions as:

$$\Sigma(x) = \sum_i f_i(x) + \sum_{\bar{\imath}} f_{\bar{\imath}}(x)$$

$$V(x) = \sum_i f_i(x) - \sum_{\bar{\imath}} f_{\bar{\imath}}(x)$$

Valence sum rule

$$V^{(1)} = \int_0^1 dx \sum_q (f_q(x) - f_{\bar{q}}(x)) = N(\text{valence quarks}) = constant$$

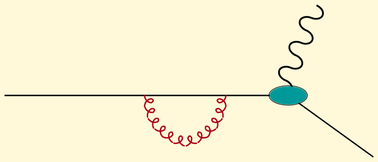
Thus $$\frac{dV^{(1)}}{dt} \equiv 0 \Rightarrow \frac{\alpha_s}{2\pi} P_{qq}^{(1)} V^{(1)} = 0$$

Since $V^{(1)}=3$, we must have $P_{qq}^{(1)}=0$, i.e. $$\int_0^1 dz\, P_{qq}(z) = 0$$

This requires to modify $P_{qq}(z)$ as follows:

$$P_{qq}(z) \to \left(\frac{1+z^2}{1-z}\right)_+ \equiv \frac{1+z^2}{1-z} - \delta(1-z)\int_0^1 dy \left(\frac{1+y^2}{1-y}\right)$$

Subtraction of the virtual singularity



$$\int_0^1 dx\, f(x)\,[g(x)]_+ \equiv \int_0^1 dx\,[f(x)-f(1)]\,g(x)$$

Momentum sum rule (exercise)

$$\int_0^1 dx\, x \left[\sum_{i,\bar{i}} f_i(x) + f_g(x) \right] \equiv \Sigma^{(2)} + f_g^{(2)} = 1$$

This implies

(1) $$P_{qq}^{(2)} + P_{gq}^{(2)} = 0$$

(2) $$P_{gg}^{(2)} + 2n_f\, P_{qg}^{(2)} = 0$$

(1) is trivially true (check!)

(2) requires a modification of P_{gg} to subtract soft virtual singularity (verify!):

$$P_{gg} \rightarrow 2C_A \left\{ \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right\} + \delta(1-x) \left[\frac{11C_A - 2n_f}{6} \right]$$

Subtraction of gluon loop in virtual diagrams

Subtraction of quark loop in virtual diagrams

General solution of the PDF evolution

$$V^{(n)}(Q^2) \overset{*}{=} V^{(n)}(\mu^2) \left[\frac{\log Q^2/\Lambda^2}{\log \mu^2/\Lambda^2} \right]^{P_{qq}^{(n)}/2\pi b_0} = V^{(n)}(\mu^2) \left[\frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} \right]^{P_{qq}^{(n)}/2\pi b_0}$$

* see footnote next page

Verify that all moments $P_{qq}^{(n)}$ are negative. Therefore as Q grows, all moments decrease. The valence distribution becomes softer and softer.

For $\Sigma^{(n)}$ and $g^{(n)}$ one needs to diagonalize the 2x2 matrix. In the case of n=2, corresponding to the momentum fraction carried by gluons and quarks, simple asymptotic solutions ($Q^2 \rightarrow \infty$) can be obtained (exercise!):

$$\begin{cases} P_{qq}^{(2)} \Sigma^{(2)} + 2n_f P_{qg}^{(2)} f_g^{(2)} = 0 & (\Sigma^{(2)} \rightarrow \text{constant at large Q}) \\ \Sigma^{(2)} + f_g^{(2)} = 1 & \text{(sum rule)} \end{cases}$$

$$\begin{cases} \Sigma^{(2)} = \dfrac{1}{1 + \frac{4C_F}{n_f}} \\ f_g^{(2)} = \dfrac{4C_F}{4C_F + n_f} \end{cases} \qquad \frac{g^{(2)}}{\Sigma^{(2)}} = \frac{4C_F}{n_f} = \frac{16}{3n_f}$$

Footnote: α_s running

$$\frac{d\alpha_s(\mu^2)}{d\log\mu^2} = \beta(\alpha_s)$$

$$\beta(\alpha_s) = -b_0\alpha_s^2\,(1 + b'\alpha_s + \ldots)$$

LO NLO

$$b_0 = \frac{11C_A - 2n_f}{12\pi}$$

$$b' = \frac{17C_A^2 - 5C_A n_f - 3C_F n_f}{2\pi(11C_A - 2n_f)}$$

$\beta(\alpha)$ for QCD is known up to NNNLO (4-loops)

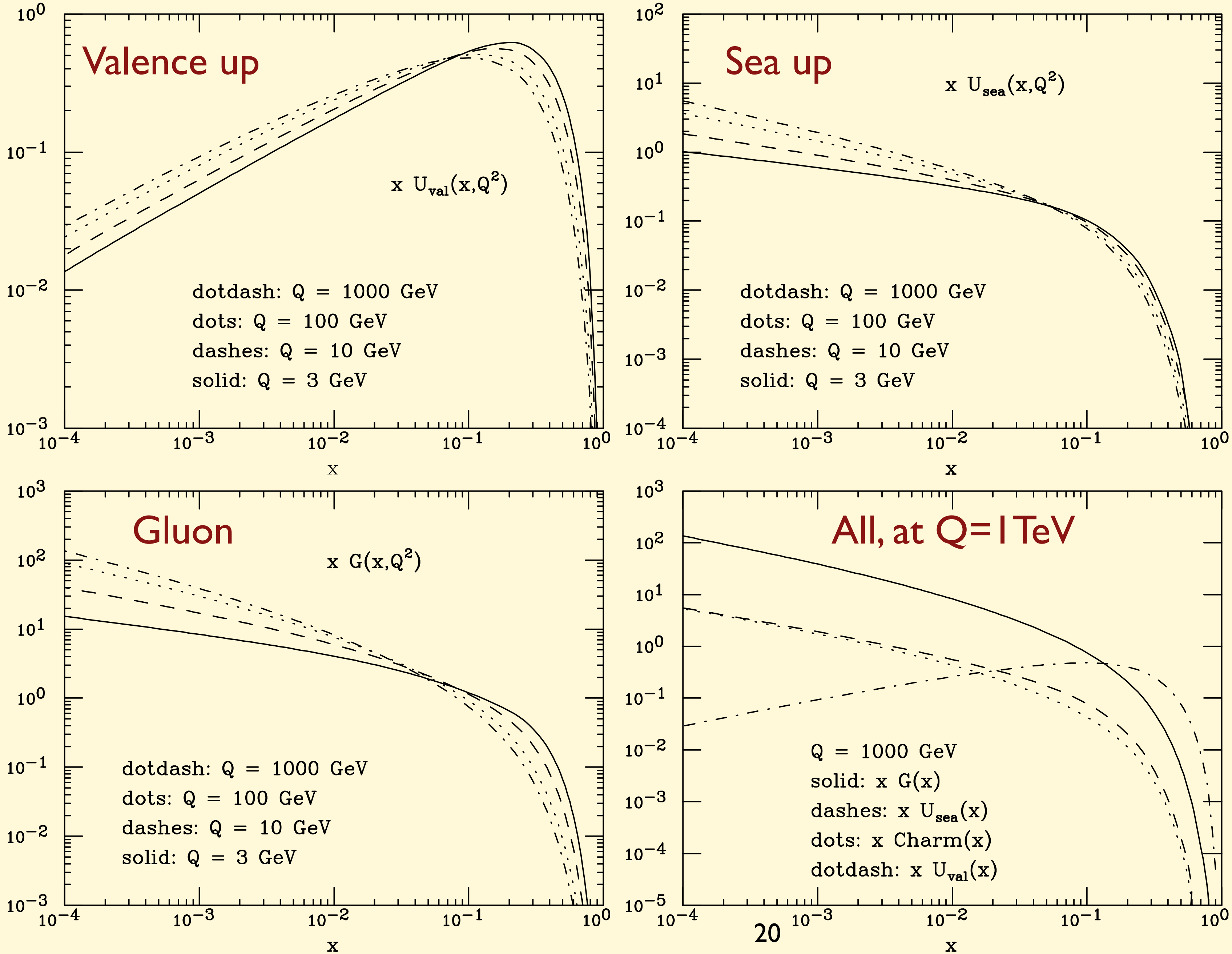
At LO

$$\alpha_s(\mu^2) = \frac{1}{b_0\,\log\mu^2/\Lambda^2}$$

At NLO

$$\alpha_s(\mu^2) = \frac{1}{b_0\,\log\mu^2/\Lambda^2}\left[1 - \frac{b'}{b_0}\frac{\log\log\mu^2/\Lambda^2}{\log\mu^2/\Lambda^2}\right]$$

Examples of PDFs and their evolution



Note:
sea ≈10% glue

Note:
charm≈up at high Q